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Global Output Feedback Regulation of Uncertain Nonlinear Systems with Unknown Time Delay

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Abstract: This paper investigates the problem of global output feedback regulation for a class of nonlinear systems with unknown time delay. It is also allowed to contain uncertain functions of all the states and input as long as the uncertainties satisfying certain bounded condition for the considered systems. In this paper, a constructive control technique has been proposed for controlling the systems. By using dynamic high-gain scaling approach and choosing an appropriate Lyapunov-Krasovskii functional, a delay-independent robust adaptive output feedback controller is constructed such that the states of the considered systems achieve global regulation. Two simulation examples are provided to demonstrate the effectiveness of the proposed design scheme.

Keywords: Adaptive control, output feedback, time-delay systems, uncertain nonlinear systems.

1. INTRODUCTION

The problem of global output feedback stabilization for nonlinear systems has received considerable attention over the past few years (see e.g., [1-4] and the references therein). Recently, the output feedback control problem has been investigated for nonlinear systems with unmeasured states dependent growth and known output function or known growth rates ([5-7]). More generally, when the growth rate is an unknown positive constant, the problem of global robust output feedback control of the uncertain system becomes much more involved and difficult. The systems were also investigated in [8-10]. Using new high-gain K-filters techniques ([11]), a constructive design procedure was proposed for a class of systems with uncertain control coefficient and unmeasured states dependent growth multiplying an unknown constant. In practice, a number of physical devices ([see 12-14]), after a change of feedback, can be described by equations with the feedforward structure. Furthermore, output feedback stabilization or regulation were also addressed by [15-18] for feedforward systems with uncertain functions involving all unmeasurable states and the assumed bounds on uncertain functions.

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On the other hand, systems with time delays are frequently encountered. Various engineering systems have the characteristics of time delay, such as turbojet engines, nuclear reactors, chemical process. Time delay usually leads to poor performances and often causes instability. So far, there have been tremendous efforts in stability analysis and robust control for these time-delay systems (see e.g., [19-21] and the references therein). In Zhang and Cheng [22] and Guan [23], output feedback controllers were constructed to stabilise a class of time-delay nonlinear systems that are dominated by a lower triangular time-delay system satisfying linear growth in unmeasured states. Recently, the output feedback control problem for feedforward nonlinear time-delay systems were also addressed by dynamic gain scaling in Krishnamurthy *et al.* [24] and Zhang *et al.* [25]. In [24,25], the growth rate is a known or partially known positive constant. However, there is few papers focused on the case that the growth rate is an unknown positive constant for the feedforward nonlinear time-delay (in the input and states) systems.

Motivated by [8,16] and [25], in this paper, a delay-independent controller for a class of nonlinear time-delay systems is constructed. To the best of our knowledge, there is no work dealing with such a class of systems satisfying Assumption 1 in the literature at present. So the proposed method expands the class of nonlinear systems that can be handled using dynamic gain scaling technique.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following single-input-single-output (SISO) time-delay systems

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + B_0 u(t) + \phi(t, x, u(t), x(t-\tau), u(t-\tau)),\\ y(t) &= C_0 x(t)\end{aligned} \quad (1)$$

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with

$$A_0 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$\phi(\cdot) = \begin{bmatrix} \phi_1(\cdot) \\ \vdots \\ \phi_{n-1}(\cdot) \\ 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix},$$

where $x(t) = [x_1(t), \ x_2(t), \ \ldots, \ x_n(t)]^T \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the states, control input, and output of system, respectively. The constant $\tau \geq 0$ is an unknown time delay of the system, and there exists a known positive constant d such that $\tau \leq d$. In this paper, we always denote $x_i(t), \ \varepsilon_i(t), \ z_i(t)$ by $x_i, \ \varepsilon_i, \ z_i$. The uncertain functions $\phi_i : \mathbb{R}^+ \times \mathbb{R}^{2(n+1)} \to \mathbb{R}, \ i = 1, \ldots, n-1,$ are continuously differentiable with respect to all the variables. We have that the following assumption for the uncertain system (1).

