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Robust Fault Tolerant Tracking Control Design for a Linearized Hypersonic Vehicle with Sensor Fault

Zhi-Feng Gao*, Jin-Xing Lin, and Teng Cao

Abstract: In this study, the robust fault tolerant tracking problem is investigated for a linearized hypersonic vehicle model with the bounded external disturbance and the sensor fault. Firstly, the nonlinear longitudinal dynamics of hypersonic vehicle is linearized as a linear time-invariant system with sensor fault, a reference model is introduced for the aim of fault tolerant tracking control. And then an observer-based fault tolerant output feedback tracking controller design approach is proposed by using the linear matrix inequalities (LMIs) technique. The asymptotic stability of the whole closed-loop system is analyzed using the well-known Lyapunov stability method. Finally, the simulation results are presented to verify the applicability of the developed fault tolerant approach.

Keywords: Fault tolerant control, hypersonic vehicle, sensor fault, tracking control.

1. INTRODUCTION

A hypersonic vehicle is a vehicle that travels at least 5 times faster than the speed-of-sound, or greater than Mach 5. A hypersonic vehicle can be regards as an airplane, missile, or spacecraft. Some hypersonic vehicles have a special type of jet engine called a supersonic combustion ramjet or scramjet to fly through the atmosphere [1]. Compare with the existing airships, space shuttles and aeroplanes, hypersonic vehicles have many advantages in launch cost, maintainability, reusability, flight performance and so on. Consequently, it is not surprising that the controller design and stability analysis problem of hypersonic vehicle have been extensively considered by many scholars [2-8]. In [2], a robust output feedback control scheme is propose for the longitudinal model of a flexible air-breathing hypersonic vehicle by combining the nonlinear observer and backstepping technique. In [3], a nonlinear controller is exploited and analyzed for the nonlinear longitudinal dynamics of hypersonic vehicle by combining the nonlinear observer and back-stepping technique. In [4], a fuzzy guaranteed cost state feedback controller is designed for a flexible air-breathing hypersonic vehicle to guarantee that the closed-loop system is asymptotically stable and the proposed performance index has an upper bound. In [5], a novel gain-scheduled switching control approach is proposed for the longitudinal motion of a flexible air-breathing hypersonic vehicle, the designed gain-scheduled controller could guarantees the closed-loop system to be asymptotically stable and satisfy a given tracking error performance criterion. In [6], the adaptive Kriging controller is investigated for the longitudinal dynamics of a generic is investigated for the longitudinal dynamics of a generic hypersonic flight vehicle. The Kriging system is used to estimate the uncertainty, which is described as the realisations of Gaussian random functions. In [7], a non-fragile output tracking control scheme is presented for a flexible hypersonic air-breathing vehicles, with guarantees the tracking error dynamics to be robustly stable. In [8], the robust flight control problem for the longitudinal dynamics of hypersonic vehicle under mismatched disturbances is investigated by using the nonlinear-disturbance-observer-based control method, the proposed method obtains not only promising robustness and disturbance rejection performance but also the property of nominal performance recovery.

As a new aerospace vehicle, the flight control system of hypersonic vehicle will inevitably be subject to all kinds of faults, which may be caused by actuators, sensors or system components [9-11]. To improve the safety and reliability of hypersonic vehicle, fault diagnosis and fault tolerant control (FTC) technology must be considered in designing the stable flight control system of hypersonic vehicle. In [12], a bank of sliding-mode observers are designed for the attitude dynamics of hypersonic vehicle with actuator faults to provide the residual signals for the aim of fault detection and isolation, and a novel fault diagnostic algorithm is proposed to estimate the actuator fault. In [13], a linear parameter-varying (LPV) model of hypersonic vehicle longitudinal dynamics with is derived by converting actuator/system component faults into equivalent sensor faults. Then a bank of LPV FDI observers is designed to track individual fault with minimum error and suppress

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Zhi-Feng Gao, Jin-Xing Lin, and Teng Cao are with the College of Automation, Nanjing University of Posts and Telecommunications, 210046 Nanjing, P. R. China (e-mails: gaozhifeng80 @gmail.com, jxlin2004@126.com, caoteng2014@126.com).
* Corresponding author.

the effects of disturbances and other fault signals. In [14], A new adaptive FTC strategy with input saturation is designed for the longitudinal model of an airbreathing hypersonic vehicle with actuator faults to guarantee that velocity and altitude track their reference trajectories. It is noting that the fault tolerant control approach described above is mainly solved to the actuator fault problem for hypersonic vehicle. To the best of our knowledge, the fault tolerant control for hypersonic vehicle with sensor fault has not been fully investigated yet, which remains challenging and motivates us to do this study.

In this paper, we design an observer-based fault tolerant control approach for a linearized dynamical system of hypersonic vehicle subject to sensor fault. Firstly, the nonlinear longitudinal dynamics of an hypersonic vehicle is introduced, which can be linearized at nominal hypersonic cruise flight condition. By considering the sensor fault, a linearized faulty model of hypersonic vehicle is given. And then, a novel fault tolerant controller design approach is proposed for the established hypersonic vehicle model by using the observer-based control technique. The existence conditions for FTC are formulated in the form of LMIs, and the FTC design problem is cast into a convex optimization problem. If the optimization problem is solvable, a desired fault tolerant controller can be readily constructed. The simulation results are provided to show the effectiveness of the proposed FTC approach.

