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# Distributed Adaptive Flocking of Robotic Fish System with a Leader of Bounded Unknown Input

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**Abstract:** A distributed leader-follower flocking problem of multiple robotic fish governed by extended second-order unicycles is studied in this paper. The multi-agent system consists of only one leader with pre-appointed and bounded speeds. A distributed flocking algorithm on the basis of the combination of consensus and attractive/repulsive functions is investigated, in which adaptive strategy is adopted to compute the weight of the velocity coupling strengths. The proposed control algorithm enables followers to asymptotically track the leader's varying velocities and approach the equilibrium distances with their neighbors. Furthermore, the arbitrarily-shaped formation flocking problem of the system can also be solved by adding the information of a desired formation topology to the potential function term. Finally, simulations are carried out to verify the effectiveness of the proposed theoretical results.

**Keywords:** Adaptive strategy, cohesive flocking, distributed control, formation flocking, robotic fish system, non-holonomic constraints.

## 1. INTRODUCTION

In nature, flocking phenomenon can be widely observed among social animals, in which a group of scattered agents maintain a cohesive or geometric formation based on neighbor information and simple rules. Therein, leaders commonly play an important role in guiding the behaviors of the group.

In the literatures on flocking [1-5], the existence of virtual leaders is a common assumption to expediently guide the flocking behaviors of the group. Beyond that, a new solution has emerged that assigns one or more real vehicles as leaders [6]. Since the real leaders participate in the energy distribution and formation construction of the whole system, it brings new challenges to the research on flocking. In this paper, we study the leader-follower flocking problem of multiple robotic fish considering its kinematic constraints.

Gu *et al*. [6] studied a leader-follower flocking problem concerning multiple leaders, and followers use the position of flocking center to keep their connections. The flocking group proposed by Gu *et al*. [6] was just able to track a specific trajectory led by group leaders, while Yu *et al*. [4] have given a distributed leader-follower algorithm considering the group consisting of one leader with time-variable speeds. However, the leader involved in Yu's work [4] is still a virtual one. Thus, it is necessary to study a distributed flocking algorithm for the multi-agent system consisting of one real leader with varying speeds. Besides, Savkin *et al*. [7] studied the formation flocking problem aiming to obtain a geometric formation for a network of unicycles with hard constraints, but they didn't involve any leaders. To this end, another task in this paper is to further extend the designed control algorithm to solve the relative formation flocking problem.

Adaptive strategy has been widely applied to synchronization of complex networks, especially for the cases include uncertain terms [8-12]. For example, Hou *et al*. [8] given a robust adaptive control approach to solve the consensus problem of multi-agent systems, whose dynamics included the uncertainties and external disturbances. In Demetriou *et al*.'s work [9], the adaptation of the consensus gains is used in the disagreement terms of local filters for a sensor network consisting of groups of sensors. Min *et al*. [10] studied adaptive consensus protocols for non-point, non-linear networked Euler-Lagrange systems with unknown parameters. Su *et al*. [11] introduced local adaptation strategies for both the weights on the velocity navigational feedback and the velocity coupling strengths that enable all agents to synchronize with the virtual leader. Liu *et al*. [12] proposed an adaptive protocol to solve the consensus problem of multi-agent systems with high-order nonlinear dynamics by using neural networks to approximate the unknown nonlinear system functions. It thus appears that adaptive control could be an effective method approaching to the unknown varying speed of the leader.

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In this paper, we present a distributed adaptive flocking algorithm to solve the cohesive flocking and geometric-formation flocking of multiple robotic fish consisting of only one leader with varying and bounded speeds. Therein, consensus algorithm is adopted for realizing the velocity alignment, while the problems of connectivity preservation and distance equilibrium are all solved by artificial potential field method [13]. Adaptive strategy is introduced to deal with the varying speeds of the leader. The stability of the closed-loop system is analyzed by means of LaSalle-Krasovskii invariance principle, provided that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time. Furthermore, by adding the information of a desired formation topology to the potential function term, the proposed flocking algorithm can be extended to solve arbitrarily shaped formation flocking problem. Finally, the simulation results are given to verify the effectiveness of the proposed algorithms.

The rest of this paper is organized as follows: Section 2 formulates the flocking problem of multiple robotic fish with one variable-speed leader. Two distributed adaptive flocking algorithms are presented, and the stability analysis of the closed-loop system is also provided in Section 3. The simulation results are given in Section 4. Finally, the conclusions are drawn in Section 5.

## 2. FORMAT PROBLEM STATEMENT

The multi-agent system under consideration consists of $N$ robotic fish traveling in a two-dimensional Euclidean space. Robotic fish was modeled as a unicycle during the research on cooperation control of multiple robotic fish [14,15]. Considering that the main propulsive force of the robotic fish comes from the latter part of its body, we make further efforts to draw the swimming robotic fish by a dynamic model named an extended second-order unicycle model, whose geometrical center and mass center don't coincide. Let $Z^+$ denote the set of positive integers, $R$ the set of real numbers, and $R^+$ the set of positive real numbers. As shown in Fig. 1, the robotic fish is modeled by the following kinematic equation

$$\begin{aligned}
&\dot{x}_i(t) = \upsilon_i(t)\cos\theta_i(t) - \omega_i(t) l_i \sin\theta_i(t),\\
&\dot{y}_i(t) = \upsilon_i(t)\sin\theta_i(t) + \omega_i(t) l_i \cos\theta_i(t),\\
&\dot{\theta}_i(t) = \omega_i(t),\\
&\dot{\upsilon}_i(t) = a_i(t),\\
&\dot{\omega}_i(t) = b_i(t)/l_i,
\end{aligned} \tag{1}$$

where $p_i(t)=[x_i(t), y_i(t)]^T \in R^2$ is the position vector, $\theta_i(t)\in R$ the heading angle, $\upsilon_i(t)\in R$ the thrusting speed, $\omega_i(t)\in R$ the rotational speed, $l_i \in R^+$ the distance between the geometrical center $C_i$ and the mass center $M_i$, $\vartheta_i(t)=\omega_i(t) l_i \in R$ the tangential speed, $a_i(t)\in R$ the thrusting acceleration, and $b_i(t)\in R$ the rotational acceleration. Here, $\theta_i(t)\in[0,2\pi)$ is measured

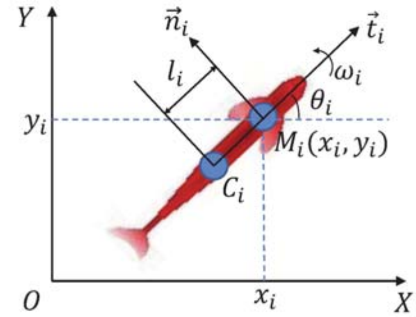



Fig. 1. The simplified model of the robotic fish.

from the x-axis in the anticlockwise rotation. In this paper, we do not consider the individual difference during the theoretical analysis. Thus, it should be noted that $l_i = l_d$, $i=1,\cdots,N$, where $l_d$ is a constant.