Assumption 1: For the uncertain functions $\phi_i(\cdot)$, there exists an unknown constant $C > 0$ such that for any $s \in (0,1]$, the following inequality holds

$$\sum_{i=1}^{n-1} s^{n-i+1} |\phi_i(\cdot)| \leq Cs^2 [\sum_{i=1}^{n} s^{n-i+1} (|x_i| + |x_i(t-\tau)|) + |u| + |u(t-\tau)|].$$

Remark 1: It is not difficult to prove that if the following condition for some unknown constant $c > 0$

$$|\phi_i(\cdot)| \leq c \left[\sum_{j=i+2}^{n} (|x_j| + |x_j(t-\tau)|) + |u| + |u(t-\tau)| \right] \quad (2)$$

is satisfied, then Assumption 1 is always satisfied, but not vice versa. So the system (1) is of a more general form than a class of feedforward systems satisfying (2).

The following Lemma is used in this paper.

Lemma 1: There exist two constant symmetric matrices $P > 0, \ Q > 0,$ and two vectors $a = (a_1, \ \ldots, \ a_n)^T$, $b = (b_1, \ \ldots, \ b_n)^T$ such that

$$\begin{cases} A^T P + PA \leq -I, \text{ and } DP + PD \geq 0, \\ B^T Q + QB \leq -2I, \text{ and } DQ + QD \geq 0, \end{cases} \quad (4)$$

where

$$A = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_1 & -b_2 & \cdots & -b_n \end{bmatrix},$$

$D = \text{diag}\{n, n-1, \ldots, 1\}$.

Similar lemmas have been announced in [16,25] and [27], and we omit its proof here.

3. MAIN RESULT

This section is devoted to the design of the observer-based controller. By appropriate choice of parameters we show that a linear-like controller is able to bring the states of the nonlinear time-delay system to the origin.

Theorem 1: For the system (1) satisfying Assumption 1, the following output feedback makes the solutions of the closed-loop system bounded and $x_i, \ 1 \leq i \leq n,$ converge to the origin:

$$\begin{cases} \dot{\hat{x}} = A_0 \hat{x} + B_0 u + T_{1L} a C_0 (x - \hat{x}), \\ u = -b^T T_{2L} \hat{x}, \\ \dot{L} = \dfrac{1}{L^2} \left(\dfrac{y - \hat{x}_1}{L^n} \right)^2, \text{ with } L(t) = 1, \text{ for } t \in [-d, 0], \end{cases} \quad (4)$$

where

$$\hat{x} = [\hat{x}_1, \ \hat{x}_2, \ \ldots, \ \hat{x}_n]^T \in \mathbb{R}^n$$

$$T_{1L} = \text{diag}\left\{ \frac{1}{L}, \frac{1}{L^2}, \ldots, \frac{1}{L^n} \right\},$$

$$T_{2L} = \text{diag}\left\{ \frac{1}{L^n}, \frac{1}{L^{n-1}}, \ldots, \frac{1}{L} \right\},$$

a and b are the appropriately chosen parameters such that Lemma 1 holds.

Proof: Consider the following rescaling transformation

$$\varepsilon_i = \frac{x_i - \hat{x}_i}{L^{n-i+1}}, \quad z_i = \frac{\hat{x}_i}{L^{n-i+1}}, \quad i = 1, 2, \ldots, n. \quad (5)$$

Then, by (5), the closed-loop system (1) and (4) can be given by a simple calculation

$$\begin{cases} \dot{\varepsilon} = \dfrac{1}{L} A\varepsilon + \Phi(\cdot) - \dfrac{\dot{L}}{L} D\varepsilon, \\ \dot{z} = \dfrac{1}{L} Bz + \dfrac{1}{L} a\varepsilon_1 - \dfrac{\dot{L}}{L} Dz, \end{cases} \quad (6)$$

where a, A, B and D are defined by Lemma 1,

$$\varepsilon = (\varepsilon_1, \ \ldots, \ \varepsilon_n)^T, \quad z = (z_1, \ \ldots, \ z_n)^T, \quad \Phi(\cdot) = T_{2L} \phi(\cdot),$$