2. PROBLEM STATEMENT

The nonlinear longitudinal equations of motion flying for hypersonic vehicle at orbital altitudes must include both an inverse-square-law gravitational model and the centripetal acceleration for the non-rotating Earth. The state space equations for the longitudinal dynamics are governed by the set of differential equations for altitude , velocity V, angle of attack α, pitch angle θ, and pitch rate q as

$$\dot{h} = V\sin(\theta-\alpha), \quad (1)$$

$$\dot{V} = (T\cos\alpha - D)/m - g\sin(\theta-\alpha), \quad (2)$$

$$\dot{\alpha} = (-T\sin\alpha - L)/(mV) + Q + \cos(\theta-\alpha)/V, \quad (3)$$

$$\dot{\theta} = q, \quad (4)$$

$$\dot{q} = M_{yy}/I_{yy}, \quad (5)$$

where $L = 0.5\rho V^2 SC_L$, $D = 0.5\rho V^2 SC_D$, $T = 0.5\rho V^2 SC_T$, $M_{yy} = 0.5\rho V^2 S\overline{c}C_M$.

The aerodynamic coefficients C_L, C_D, C_M and the thrust coefficient C_T are functions of the Mach number M , the angle of attack α, and control inputs which are elevator deflection angle δ_e and throttle setting β. The reader can refer to [14] for further details.

In straight and level flight, the nonlinear dynamic equations (1)-(5) can be linearized at different points within the flight envelope as defined by the velocity and altitude [13]. By trimming the hypersonic vehicle at the specified velocity and altitude, then numerically linearizing equations of motion at the trim point, the longitudinal dynamics of hypersonic vehicle can be described by a linear model as follows:

$$\dot{x} = Ax + Bu + \omega(t), \quad (6)$$

$$y = Cx, \quad (7)$$

where $x = [h, V, \alpha, \gamma, q]^T$ is the state vector, $u = [\beta, \delta_e]^T$ is the control vector, and $y = [h, V]^T$ is the output vector. $\omega(t)$ represents a time-dependent unknown, nonlinear external disturbance.

Given whether a fault occurs on each sensor or not, a matrix F is introduced to represent the sensor fault situation as follows [15]:

$$F = \mathrm{diag}\{f_1, f_2, \cdots f_m\}, \quad (8)$$

where $f_i = 0$ when the ith sensor is completely failure, $f_i = 1$, represents the sensor is healthy, and for partial failure, $0 < f_i < 1$.

Based on the sensor fault model (8), the following faulty longitudinal dynamics of hypersonic vehicle is given by:

$$\dot{x} = Ax + Bu + \omega(t), \quad (9)$$

$$y_F = Fy = FCx. \quad (10)$$

Remark 1: The sensor faults of flight control system are broadly divided into four categories: (a) sensor bias; (b) loss of accuracy or calibration error; (c) sensor drift; (d) frozen sensor. Note that the fault type (8) considered in this paper is loss of accuracy, which often occurs in actual flight operation and is the focus of this study.

For the purpose of tracking control of hypersonic vehicle, a reference model is introduced by

$$\dot{x}_m = A_m x_m + B_m \delta, \quad (11)$$

$$y_m = Cx_m, \quad (12)$$

where A_m is the state matrix of the reference model, which is Hurwitz. B_m is the input matrix of the reference model, δ is the reference input, x_m represents the reference state vector, y_m is the reference output vector.

The attenuation of external disturbance is guaranteed by considering the H_∞ performance index related to the tracking error $e_p = x - x_m$ as follows:

$$\int_0^t e_p^T e_p dt \le \eta^2 \int_0^t (\delta^T\delta + \omega^T\omega)dt. \quad (13)$$

Before ending this section, we recall the following lemmas which will be needed in the proof of the main results.

Lemma 1 [16]**:** For real matrices X, Y and $S = S^T > 0$ with appropriate dimensions and a positive constant γ, the following inequalities hold:

$$X^TY + Y^TX < \gamma X^TX + \gamma^{-1}Y^TY$$

and

$$X^TY + Y^TX < X^TSX + Y^TS^{-1}Y.$$

Lemma 2 [17]**:** Let a matrix $\Omega < 0$, a matrix X with appropriate dimension such that $X^T \Omega X < 0$, the following inequality holds:

$$X^T \Omega X \le -(X^T + X) - \Omega^{-1}.$$

3. ROBUST FAULT TOLERANT TRACTING CONTROL SYNTHESIS

In order to derive an observer-based fault tolerant output feedback controller, an additional observer is introduced as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y_F - \hat{y}), \tag{14}$$

$$\hat{y} = C\hat{x}. \tag{15}$$

Based on the reference model (11)-(12) and the observer model (14)-(15), the following robust fault tolerant tracking controller is given by

$$u = -K(x_m - \hat{x}). \tag{16}$$

Let us consider the observer error $e_0 = x - \hat{x}$, then we have the following observer error dynamics:

$$\dot{e}_0 = (A - LC)e_0 + L(I - F)C(e_p + x_m) + \omega. \tag{17}$$

According to the tracking error $e_p = x - x_m$, we have the following tracking error dynamics as follows:

$$\begin{aligned} \dot{e}_p &= Ax + Bu + \omega - (A_m x_m + \delta) \\ &= Ax - BK(x_m - \hat{x}) + \omega - (A_m x_m + B_m \delta) \\ &= (A + BK)e_p - BKe_0 + (A - A_m)x_m + \omega - B_m \delta. \end{aligned} \tag{18}$$

