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Multiple-model Rao-blackwellized Particle Probability Hypothesis Density Filter for Multitarget Tracking

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Abstract: Multitarget tracking (MTT) is a frequent topic in visual surveillance systems. Although the multiple-model probability hypothesis density (MM-PHD) filter plays an important role in the MTT, both computerized intractability and imprecise estimate are still inevitable. To solve the problems, a novel filter is presented in this paper. Different from the previous work, the Rao-Blackwellized particle filtering algorithm is incorporated with the MM-PHD filter to reduce computational load, where the sequence Monte Carlo method is adopted to estimate the nonlinear state of targets, and the linear state is predicted using the Kalman filter with the information embedded in the estimated nonlinear state. With respect to tracking precision, we find that the reweighting scheme can be realized for the number-estimate of both undetected targets and false alarms. The result is useful in balancing the required particle number in order to stabilize target estimates during the surveillance period. The illustrative simulation is finally provided to show the effectiveness of the proposed filter.

Keywords: False alarm, multitarget tracking, particle, probability hypothesis density filter.

1. INTRODUCTION

Multitarget tracking (MTT) is to jointly estimate the number of targets and their states in a series of the noisy and cluttered measurements. With regard to the unknown and time-varying targets, the MTT has played a major role in both military and civilian fields, such as radar tracking, sonar tracking, etc [1,2]. In the past decade, several scholars have studied this topic with a great deal of success and many papers with respect to the well established MTT algorithms have been published in the important literature [3-6]. As early as in 2003, the work conducted by Mahler demonstrated the prospective development of multitarget filters. In particular, the probability hypothesis density (PHD) filter based on the statistical first moment approximation has been receiving significant attention [3]. This filter propagates the PHD of multitarget posterior density instead of the full multitarget posterior density as well as incorporates track without consideration of the traditional measurement-to-track association [4]. Operating on the single-target state space, the integral of the PHD over a certain region is regarded as the estimated number of targets, and the associated peaks represent the estimated state of targets [5,6].

Although the PHD filter is a promising algorithm for the MMT, based on the single-model (SM) method, it is difficult to match the true target dynamics, especially the maneuvering targets. In addition, the filter cannot reflect all variations in dynamic parameters due to the errors falling outside range [7]. From analysis perspective, the multiple-model (MM) method can well match different target motions with parallel sub-filters; for example [8,9] and the references therein. In [10], the authors derived the closed-form solution to the PHD recursion for linear Gaussian jump Markov multitarget mode with available Gaussian components. In [11], the MM implementation of the particle PHD filter was applied with satisfactory performance using the sequential Monte Carlo (SMC) method. It is worth noting that the usefulness of both filters was demonstrated under one constant velocity (CV) model and/or one constant turn (CT) model. Some drawbacks are apparently related to implementation of the existing MM-PHD filters, even the individual PHD filter. To begin with, there is no closed-form solution owing to the PHD propagation involving multiple integrals [6,12]. Then, some fluctuations in number-estimate and state-estimate are inevitable due to both sensor imperfection and clutter influence. Last but not least, the extended high-dimensional state space incurs enormous computational load, such as extra required particles and long running time. To avoid intractable computation, [13] presented the particle approximation to the closed-form solution to the PHD propagation. For more results to this topic, we refer readers to [14-16]. As for the second shortcoming, the improved PHD filtering algorithm for overestimated target number was put forward in [17] that confirmed the label with biggish weight to eliminate the clutter influence. Nevertheless, to our knowledge, the last puzzle has not been dealt with up to now.

Following the work of [11], it is our intention in this article to solve the problems: (i) how the computational

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complexity can be reduced with small particle number? (ii) how the tracking accuracy of the individual PHD filter accompanied by the proposed strategy can be improved? For the purpose, we present an MM version of the Rao-Blackwellized particle (RBP) PHD filter, named the RBPPHD filter. The main contributions of this work can be summarized as follows: (i) reduced computational complexity: It is known that the RBP filtering algorithm can reduce particle number with small variance. There is a definite link between filtering algorithm and computational load. Application of the algorithm to the MM-PHD filter would likely perform better; (ii) improved tracking accuracy: On the basis of reweighting technique, the weight of the survival and newborn particles is balanced. As a result, the number of undetected targets and false alarms is corrected. Furthermore, the accuracy of state-estimate using the individual PHD filter is improved in the k-means clustering.

The remainder of this note is organized as follows: Section 2 addresses the principle of the MM-PHD filter. The improvements of the proposed filter and the particle implementation are studied in Section 3. In Section 4, the simulation is showed with results to verify tracking performance of the proposed filter. In Section 5, the conclusions are drawn by presenting the future work.

