International Journal of Control, Automation, and Systems (2015) 13(3):718-724 ISSN:1598-6446 eISSN:2005-4092
DOI 10.1007/s12555-014-0024-5 http://www.springer.com/12555

Sliding Mode Tracking Control of Mobile Robots with Approach Angle in Cartesian Coordinates

Jun Ku Lee, Yoon Ho Choi, and Jin Bae Park*

Abstract: In this paper, we propose a method for designing the sliding mode based tracking control of a mobile robot in Cartesian coordinates with an approach angle and an improved reaching law. In the proposed method, to solve the singular point problem, we consider the kinematics in Cartesian coordinates instead of the kinematics in polar coordinates. We consider the bounded disturbances of the dynamics. Next, we design a new sliding surface by using an approach angle to solve the sliding surface constraint problem. Also, we propose an improved reaching law which can reduce the chattering phenomenon and the reaching time. Then, we derive the new matrix to use the sliding mode control method to the kinematics and dynamics equations. Based on the proposed control law, we can derive the control input for the given arbitrary trajectories. We prove that the position tracking error asymptotically converges to zero by using the Lyapunov stability theory. Finally, we demonstrate the effectiveness of the proposed control system through computer simulations.

Keywords: Approach angle, Cartesian coordinates, disturbance, Lyapunov function, mobile robot, reaching law, sliding mode control, tracking control.

1. INTRODUCTION

On a tracking control of a mobile robot, there have been many researches [1-12]. However, it is still a difficult problem to design the tracking controller of a mobile robot, because a mobile robot has the nonholonomic constraint property that makes the restrictions on the movement of a mobile robot. Moreover, in real mobile robots, there exist the model uncertainties and the disturbances. In order to solve these problems, various control methodologies have been studied, including adaptive control theory, robust control theory, fuzzy theory, and switching theory. Among various control methodologies, many researchers focused on sliding mode control (SMC) [8-12] because SMC has robust response for the changing parameters. Therefore, due to those properties, SMC is suitable for the tracking controller of a mobile robot.

In the design of tracking controller for a mobile robot, some researchers have considered only the kinematic model of mobile robots with the assumption of perfect velocity tracking [2,3]. However, since it is difficult to track the velocity perfectly, there are the gap between the simulation results and the experimental results of real mobile robots. Therefore, a number of researchers consider the kinematic and dynamic models together [4,8-11].

Yang and Kim proposed a tracking control of a mobile robot, which considers the kinematic and dynamic models in polar coordinates [8]. In addition, Park *et al.* proposed a new sliding surface based SMC to improve the performance of a tracking controller [10]. However, the movement of a mobile robot is constrained in those studies. It is because those studies consider the kinematic model in polar coordinates, which has the singular point problem at the origin. In order to solve the singular point problem, Lee *et al.* proposed a kinematic model in Cartesian coordinates [11], and Dong *et al.* presented a tracking control for a mobile robot with the kinematic model in Cartesian coordinates by using SMC [2]. Also, the control input is simplified by using the backstepping method in some studies [12]. However, due to the sliding surface constraint, the control input has the constraints on the movement of a mobile robot.

Moreover, many studies have considered the chattering phenomenon in the design of tracking controller for a mobile robot. The chattering phenomenon is undesirable because it can be the cause of a damage to a controller. Therefore, many solutions have been proposed to overcome the chattering phenomenon. Gao and Hung proposed an improved reaching law with the fractional power terms of the sliding surface, which reduces the chattering phenomenon [13]. However, the proposed reaching law increases the reaching time of a controller. In order to solve the above problem, Fallaha *et al.* proposed an advanced reaching law by using an exponential terms which adapts to the variations of chattering [14]. However, the proposed reaching law has the limited maximum value, so the reaching time is not

Manuscript received January 6, 2014; revised May 12, 2014; accepted July 26, 2014. Recommended by Associate Editor Gon-Woo Kim under the direction of Editor Duk-Sun Shim.

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decreased very much.