We consider the robotic fish system with only one leader. If an agent has external control input, we call it a leader; else, we call it a follower. Without loss of generality, let leader set be $\mathbb{L}=\{1\}$ and follower set be $\mathbb{F}=\{2,\cdots,N\}$. Each agent only interacts with its neighbor due to its limited interaction capability. We suppose that interconnection with the leader is unidirectional, while interconnection with the follower is bidirectional [16]. Let $N_i(t)$ denote the neighbor set of the follower $i\in\mathbb{F}$ at time $t$, and the initial neighbor set of the follower $i$ is defined as

$$N_i(0)=\left\{ j \,\middle|\, \left\| p_i(0)-p_j(0)\right\| < D, j=1,\cdots,N, j\neq i\right\}, \tag{2}$$

where $D>0$ is a constant.

The interaction network $G(t)$ is a dynamic directed graph consisting of a vertex set $\nu=\{1,\cdots,N\}$ indexed by robotic fish and a time-varying edge set $\varepsilon(t)=\{(i,j)\,|\,(i,j)\in\mathbb{F}\times\nu, j\in N_i(t)\}$. Therein, the followers' interaction network $\hat{G}(t)$ with vertex set $\mathbb{F}$ and edge set $\hat{\varepsilon}(t)=\{(i,j)\,|\,(i,j)\in\mathbb{F}\times\mathbb{F}, j\in N_i(t)\}$ is an undirected graph. The matrix $A(t)=[a_{ij}(t)]_{N\times N}$ with definition

$$a_{ij}(t)=\begin{cases}1, & i=1,\ j=1\\ 1, & i\in\mathbb{F},\ j\in N_i(t)\\ 0, & \text{otherwise}\end{cases} \tag{3}$$

represents the coupling configuration of the network $G(t)$. Let $c_{ij}$ denote the coupling strengths between node $i$ and node $j$. Define the matrix of the weighted coupling configuration of the network $G(t)$ as follows:

$$B(t)=[b_{ij}(t)]_{N\times N} = \begin{bmatrix} c_{11}(t)a_{11}(t) & c_{12}(t)a_{12}(t) & \cdots & c_{1N}(t)a_{1N}(t)\\ c_{21}(t)a_{21}(t) & c_{22}(t)a_{22}(t) & \cdots & c_{2N}(t)a_{2N}(t)\\ \vdots & \vdots & \ddots & \vdots \\ c_{N1}(t)a_{N1}(t) & c_{N2}(t)a_{N2}(t) & \cdots & c_{NN}(t)a_{NN}(t)\end{bmatrix}, \tag{4}$$

where $b_{ij}(t)\in R$, $c_{ij}(t)\in R^+$, $i,j\in\nu$. The Laplacian matrix $L_{N-1}(t)=[l_{ij}(t)]_{(N-1)\times(N-1)}$ of the graph $\hat{G}(t)$ is

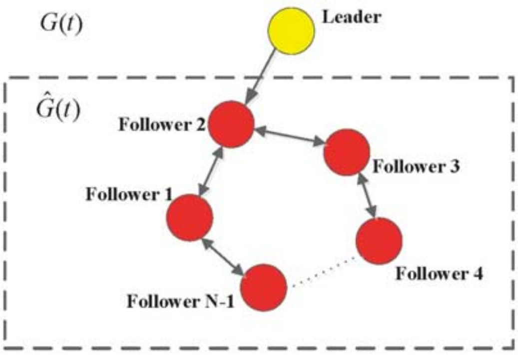



Fig. 2. An example of the interaction network.

given by

$$l_{ij}(t)=\begin{cases}-c_{ij}(t)a_{ij}(t), & i\neq j\\ \sum_{k=2,k\neq i}^{N}c_{ik}(t)a_{ik}(t), & i=j.\end{cases}\tag{5}$$

For $\hat{G}(t)$ is an undirected graph, the Laplacian matrix $L_{N-1}(t)$ is symmetric and positive semi-definite. Fig. 2 gives a sample of the interaction network for the robotic fish system.

Define that the interaction network $G(t)$ switches at $t_p$, $p=1,2,\cdots$. Thus, $G(t)$ is a fixed graph in each nonempty, bounded, and contiguous time-interval $[t_r,t_{r+1})$, where $r=0,1,\cdots$ and $t_0=0$. Given that $\hat{G}(0)$ is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time $t_0=0$. In order to preserve the connectivity of the interaction network $G(t)$, the hysteresis adding new edges to the network is introduced [17,18], such that

1) if $(i,j)\in\varepsilon(t^-)$ and $\|\hat{p}_i(t)-\hat{p}_j(t)\|<2D$, where $\hat{p}_i(t)=p_i(t)-p_i^*(t)$ and $p_i^*(t)=\int_0^t w_{i1}(t)H_i^T(t)q_1 d\tau$ $(i=1,\cdots,N)$, then $(i,j)\in\varepsilon(t)$, for $t>0$;

2) if $(i,j)\notin\varepsilon(t^-)$ and $\|p_i(t)-p_j(t)\|<D$, then $(i,j)\in\varepsilon(t)$, for $t>0$.

## 3. FLOCKING ALGORITHM

Let $q_i(t)=[\upsilon_i(t),\vartheta_i(t)]^T$ be the velocity vector of agent $i$, the model (1) can be rewritten in matrix form as

$$\begin{aligned}\dot{p}_i(t)&=H_i(t)^T q_i(t),\\ \dot{q}_i(t)&=u_i(t),\end{aligned}\tag{6}$$

where $H_i(t)=\begin{bmatrix}\cos\theta_i(t) & \sin\theta_i(t)\\ -\sin\theta_i(t) & \cos\theta_i(t)\end{bmatrix}$, and $u_i(t)=[a_i(t),$ $i=1,\ldots,N$.