By combining (17), (18) with the reference state vector xm, a new state vector for the global closed-loop control system is defined as $\tilde{x} = [e_0, e_p, x_m]^T$, after some manipulations, the augmented closed-loop control system could be described by:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\varpi, \tag{19}$$

where $\varpi = [\omega, \delta]^T$ and

$$\tilde{A} = \begin{bmatrix} A - LC & L(I-F)C & L(I-F)C \\ -BK & A + BK & A - A_m \\ 0 & 0 & A_m \end{bmatrix},$$

$$\tilde{B} = \begin{bmatrix} I & 0 \\ I & -B_m \\ 0 & B_m \end{bmatrix}.$$

Note that with the state vector $\tilde{x}$ and the disturbance vector ϖ, H_∞ performance index (13) is revised as:

$$\int_0^t \tilde{x}^T \tilde{x} dt \le \eta^2 \int_o^t \varpi^T \varpi dt. \tag{20}$$

Remark 2: In [18], Mansouri *et al*. presented an output feedback tracking controller design approach for a class of Takagi-Sugeno fuzzy systems, which is borrowed to solve the robust fault tolerant control problem of hypersonic vehicle with sensor fault. It is worth mentioning that the measured sensor is work normally in [18], and the output signal y is measured accurately, which could be used for the output feedback control design. In our study, the measured sensor is work in faulty case, the output signal y is not measured accurately owing to the effect of sensor fault. The observer-based fault tolerant controller design approach developed in this paper for the hypersonic vehicle with sensor fault is the main contribution of this study.

The objective is now to compute the controller gain K and the observer gain L from $\tilde{A}$ described in (19) to ensure the asymptotical stability of the closed-loop system (19) satisfying a H_∞ performance index (20). A straightforward result is summarized in the following theorem.

Theorem 1: Consider the closed-loop control system (19) with sensor fault model (8), if there exist a matrix $\tilde{P} = \tilde{P}^T > 0$, and a positive constant γ such that the following matrix inequality holds,

$$\begin{bmatrix} \tilde{P}\tilde{A} + \tilde{A}^T \tilde{P} + I & \tilde{P}\tilde{B} \\ \tilde{B}^T \tilde{P} & -\eta^2 I \end{bmatrix} < 0. \tag{21}$$

Then the asymptotic stability of the closed-loop control system (19) is ensured and the H_∞ performance index (20) is guaranteed with an attenuation level η.

Proof: Consider the following candidate Lyapunov function:

$$V(t) = \tilde{x}^T \tilde{P} \tilde{x} \quad \text{with} \quad \tilde{P} = \tilde{P}^T > 0. \tag{22}$$

The asymptotic stability of the closed-loop model (19) with the disturbance attenuation level η is satisfied if the following inequality holds,

$$\dot{V}(t) + \tilde{x}^T \tilde{x} - \eta^2 \varpi^T \varpi < 0. \tag{23}$$

The inequality (23) is further rewritten as the following:

$$\begin{bmatrix} \tilde{x} \\ \varpi \end{bmatrix}^T \begin{bmatrix} \tilde{P}\tilde{A} + \tilde{A}^T \tilde{P} + I & \tilde{P}\tilde{B} \\ \tilde{B}^T \tilde{P} & -\eta^2 I \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \varpi \end{bmatrix} < 0. \tag{24}$$

If the inequality (21) in Theorem 1 holds, it can be easily known that the above inequality is also right.

The goal is now to obtain a tractable LMI problem that allows solving the unknown controller gain K and observer gain L and to prove the closed-loop stability with the prescribed disturbance attenuation level η.

Theorem 2: Consider the closed-loop control system (19) with sensor fault model (8), if there exist three positive definite symmetric matrices P_1, P_3, N, two real matrices Y, Z, and a positive constant η, such that the following LMI conditions (25)-(26) are satisfied, then the asymptotic stability of the closed-loop control system (19) with the disturbance attenuation level η is guaranteed. Meanwhile, the controller gain K and the observer gain L are solved by $K = YN^{-1}$ and $L = P_1^{-1}Z$.