In this paper, we propose a method for designing the sliding mode based tracking controller of a mobile robot in Cartesian coordinates with an approach angle and an improved reaching law. In the proposed method, in order to solve the singular point problem of the kinematics in polar coordinates, we consider the kinematics in Cartesian coordinates. Also, we consider the bounded disturbance of the dynamics. In order to use SMC to the kinematics and dynamics equations, we derive the new matrix. Then, to solve the sliding surface constraint problem, we design a new sliding surface by using an approach angle. In addition, we propose an improved reaching law which can reduce the chattering phenomenon and the reaching time. By using the proposed control system, we derive the control input for the given arbitrary trajectories. We prove that the position tracking error asymptotically converges to zero by using the Lyapunov stability theory. Finally, we demonstrate the effectiveness of the proposed control system through computer simulations.

This paper is organized as follows: In Section 2, we introduce the kinematic and dynamic models of a mobile robot in Cartesian coordinates. In Section 3, we propose a new controller of a mobile robot with a new sliding surface and an improved reaching law by using an approach angle, and then prove the stability with Lyapunov function. We present several computer simulation results in Section 4, and draw conclusions in Section 5.

2. MOBILE ROBOT MODELING

We consider the kinematic and dynamic models of a mobile robot in Cartesian coordinates instead of polar coordinates. The kinematic model of a mobile robot in polar coordinates has the singular point problem. Therefore, by using the kinematic models in Cartesian coordinates, we remove the restrictions on the movement of a mobile robot. Then, by adding the dynamic models, we improve the performance of tracking controller for a mobile robot.

2.1. Kinematic model of mobile robot

In this paper, we consider a mobile robot which is composed of two driving wheels and having a driving axis at the center of the wheels as shown in Fig. 1. In order to describe the posture of a mobile robot, we define the position as (x_c, y_c), the middle point of the driving axis, and the heading angle as θ_c, the angle between the X-axis and the heading direction. Fig. 1 describes the posture of a mobile robot in Cartesian coordinates.

We define the current posture of a mobile robot as $q_c = [x_c\ y_c\ \theta_c]^T$, and then by using the defined variables in the above, the nonholonomic constraint of a mobile robot is described as follows:

$$\dot{x}_c \sin\theta_c - \dot{y}_c \cos\theta_c = 0. \tag{1}$$

We then define the linear velocity and the angular velocity of a mobile robot as v and w, respectively.

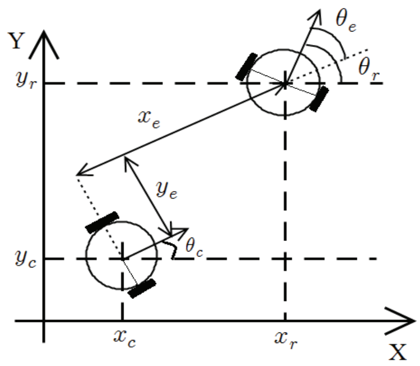

Fig. 1. Coordinate assignment for a mobile robot.

Additionally, we define the control input of current posture as $z_c = [v_c\ w_c]^T$. The kinematic model of a mobile robot in Cartesian coordinates is then expressed as follows:

$$\dot{q}_c = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} v_c \cos\theta_c \\ v_c \sin\theta_c \\ w_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & 0 \\ \sin\theta_c & 0 \\ 0 & 1 \end{bmatrix} z_c. \tag{2}$$

Similarly, we define the reference posture and the reference control input as $q_r = [x_r\ y_r\ \theta_r]^T$ and $z_r = [v_r\ w_r]^T$, respectively. With these defined terms, we derive the posture error (3), which is the difference between the current posture and the reference posture, as follows:

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_c \\ y_r - y_c \\ \theta_r - \theta_c \end{bmatrix}. \tag{3}$$

Differentiating (3) yields (4), which contains the linear velocity and angular velocity terms as follows:

$$\dot{q}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e w_c - v_c + v_r \cos\theta_e \\ -x_e w_c + v_r \sin\theta_e \\ w_r - w_c \end{bmatrix}. \tag{4}$$