The external control input of the leader is given by

$$u_1(t)=f(t),\tag{7}$$

where $f(t)=[f_1(t),f_2(t)]^T\in R^2$ is bounded, who satisfies that

$$\lim_{t\to\infty}f_1(t)=0,\tag{8}$$

$$\lim_{t\to\infty}f_2(t)=0.\tag{9}$$

In addition, the control input of the follower $i$ $(i=2,\cdots,N)$ is designed by

$$\begin{aligned}a_i(t)=&-\sum_{j\in N_i(t)}c_{ij}(t)(\hat{\upsilon}_i(t)-\hat{\upsilon}_j(t))\\ &-\sum_{j\in N_i(t)}c_{ij}(t)\dot{\hat{p}}_{ij}(t)^T\vec{t}_i(t)\\ &-\sum_{j\in N_i(t)}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)^T\vec{t}_i(t),\\ b_i(t)=&-\sum_{j\in N_i(t)}c_{ij}(t)l_d(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\ &-\sum_{j\in N_i(t)}c_{ij}(t)l_d(\hat{\omega}_i(t)-\hat{\omega}_j(t))\\ &-\sum_{j\in N_i(t)}c_{ij}(t)\dot{\hat{p}}_{ij}(t)^T\vec{n}_i(t)\\ &-\sum_{j\in N_i(t)}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)^T\vec{n}_i(t),\\ \dot{c}_{ij}(t)=&\,k_{ij}(\hat{q}_i(t)-\hat{q}_j(t))^T(\hat{q}_i(t)-\hat{q}_j(t))\\ &+k_{ij}\dot{\hat{p}}_{ij}(t)^T\dot{\hat{p}}_{ij}(t),\end{aligned}\tag{10}$$

where $\hat{\upsilon}_i(t)=\upsilon_i(t)-a_{i1}(t)\upsilon_1(t)$, $\hat{\omega}_i(t)=\omega_i(t)-a_{i1}(t)\omega_1(t)$, $\hat{\theta}_i(t)=\theta_i(t)-a_{i1}(t)\theta_1(t)$, and $\hat{p}_{ij}(t)=\hat{p}_i(t)-\hat{p}_j(t)$, $c_{ij}(0)\geq 0$, $k_{ij}$ is the weight of the adaptive parameter $c_{ij}(t)$. Here, $p_i^*(t)=\int_0^t a_{i1}(t)H_i(t)^T q_1(t)d\tau$. Thus, we have

$$\begin{aligned}\dot{\hat{p}}_i(t)&=\dot{p}_i(t)-\dot{p}_i^*(t)\\ &=H_i(t)^T q_i(t)-a_{i1}(t)H_i(t)^T q_1(t)\\ &=H_i(t)^T\hat{q}_i(t),\end{aligned}$$

where $\hat{q}_i(t)=q_i(t)-a_{i1}(t)q_1(t)$. Besides, $\vec{t}_i(t)=\begin{bmatrix}\cos\theta_i(t)\\ \sin\theta_i(t)\end{bmatrix}$ and $\vec{n}_i(t)=\begin{bmatrix}-\sin\theta_i(t)\\ \cos\theta_i(t)\end{bmatrix}$ are two unit vectors orthogonal to each other, and $\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)$ is the gradient of an artificial potential function $V(\|\hat{p}_{ij}(t)\|)$ with the following definition:

**Definition 1:** Potential $V(\|\hat{p}_{ij}(t)\|)$ is a differentiable, nonnegative, radially unbounded function of the Euclidean norm $\|\hat{p}_{ij}(t)\|$ between agent $i$ and $j$, such that

1) $V(\|\hat{p}_{ij}(t)\|)\to\infty$ as $\|\hat{p}_{ij}(t)\|\to 0$;
2) $V(\|\hat{p}_{ij}(t)\|)\to\infty$ as $\|\hat{p}_{ij}(t)\|\to 2D$;
3) $V(\|\hat{p}_{ij}(t)\|)$ attains its unique minimum when the Euclidean norm $\|\hat{p}_{ij}\|$ equals to a certain value between 0 and $2D$.

Then, we have Theorem 1 to solve the leader-follower cohesive flocking problem.

**Theorem 1:** Consider a system of $N$ agents with dynamics (1). The leader and the followers are respectively steered by control protocols (7) and (10). Suppose that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time.

Then the following statements hold:

1) The connectivity of the interaction network is preserved at all times, that is, $\varepsilon(t_r) \subseteq \varepsilon(t_{r+1})$.

2) The thrusting speed, the rotational speed, and the heading angle of each follower asymptotically become the same as those of the leader, that is, $\lim_{t\to\infty}(\upsilon_i(t)-\upsilon_j(t))=0$, $\lim_{t\to\infty}(\omega_i(t)-\omega_j(t))=0$, and $\lim_{t\to\infty}(\theta_i(t)-\theta_j(t))=0$, where $i, j\in\nu, i\neq j$.

3) The system approaches a cohesive configuration that minimizes the total potential, that is,

$$\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)=0,$$
$$\sum_{j\in N_l}\nabla_{\hat{p}_{1j}(t)}V(\|\hat{p}_{1j}(t)\|)=0.$$

**Proof:** Consider the system (1) with control input (7) and (10) on time interval $[t_r, t_{r+1})$, where the interaction network of the robotic fish system is fixed. Define that $\tilde{p}(t)=[\hat{p}_{11}(t)^T,\cdots,\hat{p}_{1N}(t)^T,\cdots,\hat{p}_{N1}(t)^T,\cdots,\hat{p}_{NN}(t)^T]^T$, $\hat{q}(t)=[\hat{q}_2(t)^T,\cdots,\hat{q}_N(t)^T]^T$, and $\hat{\theta}(t)=[\hat{\theta}_2(t),\cdots,\hat{\theta}_N(t)]^T$. Let $N_l=\{i\mid a_{i1}=1, i\in\mathbb{F}\}$ denote the set of followers who have one leader neighbor on time interval $[t_r, t_{r+1})$. Consider the following energy function as the common Lyapunov function

$$E(\tilde{p}(t),\hat{q}(t),\hat{\theta}(t))=E_1(t)+E_2(t)+E_3(t)+E_4(t), \quad (11)$$

where

$$E_1(t)=\frac{1}{2}V(t), \quad (12)$$
$$E_2(t)=\frac{1}{2}\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\hat{q}_i(t), \quad (13)$$
$$E_3(t)=\frac{1}{2}\Theta(t), \quad (14)$$
$$E_4(t)=\frac{1}{4}\sum_{i\in\mathbb{F}}\sum_{j\in N_i(t)}\frac{(c_{ij}(t)-m)^2}{k_{ij}}, \quad (15)$$

where $V(t)=\sum_{i\in\mathbb{F}}\sum_{j\in N_i(t)}V\left(\left\|\hat{p}_{ij}(t)\right\|\right)+\sum_{j\in N_l(t)}V\left(\left\|\hat{p}_{1j}(t)\right\|\right)$, and $\Theta(t)=l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i(t)}c_{ij}(t)\hat{\theta}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t))$.