$$\begin{bmatrix} \vec{\Gamma}_{11}^1 & 0 & \vec{\Gamma}_{13}^1 & P_1 & 0 & Z & 0 & 0 \\ 0 & \vec{\Gamma}_{22}^1 & 0 & 0 & 0 & 0 & N^T & \vec{\Gamma}_{28}^1 \\ * & 0 & \vec{\Gamma}_{33}^1 & 0 & \vec{\Gamma}_{35}^1 & 0 & 0 & 0 \\ * & 0 & 0 & \vec{\Gamma}_{44}^1 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & \vec{\Gamma}_{55}^1 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 & -2I & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (25)$$

where

$$\vec{\Gamma}_{11}^1 = \tfrac{1}{2}\left[(P_1A - ZC) + (P_1A - ZC)^T\right],$$
$$\vec{\Gamma}_{13}^1 = Z(I-F)C, \quad \vec{\Gamma}_{35}^1 = \tfrac{1}{2}P_3B_m,$$
$$\vec{\Gamma}_{22}^1 = \tfrac{1}{2}\left[(AN + BY) + (AN + BY)^T\right],$$
$$\vec{\Gamma}_{28}^1 = N^TC^T(I-F)^T, \quad \vec{\Gamma}_{33}^1 = \tfrac{1}{2}(P_3A_m + A_m^TP_3),$$
$$\vec{\Gamma}_{33}^1 = \tfrac{1}{2}(P_3A_m + A_m^TP_3), \quad \vec{\Gamma}_{44}^1 = \vec{\Gamma}_{55}^1 = -\tfrac{1}{2}\eta^2 I,$$

$$\begin{bmatrix} \vec{\Gamma}_{11}^2 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -2N & 0 & 0 & 0 & 0 & 0 & Y^T \\ 0 & 0 & \vec{\Gamma}_{33}^2 & \vec{\Gamma}_{34}^2 & I & -B_m & N^T & 0 \\ 0 & 0 & * & \vec{\Gamma}_{44}^2 & 0 & \vec{\Gamma}_{46}^2 & 0 & 0 \\ 0 & 0 & * & 0 & \vec{\Gamma}_{55}^2 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & \vec{\Gamma}_{66}^2 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 & -2I & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (26)$$

where

$$\vec{\Gamma}_{11}^2 = \tfrac{1}{2}\left[(P_1A - ZC) + (P_1A - ZC)^T\right],$$
$$\vec{\Gamma}_{33}^2 = \tfrac{1}{2}\left[(AN + BY) + (AN + BY)^T\right] + BB^T,$$
$$\vec{\Gamma}_{34}^2 = A - A_m, \quad \vec{\Gamma}_{44}^2 = \tfrac{1}{2}(P_3A_m + A_m^TP_3),$$
$$\vec{\Gamma}_{46}^2 = \tfrac{1}{2}P_3B_m, \quad \vec{\Gamma}_{55}^2 = \vec{\Gamma}_{66}^2 = -\tfrac{1}{2}\eta^2 I,$$

and * is denotes the transposed elements in the symmetric position.

Proof: For the convenience of FTC design, it is assumed that $\tilde{P} = \mathrm{diag}\{P_1, P_2, P_3\}$ in Theorem 1, then (21) is transformed into the following form:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & P_1L(I-F)C & P_1 & 0 \\ * & \Pi_{22} & P_2(A-A_m) & P_2 & -P_2B_m \\ * & * & P_3A_m + A_m^TP_3 & 0 & P_3B_m \\ * & * & 0 & -\eta^2 I & 0 \\ 0 & * & * & 0 & -\eta^2 I \end{bmatrix} < 0,$$

where

$$\Pi_{11} = P_1(A - LC) + (A - LC)P_1,$$
$$\Pi_{12} = P_1L(I-F)C - K^TB^TP_2,$$
$$\Pi_{22} = P_2(A + BK) + (A + BK)^TP_2 + I.$$

In the following, we rewrite the above inequality as the follow form:

$$\Gamma^1 + \Gamma^2 < 0, \quad (27)$$

where

$$\Gamma^1 = \begin{bmatrix} \Gamma_{11}^1 & \Gamma_{12}^1 & P_1L(I-F)C & P_1 & 0 \\ * & \Gamma_{22}^1 & 0 & 0 & 0 \\ * & 0 & \tfrac{1}{2}(P_3A_m + A_m^TP_3) & 0 & \tfrac{1}{2}P_3A_m \\ * & 0 & 0 & -\tfrac{1}{2}\eta^2 I & 0 \\ 0 & 0 & * & 0 & \tfrac{1}{2}\eta^2 I \end{bmatrix}$$

with

$$\Gamma_{11}^1 = \tfrac{1}{2}[P_1(A - LC) + (A - LC)P_1],$$
$$\Gamma_{12}^1 = P_1L(I-F)C,$$
$$\Gamma_{22}^1 = \tfrac{1}{2}[P_2(A + BK) + (A + BK)^TP_2 + I],$$

$$\Gamma^2 = \begin{bmatrix} \Gamma_{11}^2 & \Gamma_{12}^2 & 0 & 0 & 0 \\ * & \Gamma_{22}^2 & P_2(A-A_m) & P_2 & -P_2B_m \\ 0 & * & \tfrac{1}{2}(P_3A_m + A_m^TP_3) & 0 & \tfrac{1}{2}P_3B_m \\ 0 & * & 0 & -\tfrac{1}{2}\eta^2 I & 0 \\ 0 & * & * & 0 & \tfrac{1}{2}\eta^2 I \end{bmatrix}$$

with

$$\Gamma_{11}^2 = \tfrac{1}{2}[P_1(A - LC) + (A - LC)P_1],$$
$$\Gamma_{12}^2 = -K^TB^TP_2,$$
$$\Gamma_{22}^2 = \tfrac{1}{2}[P_2(A + BK) + (A + BK)^TP_2 + I].$$

If $\Gamma^1 < 0$ and $\Gamma^2 < 0$ hold, then the inequality (27) is guaranteed.