2.2. Dynamic model of mobile robot

With the Euler-Lagrange formulation, the dynamic model of a mobile robot under nonholonomic constraints is described as follows:

$$\begin{aligned} &M(q_c)\frac{d}{dt}\dot{q}_c + V(q_c,\dot{q}_c)\dot{q}_c + G(q_c) \\ &\quad = B(q_c)\tau - A^T(q_c)\lambda, \end{aligned} \tag{5}$$

where $q_c \in R^n$ is the generalized coordinates, $\tau \in R^r$ is the torque control input vector, $\lambda \in R^m$ is the constraint force vector, $M(q_c) \in R^{n\times n}$ is the symmetric and positive definite inertia matrix, $V(q_c,\dot{q}_c) \in R^{n\times n}$ is the centripetal and coriolis matrix, $G(q_c) \in R^n$ is the gravitation vector, $B(q_c) \in R^{n\times r}$ is the input transformation matrix, and $A(q_c) \in R^{m\times n}$ is the matrix related to nonholonomic constraints.

From the nonholonomic kinematic constraint, we derive two equations: $A(q_c)\dot{q}_c = 0$ and $A(q_c)S(q_c) = 0.$ Here, $S(q_c) \in R^{n \times r}$ is composed of linearly independent vectors in the null space of $A(q_c)$. From the relation between those two equations, we can derive

$$\dot{q}_c = S(q_c)z_c. \tag{6}$$

Using (6), we transform (5) into

$$H(q_c)\dot{z}_c + F(q_c, z_c) = \tau, \tag{7}$$

where $H(q) = (S^T(q)B(q))^{-1}S^T(q)M(q)S(q)$ and $F(q,z) = (S^T(q)B(q))^{-1}S^T(q)(M(q)\dot{S}(q)v(q,\dot{q})S(q))z.$

Assumption 1: If there are the bounded disturbances τ_d, (7) can be represented as follows:

$$H(q_c)\dot{z}_c + F(q_c, z_c) + \tau_d = \tau, \tag{8}$$

where $\tau_d = H(q_c)f$, $f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$, $|f_i| \le f_{mi}$, $i = 1,2$, and $H(q_c)$ and $F(q_c, z_c)$ are assumed to be known.
By using the computed torque method, we can derive the torque control input τ as follows:

$$\tau = H(q_c)\dot{z}_r + F(q_c, z_c) + H(q_c)u. \tag{9}$$

By substituting (9) into (8), we can derive

$$\dot{z}_c + f = \dot{z}_r + u. \tag{10}$$

Then, by transforming (10), we can derive

$$\dot{z}_c - \dot{z}_r = \begin{bmatrix} \dot{v}_c - \dot{v}_r \\ \dot{w}_c - \dot{w}_r \end{bmatrix} = u - f. \tag{11}$$

3. DESIGN OF TRACKING CONTROLLER

In this section, we describe the process for designing the tracking control of a mobile robot. First, we develop a new appropriate sliding surface to solve the sliding surface constraint problem. Second, we design an improved reaching law to reduce the chattering phenomenon and the reaching time. Third, we design a control law based on the proposed sliding surface with the proposed reaching law. Finally, we prove that the posture errors converge to zero with the Lyapunov stability theory.

3.1. Sliding surface with approach angle

The objective of a tracking controller is to make the posture error q_e converge to zero for an arbitrary reference trajectory as time elapses. However, the previous sliding mode tracking controllers have some restrictions on the movement of a mobile robot. Therefore the posture error does not converge to zero for the specific condition. It is because that in those studies, the sliding surface is defined as follows:

$$\theta_e + \tan^{-1}(v_r y_e) = 0. \tag{12}$$

In (12), we can see that even when θ_e converge to zero, y_e does not converge to zero if $v_r \approx 0$ and $|y_e| > 0$. It is because $v_r|y_e| \approx 0$. Thus, the tracking controller with

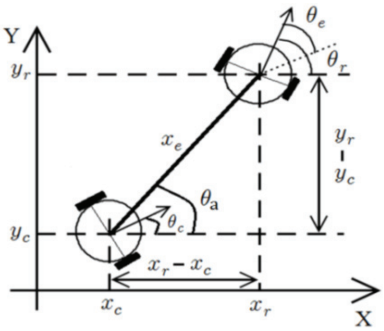

Fig. 2. Coordinate assignments for an approach angle.

