

# Robust Sliding-mode Observer-based Sensor Fault Estimation, Actuator Fault Detection and Isolation for Uncertain Nonlinear Systems

Junqi Yang\*, Fanglai Zhu, Xin Wang, and Xuhui Bu

**Abstract:** This paper deals with the issues of sensor fault estimation, actuator fault detection and isolation for a class of uncertain nonlinear systems. By taking the sensor fault vector as a part of an extended state vector, the original system with sensor faults, actuator faults and unknown inputs is transformed into an augmented singular system which is just with actuator faults and unknown inputs. For the constructed singular system, a robust sliding-mode observer is developed to simultaneously estimate the states and sensor faults of original system, and the observer gain matrices are computed in terms of linear matrix inequalities by solving an optimization problem. Then an actuator fault detector is designed to detect actuator faults when ones occur, and multiple observers used as actuator fault isolators are proposed to identify which actuator is with fault. Finally, a simulation example is given to illustrate the effectiveness of the proposed methods.

**Keywords:** Fault detection and isolation, fault estimation, linear matrix inequality, nonlinear system, sliding-mode observer.

## 1. INTRODUCTION

The field of fault diagnosis for dynamic systems has become an important topic of research in the past three decades. Any faults in actuators and/or sensors may cause performance degradation or a fatal accident. For example, faulty actuators may severely affect the overall system performance. Similarly, faulty sensors give wrong information about the system status and make the system be unstable. Among various fault detection and isolation (FDI) schemes [1-7], the observer-based approaches exploit analytic redundancy and use a mathematic model of the system to design an observer generating residual signals that provide fault signatures, and the observer residuals are nonzero when there are some failures in the systems and become zero or close to zero when there is no failure. In sliding mode observer-based FDI [8-14], Edwards *et al.* consider the application

of a particular sliding mode observer to the problem of fault detection and isolation based on the equivalent output injection concept [8]. Later, Tan and Edwards seek to relax that the first Markov parameter from the fault to the output must be full rank by using multiple sliding mode observers in cascade [9]. Chen and Saif develop an actuator fault diagnosis scheme for a general class of linear systems subject to unknown inputs that can work without the assumptions on fault diagnosis strategies, and a method which can be used to estimate the faults is proposed [10]. Next, an actuator fault diagnosis scheme is proposed by Chen and Saif for a class of affine nonlinear systems with both known and unknown inputs based on the input/output relation derived from the considered nonlinear systems and high-order sliding-mode robust differentiators [11]. Veluvolu and Soh consider the design of sliding mode observers for fault reconstruction and state estimations, and the reconstruction can be performed online with the state estimation [12]. A robust high gain observer which can estimate the states and unknown inputs/faults is developed for a special class of nonlinear systems [13]. Raoufi *et al.* present a scheme to design robust sliding mode observer with  $H_\infty$  performance for uncertain Lipschitz nonlinear systems where both faults and disturbances are considered [14]. In the adaptive observer-based FDI [15-20], Wang *et al.* propose a kind of adaptive fault estimation observer by exploiting the on-line learning ability of radial basis function neural networks to approximate the actuator fault [15]. Zhang *et al.* develop a fault detection and isolation scheme by using adaptive estimation techniques for a class of Lipschitz nonlinear systems with nonlinear and unstructured modeling uncertainty [16]. Later, Zhang presents a sensor bias fault diagnosis method for a class of Lipschitz nonlinear systems with unstructured

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modeling uncertainty in [17]. Wang and Lum provide an adaptive unknown input observer to detect and isolate aircraft actuator faults [18]. Zhang *et al.* study the problem of fault estimation using adaptive fault diagnosis observer, and a fast adaptive fault estimation approximator is proposed to improve the rapidity of fault estimation [19]. Li and Yang address the adaptive fault detection and isolation problem for linear time-invariant systems under feedback control [20]. In the descriptor observer-based FDI [21,22], paper [21] develops a robust  $H_\infty$  sliding mode descriptor observer for simultaneous state and disturbance estimation. Gao and Ding propose a robust state-space observer to simultaneously estimate descriptor system states, actuator faults, their finite time derivatives, and attenuate input disturbances in any desired accuracy [22]. Nevertheless, most work has dealt with the actuator faults without considering sensor faults [4-6,8-15,18-20,22,25], or sensor faults without considering actuator faults [17]. Many works consider both actuator faults and sensor faults with the same form [2-3].

In the present paper, we discuss not only the sensor fault estimation but also actuator fault detection and isolation for a class of uncertain nonlinear systems with unknown input, actuator and sensor faults. First, the sensor fault is regarded as a part of new augmented state vector and a new singular system is constructed. Next, a robust sliding mode observer for singular system is proposed such that states and sensor faults of original system are estimated. Second, a robust sliding mode observer is developed as fault detector to detect actuator faults. Finally, multiple observers are designed to detect each actuator fault and the purpose of the actuator fault isolation is realized. In the existing literature, papers [2] and [5] use parity space approach to detect and isolation faults for discrete system and the equivalent output injection technique is adopt to reconstruct fault signals in [8-9], while this paper designs roust singular observer and multiple robust observers so that the issues of sensor fault estimation, actuator fault detection and isolation are simultaneously achieved.

The rest of this paper is organized as follows: A general model and some preliminaries are presented in Section 2, and a singular system is constructed by augmenting the state and sensor fault vectors as a new state vector. A robust sliding mode observer is developed to provide the sensor fault estimation in Section 3. Then, a similar robust sliding mode observer used as actuator fault detector is proposed to detect actuator faults, and multiple observers are used to isolate actuator faults in Section4. Simulation results are given in Section 5. In Section 6, some conclusions are summarized.

**2. GENERAL MODEL AND PRELIMINARIES**

Consider a class of uncertain nonlinear systems subject to simultaneous actuator and sensor faults as follows:

$$\begin{cases} \dot{x} = Ax + Bu + E\Phi(x,t) + F_a f_a + D\eta, \\ y = Cx + F_s f_s, \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $y \in \mathbb{R}^p$ ,  $u \in \mathbb{R}^m$  and  $\eta \in \mathbb{R}^k$  are the state, output, known (measurable) input, and unknown input (or modeling uncertainties) vectors, respectively.  $f_a \in \mathbb{R}^{n_a}$  and  $f_s \in \mathbb{R}^{n_s}$  stand for the actuator and sensor fault vectors, respectively. The sensor fault  $f_s$  is a continuous differentiable function and can be in many forms, such as constant, time varying, even unbounded. The function  $\Phi(x,t): \mathbb{R}^n \times \mathbb{R}^+ \rightarrow \mathbb{R}^{n_f}$  is the nonlinear term and it is assumed to be continuous. Matrices  $A, B, C, D, E, F_a$  and  $F_s$  are known with appropriate dimensions. Without loss of generality, it is assumed that the output matrix  $C$  is a full-row rank matrix, the distribution matrices  $H = [F_a \ D]$  and  $F_s$  are all full-column rank matrices, and  $n \geq p \geq q$  and  $q = n_a + n_s + k$ .