Suppose that the interaction network $G(t)$ switches at $t_p$, $p=1,2,\cdots$. $G(t)$ is a fixed graph in each nonempty, bounded, and contiguous time-interval $[t_r, t_{r+1})$, $r=0,1,\cdots$. Here, $t_0=0$. Then the derivative of $E(\tilde{p},\hat{q},\hat{\theta})$ w.r.t. the time $t\in[t_r,t_{r+1})$ is

$$\frac{dE}{dt}=\frac{dE_1}{dt}+\frac{dE_2}{dt}+\frac{dE_3}{dt}+\frac{dE_4}{dt}. \quad (16)$$

We will respectively express the four parts on the right side of equation (16) in the following paragraphs.

Firstly, we have that

$$\begin{aligned}\frac{dE_1}{dt}&=\frac{1}{2}\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{p}}_{ij}(t)^T\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\&\quad+\frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{1j}(t)^T\nabla_{\hat{p}_{1j}(t)}V(\|\hat{p}_{1j}(t)\|)\\&=\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\&\quad+\frac{1}{2}\sum_{i\in N_l}\dot{\hat{p}}_{i1}(t)^T\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|)\\&\quad+\frac{1}{2}\sum_{j\in N_l}\dot{p}_{1j}(t)^T\nabla_{\hat{p}_{1j}(t)}V(\|\hat{p}_{1j}(t)\|).\end{aligned} \quad (17)$$

Due to $\dot{\hat{p}}_{ij}(t)^T=-\dot{\hat{p}}_{ji}(t)^T$ and the symmetric nature of $V(\|p_{ij}(t)\|)$, one gets

$$\begin{aligned}&\frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{1j}(t)^T\nabla_{\hat{p}_{1j}(t)}V(\|\hat{p}_{1j}(t)\|)\\&=\frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{j1}(t)^T\nabla_{\hat{p}_{j1}(t)}V(\|\hat{p}_{j1}(t)\|)\\&=\frac{1}{2}\sum_{i\in N_l}\dot{\hat{p}}_{i1}(t)^T\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|).\end{aligned} \quad (18)$$

With $\dot{\hat{p}}_{i1}(t)^T=\dot{\hat{p}}_i(t)^T-\dot{\hat{p}}_1(t)^T=\dot{\hat{p}}_i(t)^T$, we have

$$\begin{aligned}\frac{dE_1}{dt}&=\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\&\quad+\sum_{i\in N_l}\dot{\hat{p}}_{i1}(t)^T\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|)\\&=\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\&\quad+\sum_{i\in N_l}\dot{\hat{p}}_i(t)^T\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|)\\&=\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|).\end{aligned} \quad (19)$$

Secondly, since $\hat{\theta}_1(t)=0$, $\Theta(t)$ can be rewritten by

$$\begin{aligned}\Theta(t)&=l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}c_{ij}(t)\hat{\theta}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\&\quad+l_d^2\sum_{i\in N_l}c_{i1}(t)\hat{\theta}_i^2(t).\end{aligned} \quad (20)$$

Furthermore, we have

$$\begin{aligned}\frac{dE_2}{dt}&=l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}c_{ij}(t)\dot{\hat{\theta}}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\&\quad+l_d^2\sum_{i\in N_l}c_{i1}(t)\dot{\hat{\theta}}_i(t)\hat{\theta}_i(t)\\&=l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}c_{ij}(t)\dot{\hat{\theta}}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\&=l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}c_{ij}(t)\hat{\omega}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t)).\end{aligned} \quad (21)$$

Thirdly, let $\hat{q}_i^*(t)=q_i^*(t)-a_{i1}(t)q_1^*(t)=[0, l_d\hat{\theta}_i(t)]^T$, $q_i^*(t)=[0, l_d\theta_i(t)]^T$, $i=1,\cdots,N$. The control protocol (10) can be rewritten as

$$\begin{aligned}\dot{q}_i(t) &= -\sum_{j\in N_i} c_{ij}(t)(\hat{q}_i^*(t)-\hat{q}_j^*(t))\\ &\quad -\sum_{j\in N_i} c_{ij}(t)(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -H_i(t)\sum_{j\in N_i} c_{ij}(t)\dot{\hat{p}}_{ij}(t)\\ &\quad -H_i(t)\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|),\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{c}_{ij}(t) &= k_{ij}(\hat{q}_i(t)-\hat{q}_j(t))^T(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad +k_{ij}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))^T(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t)).\end{aligned}$$

Due to $\dot{\hat{q}}_i(t)=\dot{q}_i(t)-a_{i1}\dot{q}_1(t)=\dot{q}_i(t)-a_{i1}f(t)$ and $\dot{\hat{p}}_i(t)^T=\hat{q}_i(t)^T H_i(t),$ we have

$$\begin{aligned}\frac{dE_3}{dt} &= \sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\dot{\hat{q}}_i(t)\\ &= \sum_{i\in\mathbb{F}}\hat{q}_i(t)^T(\dot{q}_i-a_{i1}f(t))\\ &= \sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\Big(-\sum_{j\in N_i}c_{ij}(t)(\hat{q}_i^*(t)-\hat{q}_j^*(t))\\ &\quad -\sum_{j\in N_i}c_{ij}(t)(\hat{q}_i(t)-\hat{q}_j(t))-H_i(t)\sum_{j\in N_i}c_{ij}(t)\dot{\hat{p}}_{ij}(t)\\ &\quad -H_i(t)\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)-a_{i1}f(t)\Big)\\ &= -\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}c_{ij}(t)(\hat{q}_i^*(t)-\hat{q}_j^*(t))\\ &\quad -\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}c_{ij}(t)(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T H_i(t)\sum_{j\in N_i}c_{ij}(t)\dot{\hat{p}}_{ij}(t)\\ &\quad -\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T H_i(t)\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\ &\quad -\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T a_{i1}f(t)\\ &= -l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}c_{ij}(t)\hat{\omega}_i(t)(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\ &\quad -\sum_{i\in\mathbb{F}}\sum_{j\in N_i}c_{ij}(t)\hat{q}_i(t)^T(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -\sum_{i\in\mathbb{F}}\sum_{j\in N_i}c_{ij}(t)\dot{\hat{p}}_i(t)^T(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))\\ &\quad -\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)\\ &\quad -\sum_{i\in\mathbb{F}}a_{i1}\hat{q}_i(t)^T f(t).\end{aligned}$$

Finally, we have

$$\begin{aligned}\frac{dE_4}{dt} &= \frac{1}{2}\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\frac{(c_{ij}(t)-m)\dot{c}_{ij}(t)}{k_{ij}}\\ &= \frac{1}{2}\sum_{i\in\mathbb{F}}\sum_{j\in N_i}(c_{ij}(t)-m)((\hat{q}_i(t)\\ &\quad -\hat{q}_j(t))^T(\hat{q}_i(t)-\hat{q}_j(t)))\\ &\quad +\frac{1}{2}\sum_{i\in\mathbb{F}}\sum_{j\in N_i}(c_{ij}(t)-m)(\dot{\hat{p}}_i(t)\\ &\quad -\dot{\hat{p}}_j(t))^T(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))\\ &= \sum_{i\in\mathbb{F}}\sum_{j\in N_i}(c_{ij}(t)-m)\hat{q}_i(t)^T(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad +\sum_{i\in\mathbb{F}}\sum_{j\in N_i}(c_{ij}(t)-m)\dot{\hat{p}}_i(t)^T(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t)).\end{aligned} \tag{23}$$