the sliding surface (12) has the surface constraint. To solve this problem, with respect the newly defined coordinate as shown in Fig. 2, we introduce an approach angle θ_a as follows:

$$\theta_a = \begin{cases} \tan^{-1}\left(\dfrac{y_r - y_c}{x_r - x_c}\right) & \text{where } y_r \ne y_c \text{ and } x_r \ne x_c \\ \theta_r & \text{where } y_r = y_c \text{ and } x_r = x_c. \end{cases} \tag{13}$$

Then as shown in Fig. 2, if the heading angle θ_c of a mobile robot converges to an approach angle θ_a, then y_e converges to zero as follows:

$$\theta_c \to \theta_a \Rightarrow y_e \to 0. \tag{14}$$

Since the heading angle θ is tangential of trajectory, the heading angle error θ_e converges to zero as y_r and x_r converge to y_c and x_c, respectively. Then, we define the first order filters as follows:

$$s_x = \dot{x}_e + x_e, \quad s_{th} = \dot{\theta}_{ca} + \theta_{ca}, \tag{15}$$

where $\theta_{ca} = \theta_a - \theta_c$. According to (15), if s_x and s_{th} converge to zero, then x_e and θ_{ca} converge to zero.

By using (14) and (15), we design a new sliding surface, which does not have the sliding surface constraint, as follows:

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} s_x \\ s_{th} \end{bmatrix} = \begin{bmatrix} \dot{x}_e + x_e \\ \dot{\theta}_{ca} + \theta_{ca} \end{bmatrix}. \tag{16}$$

3.2. Design of reaching law

After designing the sliding surface, we should design the control law that makes each state reach the sliding surface. To do so, we design the control law with the following reaching condition:

$$S \cdot \dot{S} < 0 \quad \text{for any } t. \tag{17}$$

Here, $\dot{S}$ is typically chosen to satisfy the above condition as follows:

$$\dot{S} = -K \cdot \text{sgn}(S), \tag{18}$$

where K is the positive constant. From the above reaching law, we derive the reaching time t_r, which is the necessary time for each state to converge to the sliding surface, as follows:

$$t_r = \frac{|S(0)|}{K}. \tag{19}$$

However, a controller with typical reaching law has the large t_r and the chattering phenomenon. In order to decrease the chattering, some studies proposed the reaching law with sat(S), instead of sgn(S). However, with this reaching law, there exist a tracking errors, and does not decrease t_r. In the other studies, to decrease t_r, the reaching law is proposed as follows:

$$\dot{S} = -K|S|^{\alpha}\,\mathrm{sgn}(S), \tag{20}$$

where $K, \alpha > 0$ and $t_r = \frac{|S(0)|}{K|S|^{\alpha}}$. With a large α, the proposed reaching law decreases t_r, but it makes the large tracking errors. To solve this dilemma, we propose an improved reaching law as follows:

$$\dot{S} = -K \cdot R(S) \cdot \mathrm{sgn}(S), \tag{21}$$

where $R(S) = \frac{|S|^{\alpha}}{P + (1-P)^{-\beta|S|^{\gamma}}}$. Here, P is a strictly positive offset less than 1. α, β and γ are a strictly positive integer. With the above conditions, $R(S)$ is always strictly positive. Therefore, $R(S)$ does not affect the stability of controller. Also the proposed reaching law has the following properties.

With a large $|S|$, $R(S)$ converges to $|S|^{\alpha}/P$, it means that each states converge to the proposed sliding surface with the $t_r = P\frac{|S(0)|}{K|S|^{\alpha}}$. Furthermore, the proposed reaching law does not need to have the large α because t_r can be controlled by P. Therefore, with the proposed reaching law, we can reduce the reaching time, the chattering phenomenon, and the tracking errors by selecting an appropriate α and P.