**Assumption 1:** The nonlinear function vector  $\Phi(x,t)$  satisfies Lipschitz conditions, i.e.,

$$\|\Phi(x,t) - \Phi(\hat{x},t)\| \leq L_f \|x - \hat{x}\|, \quad \forall x, \hat{x} \in \mathbb{R}^n,$$

where  $L_f$  is the Lipschitz constant.

**Assumption 2:** For every complex number  $s$  with nonnegative real part, the rank condition

$$\text{rank} \begin{bmatrix} sI - A & 0 & H \\ C & F_s & 0 \end{bmatrix} = n + q \quad (2)$$

holds.

**Assumption 3:** The actuator fault  $f_a$  and unknown input  $\eta$  are all bounded in norm. Specially, there exist positive scalars  $\mu_1, \mu_2$  and  $\mu_3$  such that  $\|\varphi\| \leq \mu_1, \|\eta\| \leq \mu_2$  and  $\|f_a\| \leq \mu_3$  hold, where  $\varphi = [f_a^T \ \eta^T]^T$ .

If an augmented state vector  $z = [x^T \ f_s^T]^T$  which can deal with the sensor fault  $f_s$  is introduced, then we obtain the following augmented singular system

$$\begin{cases} M\dot{z} = A_s z + Bu + E\Phi(x,t) + F_a f_a + D\eta, \\ y = C_s z, \end{cases} \quad (3)$$

where  $M = [I_n \ 0_{n \times n_s}]$ ,  $A_s = [A \ 0_{n \times n_s}]$  and  $C_s = [C \ F_s]$ . If a new matrix is set as

$$W = \begin{bmatrix} M \\ C_s \end{bmatrix} = \begin{bmatrix} I_n & 0_{n \times n_s} \\ C & F_s \end{bmatrix},$$

then it is easy to check that  $W$  is a full-column matrix since  $F_s$  is full-column rank matrix. So, there exist matrices  $U \in \mathbb{R}^{(n+n_s) \times n}$  and  $V \in \mathbb{R}^{(n+n_s) \times p}$  such that

$$[U \ V]W = [U \ V] \begin{bmatrix} M \\ C_s \end{bmatrix} = UM + VC_s = I_{n+n_s} \quad (4)$$

holds and is with a general solution given by

$$[U \ V] = W^+ + \Gamma(I_{n+p} - WW^+),$$

where  $\Gamma$  is an arbitrary  $(n+n_s) \times (n+p)$  matrix, and  $W^+ = (W^T W)^{-1} W^T$ .  $\Gamma(I_{n+p} - WW^+)$  represents the freedom left in  $[U \ V]$  after satisfying  $[U \ V]W = I_{n+n_s}$ . Specially, a special solution of (4) is given as

follows:

$$\begin{aligned} [U \ V] &= (W^T W)^{-1} W^T = (W^T W)^{-1} \begin{bmatrix} M \\ C_s \end{bmatrix}^T \\ &= (W^T W)^{-1} \begin{bmatrix} I_n & C^T \\ 0_{n_s \times n} & F_s^T \end{bmatrix}. \end{aligned}$$

So, we can obtain that

$$U = (W^T W)^{-1} \begin{bmatrix} I_n \\ 0_{n_s \times n} \end{bmatrix}, \quad V = (W^T W)^{-1} \begin{bmatrix} C^T \\ F_s^T \end{bmatrix}.$$

**Lemma 1:** The triple  $\{U A_s, C_s, UH\}$  is minimum phase, i.e., the invariant zeros of the triple  $\{U A_s, C_s, UH\}$  are all in the open left-hand complex plant, or

$$\text{rank} \begin{bmatrix} sI - U A_s & UH \\ C_s & 0 \end{bmatrix} = n + q \quad (5)$$

holds for every complex number  $s$  with nonnegative real part if and only if (2) holds for every complex number  $s$  with nonnegative real part.

**Proof:** For every complex number  $s$  with nonnegative real part, if we set matrices

$$\begin{aligned} \Pi_1 &= \begin{bmatrix} (W^T W)^{-1} & 0 \\ 0 & I_p \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I_n & 0 & -sC^T \\ 0 & I_{n_s} & -sF_s^T \\ 0 & 0 & I_p \end{bmatrix}, \\ \Pi_3 &= \begin{bmatrix} sI_n + sC^T C - A & sC^T F_s \\ sF_s^T C & sF_s^T F_s \end{bmatrix}, \end{aligned}$$

then there is

$$\begin{aligned} &\text{rank} \begin{bmatrix} sI - U A_s & UH \\ C_s & 0 \end{bmatrix} \\ &= \text{rank} \left( \Pi_1 \begin{bmatrix} s(W^T W) - \begin{bmatrix} A_s \\ 0 \end{bmatrix} & \begin{bmatrix} H \\ 0 \end{bmatrix} \\ C_s & 0 \end{bmatrix} \right) \\ &= \text{rank} \begin{bmatrix} s(W^T W) - \begin{bmatrix} A_s \\ 0 \end{bmatrix} & \begin{bmatrix} H \\ 0 \end{bmatrix} \\ C_s & 0 \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} s(W^T W) &= s \begin{bmatrix} I_n & 0 \\ C & F_s \end{bmatrix}^T \begin{bmatrix} I_n & 0 \\ C & F_s \end{bmatrix} \\ &= s \begin{bmatrix} I_n + C^T C & C^T F_s \\ F_s^T C & F_s^T F_s \end{bmatrix}. \end{aligned}$$

Thus, the following rank equation can be obtained

$$\text{rank} \begin{bmatrix} sI - U A_s & UH \\ C_s & 0 \end{bmatrix}$$

$$\begin{aligned} &= \text{rank} \begin{bmatrix} \Pi_3 & \begin{bmatrix} H \\ 0_{n_s \times (q-n_s)} \end{bmatrix} \\ [C \ F_s] & 0 \end{bmatrix} \\ &= \text{rank} \left( \Pi_2 \begin{bmatrix} \Pi_3 & \begin{bmatrix} H \\ 0_{n_s \times (q-n_s)} \end{bmatrix} \\ [C \ F_s] & 0 \end{bmatrix} \right) \\ &= \text{rank} \begin{bmatrix} sI_n - A & 0 & H \\ 0 & 0 & 0 \\ C & F_s & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sI_n - A & 0 & H \\ C & F_s & 0 \end{bmatrix}. \end{aligned}$$

From the above rank equation, one can deduces that (5) holds for every complex number  $s$  with nonnegative real part if and only if (2) holds for every complex number  $s$  with nonnegative real part.