2. PRELIMINARIES

In the MTT, the collections of the target state and measurement at scan k are given by

$$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}), \tag{1}$$

$$\mathbf{z}_k = h_{k-1}(\mathbf{x}_{k-1}, \mathbf{e}_{k-1}). \tag{2}$$

where f_{k-1} and h_{k-1} denote the known transition and observation functions; $\mathbf{x}_k$ and $\mathbf{z}_k$ are the state and measurement vectors; $\mathbf{u}_{k-1}$ and $\mathbf{e}_{k-1}$ are the known state and measurement noises [18-20]. If $\mathbf{x}_k$ and $\mathbf{z}_k$ can be represented by the random finite set (RFS), the standard MM-PHD filter is competent. Especially in the cluttered environment, the filter has been shown to offer improvements in tracking performance against some conventional filters. To simplify presentation, we make two remarks in order: (i) without respect to target spawning, target motions are statistically independent; targets disappear from the scene; targets appear in the scene independently of existing targets; (ii) the detection probability is a positive constant p_D regardless of $\mathbf{x}_k$. Then the filtering process can be summarized as follows:

Assume the initial PHD $D_{k-1}(\mathbf{x}_{k-1}, r_{k-1} = v \mid \mathbf{Z}_{1:k-1})$ is derived by the available measurements $\mathbf{Z}_{1:k-1}$ from the previous $k-1$ scans, where r_{k-1} denotes the model index; v is the previous target model [9,11]. As each model-matched PHD filter is fed with different PHDs, the predicted PHD under the current model u at scan k is given by

$$D_{k|k-1}(\mathbf{x}_k, r_k = u \mid \mathbf{Z}_{1:k-1}) = b_k(\mathbf{x}_k, r_k = u) + \int \begin{pmatrix} p_{S,k|k-1}(\mathbf{x}_{k-1}) f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}, r_k = u) \\ \times \sum_{v=1}^{N_r} D_{k-1}(\mathbf{x}_{k-1}, r_{k-1} = v \mid \mathbf{Z}_{1:k-1}) \\ \times f_{k|k-1}(r_k = u \mid r_{k-1} = v) \end{pmatrix} \mathrm{d}\mathbf{x}_{k-1}, \tag{3}$$

where $f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}, r_k = u)$ is the Markov target transition probability; $f_{k|k-1}(r_k = u \mid r_{k-1} = v)$ is the Markov model transition probability; $b_k(\mathbf{x}_k, r_k = u)$ is the PHD of the spontaneous newborn targets; $p_{S,k|k-1}(\mathbf{x}_{k-1})$ is the probability of the survival targets with $\mathbf{x}_{k-1}$; N_r is the number of target models.

As we know, the sensor collects an average number λ of the Poisson-distributed false alarms, the spatial distribution of which is governed by the probability density $c(\mathbf{z}_k)$. We are to derive an equation for the updated PHD under the model u at scan k

$$D_k(\mathbf{x}_k, r_k = u \mid \mathbf{Z}_{1:k}) = \begin{pmatrix} 1 - p_D \\ + \sum_{\mathbf{z}_k \in \mathbf{Z}_k} \dfrac{p_D g_k(\mathbf{z}_k \mid \mathbf{x}_k)}{\lambda c(\mathbf{z}_k) + \int \begin{matrix} p_D g_k(\mathbf{z}_k \mid \mathbf{x}_k) \\ \times D_{k|k-1}(\mathbf{x}_k, r_k = u \mid \mathbf{Z}_{1:k-1}) \end{matrix} \mathrm{d}\mathbf{x}_k} \end{pmatrix} \times D_{k|k-1}(\mathbf{x}_k, r_k = u \mid \mathbf{Z}_{1:k-1}). \tag{4}$$

Based on the physical meanings of the PHD, the integral of (4) over the surveillance region is regarded as the expected number of targets, that is

$$N_k(r_k = u) = \int D_k(\mathbf{x}_k, r_k = u \mid \mathbf{Z}_{1:k}) \mathrm{d}\mathbf{x}_k. \tag{5}$$

We have in hand N_r PHD filters running in parallel. As each filter represents a different model, the total number of targets is estimated by

$$\hat{N}_k = \sum_{u=1}^{N_r} N_k(r_k = u). \tag{6}$$

Finally, the k-means clustering has been employed for state-estimate, which can efficiently partition the particle representation into the number of clusters given by the integer approximation of number-estimate. The center of each cluster represents a local maximum of intensity function as well as gives the state-estimate of a target. For the MM-PHD filter, $\hat{N}_k$ associated maximum peaks are the state-estimate.

Remark 1: We summarize the characteristics of the MM-PHD filtering process under duplicate: (i) consider the target state (position, velocity, etc.) contains at least four dimensions, the target dynamic model and the model index can be regarded as an intractable MM structure; (ii) the sensor may not detect all the adjoining targets due to $p_D \neq 1$, which may bring about the undetected targets when trajectories are close. Besides, some false alarms may appear when estimating targets.

3. MM-RBPPHD FILTER

As we know, the PHD can be approximated by a weighted set of particles, from which the state-estimate of the individual target is generated by the k-means clustering, and the estimated number of targets is given by the sum of weights. From this viewpoint, we develop the proposed filter with considerable modifications in this section.