According to Lemma 1, it is easily known that the following inequality holds,

$$\begin{aligned} & e_0^TP_1L(I-F)Ce_p + e_P^TC^T(I-F)L^TP_1e_0 \\ & \le e_0^T(P_1L)(P_1L)^Te_0 + e_p^TC^T(I-F)^T(I-F)Ce_p \\ & = e_0^T(P_1L)(P_1L)^Te_0 + e_P^TC^T(I-F)^2Ce_P. \end{aligned}$$

Substituting the above inequality into $\dot{V}, \Gamma^1 < 0$ is further revised as the following form:

$$\tilde{\Gamma}^1 = \begin{bmatrix} \tilde{\Gamma}_{11}^1 & 0 & P_1L(I-F)C & P_1 & 0 \\ 0 & \Gamma_{22}^1 & 0 & 0 & 0 \\ * & 0 & \tfrac{1}{2}(P_3A_m + A_m^TP_3) & 0 & \tfrac{1}{2}P_3A_m \\ P_1 & 0 & 0 & -\tfrac{1}{2}\eta^2 I & 0 \\ 0 & 0 & * & 0 & \tfrac{1}{2}\eta^2 I \end{bmatrix} < 0, \quad (28)$$

where

$$\tilde{\Gamma}_{22}^{1}=\tfrac{1}{2}[P_2(A+BK)+(A+BK)^T P_2+I]$$
$$\quad +C^T(I-F)^2C,$$
$$\tilde{\Gamma}_{11}^{1}=\tfrac{1}{2}[P_1(A-LC)+(A-LC)P_1]+(P_1L)(P_1L)^T.$$

Let $P_1L=Z$, $P_2^{-1}=N$ and $KP_2^{-1}=KN=Y$, then pre-post multiply of $\tilde{\Gamma}^1<0$ by $\mathrm{diag}\{I,N,I,I,I\}$, we have the following inequality,

$$\bar{\Gamma}^1=\begin{bmatrix}\bar{\Gamma}_{11}^1 & 0 & Z(I-F)C & P_1 & 0\\ 0 & \bar{\Gamma}_{22}^1 & 0 & 0 & 0\\ * & 0 & \frac{1}{2}(P_3A_m+A_m^TP_3) & 0 & \frac{1}{2}P_3A_m\\ * & 0 & 0 & \frac{1}{2}\eta^2 I & 0\\ 0 & 0 & * & 0 & \frac{1}{2}\eta^2 I\end{bmatrix}<0, \quad (29)$$

where

$$\bar{\Gamma}_{22}^{1}=\tfrac{1}{2}\left[(AN+BK)+(AN+BK)^T+N^2\right]$$
$$\quad +N^TC^T(I-F)^2CN,$$
$$\bar{\Gamma}_{11}^{1}=\tfrac{1}{2}\left[(P_1A-ZC)+(P_1A-ZC)^T\right]+ZZ^T.$$

By applying the Schur complement to the inequality (29), it is rewritten as the following form:

$$\begin{bmatrix}\vec{\Gamma}_{11}^1 & 0 & \vec{\Gamma}_{13}^1 & P_1 & 0 & Z & 0 & 0\\ 0 & \vec{\Gamma}_{22}^1 & 0 & 0 & 0 & 0 & N^T & \vec{\Gamma}_{28}^1\\ * & 0 & \vec{\Gamma}_{33}^1 & 0 & \frac{1}{2}P_3B_m & 0 & 0 & 0\\ * & 0 & 0 & \vec{\Gamma}_{44}^1 & 0 & 0 & 0 & 0\\ 0 & 0 & * & 0 & \vec{\Gamma}_{55}^1 & 0 & 0 & 0\\ * & 0 & 0 & 0 & 0 & -I & 0 & 0\\ 0 & * & 0 & 0 & 0 & 0 & -2I & 0\\ 0 & * & 0 & 0 & 0 & 0 & 0 & -I\end{bmatrix}<0, \quad (30)$$

where

$$\vec{\Gamma}_{11}^{1}=\tfrac{1}{2}\left[(P_1A-ZC)+(P_1A-ZC)^T\right],$$
$$\vec{\Gamma}_{28}^{1}=N^TC^T(I-F)^T,$$
$$\vec{\Gamma}_{33}^{1}=\tfrac{1}{2}(P_3A_m+A_m^TP_3),$$
$$\vec{\Gamma}_{22}^{1}=\tfrac{1}{2}\left[(AN+BY)+(AN+BY)^T\right],$$
$$\vec{\Gamma}_{13}^{1}=Z(I-F)C,$$
$$\vec{\Gamma}_{44}^{1}=\vec{\Gamma}_{55}^{1}=-\tfrac{1}{2}\eta^2 I.$$

Meanwhile, it is noted that the following inequality holds according to Lemma 1,

$$-e_0^TK^TB^TP_2e_P-e_p^TP_2BKe_0$$
$$\quad \le e_0^TK^TKe_0+e_P^TP_2BB^TP_2e_P.$$