Thus, $dE/dt$ can be simplified by

$$\begin{aligned}\frac{dE}{dt} &= -m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -m\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))\\ &\quad -\sum_{i\in\mathbb{F}}a_{i1}\hat{q}_i(t)^T f(t)\\ &= -m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -m\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))\\ &\quad -\sum_{i\in N_l}\hat{q}_i(t)^T f(t).\end{aligned} \tag{24}$$

Owing to $\hat{q}_1=0,$ one gets

$$\begin{aligned}&-m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}(\hat{q}_i(t)-\hat{q}_j(t))\\ &= -m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i/\{1\}}(\hat{q}_i(t)-\hat{q}_j(t))\\ &\quad -m\sum_{i\in N_l}\hat{q}_i(t)^T\hat{q}_i(t)\\ &= -m\hat{q}(t)^T(L_{N-1}\otimes I_2)\hat{q}(t)\\ &\quad -m\sum_{i\in N_l}\hat{q}_i(t)^T\hat{q}_i(t)\le 0,\end{aligned} \tag{25}$$

where $I_2$ denotes the 2×2 identity matrix, and $L_{N-1}$ is symmetric and positive semi-definite. Let **0** denote the zero vector. Besides, $-m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}(\hat{q}_i(t)-\hat{q}_j(t))=0$ if and only if $\hat{q}_i(t)=\hat{q}_j(t)$ for all $i\in\mathbb{F}$ and $j\in N_i$. Similarly, due to $\dot{\hat{p}}_1(t)=\mathbf{0},$ we have

$$\begin{aligned}&-m\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_i(t))\\ &= -m\dot{\hat{p}}(t)^T(L_{N-1}\otimes I_2)\dot{\hat{p}}(t)-m\sum_{i\in N_l}\dot{\hat{p}}_i(t)^T\dot{\hat{p}}_i(t)\le 0,\end{aligned} \tag{26}$$

where

$\hat{p}(t)=[\hat{p}_2(t)^T,\cdots,\hat{p}_N(t)^T]^T,$ and
$-m\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))=0$

if and only if $\dot{\hat{p}}_i(t)=\dot{\hat{p}}_j(t)$ for all $i\in\mathbb{F}$ and $j\in N_i$.

If $\hat{q}_i(t)=\hat{q}_j(t)$ and $\dot{\hat{p}}_i(t)=\dot{\hat{p}}_j(t),$ we have

$$\frac{dE}{dt} = -m\hat{q}(t)^T(L_{N-1}\otimes I_2)\hat{q}(t) - m\sum_{i\in N_l}\hat{q}_i(t)^T\hat{q}_i(t) - m\dot{\hat{p}}(t)^T(L_{N-1}\otimes I_2)\dot{\hat{p}}(t) - m\sum_{i\in N_l}\dot{\hat{p}}_i(t)^T\dot{\hat{p}}_i(t) - \sum_{i\in N_l}\hat{q}_i(t)^T f(t) = 0. \quad (27)$$

If $\hat{q}_i(t)\neq\hat{q}_j(t)$ or $\dot{\hat{p}}_i(t)\neq\dot{\hat{p}}_j(t)$, then we have $-m\sum_{i\in\mathbb{F}}\hat{q}_i(t)^T\sum_{j\in N_i}(\hat{q}_i(t)-\hat{q}_j(t))<0$ or $-m\sum_{i\in\mathbb{F}}\dot{\hat{p}}_i(t)^T\sum_{j\in N_i}(\dot{\hat{p}}_i(t)-\dot{\hat{p}}_j(t))<0$. Since $f(t)$ is bounded, as long as the positive constant $m$ is sufficiently large, then we have

$$dE/dt = -m\hat{q}(t)^T(L_{N-1}\otimes I_2)\hat{q}(t) - m\sum_{i\in N_l}\hat{q}_i(t)^T\hat{q}_i(t) - m\dot{\hat{p}}(t)^T(L_{N-1}\otimes I_2)\dot{\hat{p}}(t) - m\sum_{i\in N_l}\dot{\hat{p}}_i(t)^T\dot{\hat{p}}_i(t) - \sum_{i\in N_l}\hat{q}_i(t)^T f(t) < 0. \quad (28)$$

Therefore, we have $dE/dt\leq 0$. The initial potential energy of the system is finite, and the initial speeds and initial heading angles of all agents are also finite. Thus, the initial energy $E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0))$ of the system is finite, and the supremum of $E$ is obviously its initial value $E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0))$. Then, the potential energy $V$ of the system is finite, and the potential energy $V(\|\hat{p}_{ij}(t)\|)$ between agent $i$ and $j$ is finite.

If for some $(i,j)\in\varepsilon$, $\|\hat{p}_{ij}\|\to 2D$, then according to rule (2) of definition 1, one gets $V(\|\hat{p}_{ij}\|)\to\infty$, which violates the conclusion that $V(\|\hat{p}_{ij}\|)$ remains finite. It follows that $\|\hat{p}_{ij}\|<2D$ for all $(i,j)\in\varepsilon$ and $t\in[t_r, t_{r+1})$. Hence, whenever there is a link between two agents, the link is never lost during each time-interval $[t_r,t_{r+1})$, that is, $\varepsilon(t_r)\subseteq\varepsilon(t_{r+1})$. Note that when $\|\hat{p}_{ij}(t)\|<2D$, one gets $\|p_{ij}\|<R=2D+\|p_{ij}^*\|$. It is obvious that $R>D$, which ensures that if an edge $(i,j)\notin\varepsilon$ is added to $\varepsilon$, the associated potential $V(\|\hat{p}_{ij}\|)$ is bounded and hence, so is the new potential $V$. Consequently, the connectivity of the communication network can be preserved all the time. Conclusion 1 of Theorem 1 is proved.