and we have $u = -b^T z$.
Consider an observer Lyapunov function $V_\varepsilon : \mathbb{R}^n \to \mathbb{R}^+$ and a controller Lyapunov function $V_z : \mathbb{R}^n \to \mathbb{R}^+$ given by

$$V_\varepsilon = \varepsilon^T P \varepsilon, \quad V_z = z^T Q z, \quad (7)$$

where P and Q are constant matrices chosen as in Lemma 1.
Observe that by construction, $\dot{L} = \frac{\varepsilon_1^2}{L^2} \geq 0$ and hence $L(t) \geq L(t-\tau) \geq 1$, for $\forall t \in [0, t_f)$. Then, using (6) and

(3), we obtain that the derivatives of V_ε and V_z can be bounded as

$$\dot{V}_\varepsilon \le -\frac{1}{L}\|\varepsilon\|^2 + 2\varepsilon^T P\Phi(\cdot), \tag{8}$$

$$\dot{V}_z \le -2\frac{1}{L}\|z\|^2 + 2\frac{1}{L}z^T Qa\varepsilon_1. \tag{9}$$

Applying Assumption 1, $u = -b^T z$, and $2ab \le a^2 + b^2$, we have

$$\begin{aligned}
&|2\varepsilon^T P\Phi(\cdot)| \\
&\le 2\|\varepsilon\|\|P\|\left(|\frac{\phi_1(\cdot)}{L^n}| + \cdots + |\frac{\phi_{n-1}(\cdot)}{L^2}|\right) \\
&\le 2C\|\varepsilon\|\|P\|\frac{1}{L^2}[\sum_{i=1}^n (|\frac{x_i}{L^{n-i+1}}| + |\frac{x_i(t-\tau)}{L^{n-i+1}}|) \\
&\quad + |u| + |u(t-\tau)|] \\
&\le 2C\|\varepsilon\|\|P\|\frac{1}{L^2}[\sum_{i=1}^n (|\varepsilon_i| + |z_i| + |\varepsilon_i(t-\tau)| \\
&\quad + |z_i(t-\tau)|) + |u| + |u(t-\tau)|] \\
&\le 2C\|\varepsilon\|\|P\|\frac{1}{L^2}[\sqrt{n}\|\varepsilon\| + \sqrt{n}\|z\| \\
&\quad + \sqrt{n}\|\varepsilon(t-\tau)\| + \sqrt{n}\|z(t-\tau)\| \\
&\quad + \|b\|\|z\| + \|b\|\|z(t-\tau)\|] \\
&\le \frac{C\mu_1}{L^2}(\|\varepsilon(t-\tau)\|^2 + \|z(t-\tau)\|^2) \\
&\quad + \frac{C\mu_1}{L^2}(\|\varepsilon\|^2 + \|z\|^2) \\
&\le \frac{C\mu_1}{L^2(t-\tau)}(\|\varepsilon(t-\tau)\|^2 + \|z(t-\tau)\|^2) \\
&\quad + \frac{C\mu_1}{L^2}(\|\varepsilon\|^2 + \|z\|^2),
\end{aligned} \tag{10}$$

where $\mu_1 = 5(\sqrt{n} + \|b\|)\|P\|$ is a known constant. Note that

$$\begin{aligned}
|2\frac{1}{L}z^T Qa\varepsilon_1| &\le \frac{1}{L}\|z\|^2 + \frac{1}{L}\|Qa\|^2\varepsilon_1^2 \\
&= \frac{1}{L}\|z\|^2 + \frac{1}{L}\mu_2\varepsilon_1^2
\end{aligned} \tag{11}$$

$$\le \frac{1}{L}\|z\|^2 + \frac{1}{L}\mu_2\|\varepsilon\|^2, \tag{12}$$

where $\mu_2 = \|Qa\|^2$ is a known constant.
Substituting (10) and (12) into (8) and (9), respectively, result in

$$\begin{aligned}
\dot{V}_\varepsilon &\le -\frac{1}{L}\|\varepsilon\|^2 + \frac{C\mu_1}{L^2}(\|\varepsilon\|^2 + \|z\|^2) \\
&\quad + \frac{C\mu_1}{L^2(t-\tau)}(\|\varepsilon(t-\tau)\|^2 + \|z(t-\tau)\|^2),
\end{aligned} \tag{13}$$

$$\dot{V}_z \le -\frac{1}{L}\|z\|^2 + \frac{1}{L}\mu_2\|\varepsilon\|^2. \tag{14}$$