**Assumption 4:** For the triple  $\{U A_s, C_s, UH\}$ , the matrix  $(UH)$  is full-column rank, and the following rank condition

$$\text{rank}(C_s UH) = \text{rank}(UH)$$

holds.

**Remark 1:** The rank conditions (2) in Assumption 2 together with Assumption 4 are the sufficient and necessary conditions of unknown input observer design which can be found in the paper [23]. The first condition, the equation (2), is a natural one and assumption 4 is strict for unknown input observer design, and some approaches are proposed to deal with the assumption 4, please see the papers [26] and [27].

**Lemma 2** [23]: Assumptions 2 and 4 hold if and only if for some symmetric positive definite matrix  $Q \in \mathbb{R}^{(n+n_s) \times (n+n_s)}$ , there exist matrices  $L \in \mathbb{R}^{(n+n_s) \times p}$ ,  $G = [G_1^T \ G_2^T]^T \in \mathbb{R}^{(q-n_s) \times p}$  and a symmetric positive definite matrix  $P \in \mathbb{R}^{(n+n_s) \times (n+n_s)}$  such that

$$\begin{cases} (U A_s - L C_s)^T P + P (U A_s - L C_s) = -Q \\ (UH)^T P = G C_s \end{cases} \quad (6)$$

hold, where  $G_1 \in \mathbb{R}^{n_a \times p}$  and  $G_2 \in \mathbb{R}^{k \times p}$ .

**Remark 2:** The paper [23] discussed the way of finding the matrices  $L$ ,  $G$  and  $P$  satisfying (6) in detail. A computing way of them by solving the following optimization problem with LMI constraint is given as follows:

$$\begin{cases} \min \delta \\ P > I \\ P(U A_s) + \Gamma C_s + (P(U A_s) + \Gamma C_s)^T < 0 \\ \begin{bmatrix} \delta I & (UH)^T P - G C_s \\ ((UH)^T P - G C_s)^T & \delta I \end{bmatrix} > 0 \end{cases} \quad (7)$$

and  $L = -P^{-1}\Gamma$ .

**Lemma 3** [24]: Let  $X$  and  $Y$  be two real constant matrices of appropriate dimensions, then the following inequality

$$X^T Y + Y^T X \leq \nu X^T X + \frac{1}{\nu} Y^T Y$$

holds for any scalar  $\nu > 0$ .

### 3. SENSOR FAULT ESTIMATION BASED ON ROBUST SLIDING-MODE OBSERVER

In this section, a robust sliding-mode observer is proposed to estimate the states of augmented singular system (3), and the sensor faults of original system (1) can also simultaneously estimated since the states of augmented singular system (3) consist of the states and sensor fault vectors of system (1).

For the augmented singular system (3), we consider the following robust sliding-mode observer which is used to estimate the states of system (3)

$$\begin{cases} \dot{\xi} = N\xi + L_1 y + UBu + UE\Phi(\hat{x}, t) + \delta_1(y, \hat{z}, t), \\ \dot{\hat{z}} = \xi + Vy, \end{cases} \quad (8)$$

where  $N = UA_s - LC_s$ ,  $L_1 = L + NV$ , and matrix  $L$  is an observer gain which will be determined later. A sliding-mode control law is given as

$$\delta_1(y, \hat{z}, t) = \sigma_1 \frac{(UH)G(y - C_s \hat{z})}{\|G(y - C_s \hat{z})\|}, \quad (9)$$

where  $\sigma_1$  is a positive scalar and is selected to be large enough to satisfy  $\sigma_1 > \mu_1$ .

**Theorem 1:** Under Assumptions 1~4, the observer system (8) with sliding-mode control law (9) is able to asymptotically estimate the states of singular system (3) if the equation (6) and the following inequality

$$\begin{bmatrix} \Lambda & P(UE) & T^T \\ (UE)^T P & -\nu I & 0 \\ T & 0 & -\frac{1}{\nu L_f^2} I \end{bmatrix} < 0 \quad (10)$$

holds for any scalar  $\nu > 0$ , where  $\Lambda = (UA_s - LC_s)^T P + P(UA_s - LC_s)$ .

**Proof:** If the state error is set as  $e = z - \hat{z}$ , then the error dynamics between systems (3) and (8) can be obtained as follows:

$$\begin{aligned} \dot{e} &= \dot{z} - \dot{\hat{z}} = \dot{z} - \dot{\xi} - Vy = (I - VC_s)\dot{z} - \dot{\xi} = UM\dot{z} - \dot{\xi} \\ &= UA_s z + UE\Phi(x, t) + UF_a f_a + UD\eta - N\xi - L_1 y \\ &\quad - UE\Phi(\hat{x}, t) - \delta_1(y, \hat{z}, t) \\ &= UA_s z + UE\Phi(x, t) + UH\varphi - N\xi - L_1 y \\ &\quad - UE\Phi(\hat{x}, t) - \delta_1(y, \hat{z}, t) \\ &= UA_s z + UE\Phi(x, t) + UH\varphi - N(\hat{z} - Vy) \end{aligned}$$

$$\begin{aligned} &-L_1 y - UE\Phi(\hat{x}, t) - \delta_1(y, \hat{z}, t) \\ &= UA_s z + UE\Phi(x, t) + UH\varphi - N(z - e - Vy) \\ &\quad -L_1 y - UE\Phi(\hat{x}, t) - \delta_1(y, \hat{z}, t) \\ &= Ne + (UA_s - N + NVC_s - L_1 C_s)z + UE\tilde{\Phi} \\ &\quad + UH\varphi - \delta_1(y, \hat{z}, t), \end{aligned}$$

where  $\tilde{\Phi} = \Phi(x, t) - \Phi(\hat{x}, t)$  and  $\varphi = [f_a^T \quad \eta^T]^T$ . So we obtain

$$\dot{e} = Ne + UE\tilde{\Phi} + UH\varphi - \delta_1(y, \hat{z}, t) \quad (11)$$

since  $N = UA_s - LC_s$  and  $L_1 = L + NV$  hold.