3.1. RBP filtering step

In the MM-PHD filter, the large number of particles required to approximate the posterior density render the difficult in extended high-dimensional state space. As a result, the extra computational load inevitably stems from the numerous particles. Furthermore, the computing time is on a drastic increase. In this case, the RBP filtering algorithm is competent. Owing to the position information accounting for only a proportion of state, there is no nonlinear problem on velocity-estimate. Accordingly, the target state has the characteristic of linear/nonlinear, that is, the hybrid Markov structure. We will focus on the RBP filtering algorithm, such as with some components having linear dynamics that are estimated using the finite-dimensional optimal filter such as the Kalman filter (KF) conditional on other components in the SMC framework, like the particle filter (PF) [21,22]. To reduce computational cost, we incorporate the RBP filtering algorithm with the MM-PHD filter, i.e., the MM-RBPPHD filter.

Consider the independent nonlinear and linear states $\mathbf{x}_k^n$ and $\mathbf{x}_k^l$, the whole state vector at scan k can be given by $\mathbf{x}_k = [\mathbf{x}_k^n \quad \mathbf{x}_k^l]^{\mathrm{T}}$, where $[\cdot]^{\mathrm{T}}$ denotes the transpose of matrix. We rewrite the hybrid Markov structure as

$$\mathbf{x}_k^n = f_{k-1}(\mathbf{x}_{k-1}^n) + \mathbf{A}_{k-1}^n(\mathbf{x}_{k-1}^n)\mathbf{x}_{k-1}^l + \mathbf{B}_{k-1}^n(\mathbf{x}_{k-1}^n)\mathbf{u}_{k-1}^n, \quad (7)$$

$$\mathbf{x}_k^l = \mathbf{A}_{k-1}^l(\mathbf{x}_{k-1}^n)\mathbf{x}_{k-1}^l + \mathbf{B}_{k-1}^l(\mathbf{x}_{k-1}^n)\mathbf{u}_{k-1}^l, \quad (8)$$

$$\mathbf{z}_k = h_{k-1}(\mathbf{x}_{k-1}^n) + \mathbf{e}_{k-1}. \quad (9)$$

In (7)-(9), $\mathbf{A}_{k-1}^n$ and $\mathbf{A}_{k-1}^l$ are the transition matrices of the nonlinear and linear states; $\mathbf{B}_{k-1}^n$ and $\mathbf{B}_{k-1}^l$ are the input matrices of the nonlinear and linear noises; the noises follow the Gaussian distribution, that is, $\mathbf{u}_k^n \sim \mathcal{N}(0,[\mathbf{Q}_k^n \quad \mathbf{S}_k])$, $\mathbf{u}_k^l \sim \mathcal{N}(0,[\mathbf{S}_k^{\mathrm{T}} \quad \mathbf{Q}_k^l])$ and $\mathbf{e}_k \sim \mathcal{N}(0,\mathbf{R}_k^n)$, where $\mathcal{N}(\cdot)$ denotes the normal probability density function; $\mathbf{Q}_k^n$, $\mathbf{Q}_k^l$ and $\mathbf{R}_k$ are the variances; $\mathbf{S}_k$ is the covariance [23,24]. We consider (7) and (8) are linear and Gaussian when $\mathbf{x}_k^l$ and $(\mathbf{x}_k^n - f_{k-1}(\mathbf{x}_{k-1}^n))$ are seen as the state and measurement vectors, and therefore are estimated using the KF. On the other hand, the PF is used to estimate the nonlinear state $\mathbf{x}_k^n$.

3.2. Prediction step

First, for the nonlinear state, we have the particle representation of the posterior PHD as

$$D_{k-1}\left(\mathbf{x}_{k-1}, r_{k-1} \mid \mathbf{Z}_{1:k-1}\right) = \sum_{i=1}^{L_{k-1}} \mathbf{w}_{k-1}^{(i)} \delta\left(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{n,(i)}, r_{k-1} - r_{k-1}^{(i)}\right), \quad (10)$$

where $\delta(\cdot)$ is the Dirac Delta function; $r_{k-1}^{(i)}$ is the index of target models; L_{k-1} is the predicted number of particles.

Draw a set of particles $\{\mathbf{x}_{k-1}^{n,(i)}, \mathbf{w}_{k-1}^{(i)}, r_{k-1}^{(i)}\}_{i=1}^{L_{k-1}}$ from (10), where $\sum_{i=1}^{L_{k-1}} \mathbf{w}_{k-1}^{(i)}$ is the expected number of targets at scan $k-1$. Consider the newborn and survival targets, at scan k, the weight approximation of the predicted PHD in (3) can be written as

$$D_{k|k-1}\left(\mathbf{x}_{k-1}, r_k \mid \mathbf{Z}_{1:k-1}\right) = \sum_{i=1}^{L_{k-1}+J_k} \mathbf{w}_{k|k-1}^{(i)} \delta\left(\mathbf{x}_{k-1} - \mathbf{x}_{k|k-1}^{n,(i)}, r_{k-1} - r_{k|k-1}^{(i)}\right), \quad (11)$$

where J_k is the number of the newborn particles.