3.3. Design of control law

In this subsection, we design an appropriate control law which makes the proposed sliding surface (16) converges to zero with the proposed reaching law (21). In the designing process for control law, we consider the underactuated problem of a mobile robot. To solve this problem, we derive a new matrix, which is used to apply SMC to the kinematics and dynamics equations, from (2) as follows:

$$P(x,y,\theta)\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix}$$

$$\Rightarrow \begin{cases} p_1\dot{x} + p_2\dot{y} + p_3\dot{\theta} = v \\ p_4\dot{x} + p_5\dot{y} + p_6\dot{\theta} = w \end{cases}$$

$$\Rightarrow \begin{cases} p_1 = \cos\theta, \quad p_1 = \sin\theta, \quad p_1 = 0 \\ p_4 = -\sin\theta, \quad p_4 = \cos\theta, \quad p_4 = 1 \end{cases}$$

$$\therefore P(x,y,\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 1 \end{bmatrix}. \tag{22}$$

By using the above derived matrix, we modify (16) and (4) as follows:

$$S = P(x_c, y_c, \theta_c)\begin{bmatrix} s_x\cos\theta \\ s_x\sin\theta \\ s_{th} \end{bmatrix} = \begin{bmatrix} \dot{x}_e + x_e \\ \dot{\theta}_{ca} + \theta_{ca} \end{bmatrix}$$
$$= \begin{bmatrix} y_e w_c + v_r\cos\theta_e + x_e \\ \dot{\theta}_a + \theta_{ca} \end{bmatrix} - \begin{bmatrix} v_c \\ w_c \end{bmatrix}, \tag{23}$$

$$\begin{bmatrix} v_r \\ w_r \end{bmatrix} = P(x_r, y_r, \theta_r)\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix}. \tag{24}$$

Next, by differentiating (23) and (24), we obtain

$$\begin{bmatrix} \dot{v}_c \\ \dot{w}_c \end{bmatrix} = -\dot{S} + \frac{d}{dt}\begin{bmatrix} y_e w_c + v_r\cos\theta_e + x_e \\ \dot{\theta}_a + \theta_{ca} \end{bmatrix}, \tag{25}$$

$$\begin{bmatrix} \dot{v}_r \\ \dot{w}_r \end{bmatrix} = \dot{P}(x_r, y_r, \theta_r)\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix} + P(x_r, y_r, \theta_r)\frac{d}{dt}\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix}. \tag{26}$$

Then, by substituting (25) and (26) into (11), we then obtain

$$-\dot{S} + \frac{d}{dt}\begin{bmatrix} y_e w_c + v_r\cos\theta_e + x_e \\ \dot{\theta}_a + \theta_{ca} \end{bmatrix}$$
$$-\dot{P}(x_r, y_r, \theta_r)\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix} - P(x_r, y_r, \theta_r)\frac{d}{dt}\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix} = u - f. \tag{27}$$

Then, by substituting (21) and $l \cdot \mathrm{sgn}(S)$ into (27) for bounded disturbances, we derive the control input u as follows:

$$u = \left(K \cdot R(S) + l\right)\mathrm{sgn}(S) + \frac{d}{dt}\begin{bmatrix} y_e w_c + v_r\cos\theta_e + x_e \\ \dot{\theta}_a + \theta_{ca} \end{bmatrix}$$
$$-\dot{P}_r(x_r, y_r, \theta_r)\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix} - P_r(x_r, y_r, \theta_r)\frac{d}{dt}\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix}, \tag{28}$$

where

$$l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}, \quad \mathrm{sgn}(S) = \begin{bmatrix} sgn(s_x) \\ sgn(s_{th}) \end{bmatrix},$$
$$(f_1 < l_1,\ f_2 < l_2 \ \text{ and } \ k_1, k_2 > 0).$$

3.4. Stability analysis

In this paper, we use the Lyapunov stability theory to analyze the stability of the proposed tracking control system.

Theorem 1: The control input (28) stabilizes the sliding surface (23) under Assumption 1. Thus, the posture errors q_e converge to zero.