Consider the Lyapunov function candidate  $V_L = e^T P e$ , the derivative of  $V_L$  along (11) is

$$\begin{aligned} \dot{V}_L &= e^T ((UA_s - LC_s)^T P + P(UA_s - LC_s))e \\ &\quad + e^T P(UE)\tilde{\Phi} + \tilde{\Phi}^T (UE)^T P e + 2e^T P(UH)\varphi \\ &\quad - 2e^T P\delta_1(y, \hat{z}, t). \end{aligned} \quad (12)$$

Based on the second equation of (6), we have

$$\begin{aligned} 2e^T P(UH)\varphi &= 2e^T C_s^T G^T \varphi \\ &\leq 2\|GC_s e\|\|\varphi\| \leq 2\mu_1 \|GC_s e\|. \end{aligned} \quad (13)$$

Form the sliding-mode control law (9), there is

$$\begin{aligned} -2e^T P\delta_1(y, \hat{z}, t) &= -2\sigma_1 \frac{e^T P(UH)G(y - C_s \hat{z})}{\|G(y - C_s \hat{z})\|} \\ &= -2\sigma_1 \frac{e^T C_s^T G^T G(y - C_s \hat{z})}{\|G(y - C_s \hat{z})\|} \\ &= -2\sigma_1 \|GC_s e\|. \end{aligned} \quad (14)$$

Consider the Lipschitz condition in Assumption 1, the following inequality

$$\begin{aligned} \|\tilde{\Phi}\| &= \|\Phi(x, t) - \Phi(\hat{x}, t)\| \leq L_f \|x - \hat{x}\| \\ &= L_f \|T(z - \hat{z})\| = L_f \|Te\| \end{aligned}$$

holds since  $z = [x^T \quad f_s^T]^T$ , where  $T = [I_n \quad 0_{n \times n_s}]$ . The above inequality can be written as the equivalent form to

$$\tilde{\Phi}^T \tilde{\Phi} \leq L_f^2 e^T T^T T e. \quad (15)$$

Based on Lemma 3 and (15), one can obtain

$$\begin{aligned} &e^T P(UE)\tilde{\Phi} + \tilde{\Phi}^T (UE)^T P e \\ &\leq \nu \tilde{\Phi}^T \tilde{\Phi} + \frac{1}{\nu} e^T P(UE)(UE)^T P e \\ &\leq \nu L_f^2 e^T T^T T e + \frac{1}{\nu} e^T P(UE)(UE)^T P e. \end{aligned} \quad (16)$$

Equations (12)~(14) and (16) together yields

$$\begin{aligned} \dot{V}_L &\leq e^T ((UA_s - LC_s)^T P + P(UA_s - LC_s))e \\ &\quad + \nu L_f^2 e^T T^T T e + \frac{1}{\nu} e^T P(UE)(UE)^T P e \\ &\quad + 2(\mu_1 - \sigma_1) \|GC_s e\|. \end{aligned}$$

Thus, if one sets  $\Lambda = (UA_s - LC_s)^T P + P(UA_s - LC_s)$ , then there is

$$\dot{V}_L < e^T \left( \Lambda + \nu L_f^2 T^T T + \frac{1}{\nu} P(UE)(UE)^T P \right) e$$

since  $\sigma_1 > \mu_1$ . By applying Schur complement to the matrix inequality (10), there is  $\dot{V}_L < 0$ . So, the equilibrium point of zero of the observer error dynamics system (11) is asymptotically stable.

Theorem 1 provides a LMI solution to the nonlinear robust sliding-mode observer (8). Since the algebraic equation (6) can be solved by the optimization problem (7) with LMI constraint, so we can derive the following Corollary 1 from (7) and (10) based on the well-known Schur complement Lemma.

**Corollary 1:** Under Assumptions 1~4, the observer system (8) exists if the following optimization problem with LMI constraint

$$\begin{cases} \min \delta \\ P > I \\ \begin{bmatrix} \Omega & P(UE) & T^T \\ (UE)^T P & -\nu I & 0 \\ T & 0 & -\frac{1}{\nu L_f^2} I \end{bmatrix} < 0 \\ \begin{bmatrix} \delta I & (UH)^T P - GC_s \\ ((UH)^T P - GC_s)^T & \delta I \end{bmatrix} > 0 \end{cases} \quad (17)$$

is feasible for any scalar  $\nu > 0$ , and gain matrix  $L = -P^{-1}\Gamma$ , where  $\Omega = P(UA_s) + \Gamma C_s + (P(UA_s) + \Gamma C_s)^T$ .

After the estimated state  $\hat{z}$  is derived from the robust sliding-mode observer (8), it is easy to obtain that the estimated state  $\hat{x}$  and sensor fault  $\hat{f}_s$  are described as

$$\hat{x} = [I_n \quad 0_{n \times n_s}] \hat{z}, \quad \hat{f}_s = [0_{n_s \times n} \quad I_{n_s}] \hat{z} \quad (18)$$

since the fact that  $\hat{z} = [\hat{x}^T \quad \hat{f}_s^T]^T$ .

#### 4. ACTUATOR FAULT DETECTION AND ISOLATION

The robust observer design methods of the previous section simultaneously provide the estimations of both the states and sensor faults of original system (1). However, the actuator faults and unknown inputs are eliminated by designing the robust sliding-mode control law (9). That is to say, the observer (8) is robust to actuator faults and unknown inputs, which makes the detection and isolation of actuator faults to be difficult. In this section, the detectors and isolators of actuator faults are developed based on the sliding-mode technique.

**Theorem 2:** Under Assumptions 1~4, if there is without actuator faults ( $f_a = 0$ ), the observer system (19)

with sliding-mode control law (20) is able to asymptotically estimate the states of singular system (3).

$$\begin{cases} \dot{\xi}_1 = N \xi_1 + L_1 y + U B u + U E \Phi(\hat{x}, t) + \delta_2(y, \hat{z}_1, t), \\ \dot{\hat{z}}_1 = \xi_1 + V y, \end{cases} \quad (19)$$

where  $N = UA_s - LC_s$ ,  $L_1 = L + NV$ , and matrix  $L$  is an observer gain which will be determined later. A sliding-mode control law is given as

$$\delta_2(y, \hat{z}_1, t) = \sigma_2 \frac{(UD)G_2(y - C_s \hat{z}_1)}{\|G_2(y - C_s \hat{z}_1)\|}, \quad (20)$$

where  $\sigma_2$  is a positive scalar and is selected to be large enough to satisfy  $\sigma_2 > \mu_2$ .