Construct the Lyapunov-Krasovskii functional

$$\begin{aligned}
V_0 &= \int_{t-\tau}^t \frac{C\mu_1(\mu_2+1)}{L^2(s)}(\|\varepsilon(s)\|^2 + \|z(s)\|^2)\mathrm{d}s \\
&\quad + (\mu_2+1)V_\varepsilon + V_z.
\end{aligned} \tag{15}$$

From (13) and (14), we get

$$\dot{V}_0 \le -\frac{1}{L^2}[L - 2C\mu_1(\mu_2+1)](\|\varepsilon\|^2 + \|z\|^2). \tag{16}$$

It is easy to prove that the solution (L, ε, z) of the closed-loop systems (4)-(6) exists and is unique on the maximally extended interval $[0, T_f)$. In what follows, using (16), we will first prove that the states variables (L, ε, z) are bounded on $[0, T_f)$.

Firstly, we claim that L is bounded on $[0, T_f)$, which can be proved by a contradiction argument. Supposed that $\lim_{t\to T_f} L(t) = +\infty$. Recall that $\dot{L} = \frac{\varepsilon_1^2}{L^2} \ge 0$, then, there exists a finite time $t_1 \in [0, T_f)$, such that

$$L(t) \ge 2C\mu_1(\mu_2+1) + 1, \quad \text{for} \quad \forall t \in [t_1, T_f).$$

From (16), we obtain

$$\dot{V}_0 \le -\frac{1}{L^2}(\|\varepsilon\|^2 + \|z\|^2), \quad \text{for} \quad \forall t \in [t_1, T_f).$$

Then

$$\begin{aligned}
+\infty = L(T_f) - L(t_1) &= \int_{t_1}^{T_f} \dot{L}(t)\mathrm{d}t \\
&= \int_{t_1}^{T_f} \frac{\varepsilon_1^2(t)}{L^2(t)}dt \le V_0(t_1) = \text{constant},
\end{aligned}$$

which leads to a contradiction and thus L is bounded on $[0, T_f)$ and $\lim_{t\to T_f} L(t) < +\infty$. Since $\dot{L} = \frac{\varepsilon_1^2}{L^2}$, then $\int_0^t \frac{\varepsilon_1^2(t)}{L^2(t)}\mathrm{d}t$ and $\int_0^t \varepsilon_1^2(t)\mathrm{d}t$ are bounded on $[0, T_f)$.

Secondly, we prove that z is bounded on $[0, T_f)$. Consider the Lyapunov function $V_z = z^T Qz$ for the z–dynamic system of (6). By (3), (6) and (11), we know that

$$\begin{aligned}
\dot{V}_z &\le -\frac{1}{L}\|z\|^2 + \frac{1}{L}\mu_2\varepsilon_1^2 \\
&= -\frac{1}{L}\|z\|^2 + \mu_2 L\dot{L}, \quad \text{for} \quad \forall t \in [0, T_f).
\end{aligned}$$

Hence, for $\forall t \in [0, T_f)$

$$\begin{aligned}
\lambda_{\min}(Q)\|z(t)\|^2 - z^T(0)Qz(0) &\le -\int_0^t \frac{1}{L(t)}\|z(t)\|^2\,\mathrm{d}t \\
&\quad + \frac{\mu_2}{2}[L^2(t) - 1] \le \frac{\mu_2}{2}[L^2(t) - 1].
\end{aligned} \tag{17}$$

Furthermore, for $\forall t \in [0, T_f)$, one has

$$\int_0^t \frac{1}{L(t)}\|z(t)\|^2\,dt \le z^T(0)Qz(0) + \frac{\mu_2}{2}[L^2(t) - 1]. \tag{18}$$

Since L is bounded on $[0,T_f)$, we can conclude from (17) and (18) that z and $\int_0^t \frac{1}{L(t)}\|z(t)\|^2\,\mathrm{d}t$ are bounded on $[0,T_f)$. Furthermore, $\int_0^t \|z(t)\|^2\,\mathrm{d}t$ is also bounded on $[0,T_f)$.