Proof: We use the Lyapunov stability theory to analyze the stability of the proposed sliding surface and the control law. For this purpose, we choose the Lyapunov function as follows:

$$V = \frac{1}{2} S^T S. \tag{29}$$

Differentiating (29), we obtain

$$\dot{V} = S^T \dot{S} = S\left\{ -\dot{P}(x_r, y_r, \theta_r)\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} - P(x_r, y_r, \theta_r)\frac{d}{dt}\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} \right.$$
$$\left. + \frac{d}{dt}\begin{bmatrix} y_e w_c + v_r \cos\theta_e + x_e \\ \dot{\theta}_a + \theta_{ca} \end{bmatrix} - u + f \right\}. \tag{30}$$

By applying (28) to (30), we obtain

$$\begin{aligned} \dot{V} = S^T \dot{S} &= S\{[-K \cdot R(S) - l] \cdot \mathrm{sgn}(S) + f\} \\ &= -K \cdot R(S)|S| - l|S| + f \cdot S, \end{aligned} \tag{31}$$

where $-l|S| + f \cdot S \le 0$ and $R(S) > 0$. Since V is positive definite and $\dot{V}$ is negative definite, the sliding surface S converges to zero by the Lyapunov stability theory. Then, from (16), we derive

$$s_1^2 + s_2^2 = s_x^2 + s_{th}^2 = 0. \tag{32}$$

According to (32), s_x and s_{th} converge to zero, which means that x_e and θ_{ca} converge to zero. Since y_e converges to zero as θ_{ca} converge to zero, the position error (x_e, y_e) converges to zero. In addition, the heading angle error θ_e converges to zero because the heading angle is tangential of trajectory. In conclusion, with the control input u, the posture error (x_e, y_e, θ_e) converges to zero, and then a mobile robot exactly follow a given reference trajectory.

4. SIMULATION RESULTS

In this section, we present the simulation results for the given reference trajectory to demonstrate the effectiveness of the proposed tracking controller. For the simulation, we select the design parameter values as $k_1 = k_2 = 5$, $\alpha = 0.3$, $\beta = 0.3$, $\gamma = 0.2$, $P = 0.3$, and $\alpha = 1.5$ for the reaching law (20). Then we select the initial point of a mobile robot as $[x_c, y_c, \theta_c] = [0, 0, 0]$ to show the improvement of singularity at the origin with the proposed control system. Then, the input disturbances for a mobile robot control system are chosen to be Gaussian random noises with variance = 0.5 and mean = 0, which are assumed that the upper bounds are $f_{m1} = f_{m2} = 0.5(N)$. The reference trajectory is generated by using the kinematic model (2) with the reference velocities v_r and w_r, which are depending on the following conditions:

Condition 1: For the sliding surface constraints, we select the initial conditions of the reference trajectory as $[x_r, y_r, \theta_r] = [5,\ 3,\ \frac{\pi}{4}]$, the linear velocity as $v_r = 0$ m/s, and the angular velocity as $w_r = 0$ m/s.

Condition 2: For the complicated line, we select the initial conditions of the reference trajectory as $[x_r, y_r, \theta_r] = [5,\ 3,\ \frac{\pi}{4}]$, and the linear and angular velocities as follows:

$$\begin{aligned} &0\,\mathrm{s} \le t < 3\,\mathrm{s}: \ v_r = 5\,\mathrm{m/s},\ w_r = 0\,\mathrm{m/s}, \\ &3\,\mathrm{s} \le t < 7.7\,\mathrm{s}: \ v_r = 1.5\,\mathrm{m/s},\ w_r = -0.\mathrm{m/s}, \\ &7.7\,\mathrm{s} \le t < 10\,\mathrm{s}: \ v_r = 3\,\mathrm{m/s},\ w_r = 0\,\mathrm{m/s}, \\ &10\,\mathrm{s} \le t < 15\,\mathrm{s}: \ v_r = 1.5\,\mathrm{m/s},\ w_r = 0.5\,\mathrm{m/s}, \\ &15\,\mathrm{s} \le t < 17\,\mathrm{s}: \ v_r = 3\,\mathrm{m/s},\ w_r = 0\,\mathrm{m/s}. \end{aligned}$$

Then, to show the effectiveness of proposed reaching law, we present three simulation results to Condition 2; one is with the typical reaching law (18), other is with the reaching law (20), and the other is with the proposed reaching law (21).