**Proof:** If there is without actuator faults ( $f_a = 0$ ), then the error dynamics between systems (3) and (19) can be obtained by the similar way to Theorem 1 as follows

$$\dot{e}_1 = N e_1 + U E \tilde{\Phi} + U D \eta - \delta_2(y, \hat{z}_1, t),$$

where  $e_1 = z - \hat{z}_1$  is the observer error. Note that

$$\begin{aligned} 2e_1^T P(UD)\eta &= 2e_1^T C_s^T G_2^T \eta \\ &\leq 2\|G_2 C_s e_1\| \|\eta\| \leq 2\mu_2 \|G_2 C_s e_1\| \end{aligned}$$

and

$$\begin{aligned} -2e_1^T P \delta_2(y, \hat{z}_1, t) &= -2\sigma_2 \frac{e_1^T P(UD)G_2(y - C_s \hat{z}_1)}{\|G_2(y - C_s \hat{z}_1)\|} \\ &= -2\sigma_2 \frac{e_1^T C_s^T G_2^T G_2(y - C_s \hat{z}_1)}{\|G_2(y - C_s \hat{z}_1)\|} \\ &= -2\sigma_2 \|G_2 C_s e_1\|, \end{aligned}$$

it is easy to derive the following proof of Theorem 2 from that of Theorem 1.

In this paper,  $n_a$  adaptive robust sliding-mode observers whose numbers equal to ones of actuator components are developed to isolate the actuator fault  $f_a$  based on multi-observer idea, while the unknown input  $\eta$  is eliminated by designing sliding-mode control law. In fact, we assume that the distribution matrices of actuator fault vector and unknown input vector are column linear independent, which means that the filter direction of the actuator fault is different from that of the unknown input. So, the sliding-mode control law (20) just affects unknown input  $\eta$ , and the observer output error  $e_{out} = y - C_s \hat{z}_1$  is with robust character to unknown input  $\eta$  instead of actuator fault  $f_a$ . Thus,  $e_{out}$  is sensitive to actuator faults, and will asymptotically approach to zero or the small neighborhood of zero when  $f_a = 0$ . So the robust sliding-mode observer (19) with sliding mode control law (20) can be regarded as an actuator fault detector, and the decision logic for actuator fault detection is described as

$$J_a(t) = \begin{cases} \|e_{out}\| \leq \Theta, & \text{actuator fault free} \\ \|e_{out}\| > \Theta, & \text{actuator fault occur,} \end{cases}$$

where  $\Theta$  is a fixed threshold selected to be a little larger than the maximum residual when the system is without actuator fault, one can determine whether the system is affected by actuator faults or not.

Although the proposed actuator fault detector (19) can alert us to the occurrence of the actuator faults when at least one actuator fault occurs, it can neither tell which actuator is with fault, nor point out when an actuator is with fault. To overcome these drawbacks, the multi-observer idea has been considered [25]. Next,  $n_a$  adaptive robust sliding-mode observers which can isolate actuator faults will be designed. The observer output error of  $\tau$  th observer is just impacted by  $\tau$  th actuator fault, but it is not impacted by the unknown input and other actuator faults, where  $\tau = 1, 2, \dots, n_a$ .

If we introduce the following matrices or vectors

$$\begin{aligned} f'_{a,\tau} &= [f_{a,1} \ \cdots \ f_{a,\tau-1} \ f_{a,\tau+1} \ \cdots \ f_{a,n_a}]^T, \\ F'_{a,\tau} &= [F_{a,1} \ \cdots \ F_{a,\tau-1} \ F_{a,\tau+1} \ \cdots \ F_{a,n_a}], \\ G'_{a,\tau} &= [G_{1,1}^T \ \cdots \ G_{1,\tau-1}^T \ G_{1,\tau+1}^T \ \cdots \ G_{1,n_a}^T]^T, \end{aligned}$$

where  $\tau = 1, 2, \dots, n_a$ , and  $f_{a,\tau}$ ,  $F_{a,\tau}$  and  $G_{1,\tau}$  are the  $\tau$  th actuator fault of  $f_a$ , the  $\tau$  th column vector of  $F_a$  and the  $\tau$  th row vector of  $G_1$ , respectively.

**Theorem 3:** Under Assumption 1~4, for  $\tau$  th actuator fault ( $\tau = 1, 2, \dots, n_a$ ), one can construct the following robust sliding-mode observer

$$\begin{cases} \dot{\xi}_{ar} = N\xi_{ar} + L_1y + UB u + UE\Phi(\hat{x}, t) \\ \quad + \delta_{\tau 1}(y, \hat{z}_{ar}, t) + \delta_{\tau 2}(y, \hat{z}_{ar}, t), \\ \hat{z}_{ar} = \xi_{ar} + Vy, \end{cases} \quad (21)$$

with sliding-mode control law

$$\begin{aligned} \delta_{\tau 1}(y, \hat{z}_{ar}, t) &= \sigma_{\tau 1} \frac{(UD)G_2(y - C_s \hat{z}_{ar})}{\|G_2(y - C_s \hat{z}_{ar})\|}, \\ \delta_{\tau 2}(y, \hat{z}_{ar}, t) &= \sigma_{\tau 2} \frac{(UF'_{a,\tau})G'_{a,\tau}(y - C_s \hat{z}_{ar})}{\|G'_{a,\tau}(y - C_s \hat{z}_{ar}t)\|}, \end{aligned}$$

where  $\sigma_{\tau 1}$  and  $\sigma_{\tau 2}$  are all positive scalars and are selected to be large enough to satisfy  $\sigma_{\tau 1} > \mu_2$  and  $\sigma_{\tau 2} > \mu_3$ .

**Proof:** When the  $\tau$  th actuator is without fault, the system (3) is equivalent to

$$\begin{cases} M\dot{z} = A_s z + Bu + E\Phi(x, t) + F'_{a,\tau} f'_{a,\tau} + D\eta, \\ y = C_s z. \end{cases} \quad (22)$$