Let $\alpha_k(\cdot \mid r_{k-1})$ and $\beta_k(\cdot)$ denote the model probability mass functions of the survival and newborn targets, $r_{k-1}^{(i)}$ is generated by the importance sampling from the following proposal density

$$r_{k|k-1}^{(i)} \sim \begin{cases} \alpha_k(\cdot \mid r_{k-1}), & i = 1,\cdots,L_{k-1} \\ \beta_k(\cdot), & i = L_{k-1}+1,\cdots,L_{k-1}+J_k. \end{cases} \quad (12)$$

Simultaneously, we have in hand the predicted particle

$$\mathbf{x}_{k|k-1}^{(i)} \sim \begin{cases} q_k(\cdot \mid \mathbf{x}_{k-1}, r_{k|k-1}, \mathbf{Z}_k), & i = 1,\cdots,L_{k-1} \\ p_k(\cdot \mid r_{k|k-1}, \mathbf{Z}_k), & i = L_{k-1}+1,\cdots,L_{k-1}+J_k, \end{cases} \quad (13)$$

where $q_k(\cdot \mid \mathbf{x}_{k-1}, r_{k|k-1}, \mathbf{Z}_k)$ and $p_k(\cdot \mid r_{k|k-1}, \mathbf{Z}_k)$ denote the proposal distributions of the survival and newborn targets [25,26].

For the survival targets, $\mathbf{x}_{k|k-1}^{n,(i)}$ can be obtained as follows:

$$\mathbf{x}_{k|k-1}^{n,(i)} \mid \mathbf{x}_{k-1}^{n,(i)}, r_{k|k-1} \sim \mathcal{N}(f_{k-1}(\mathbf{x}_{k-1}^{n,(i)}, r_{k|k-1}) + \mathbf{A}_{k-1}^n \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}, \mathbf{R}^{n,(i)}), \quad (14)$$

where

$$\mathbf{R}_k^{n,(i)} = \mathbf{A}_{k-1}^n \mathbf{P}_{k-1|k-2}^{l,(i)} \left(\mathbf{A}_{k-1}^n\right)^{\mathrm{T}} + \mathbf{B}_{k-1}^n \mathbf{Q}_{k-1|k-2}^n \left(\mathbf{B}_{k-1}^n\right)^{\mathrm{T}};$$

$\mathbf{P}_{k-1|k-2}^{l,(i)}$ is the covariance. Furthermore, $\mathbf{x}_{k|k-1}^{n,(i)}$ is partly determined by the linear state $\hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}$.

Define the Kalman gain as

$$\mathbf{K}_{k-1} = \mathbf{P}_{k-1|k-2}^{l,(i)} (\mathbf{A}_{k-1}^n)^{\mathrm{T}} (\mathbf{A}_{k-1}^n \mathbf{P}_{k-1|k-2}^{l,(i)} (\mathbf{A}_{k-1}^n)^{\mathrm{T}} + \mathbf{B}_{k-1}^n \mathbf{Q}_{k-1}^n (\mathbf{B}_{k-1}^n)^{\mathrm{T}})^{-1}. \quad (15)$$

Then we have the predicted linear state

$$\hat{\mathbf{x}}_{k|k-1}^{l,(i)} = \mathbf{A}_{k-1}^l (\hat{\mathbf{x}}_{k-1|k-2}^{l,(i)} + \mathbf{K}_{k-1} (\mathbf{x}_{k|k-1}^{n,(i)} - f_{k-1}(\mathbf{x}_{k-1}^{n,(i)}, r_{k|k-1})) - \mathbf{A}_{k-1}^n \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}). \quad (16)$$

The predicted covariance is

$$\mathbf{P}_{k|k-1}^{l,(i)} = \mathbf{A}_{k-1}^l (\mathbf{P}_{k-1|k-2}^{l,(i)} - \mathbf{K}_{k-1} \mathbf{A}_{k-1}^n \mathbf{P}_{k-1|k-2}^{l,(i)}) (\mathbf{A}_{k-1}^l)^{\mathrm{T}} + \mathbf{B}_{k-1}^l \mathbf{Q}_{k-1}^l (\mathbf{B}_{k-1}^l)^{\mathrm{T}}. \quad (17)$$

Given that the nonlinear state and the prior distribution of the linear state are uncorrelated, the newborn targets are also given by $\mathbf{x}_{k|k-1}^{n,(i)}$.

With regard to the survival and newborn targets, we obtain the weight of the predicted particles

$$\mathbf{w}_{k|k-1}^{(i)}=\begin{cases}\dfrac{f_{k|k-1}\left(r_{k|k-1}^{(i)}\mid r_{k-1}^{(i)}\right)}{\alpha_k\left(r_{k|k-1}^{(i)}\mid r_{k-1}^{(i)}\right)}\\ \times\dfrac{p_{S,k|k-1}\left(\mathbf{x}_{k|k-1}^{n,(i)}\right)f_{k|k-1}\left(\mathbf{x}_{k|k-1}^{n,(i)}\mid \mathbf{x}_{k-1}^{n,(i)},r_{k-1}^{(i)}\right)\mathbf{w}_{k-1}^{(i)}}{q_k\left(\mathbf{x}_{k|k-1}^{n,(i)}\mid \mathbf{x}_{k-1}^{n,(i)},r_{k-1}^{(i)},\mathbf{Z}_k\right)},\\ i=1,\cdots,L_{k-1}\\ \dfrac{f_k\left(r_{k|k-1}^{(i)}\right)}{\beta_k\left(r_{k|k-1}^{(i)}\right)}\dfrac{b_k\left(\mathbf{x}_{k|k-1}^{n,(i)},r_{k-1}^{(i)}\right)}{J_k p_k\left(\mathbf{x}_{k|k-1}^{n,(i)}\mid r_{k-1}^{(i)},\mathbf{Z}_k\right)},\\ i=L_{k-1}+1,\cdots,L_{k-1}+J_k.\end{cases} \tag{18}$$

where $f_k(r_{k|k-1}^{(i)})$ denotes the model distribution of the newborn targets.