Fig. 3 depicts the tracking result of a controlled mobile robot with Condition 1 for the sliding surface constraints of previous study. As shown in Fig. 3, a mobile robot follows the given reference trajectory exactly from the origin point. Also, the posture error converges to zero, as shown in Fig. 4. From the results of Figs. 3 and 4, we verify that the sliding surface constraint and the singular point problems are solved with the proposed tracking controller.

For Condition 2, Figs. 5-8 depict the tracking control results with the reaching laws (18), (20) and (21). Then, by comparing Fig. 5-8, we can see the effectiveness of the proposed tracking controller for a mobile robot. As shown in Fig. 6, the tracking controller with a typical

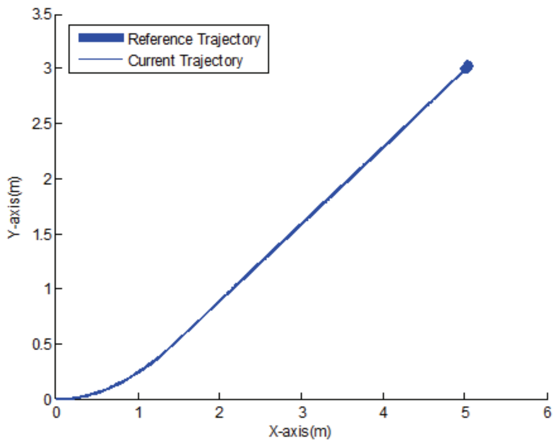

Fig. 3. Tracking control result for Condition 1.

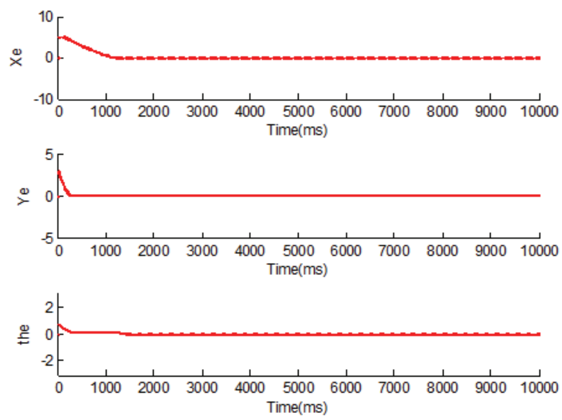

Fig. 4. Tracking control errors for Condition 1.

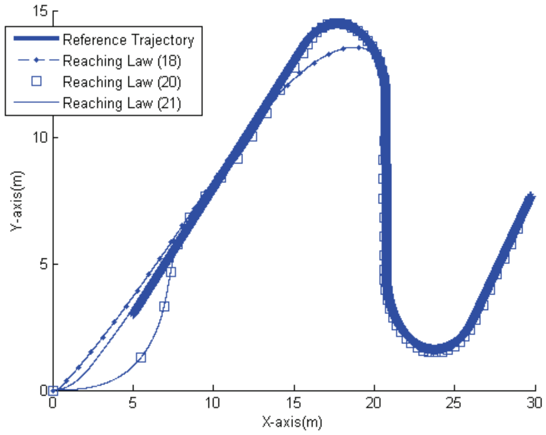

Fig. 5. Tracking control result for Condition 2.

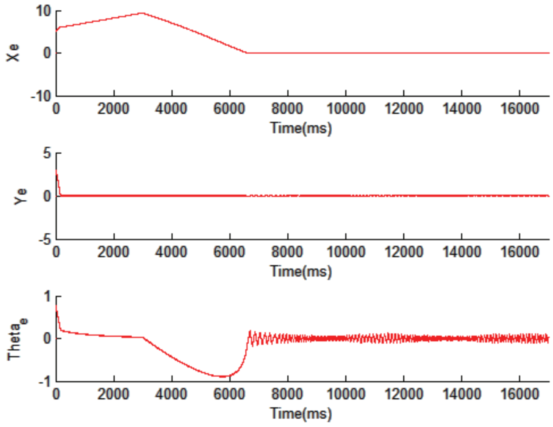

Fig. 6. Tracking control errors for Condition 2 with the reaching law (18).