The error dynamic equation between (21) and (22) can be written as

$$\begin{aligned} \dot{e}_\tau &= Ne_\tau + UE\tilde{\Phi} + UD\eta + UF'_{a,\tau} f'_{a,\tau} \\ &\quad - \delta_{\tau 1}(y, \hat{z}_{ar}, t) - \delta_{\tau 2}(y, \hat{z}_{ar}, t), \end{aligned}$$

where  $e_{ar} = z - \hat{z}_{ar}$ . Consider the Lyapunov function  $V_{ar} = e_{ar}^T P e_{ar}$  and notice that

$$\begin{aligned} 2e_{ar}^T P(UD)\eta &\leq 2\|G_2 C_s e_{ar}\| \|\eta\| \leq 2\mu_2 \|G_2 C_s e_{ar}\|, \\ 2e_{ar}^T P(UF'_{a,\tau}) f'_{a,\tau} &= 2e_{ar}^T C_s^T G_{a,\tau}^T f'_{a,\tau} \\ &\leq 2\|G'_{a,\tau} C_s e_{ar}\| \|f'_{a,\tau}\| \leq 2\mu_3 \|G'_{a,\tau} C_s e_{ar}\|, \\ -2e_{ar}^T P\delta_{\tau 1}(y, \hat{z}_{ar}, t) &= -2\sigma_{\tau 1} \frac{e_{ar}^T P(UD)G_2(y - C_s \hat{z}_{ar})}{\|G_2(y - C_s \hat{z}_{ar})\|} \\ &= -2\sigma_{\tau 1} \frac{e_{ar}^T C_s^T G_2^T G_2(y - C_s \hat{z}_{ar})}{\|G_2(y - C_s \hat{z}_{ar})\|} = -2\sigma_{\tau 1} \|G_2 C_s e_{ar}\| \end{aligned}$$

and

$$\begin{aligned} -2e_{ar}^T P\delta_{\tau 2}(y, \hat{z}_{ar}, t) &= -2\sigma_{\tau 2} \frac{e_{ar}^T P(UF'_{a,\tau})G'_{2,\tau}(y - C_s \hat{z}_{ar})}{\|G'_{2,\tau}(y - C_s \hat{z}_{ar})\|} \\ &= -2\sigma_{\tau 2} \frac{e_{ar}^T C_s^T G_{a,\tau}^T G'_{2,\tau}(y - C_s \hat{z}_{ar})}{\|G'_{2,\tau}(y - C_s \hat{z}_{ar})\|} \\ &= -2\sigma_{\tau 2} \|G'_{2,\tau} C_s e_{ar}\|, \end{aligned}$$

the following proof is similar to that of Theorem 1.

The robust sliding-mode observer (21) is the  $\tau$  th fault detector for the  $\tau$  th actuator fault. So, based on observer output error  $e_{out,\tau} = y - C_s \hat{z}_\tau$ , the following decision logic for detecting the  $\tau$  th actuator fault is described as

$$J_{a,\tau}(t) = \begin{cases} \|e_{out,\tau}\| \leq \Theta_\tau, & \text{the } \tau\text{th actuator fault free} \\ \|e_{out,\tau}\| > \Theta_\tau, & \text{the } \tau\text{th actuator fault occurs} \end{cases}$$

where  $\Theta_\tau$  ( $\tau = 1, 2, \dots, n_a$ ) are fixed thresholds selected to be a little larger than the maximum residual.

**Remark 3:** In [2], the fault detection problem is discussed for linear discrete system with the same actuator and sensor fault information, while an actuator fault estimation approach for nonlinear descriptor system is developed in [15]. A fault detection and isolation approach for actuator stuck faults is addressed in [20]. The results of the above papers are derived under the assumptions that the linear or nonlinear system is just with actuator faults [15,20] or with the same actuator and sensor fault signals [2]. Our paper tries to deal with the issues of sensor fault estimation, actuator fault detection and isolation when the nonlinear system is with unknown input, the different actuator and sensor fault signals. So, to some extent, our work is an improvement of papers [2,15,20].

## 5. SIMULATION

### 5.1. Model expression

In this section, an illustrative example will be given to verify the proposed methods. Consider the modified system (1) with the following state-space matrices for an aircraft model [28]

$$A = \begin{bmatrix} -1.05 & -2.55 & 0 & 0 & -169 & -0.0091 \\ 2.55 & -0.05 & 0 & 0 & 67.09 & 0.0017 \\ 0 & 0 & -77.53 & 39.57 & 0 & 0 \\ 0 & 0 & 0 & -20.2 & 0 & 0 \\ 0 & 0 & -8.8 & 0 & -20.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4.49 \\ 0 \\ 0 \end{bmatrix}, \quad D = E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad F_a = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.01 & 0.09 & 0.07 & 0 & 0 & 0 \\ -4.8 & -5.9 & 0 & 0 & -9.51 & -0.26 \\ 3 & 0.9 & -0.6 & 0 & 0 & 0 \\ 2.6 & -7 & 1 & 0 & 0 & 0 \end{bmatrix},$$

and  $F_s = [1 \ 0 \ 1 \ 1]^T$ . The nonlinearity is  $\Phi(x, t) = 0.5(|x_1 + 1| - |x_1 - 1|)$  with Lipschitz constant  $L_f = 1$ . The input uncertainty  $\eta$ , actuator fault  $f_a$  and sensor fault  $f_s$  have been added to demonstrate the proposed method, and they are assumed to be

$$\eta = 2.5 \sin(2.5t), \quad f_s = 1.5 \cos(3.6t + 4.8), \quad f_a = \begin{bmatrix} f_{a1} \\ f_{a2} \end{bmatrix},$$

where

$$f_{a1} = \begin{cases} 3, & 9 \leq t \leq 14 \\ 0, & \text{other,} \end{cases} \quad f_{a2} = \begin{cases} 5, & 12 \leq t \leq 18 \\ 0, & \text{other.} \end{cases} \quad (23)$$

The augmented singular system (3) can be obtained by introducing new state vector  $z = [x^T \ f_s^T]^T$ . The following matrices  $U$  and  $V$  can be computed such (4) holds as follows:

$$U = \begin{bmatrix} 0.4993 & 0.1892 & 0.2441 \\ 0.1892 & 0.116 & 0.1255 \\ 0.2441 & 0.1255 & 0.1773 \\ 0 & 0 & 0 \\ -0.3651 & -0.1655 & -0.1987 \\ -0.01 & -0.0045 & -0.0054 \\ -0.1535 & 0.0848 & 0.0864 \\ 0 & -0.3651 & -0.01 \\ 0 & -0.1655 & -0.0045 \\ 0 & -0.1987 & -0.0054 \\ \leftarrow 1 & 0 & 0 \\ 0 & 0.2953 & -0.0193 \\ 0 & -0.0193 & 0.9995 \\ 0 & 0.0246 & 0.0007 \end{bmatrix},$$

$$V = \begin{bmatrix} -0.1144 & -0.0384 & 0.0501 & 0.0644 \\ 0.1059 & -0.0174 & 0.0039 & -0.1098 \\ 0.1125 & -0.0209 & -0.1325 & 0.02 \\ 0 & 0 & 0 & 0 \\ -0.0079 & -0.0741 & -0.0273 & -0.0352 \\ -0.0002 & -0.002 & -0.0007 & -0.001 \\ 0.9444 & 0.0026 & 0.0297 & 0.0259 \end{bmatrix}.$$