3.3. Updated step

With the measurement $\mathbf{z}_k$ at scan k, the updated particle weight is given by

$$\tilde{\mathbf{w}}_k^{(i)}=\left(1-p_D+\sum_{\mathbf{z}_k\in\mathbf{Z}_k}\frac{p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(i)})\mathbf{w}_{k|k-1}^{(i)}}{\lambda c(\mathbf{z}_k)+\sum_{j=1}^{L_{k-1}+J_k}p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(j)})\mathbf{w}_{k|k-1}^{(j)}}\right)\mathbf{w}_{k|k-1}^{(i)}. \tag{19}$$

Remark 2: In (19), the particle weight contains the undetected component $(1-p_D)\mathbf{w}_{k|k-1}^{(i)}$ and the detected component

$$\sum_{\mathbf{z}_k\in\mathbf{Z}_k}\frac{p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(i)})\mathbf{w}_{k|k-1}^{(i)}}{\lambda c(\mathbf{z}_k)+\sum_{j=1}^{L_{k-1}+J_k}p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(j)})\mathbf{w}_{k|k-1}^{(j)}}\mathbf{w}_{k|k-1}^{(i)}.$$

In order to correct the number of the undetected targets and false alarms, it is necessary to decrease the weight of the newborn particles to eliminate the potential false alarms, and therefore utilize the excess weights to compensate the undetected components of the survival targets.

First, we define $\mathbf{w}_{B,k}^{(i)}$ is the modified weight of the newborn particles. Note that the detection threshold ε for the newborn particles, which is usually accredited that the newborn targets have been detected when $\tilde{\mathbf{w}}_k^{(i)}>\varepsilon$. We attempt to decrease the modified weight $\mathbf{w}_{B,k}^{(i)}$ to

$$\sum_{\mathbf{z}_k\in\mathbf{Z}_k}\frac{p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(i)})\mathbf{w}_{k|k-1}^{(i)}}{\lambda c(\mathbf{z}_k)+\sum_{j=1}^{L_{k-1}+J_k}p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(j)})\mathbf{w}_{k|k-1}^{(j)}}\mathbf{w}_{k|k-1}^{(i)}.$$

On the other hand, $\mathbf{w}_{B,k}^{(i)}$ remains unchanged when $\tilde{\mathbf{w}}_k^{(i)}\le\varepsilon$. Then we have

$$\mathbf{w}_{B,k}^{(i)}=\begin{cases}\displaystyle\sum_{\mathbf{z}_k\in\mathbf{Z}_k}\frac{p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(i)})\mathbf{w}_{k|k-1}^{(i)}}{\lambda c(\mathbf{z}_k)+\sum_{j=1}^{L_{k-1}+J_k}p_D g_k(\mathbf{z}_k\mid \mathbf{x}_k^{n,(j)})\mathbf{w}_{k|k-1}^{(j)}}\mathbf{w}_{k|k-1}^{(i)}, & \tilde{\mathbf{w}}_k^{(i)}>\varepsilon\\ \tilde{\mathbf{w}}_k^{(i)}, & \tilde{\mathbf{w}}_k^{(i)}\le\varepsilon.\end{cases} \tag{20}$$

Let us turn attention to the modified weight $\mathbf{w}_{S,k}^{(i)}$ of the survival particles. To guarantee accurate number-estimate, the sum of $(1-p_D)\mathbf{w}_{k|k-1}^{(i)}$ from the newborn particles in the case of $\tilde{\mathbf{w}}_k^{(i)}>\varepsilon$ will be reassigned to all the survival particles in proportion, that is

$$\mathbf{w}_{S,k}^{(i)}=\left(1+\frac{\sum_{i=L_{k-1}+1,\tilde{\mathbf{w}}_k^{(i)}>\varepsilon}^{L_{k-1}+J_k}\left(1-p_D\right)\mathbf{w}_{k|k-1}^{(i)}}{\sum_{i=1}^{L_{k-1}}\tilde{\mathbf{w}}_k^{(i)}}\right)\tilde{\mathbf{w}}_k^{(i)}. \tag{21}$$

Given that $\mathbf{w}_k^{(i)}$ contains $\mathbf{w}_{B,k}^{(i)}$ and $\mathbf{w}_{S,k}^{(i)}$, the estimated number of targets in (5) can be rewritten as

$$N_k=\text{round}\left(\sum_{i=1}^{L_{k-1}}\mathbf{w}_{S,k}^{(i)}\right)+\text{round}\left(\sum_{i=L_{k-1}+1}^{L_{k-1}+J_k}\mathbf{w}_{B,k}^{(i)}\right), \tag{22}$$

where $\text{round}(\cdot)$ denotes integer approximation. Here, N_k is no longer determined by

$$\text{round}\left(\sum_{i=1}^{L_{k-1}+J_k}\mathbf{w}_k^{(i)}\right)\ \text{but by}\ \text{round}\left(\sum_{i=1}^{L_{k-1}}\mathbf{w}_{S,k}^{(i)}\right)\ \text{and}$$

$$\text{round}\left(\sum_{i=L_{k-1}+1}^{L_{k-1}+J_k}\mathbf{w}_{B,k}^{(i)}\right).$$

It means the estimated number of targets is more accurate.