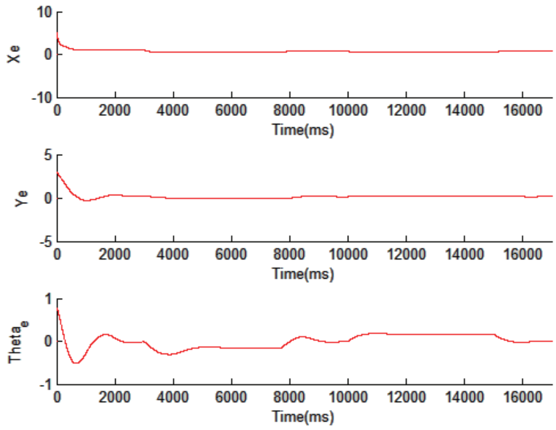

Fig. 7. Tracking control errors for Condition 2 with the reaching law (20).

reaching law (18) takes about 6.8 seconds to make a mobile robot follow a given reference trajectory, and there exist the severe chattering phenomenon. And as shown in Fig. 7, the tracking controller with the reaching law (20) has the small reaching time, about 1.0 seconds, but there exist errors in the heading angle even after converged.

Then, Fig. 8 depicts the tracking control result for the proposed controller with the control input signal as

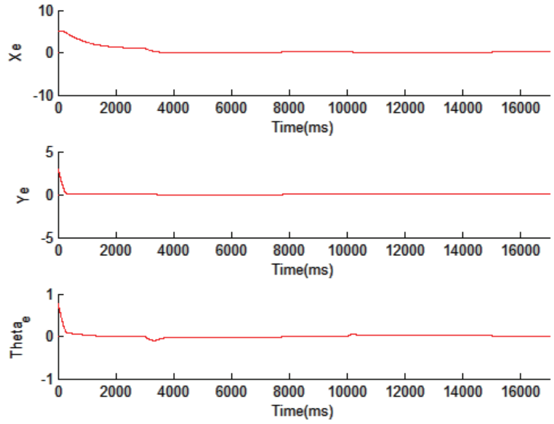

Fig. 8. Tracking control errors for Condition 2 with the reaching law (21).

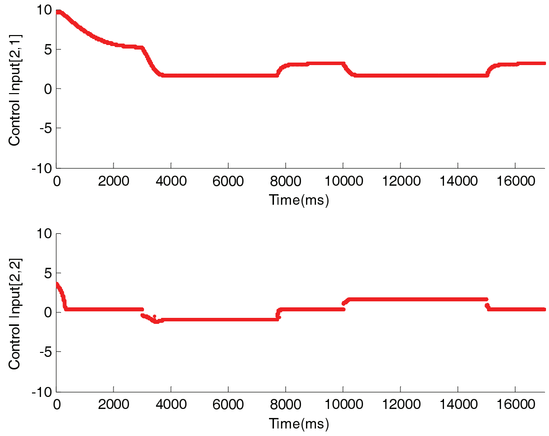

Fig. 9. Control input signal U for Condition 2 with the proposed controller.

shown in Fig. 9. As shown in Fig. 8, the proposed controller takes 1.0 seconds to make the posture errors converge to zero, and there does not exist the large chattering and errors in the heading angle. From the simulation results, we verify that the proposed tracking controller make a mobile robot follow the arbitrary trajectories exactly with the decreased reaching time, chattering phenomenon and the unrestricted movement.

5. CONCLUSION

In this paper, we proposed a method for designing the sliding mode based tracking controller of a mobile robot in Cartesian coordinates with an approach angle and an improved reaching law. In the proposed method, in order to solve the singular point problem of the kinematics in polar coordinates, we considered the kinematics in Cartesian coordinates. Also, we considered the bounded disturbance of the dynamics. In order to use SMC to the kinematics and dynamics equations, we derived the new matrix. Then, to solve the sliding surface constraint problem, we designed a new sliding surface by using an approach angle. In addition, we proposed an improved reaching law which reduces the chattering phenomenon and the reaching time. By using the proposed control system, we derived the control input for the given arbitrary trajectories. We proved that the position tracking error asymptotically converges to zero by using the Lyapunov stability theory. Finally, we demonstrated the effectiveness of the proposed control system through computer simulations.

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