Assume that there are $m_r\in Z^+$ new links being added to the interaction network at switching time $t_r$, $r=1,2,\cdots$. We have supposed that the initial interaction network should satisfy the system topology is a leader-follower connected graph. Let $G_1$ denote the initial interaction network of the system, and $G_c$ denote the set of all graphs meeting the proposed two conditions on the vertices. The proposed control algorithm can guarantee that the sequence of switching topologies $G_{r+1}$ within $[t_r,t_{r+1})$ consists of such graphs satisfying $G_{r+1}\in G_c$. The number of the vertices is finite, thus $G_c$ is a finite set. Assume that there are at most $M\in Z^+$ new links that can be added to the initial interaction network $G_1$. Clearly, $0<m_r\leq M$ and $r\leq M$. Therefore, the number of switching times of the system is finite, and the interaction network $G(t)$ eventually becomes fixed. Suppose that the last switching time is $t_f$, the following discussions are restricted on the time interval $[t_f,\infty)$.

Note that the distance between neighbors is no longer than $D$. Hence, the set

$$B=\{\tilde{p}\in D_p,\hat{q},\hat{\theta}\mid E(\tilde{p},\hat{q},\hat{\theta})\leq E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0))\} \quad (29)$$

is positively invariant, where $D_p=\{\tilde{p}\mid\|\hat{p}_{ij}\|\in(0,2D), \forall(i,j)\in\varepsilon\}$. Since $G(t)$ is connected for all $t\geq 0$, one gets $\|\hat{p}_{ij}\|<2(N-1)D$ for all $i$ and $j$. Since

$$E(\tilde{p}(t),\hat{q}(t),\hat{\theta}(t))\leq E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0)),$$

we have $\hat{q}^T\hat{q}\leq 2E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0))$, i.e.,

$$\|\hat{q}\|\leq\sqrt{2E(\tilde{p}(0),\hat{q}(0),\hat{\theta}(0))}.$$

In addition, $\hat{\theta}_i\in(-2\pi,2\pi)$, $i=1,\cdots,N$. Thus, the set $B$ is closed and bounded, hence compact. Note that the system (1) with control input (7) and (10) is an autonomous system on the concerned time interval.

Then, according to the LaSalle-Krasovskii invariance principle [19], the trajectories of the followers will converge to the invariant set $S=\{\tilde{p},\hat{q},\hat{\theta}\mid dE/dt=0\}$. Clearly, $dE/dt=0$ if and only if $\hat{q}_i(t)=\hat{q}_j(t)$ and $\dot{\hat{p}}_i(t)=\dot{\hat{p}}_j(t)$ for all $i\in\mathbb{F}$ and $j\in N_i$. Then, we have $\hat{q}_i(t)=\hat{q}_j(t)=\hat{q}_1(t)$ for $i\in N_l$, $j\in N_i$. With $\hat{q}_1(t)=0$, one gets $q_i(t)=q_1(t)$ and $q_j(t)-a_{j1}q_1(t)=0$. If $j\notin N_l$, one gets $q_j(t)=0$, which means that agent $j$ stops moving. The state of agent $j$ cannot be maintained, because its neighbor $i$ always follows the moving leader and the distance between agent $i$ and agent $j$ will change to cause the control protocol (10) to work. Thus, there is only $j\in N_l$. As mentioned above, followers asymptotically approach a configuration that every follower has one leader neighbor, that is, $N_l=\mathbb{F}$. Discussion about $\dot{\hat{p}}_i(t)=\dot{\hat{p}}_j(t)$ is similar. In a word, $dE/dt=0$ means $q_i(t)=q_1(t)$ and $\dot{p}_i(t)=\dot{p}_1(t)$ for $i=2,\cdots,N$.

According to (1), $\dot{p}_i(t)=\dot{p}_1(t)$ is equivalent to

$$\begin{cases}\upsilon_i(t)\cos\theta_i(t)-\omega_i(t)l_d\sin\theta_i(t)\\ \quad=\upsilon_1(t)\cos\theta_1(t)-\omega_1(t)l_d\sin\theta_1(t)\\ \upsilon_i(t)\sin\theta_i(t)+\omega_i(t)l_d\cos\theta_i(t)\\ \quad=\upsilon_1(t)\sin\theta_1(t)+\omega_1(t)l_d\cos\theta_1(t).\end{cases} \quad (30)$$

In terms of $q_i(t)=q_1(t)$ and $l_d>0$, (30) can be explicitly expressed by

$$\begin{cases}(\upsilon_1^2(t)+l_d^2\omega_1^2(t))\sin\theta_i(t)\\ \quad=(\upsilon_1^2(t)+l_d^2\omega_1^2(t))\sin\theta_1(t)\\ (\upsilon_1^2(t)+l_d^2\omega_1^2(t))\cos\theta_i(t)\\ \quad=(\upsilon_1^2(t)+l_d^2\omega_1^2(t))\cos\theta_1(t).\end{cases} \quad (31)$$

With $q_1(t)\neq \mathbf{0}$, thus we have $\theta_i(t)=\theta_1(t)\in[0,2\pi)$. As previously mentioned, the thrusting speed, the rotational speed, and the heading angle of each follower asymptotically become the same with those of the leader, that is, $\lim_{t\to\infty}(\upsilon_i(t)-\upsilon_j(t))=0$, $\lim_{t\to\infty}(\omega_i(t)-\omega_j(t))=0$, and $\lim_{t\to\infty}(\theta_i(t)-\theta_j(t))=0$, where $i,j\in\nu, i\neq j$. Conclusion 2 of Theorem 1 is proved.

Due to $\lim_{t\to\infty} f_1(t)=0$ and $\lim_{t\to\infty} f_2(t)=0$, that is, $\lim_{t\to\infty}\dot{q}_1(t)=\mathbf{0}$, one gets $\dot{q}_i(t)=\dot{q}_1(t)=\mathbf{0}$ in steady state, $i\in\mathbb{F}$. With $\hat{\theta}_i(t)=\hat{\theta}_j(t)$, $\hat{q}_i(t)=\hat{q}_j(t)$, and $\dot{\hat{p}}_i(t)=\dot{\hat{p}}_j(t)$, $\dot{q}_i(t)=\mathbf{0}$ can be simplified as

$$\begin{cases}-\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)^T\vec{t}_i(t)=0\\ -\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)^T\vec{n}_i(t)=0.\end{cases}\tag{32}$$

For $\vec{t}_i(t)=\begin{bmatrix}\cos\theta_i(t)\\ \sin\theta_i(t)\end{bmatrix}$ and $\vec{n}_i(t)=\begin{bmatrix}-\sin\theta_i(t)\\ \cos\theta_i(t)\end{bmatrix}$ are unit vectors orthogonal to each other, as well as $l_d\neq 0$, (32) is equivalent to

$$\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)=0,\quad i\in\mathbb{F},\tag{33}$$

where $N_i=\mathbb{L}\cup\mathbb{F}/\{i\}$.
Further, we get

$$\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)=0,\tag{34}$$

i.e., $\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)+\sum_{i\in N_l}\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|)=0$. Owing to $\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}(t)}V(\|\hat{p}_{ij}(t)\|)=0$, one obtains $\sum_{i\in N_l}\nabla_{\hat{p}_{i1}(t)}V(\|\hat{p}_{i1}(t)\|)=0$, i.e.,

$$\sum_{j\in N_l}\nabla_{\hat{p}_{1j}(t)}V(\|\hat{p}_{1j}(t)\|)=0,\tag{35}$$

where $N_l=\mathbb{F}$. (33) and (35) mean that the total potential of the system is minimized. Thus, conclusion 3 of Theorem 1 is proved.