Finally, we show that ε is bounded on $[0,T_f)$. Defining constant L^* satisfying $L^* \geq \max\{L(T_f), 2C\mu_1+3\}$, where μ_1 is defined by (10), we introduce the transformation of coordinates

$$\zeta_i = \frac{x_i - \hat{x}_i}{(L^*)^{n-i+1}}, \quad i = 1,2,\ldots,n. \tag{19}$$

Accordingly, the dynamic system of ζ is given by

$$\dot{\zeta} = \frac{1}{L^*}A\zeta + \frac{1}{L^*}a\zeta_1 - \frac{1}{L}\Gamma a\zeta_1 + \Phi^*(\cdot), \tag{20}$$

where

$$\zeta = \left(\zeta_1, \;\ldots,\; \zeta_n\right)^T, \; \Gamma = \mathrm{diag}\left\{1, \frac{L^*}{L}, \ldots, \left(\frac{L^*}{L}\right)^{n-1}\right\},$$

$$\text{and} \quad \Phi^*(\cdot) = \left[\frac{\phi_1(\cdot)}{(L^*)^n}, \; \frac{\phi_2(\cdot)}{(L^*)^{n-1}}, \; \ldots, \frac{\phi_{n-1}(\cdot)}{(L^*)^2}, \; 0\right]^T.$$

a and A are defined by Lemma 1.
Let us define the function $V_\zeta = \zeta^T P\zeta$. Its time derivative along (20) is

$$\dot{V}_\zeta \leq -\frac{1}{L^*}\|\zeta\|^2 + 2\frac{1}{L^*}\zeta^T Pa\zeta_1 - 2\frac{1}{L}\zeta^T P\Gamma a\zeta_1 + 2\zeta^T P\Phi^*(\cdot).$$

Note that

$$|2\frac{1}{L^*}\zeta^T Pa\zeta_1| \leq \frac{1}{L^{*2}}\|\zeta\|^2 + \|Pa\|^2\,\zeta_1^2,$$

$$|2\frac{1}{L}\zeta^T P\Gamma a\zeta_1| \leq \frac{1}{L^{*2}}\|\zeta\|^2 + \|P\Gamma a\frac{L^*}{L}\|^2\,\zeta_1^2.$$

From Assumption 1 and the fact that $L^* \geq L(T_f) \geq L(t) \geq 1$, following the procedure of (10), we obtain

$$|2\zeta^T P\Phi^*(\cdot)| \leq \frac{C\mu_1}{L^{*2}}(\|\zeta(t-\tau)\|^2 + \|z(t-\tau)\|^2) + \frac{C\mu_1}{L^{*2}}(\|\zeta\|^2 + \|z\|^2).$$

Using the estimations above, we obtain that the time derivative of V_ζ can be bounded as

$$\begin{aligned}\dot{V}_\zeta \leq& -\frac{1}{L^{*2}}(L^* - C\mu_1 - 2)\|\zeta\|^2 + \frac{C\mu_1}{L^{*2}}\|z\|^2 \\ &+ (\|Pa\|^2 + \|P\Gamma a\frac{L^*}{L}\|^2)\zeta_1^2 \\ &+ \frac{C\mu_1}{L^{*2}}(\|\zeta(t-\tau)\|^2 + \|z(t-\tau)\|^2).\end{aligned} \tag{21}$$

Choose the Lyapunov-Krasovskii functional

$$V_1 = V_\zeta + \int_{t-\tau}^t \frac{C\mu_1}{L^{*2}}(\|\zeta(s)\|^2 + \|z(s)\|^2)\mathrm{d}s.$$