It is easy to check that the rank condition  $\text{rank}(C_s UH) = \text{rank}(UH) = 3$  is satisfied. So, by applying Matlab's LMI toolbox to optimization problem (17) with LMI constraint, matrices  $L$  and  $G$  which satisfy the matrix equation (6) are obtained

$$L = \begin{bmatrix} -497.2771 & -348.6655 & 137.1140 & 25.7299 \\ -177.6757 & -146.5706 & 52.5529 & 8.4863 \\ -250.6020 & -186.5213 & 71.0579 & 12.2715 \\ 360.4755 & -614.4401 & -8.7195 & 28.2278 \\ 371.0311 & 258.7822 & -102.3267 & -18.5677 \\ -22.8407 & 18.8910 & 1.0563 & 0.6039 \\ 164.8641 & 51.9838 & -32.6106 & -8.6171 \end{bmatrix},$$

$$G = \begin{bmatrix} -12.2774 & -35.5057 & -70.7908 & 55.1998 \\ -56.3722 & -11.3010 & 2.2161 & 8.0388 \\ 41.8914 & -1.2764 & -2.8300 & -4.0131 \end{bmatrix},$$

where the scalar is set as  $\nu = 1000$ . So, it is easy to derive the observer gains  $N$  and  $L_1$  from the matrix  $L$ .

### 5.2. State and sensor fault estimation

By Theorem 1, we know that the robust observer (8) can asymptotically estimate the state  $z$  of singular system (3). So, the states and unknown inputs of original system (1) can be obtained since  $z = [x^T \ f_s^T]^T$ . In the simulation, the initial states and initial estimation of the states are set as  $x(0) = [4.8 \ -5.6 \ 2.3 \ -5.8 \ 2.6 \ 5.7]^T$  and  $\xi(0) = [6.5 \ 9.3 \ -6.9 \ -5.8 \ 3.6 \ 5.4 \ 2.6]^T$ , respectively, and the positive scalar  $\sigma_1$  is set as 10. The state estimation error curves are given in Fig. 1, and the sensor fault estimation is also shown in Fig. 2. From Figs. 1 and 2, we see that the performance of the proposed methods is satisfactory.

### 5.3. Actuator fault detection and isolation

In the above section, the robust sliding-mode control law (9) can simultaneously eliminate the effects of actuator faults and unknown inputs such that both states and sensor faults of original system (1) can be estimated. In order to further detect and isolate the actuator faults when ones occur, a robust sliding-mode observer is designed as fault detector and multiple observers are given as fault isolators.

By Theorem 2, we know that, when the system is free from actuator faults ( $f_a = 0$ ), the observer system given by (19) and (20) is a robust sliding-mode observer

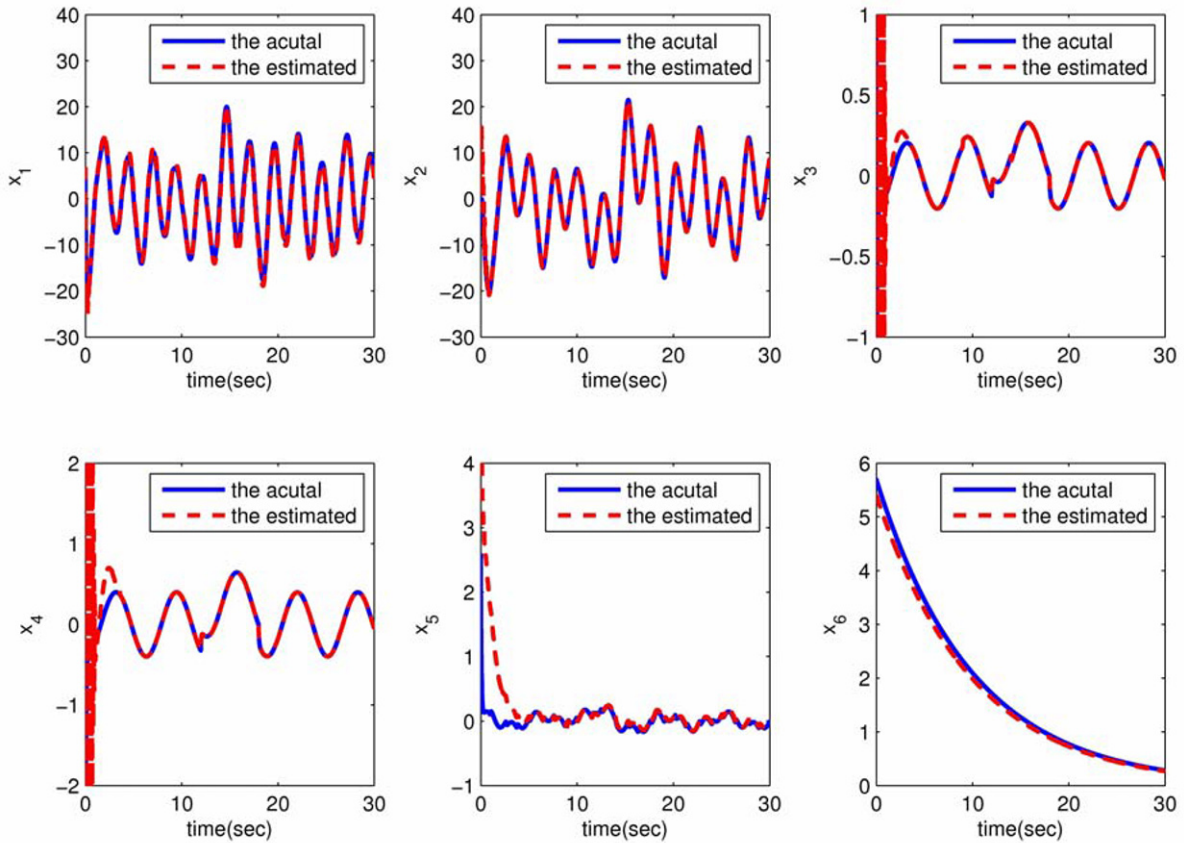


Fig. 1. Estimation curves of system state  $x$ .