Substituting the above inequality into $\dot{V},\Gamma^2<0$ is further revised as the following form:

$$\tilde{\Gamma}^2=\begin{bmatrix}\tilde{\Gamma}_{11}^2 & 0 & 0 & 0 & 0\\ 0 & \Gamma_{22}^2 & P_2(A-A_m) & P_2 & -P_2B_m\\ 0 & * & \frac{1}{2}(P_3A_m+A_m^TP_3) & 0 & \frac{1}{2}P_3A_m\\ 0 & * & 0 & -\frac{1}{2}\eta^2 I & 0\\ 0 & * & * & 0 & -\frac{1}{2}\eta^2 I\end{bmatrix}<0, \quad (31)$$

where

$$\tilde{\Gamma}_{11}^{2}=\tfrac{1}{2}[P_1(A-LC)+(A-LC)P_1]+K^TK,$$
$$\tilde{\Gamma}_{22}^{2}=\tfrac{1}{2}\left[P_2(A+BK)+(A+BK)^TP_2+I\right]+P_2BB^TP_2.$$

According to the definition of Z, N, Y described above, pre-post multiply of $\tilde{\Gamma}^2<0$ by $\mathrm{diag}\{N,N,I,I,I\}$ the inequality (31) is transformed into the following:

$$\bar{\Gamma}^2=\begin{bmatrix}\bar{\Gamma}_{11}^2 & 0 & 0 & 0 & 0\\ 0 & \bar{\Gamma}_{22}^2 & A-A_m & I & -B_m\\ 0 & * & \frac{1}{2}(P_3A_m+A_m^TP_3) & 0 & \frac{1}{2}P_3A_m\\ 0 & * & 0 & -\frac{1}{2}\eta^2 I & 0\\ 0 & * & \frac{1}{2}B_m^TP_3 & 0 & -\frac{1}{2}\eta^2 I\end{bmatrix}<0, \quad (32)$$

where

$$\bar{\Gamma}_{11}^{2}=\tfrac{1}{2}N\left[(P_1A-ZC)+(P_1A-ZC)^T\right]N+Y^TY,$$
$$\bar{\Gamma}_{22}^{2}=\tfrac{1}{2}\left[(AN+BK)+(AN+BK)^T+N^2\right]+BB^T.$$

In terms of Lemma 2, it is found that the following inequality holds,

$$\tfrac{1}{2}N\left[(P_1A-ZC)+(P_1A-ZC)^T\right]N$$
$$\quad \le -2N-\left[\tfrac{1}{2}(P_1A-ZC)+\tfrac{1}{2}(P_1A-ZC)^T\right]^{-1}. \quad (33)$$

Substituting (33) into (32), then applying the Schur complement to (32), the following inequality is obtained:

$$\begin{bmatrix}\vec{\Gamma}_{11}^2 & I & 0 & 0 & 0 & 0 & 0\\ * & \vec{\Gamma}_{22}^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \vec{\Gamma}_{33}^2 & A-A_m & I & -B_m & N^T\\ 0 & 0 & * & \vec{\Gamma}_{44}^2 & 0 & \frac{1}{2}P_3B_m & 0\\ 0 & 0 & * & 0 & \vec{\Gamma}_{55}^2 & 0 & 0\\ 0 & 0 & * & * & 0 & \vec{\Gamma}_{66}^2 & 0\\ 0 & 0 & * & 0 & 0 & 0 & -2I\end{bmatrix}<0 \quad (34)$$

with

$$\vec{\Gamma}_{11}^{2}=\tfrac{1}{2}\left[(P_1A-ZC)+(P_1A-ZC)^T\right],$$
$$\vec{\Gamma}_{22}^{2}=-2N+Y^TY,$$

$$\vec{\Gamma}_{33}^2 = \frac{1}{2}\left[(AN+BY)+(AN+BY)^T\right]+BB^T,$$
$$\vec{\Gamma}_{44}^2 = \frac{1}{2}(P_3A_m + A_m^TP_3), \quad \vec{\Gamma}_{55}^2 = \vec{\Gamma}_{66}^2 = -\frac{1}{2}\eta^2 I.$$

In the position, we use the Schur complement again to (34) for eliminating Y^TY and get the following inequality,

$$\begin{bmatrix} \vec{\Gamma}_{11}^2 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -2N & 0 & 0 & 0 & 0 & N^T & Y^T \\ 0 & 0 & \vec{\Gamma}_{33}^2 & \vec{\Gamma}_{34}^2 & I & -B_m & 0 & 0 \\ 0 & 0 & * & \vec{\Gamma}_{44}^2 & 0 & \vec{\Gamma}_{46}^2 & 0 & 0 \\ 0 & 0 & * & 0 & \vec{\Gamma}_{55}^2 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & \vec{\Gamma}_{66}^2 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 & -2I & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{35}$$

where $\vec{\Gamma}_{34}^2 = A - A_m$, $\vec{\Gamma}_{46}^2 = \frac{1}{2}P_3B_m$.