3.4. Estimation step

In this step, a new set of particles with the associated weights $\{\mathbf{x}_k^{n,(i)},\mathbf{w}_k^{(i)}/N_k,r_k^{(i)}\}_{i=1}^{L_k}$ are resampled from the set $\{\mathbf{x}_{k|k-1}^{n,(i)},\mathbf{w}_k^{(i)}/N_k,r_{k|k-1}^{(i)}\}_{i=1}^{L_{k-1}+J_k}$. Then the particle weight is scaled by N_k to $\{\mathbf{x}_k^{n,(i)},\mathbf{w}_k^{(i)},r_k^{(i)}\}_{i=1}^{L_k}$ so that all the weights keep the same as before [4,27].

Owing to the updated posterior PHD in (4), we have

$$D_k\left(\mathbf{x}_k,r_k\mid \mathbf{Z}_{1:k}\right)=\sum_{i=1}^{L_k}\mathbf{w}_k^{(i)}\delta\left(\mathbf{x}_k-\mathbf{x}_k^{n,(i)},r_k-r_k^{(i)}\right). \tag{23}$$

Finally, at scan k, the total number of targets under N_r parallel models is estimated using (6). Furthermore, $\hat{N}_k$ local maximum peaks of the PHD present the state-estimate based on the k-means clustering.

4. SIMULATION RESULTS AND DISCUSSIONS

To validate tracking performance, the numerical studies for the proposed filter scheme is presented in this section. We consider a particular scenario in the surveillance region $[-100,1000]\times[-100,1600]\ \text{m}^2$. The experimental environment was: Intel™ Core™ CPU @ 2.9 GHz, 4 GB Memory, and MATLAB™ v7.8.

4.1. Scenario

As maneuvering target has various dynamics, we define the state equations of the CV model and the CT model as

$$\mathbf{x}_k=\begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix}\mathbf{x}_{k-1}+\begin{bmatrix}1/2&0\\1&0\\0&1/2\\0&1\end{bmatrix}\mathbf{u}_{k-1}, \tag{24}$$

$$\mathbf{x}_k=\begin{bmatrix}1&\sin\omega_{k-1}/\omega_{k-1}&0&-(1-\cos\omega_{k-1})/\omega_{k-1}\\0&\cos\omega_{k-1}&0&-\sin\omega_{k-1}\\0&(1-\cos\omega_{k-1})/\omega_{k-1}&1&\sin\omega_{k-1}/\omega_{k-1}\\0&\sin\omega_{k-1}&0&\cos\omega_{k-1}\end{bmatrix}\mathbf{x}_{k-1}+\begin{bmatrix}1/2&0\\1&0\\0&1/2\\0&1\end{bmatrix}\mathbf{u}_{k-1}, \tag{25}$$

where $\mathbf{x}_k=[x_k,\dot{x}_k,y_k,\dot{y}_k]^{\mathrm{T}}$ contains the planar position $\mathbf{x}_k^n=(x_k,y_k)$ and the velocity $\mathbf{x}_k^l=(\dot{x}_k,\dot{y}_k)$; ω_{k-1} is the turn rate;

$$\mathbf{u}_{k-1}\sim\mathcal{N}\left(0,\begin{bmatrix}0.01^2&0\\0&0.01^2\end{bmatrix}\right).$$

According to the RBP filtering algorithm, we rewrite the CV model $(r_k=1)$ as

$$\mathbf{x}_k^n=\mathbf{x}_{k-1}^n+\mathbf{x}_{k-1}^l+\begin{bmatrix}1/2&0\\0&1/2\end{bmatrix}\mathbf{u}_{k-1}, \tag{26}$$

$$\mathbf{x}_k^l=\mathbf{x}_{k-1}^l+\mathbf{u}_{k-1}. \tag{27}$$

Similarly, the CT model $(r_k=2)$ is rewritten as

$$\mathbf{x}_k^n=\mathbf{x}_{k-1}^n+\begin{bmatrix}\sin\omega_{k-1}/\omega_{k-1}&-(1-\cos\omega_{k-1})/\omega_{k-1}\\(1-\cos\omega_{k-1})/\omega_{k-1}&\sin\omega_{k-1}/\omega_{k-1}\end{bmatrix}\mathbf{x}_{k-1}^l+\begin{bmatrix}1/2&0\\0&1/2\end{bmatrix}\mathbf{u}_{k-1}, \tag{28}$$

$$\mathbf{x}_k^l=\begin{bmatrix}\cos\omega_{k-1}&-\sin\omega_{k-1}\\\sin\omega_{k-1}&\cos\omega_{k-1}\end{bmatrix}\mathbf{x}_{k-1}^l+\mathbf{u}_{k-1}. \tag{29}$$

The RBP-based models can be incorporated to describe the target dynamics, and what's more, the particle dimension is cut in half using $\mathbf{x}_k^n$ and $\mathbf{x}_k^l$.