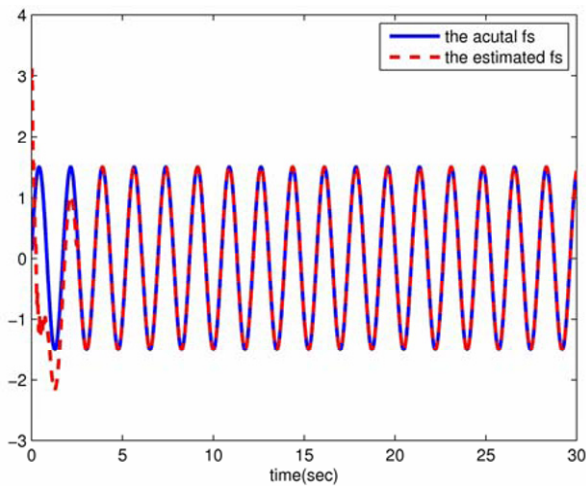


Fig. 2. Estimation of sensor fault  $f_s$ .

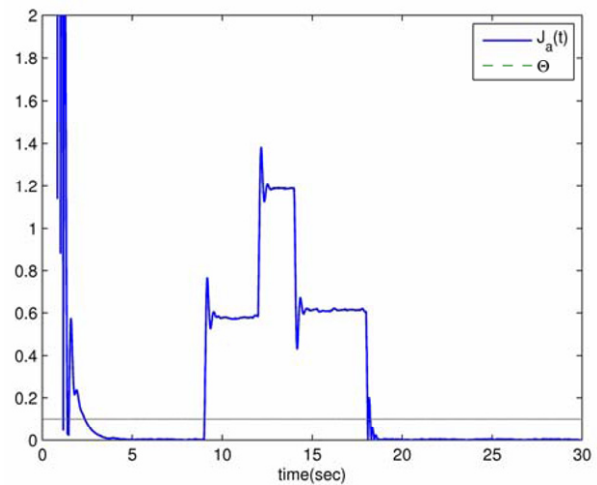


Fig. 3. Actuator fault detection.

which is an actuator fault detector. If the scalar  $\sigma_2$  and initial state are set as  $\sigma_2=8$  and  $\zeta_1(0) = \zeta(0)$ , respectively, and the actuator faults are assumed in (23). The alarm threshold is given as  $\Theta=0.1$ . The norm of the output error of detection observer and the threshold are both plotted in Fig. 3 which shows that actuator fault appears during the time of about 9-18s. Fig. 3, however, can neither tell us how many actuators are with faults nor which actuator is with fault.

In order to identify which actuator is with fault and reach the purpose of actuator fault isolation, multiple

robust sliding-mode observers are proposed in Theorem 3. For this simulation, the number of the designed isolation observer is 2 since  $n_a=2$ . The positive scalars are selected as  $\sigma_{\tau 1}=10$  and  $\sigma_{\tau 2}=5$ , and the initial state is set as  $\zeta_{at}(0) = \zeta(0)$ , where  $\tau=1,2$ . The thresh-olds for isolating actuator fault  $f_{a1}$  and  $f_{a2}$  are set as  $\Theta_1 = \Theta_2 = 0.05$ . Figs. 4 and 5 show the performances of the detection of actuator fault  $f_{a1}$  and  $f_{a2}$ , respectively. From Fig. 4, it is told that the first actuator fault occurs during the time of about 9-14s, and Fig. 5 shows that the second actuator is with fault during the time of about 12-18 s.



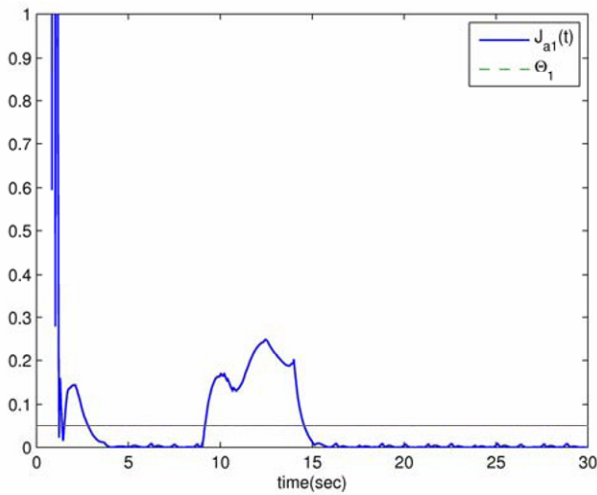


Fig. 4. Fault detection of  $f_{a1}$ .

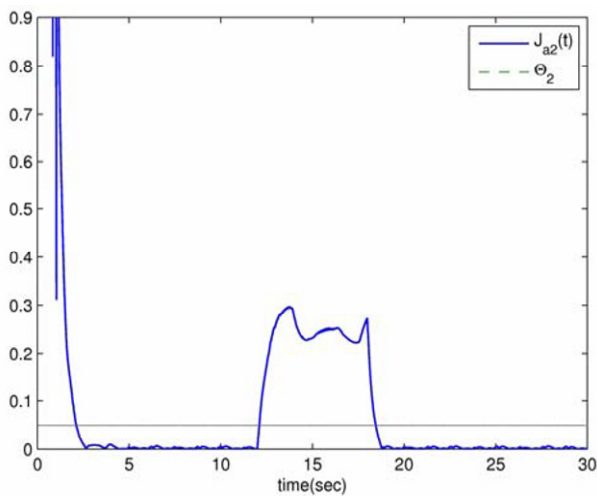


Fig. 5. Fault detection of  $f_{a2}$ .

**Remark 4:** In [20], the actuator fault detection and isolation problem is also discussed and the sensor fault is not occurred. Our paper considers not only actuator fault but also sensor fault, and the sensor fault is estimated as the state of constructed singular system. Moreover, for the actuator fault detection results, we can see from Fig. 1 of paper [20] and Fig. 3 of this one that the detection effectiveness of the proposed fault detection method is timelier.

## 6. CONCLUSIONS

In this paper, the issues of simultaneous sensor fault estimation, actuator fault detection and isolation for a class of uncertain nonlinear are discussed. An augmented state vector is introduced to deal with sensor faults, and a robust observer with sliding mode law which is adopt to attenuate unknown inputs and actuator faults is developed to estimate states and sensor faults of the original system. A robust sliding mode observer used as fault detector is proposed to detect actuator faults, and multiple observers are considered to achieve the purpose of actuator fault isolation. Our methods can not only

estimate the states and sensor faults of the original system but also detect and isolate actuator faults, while the unknown inputs are dealt with sliding mode technique.

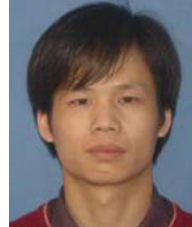
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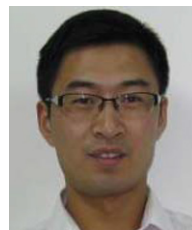
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