From the result of Theorem 2, it is easily known that the inequalities (30) and (35) hold, therefore, it is easily found that $\Gamma^1 < 0$ and $\Gamma^2 < 0$ hold, and then the closed-loop control system (19) in sensor fault case is asymptotically stable with the H_∞ disturbance attenuation level η under the observer-based fault tolerant tracking control (16), which completes the proof.

Remark 3: The controller designed in Theorem 2 are suitable to both the sensor with fault $(F \neq I)$ and those without fault $(F = I)$, with an advantage that no changes are needed to make on structures and/or parameters of the controllers to guarantee the asymptotical stability and satisfy the desired performance index of the closed-loop control system (19).

Remark 4: In [19], an observer-based FTC scheme is proposed for a hypersonic vehicle dynamical system with both parameter uncertainty and actuator fault. It is noted that the FTC approach developed in [19] can not deal with the effect of sensor fault considered in this study effectively. Furthermore, the tracking control problem is not studied in [19]. To solve the FTC tracking problem of hypersonic vehicle with sensor fault, we design an observer-based FTC approach by using both linear matrix inequality and reference model tracking techniques, and the result obtained in this paper can be regarded as the extension and supplement of [19].

Remark 5: In the proof of Theorem 2, we divide a big matrix into two matrices for easily solving both the controller gain K and the observer gain L by using Matlab LMI toolbox, therefore, the coupling problem of the controller gain K and the observer gain L caused by the sensor fault parameter F has been resolved in this study.

4. SIMULATION RESULTS

In this section, simulation results are presented to demonstrate the effectiveness of the proposed techniques. The hypersonic vehicle model parameters are borrowed from [20]. The equilibrium point of the nonlinear vehicle dynamics described by the system in Table 1 of [20]. By using those parameters, the matrices A, B and C are easily obtained, which are omitted here owing to the limitation of page space.

The main control objective is to track a step signal (predefined) with respect to a trim condition, so the reference input δ of reference model is chosen as a step input. Each command will pass through a prefilter as

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2},$$

where ξ denotes damping ratio, ω_n stands for natural frequency, and they are assumed to be 0.95 and 0.03 rad/s, respectively. The output of the prefilter is defined as a reference input of the reference model. Based on the method proposed in [13], the matrices A_m and B_m of the reference model are chosen as

$$A_m = \begin{bmatrix} 0 & 0 & -7702 & 7702 & 0 \\ -3.162 & -0.62 & 18253.45 & -21295.5 & -441.1 \\ 1.434 & 1.633 & 0.881 & -1.055 & 0.949 \\ 0 & 0 & 0 & 0 & 1 \\ -0.0334 & 0.0156 & 210.91 & -236.47 & -8.23 \end{bmatrix},$$

$$B_m = \begin{bmatrix} 0 & 3.165 & 1.437\times10^{-4} & 0 & 0.0336 \\ 0 & 0.599 & -1.65\times10^{-4} & 0 & -0.0158 \end{bmatrix}^T.$$

According to [15], the external disturbance $\omega(t)$ is assumed to be bounded, which can be regarded as a gust of wind in aerospace. Here we chose

$$\omega(t) = [2\sin t, 3\cos t, 0.3\sin t, 0.3\sin t, 0.5\sin t]^T.$$

Here we select the sensor fault matrix $F = \mathrm{diag}\{0.5, 0.5\}$, the disturbance attenuation level $\eta = 0.8$. By solving LMIs (25)-(26), one obtain the controller gain K and observer gain L as follows:

$$K = \begin{bmatrix} 0.1235 & -0.6922 & 0.9138 & 0.2788 & -0.4458 \\ -0.7860 & 0.8388 & 2.0596 & -1.1087 & -1.0354 \end{bmatrix},$$

$$L = \begin{bmatrix} -1.022 & 2.147 & -3.772 & 0.2231 & 0.0014 \\ 0.2105 & 2.7812 & 3.0006 & 1.6640 & -0.0073 \end{bmatrix}^T.$$

In the process of simulation, the reference command for altitude is chosen as 40000m and it will be switched to 20000m at the 250th seconds. The reference command for velocity is chosen as 3500m/s, and it will be switched to 1500m/s at 250th seconds. Fig. 1 is the altitude tracking response curve of hypersonic vehicle in sensor fault case, Fig. 2 is the velocity tracking response curve of hypersonic vehicle in sensor fault case, Fig. 3 is the altitude tracking error response curve, and Fig. 4 is the velocity tracking error response curve. It can be seen from Figs. 1-4 that the closed-loop control system has the satisfactory altitude tracking and velocity tracking in sensor fault case. Therefore, the simulation results

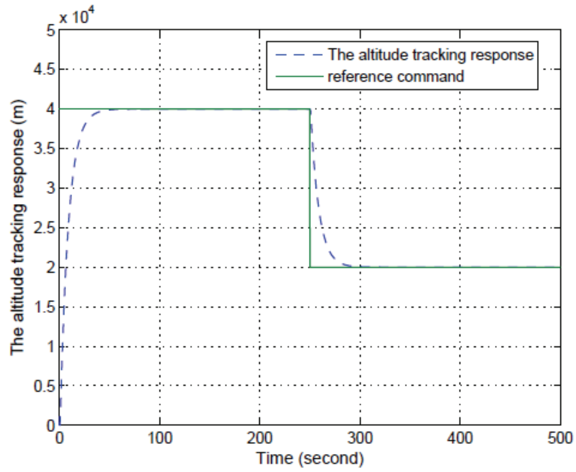

Fig. 1. The altitude tracking response in sensor fault case.