According to the above analysis, the whole system stabilizes the inter-agent distances when the total potential of the system is minimized. Thus, any desired rigid formations can be obtained, provided that the potential function reaches its minimal value at the point of the desired inter-agent distance. Given a desired geometric pattern $\chi$ that has $N$ vertices $p_i^d=[x_i^d,y_i^d]^T$, $i=1,\cdots,N$. The follower's control protocol is revised by

$$\begin{aligned}a_i(t)=&-\sum_{j\in N_i(t)}c_{ij}(t)(\hat{\upsilon}_i(t)-\hat{\upsilon}_j(t))\\&-\sum_{j\in N_i(t)}c_{ij}(t)\dot{\hat{p}}_{ij}(t)^T\vec{t}_i(t)\\&-\sum_{j\in N_i(t)}\nabla_{\tilde{p}_{ij}(t)}V(\|\tilde{p}_{ij}(t)\|)^T\vec{t}_i(t),\\ b_i(t)=&-\sum_{j\in N_i(t)}c_{ij}(t)l_d(\hat{\theta}_i(t)-\hat{\theta}_j(t))\\&-\sum_{j\in N_i(t)}c_{ij}(t)l_d(\hat{\omega}_i(t)-\hat{\omega}_j(t))\\&-\sum_{j\in N_i(t)}c_{ij}(t)\dot{\hat{p}}_{ij}(t)^T\vec{n}_i(t)\\&-\sum_{j\in N_i(t)}\nabla_{\tilde{p}_{ij}(t)}V(\|\tilde{p}_{ij}(t)\|)^T\vec{n}_i(t),\\ \dot{c}_{ij}(t)=&\,k_{ij}(\hat{q}_i(t)-\hat{q}_j(t))^T(\hat{q}_i(t)-\hat{q}_j(t))\\&+k_{ij}\dot{\hat{p}}_{ij}(t)^T\dot{\hat{p}}_{ij}(t),\end{aligned}\tag{36}$$

where $\|\tilde{p}_{ij}(t)\|=\frac{\|\hat{p}_{ij}(t)\|}{\|p_{ij}^d\|}$, and the potential function $V(\|\tilde{p}_{ij}(t)\|)$ is required to reach its minimal value at the point of $\|\tilde{p}_{ij}(t)\|=1$. Then, we have the following Corollary 1.

**Corollary 1:** Consider a system of $N$ agents with dynamics (1). The leader and the followers are respectively steered by control protocols (7) and (36). Suppose that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time. Then the following statements hold:

1) The connectivity of the interaction network is preserved at all times, that is, $\varepsilon(t_r)\subseteq\varepsilon(t_{r+1})$.

2) The thrusting speed, the rotational speed, and the heading angle of each follower asymptotically become the same as those of the leader, that is, $\lim_{t\to\infty}(\upsilon_i(t)-\upsilon_j(t))=0$, $\lim_{t\to\infty}(\omega_i(t)-\omega_j(t))=0$, and $\lim_{t\to\infty}(\theta_i(t)-\theta_j(t))=0$, where $i,j\in\nu, i\neq j$.

3) The system approaches a desired geometric configuration $\chi$ that minimizes the total potential, that is,

$$\sum_{j\in N_i}\nabla_{\tilde{p}_{ij}(t)}V(\|\tilde{p}_{ij}(t)\|)=0,\ \text{ and}$$
$$\sum_{j\in N_l}\nabla_{\tilde{p}_{1j}(t)}V(\|\tilde{p}_{1j}(t)\|)=0.$$

The proof of Corollary 1 is similar to that of Theorem 1. Thus, it is omitted here.

## 4. SIMULATION VALIDATION

In this section, we will show the simulation results of the proposed theoretical results. According to definition 1, we design the following potential function

$$V(\|\hat{p}_{ij}\|)=\frac{b}{\|\hat{p}_{ij}\|^2}-a\ln(4D^2-\|\hat{p}_{ij}\|^2)+c,\tag{37}$$

where $a$ and $b$ are positive constants, $c$ is constant. Therein, the first part is the repulsive potential term, and the second part is the attractive potential term, and $c$ is just used to guarantee the potential function positive at all times. Then, the gradient of the potential function is obtained by

$$\nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) = 2\hat{p}_{ij}\left(-\frac{b}{\|\hat{p}_{ij}\|^4} + \frac{a}{4D^2 - \|\hat{p}_{ij}\|^2}\right). \quad (38)$$

Specially, it should be noted that parameters $a$ and $b$ at least satisfy the constraint condition $\frac{\sqrt{b^2+16abD^2}-b}{2a} < D^2$. Besides, the formation flocking task requires that the potential function $V(\|\tilde{p}_{ij}(t)\|)$ reaches its minimal value at the point of $\|\tilde{p}_{ij}(t)\| = \frac{\|\hat{p}_{ij}(t)\|}{\|p_{ij}^d\|} = 1$. Thus, the parameter $a$ and $b$ should also satisfy another constraint condition $a = b(4D^2 - 1)$. Fig. 3 gives an example of the potential function, which satisfies the three rules of Definition 1.

Agents with generic initial conditions are employed, on condition that the initial interaction network among the followers is an undirected connected graph and there exists at least one follower having a leader neighbor at the initial time. According to Theorem 1 and Corollary 1, the connectivity of the interaction network can be preserved at all times.

Firstly, we assign five agents to achieve the leader-follower cohesive flocking task. Leader swims with bounded varying speeds that $f_1(t) = 0.1/(t+1)$ and $f_2(t) = 0.1/(t+1)$, while followers try to track the velocities of the leader and construct a cohesive formation together with the leader at the same time. The simulation results are shown in Fig. 4.

In Fig. 4(a), the green star point is the initial position of each agent, while the centre of the colored ball denotes the final position of each agent at time $t$ = 200s, which has been enlarged for more details. The rough red circle line denotes the trajectory of the leader. It is clear to see that the five agents asymptotically converge to a stable flocking formation. Figs. 4(b) and (c) tell us that the thrusting speed, the rotational speed, and the heading angles of the followers asymptotically track those of the leader. Fig. 4(d) gives the distances between any two agents. The distances are all asymptotically stable.