Next, the measurement equation is given by

$$\mathbf{z}_k=\begin{bmatrix}\arctan(y_k/x_k)\\\sqrt{x_k^2+y_k^2}\end{bmatrix}+\mathbf{e}_{k-1}, \tag{30}$$

where $\mathbf{e}_{k-1}\sim\mathcal{N}\left(0,\begin{bmatrix}0.1^2&\\&0.1^2\end{bmatrix}\right)$.

At scan k, the probabilities of the survival and newborn targets are 0.95 and 0.33. The sensor, located on (450,750) m, has the detection probability 0.95. The clutter is modeled as a Poisson RFS in the surveillance area and the average number returns per unit hyper volume is 0.5. The initial model probabilities for the CV and CT models are equivalent; the particle number per target is 500. The detection threshold is 0.55. We utilize the Wasserstein distance as a valid multitarget miss-distance between the true and estimated sets of multitarget states to evaluate the tracking performance.

4.2. Simulation results

Three targets randomly move in the surveillance region for 55 s. Target 1 travels from the original position (0,0) m at a CV with velocity of (40,20) m s^{-1} during $1^{\text{st}}-6^{\text{th}}$ s; after executing a 9 ° s^{-1} left turn for 9 s, it returns to the initial CV dynamics until 40^{th} s. Target 2 keeps a CV with velocity of (20,20) m s^{-1} from the position (0,700) m during $16^{\text{th}}-55^{\text{th}}$ s. Target 3 travels during $11^{\text{th}}-29^{\text{th}}$ s at a CV with velocity of (20,40) m s^{-1} from the position (100,120) m and then follows by a right turn of 9° s^{-1} for 11 s.

Fig. 1 demonstrates the measurements and the true trajectories of three-target. In this figure, we can see that Targets 1 and 3 are maneuvering and Target 2 is non-maneuvering. In addition, some unexpected false alarms occur when they are close to the true target trajectories.

Fig. 2 shows the true tracks and estimates versus time by the clutter suppression in $x-y$ coordinate. From this figure, it can be seen that both the standard MM-PHD filter and the proposed filter can track multitarget. From

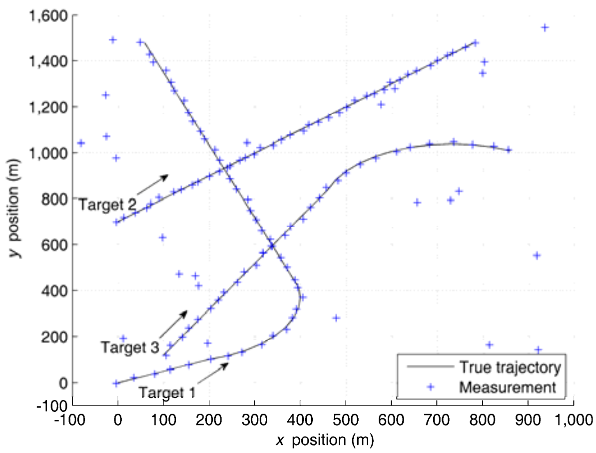

Fig. 1. Target trajectories and measurements.

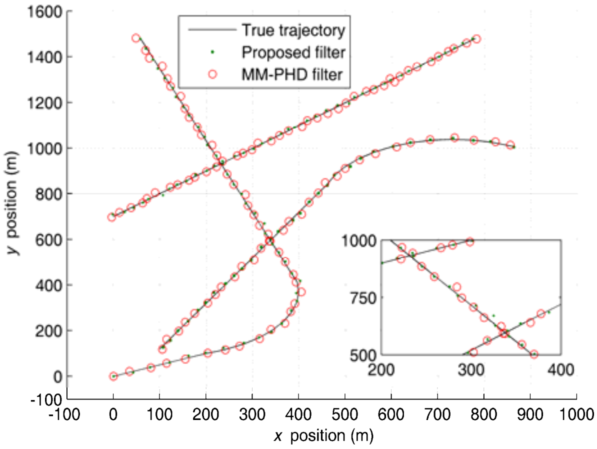

Fig. 2. *x*-*y* coordinate of position-estimate.

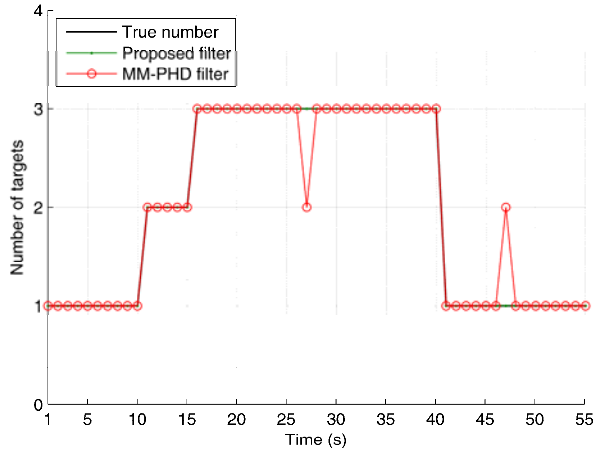

Fig. 4. Target number-estimate.