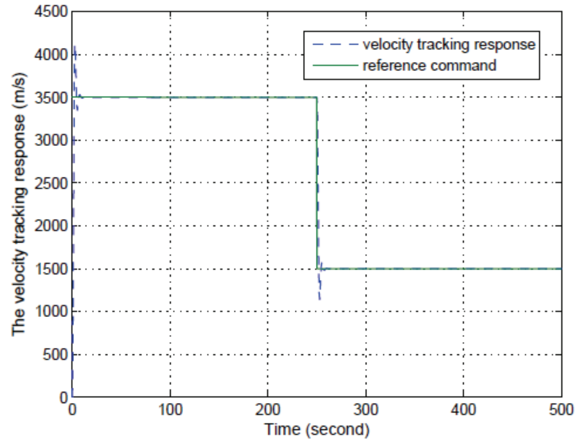

Fig. 2. The velocity tracking response in sensor fault case.

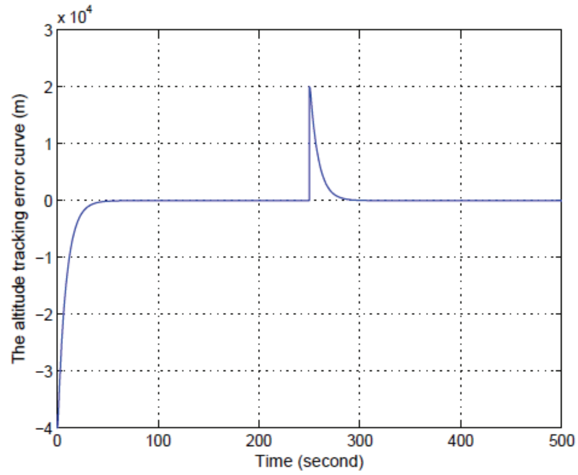

Fig. 3. The altitude tracking error curve in sensor fault case.

demonstrate that the proposed robust fault tolerant control scheme could deal with the effect of sensor fault effectively, and guarantees the robust asymptotical stability of the closed-loop control system in the event of sensor faults.

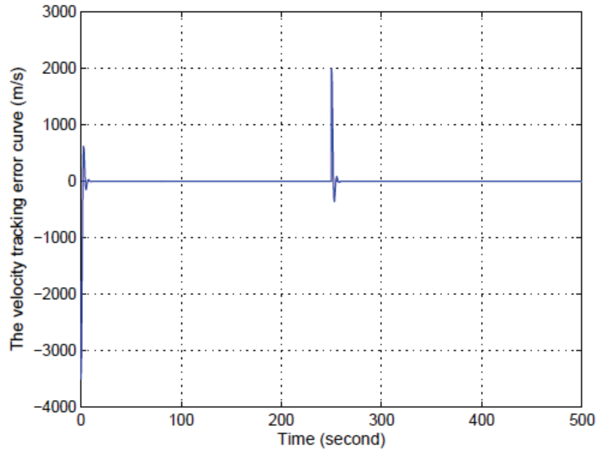

Fig. 4. The velocity tracking error curve in sensor fault case.

5. CONCLUSION

In this paper, the robust fault tolerant tracking control problem is investigated for the longitudinal dynamical systems of hypersonic vehicle with sensor fault. Firstly, the nonlinear dynamics of hypersonic vehicle is linearized at a trim point. By establishing the sensor fault model and state-space observer, a robust fault tolerant control scheme is proposed in the frame of Lyapunov stability theory. Meanwhile, it can be easily proved that the closed-loop control system for the dynamics of hypersonic vehicle is asymptotically stable in sensor fault case. The main results are provided in the form of linear matrix inequalities (LMIs), which can be readily solved via the Matlab LMI toolbox. Finally, simulation results are given to illustrate the effectiveness of the proposed approach.

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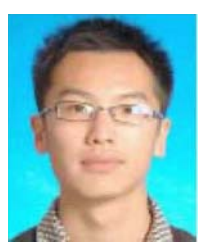
Zhi-Feng Gao received his Ph.D. degree from Nanjing University of Aeronautics and Astronautics, Nanjing, China in 2011. He joined College of Automation, Nanjing University of Posts and Telecommunications in 2011. Currently he is an associate professor. His research interests include robust control, fault diagnosis, fault tolerant control and their applications in aeronautics and astronautics.

Jin-Xing Lin received his Ph.D. degree from Southeast University, Nanjing, China in 2008. He was doing postdoctoral research at the School of Automation, Southeast University from 2008 to 2010. He joined College of Automation, Nanjing University of Posts and Telecommunications in 2010. Currently, he is an associate professor. His research interests include singular time-delay systems and switched systems.



Teng Cao received his B.Eng. degree in Automatic Control from Tongda College, Nanjing University of Posts and Telecommunications, Nanjing, China in 2014. He is currently an M.S. candidate with College of Automation in Nanjing University of Posts and Telecommunications. Her research interests include robust reliable control and its applications in flight control systems.