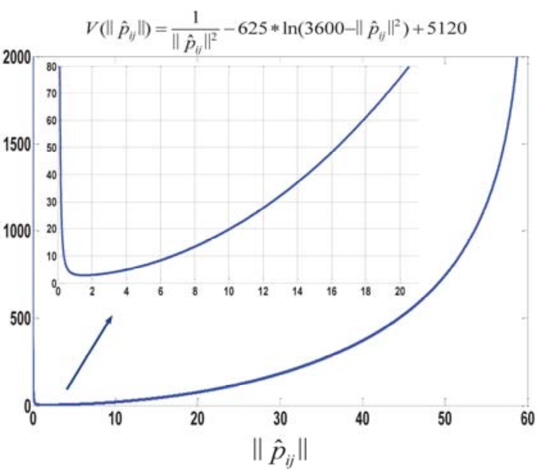
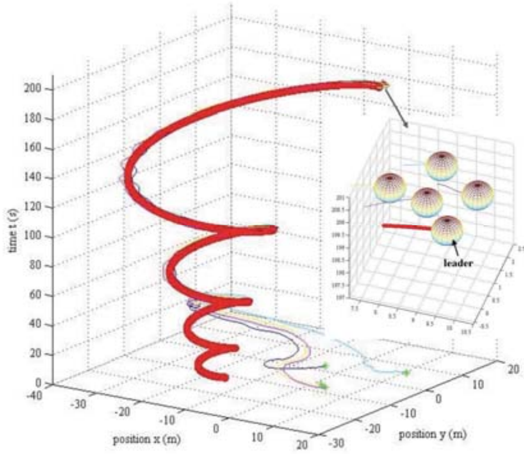
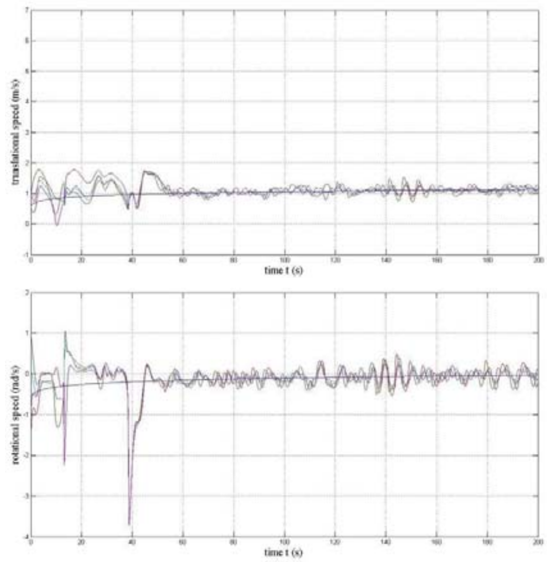


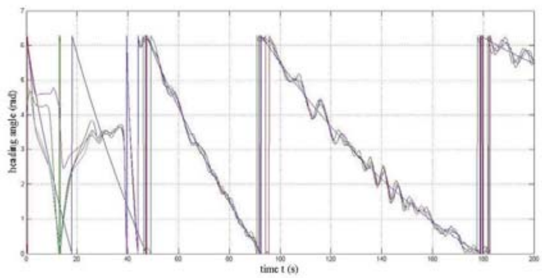

Fig. 3. The potential function (37). For example, $a$ = 625, $b$ =1, $c$ = 5120, and $D$ = 30.



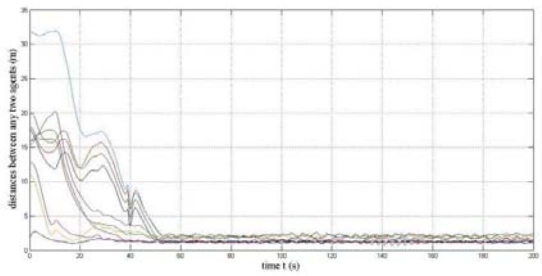

(a) Trajectories of the five agents.



(b) The thrusting speeds and the rotational speeds of the five agents with time $t$.



(c) The heading angles of the five agents with time $t$.



(d) The distances between any two agents with time $t$.

Fig. 4. Simulation results of the leader-follower cohesive flocking task.

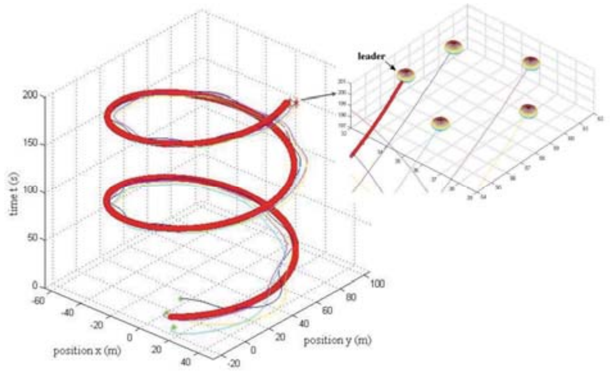



Fig. 5. Simulation results of the leader-follower ring-shaped formation flocking task.

Besides, the initial adjacency matrix of the system is

$$A(0)=\begin{bmatrix}1&0&0&0&0\\1&0&1&1&1\\1&1&0&1&1\\1&1&1&0&1\\0&1&1&1&0\end{bmatrix}, \tag{39}$$

which indicates that the followers' interaction network is a complete graph, and only part of the followers have a leader neighbor. We output the adjacency matrix of the whole system in real time, according to which we conclude that the connectivity of the system is preserved at all times. Finally, the interaction network of the whole system is fixed, which can be expressed by the following adjacency matrix

$$A(200)=\begin{bmatrix}1&0&0&0&0\\1&0&1&1&1\\1&1&0&1&1\\1&1&1&0&1\\1&1&1&1&0\end{bmatrix}. \tag{40}$$

Secondly, we verify the leader-follower formation flocking algorithm by five gents similarly. For the sake of simplicity, we only show the trajectory of the five agents to complete the leader-follower ring-shaped flocking task in Fig. 5, and the final position of the five agents has also been enlarged for more details.

## 5. CONCLUSION

A distributed leader-follower adaptive flocking problem of multiple robotic fish governed by extended second-order unicycles has been investigated in this paper. The system is consisted of only one leader with varying but bounded speeds. Two leader-follower adaptive flocking algorithms are proposed with the combination of consensus and attraction/repulsion functions to respectively solve the cohesive flocking problem and the formation flocking problem. Provided that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time, the followers asymptotically converge to a cohesive flocking configuration or formation flocking configuration together with a variable-speed leader. The stability of the closed-loop system is analyzed based on LaSalle-Krasovskii invariance principle. Finally, simulation results verify the effectiveness of the proposed control algorithms.

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