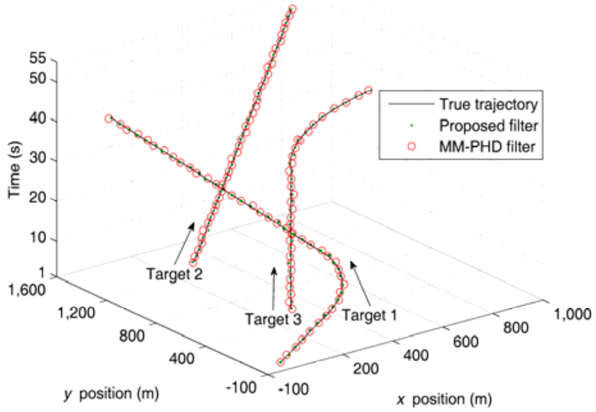

Fig. 3. *x*-*y*-*t* coordinate of position-estimate.

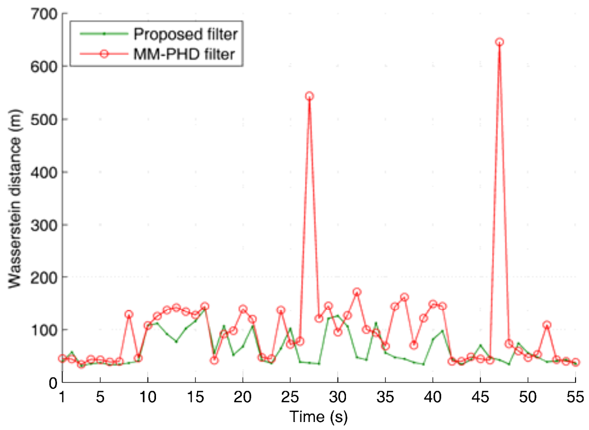

Fig. 5. Wasserstein distance.

the enlarged scale, we can clearly observe the estimated junction points of three trajectories using both filters.

Fig. 3 shows $x-y-t$ coordinate of position-estimate. In this figure, it can be seen that Target 1 is close to Targets 3 and 2 on 21st s and 27th s in x coordinate. Target 2 is nearing Target 1 in y coordinate at 27th s and then makes a stealthy approach to Target 3 during 33rd – 36th s. We find Targets 1 and 2 are close on 27th s.

Fig. 4 illustrates the target number-estimate. Note that the MM-PHD filter has unstable number of targets because it underestimates on 27th s and overestimates on 47th s. The primary cause of the former is that Target 1 is missing when it is very close to Target 2 on 27th s. In contrast, the reason for the latter is that a false alarm happens near Target 2 on 47th s. The further investigations indicate that the inaccurate number-estimate using the MM-PHD filter originates from sensor imperfection and clutter interference. For comparison, the proposed filter has its own advantage. When the newborn targets appear, the excessive particle weights are shifted to others with certain percentage.

Fig. 5 shows the Wasserstein distance of $x-y$ position. As seen, the MM-PHD filter has higher peaks when errors happen. Furthermore, it has errors at the time of model switching. Compared to the MM-PHD filter, the proposed filter has superior performance. It not only corrects the number of the undetected targets and the false alarms but also cuts down the maneuvers with small variance. As a result, the position deviations are lower during the surveillance period.

Table 1. Tracking performance.

	Target number	Wasserstein distance (m)	Running time (s)
MM-PHD filter (500 particles)	2.1891	104.1387	2.0016
MM-PHD filter (1000 particles)	2.1175	97.4072	3.8741
Proposed filter (500 particles)	2.0364	80.2573	3.1658

In addition, 100 Monte Carlo runs are performed to evaluate tracking performance under the same environment. Table 1 shows the performance of both filters. Under the particle number 500, it can be found that the proposed filter provides more accurate number-estimate with 77.07% miss-distance of the MM-PHD filter. As illustrated in this table, when 1000 particles are used in the MM-PHD filter, the tracking performance does not yet measure up to the proposed filter with 500 particles. The reason can be explained that the proposed filter reduces the particle number and the dimension by almost half. In view of the facts, the proposed filter is more acceptable for the MTT.

5. CONCLUSION

This paper has developed an MM-RBPPHD filter for the MTT in the noisy set of measurements. The challenges are to deal with computational load and imprecise estimate of the existing MM-PHD filter. Our work employs the RBPPHD filter in the overall filtering process, where the extra weights of the newborn particles are shifted to find the undetected targets. Above all, we derive the MM version of the RBPPHD filter that reduces particle number and saves running time. The numerical studies show that the proposed filter has great improvement on both number-estimate and state-estimate. As future developments of this research, we want to save running time under the current tracking precision.

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