Then, we have

$$\begin{aligned}\dot{V}_1 \leq& -\frac{1}{L^{*2}}(L^* - 2C\mu_1 - 2)\|\zeta\|^2 + \frac{2C\mu_1}{L^{*2}}\|z\|^2 \\ &+ (\|Pa\|^2 + \|P\Gamma a\frac{L^*}{L}\|^2)\zeta_1^2 \\ \leq& -\frac{1}{L^{*2}}\|\zeta\|^2 + \mu_3\|z\|^2 + \mu_3\varepsilon_1^2,\end{aligned} \tag{22}$$

where μ_3 is a constant. Since L is bounded on $[0,T_f)$ and L^* is a constant, there exists a suitable constant μ_3 depending on the unknown parameter C such that $\mu_3 \geq \max\{\frac{2C\mu_1}{L^{*2}}, (\|Pa\|^2 + \|P\Gamma a\frac{L^*}{L}\|^2)\}$.
From (22), we have the following inequality for $\forall t \in [0,T_f)$

$$\lambda_{\min}(P)\|\zeta(t)\|^2 \leq V_1(0) + \mu_3\int_0^t \|z(t)\|^2\,dt + \mu_3\int_0^t \varepsilon_1^2(t)\mathrm{d}t, \tag{23}$$

and

$$\frac{1}{L^{*2}}\int_0^t \|\zeta(t)\|^2\,\mathrm{d}t \leq V_1(0) + \mu_3\int_0^t \|z(t)\|^2\,\mathrm{d}t + \mu_3\int_0^t \varepsilon_1^2(t)\mathrm{d}t. \tag{24}$$

It follows from (23) and (24) that the boundedness of $\int_0^t \varepsilon_1^2(t)\mathrm{d}t$ and $\int_0^t \|z(t)\|^2\,\mathrm{d}t$ on $[0, T_f)$ implies the boundedness of $\zeta(t)$, $\frac{1}{L^{*2}}\int_0^t \|\zeta(t)\|^2\,\mathrm{d}t$ and $\int_0^t \|ta(t)\|^2\,\mathrm{d}t$. Furthermore, by (19), (5) and L is bounded on $[0,T_f)$, we get the boundedness of ε and $\int_0^t \|\varepsilon(t)\|^2\,\mathrm{d}t$ on $[0,T_f)$. So far, we can conclude that $T_f = +\infty$, which follows again by a contradiction argument. Suppose $T_f < +\infty$, then T_f would be the finite escape time of the closed-loop systems, i.e., $\lim_{t\to T_f}\sup\|(L(t), \varepsilon^T(t), z^T(t))\| = +\infty$. This clearly contradicts to L, ε and z are bounded on the maximal interval $[0, T_f)$, and hence also bounded at $t = T_f$ due to the continuity of the solution trajectories. Then, L, ε and z are well defined and L is bounded on $[0,+\infty)$. Since L is bounded on $[0,+\infty)$ and $\dot{L} = \frac{\varepsilon_1^2}{L^2} \geq 0$, we have $\lim_{t\to+\infty} L(t) = \bar{L}$, then $\int_0^{+\infty} \frac{\varepsilon_1^2(t)}{L^2(t)}\mathrm{d}t$ is bounded. Accordingly, $\int_0^{+\infty} \varepsilon_1^2(t)\mathrm{d}t$ is bounded. From (18) and (17), it is apparent that $\int_0^{+\infty} \|z(t)\|^2\,\mathrm{d}t$ and z are bounded on $[0,+\infty)$. From (23), (24), (19) and (5), we can conclude that ζ, $\int_0^{+\infty} \|\zeta(t)\|^2\,\mathrm{d}t, \varepsilon$ and $\int_0^{+\infty} \|\varepsilon(t)\|^2\,\mathrm{d}t$ are bounded on $[0,+\infty)$. Then, L, ε and z are well defined and all bounded on $[0,+\infty)$. Furthermore, we can obtain $\varepsilon \in L_2$, $\dot{\varepsilon} \in L_\infty$ and $z \in L_2$, $\dot{z} \in L_\infty$. By the Barbalat's Lemma, we have $\lim_{t\to+\infty} z(t) = \lim_{t\to+\infty} \varepsilon(t) = 0$, which together with (5) results in $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \hat{x}(t) = 0$.

Remark 2: It is worth pointing out that Theorem 1 also holds when the delay τ is a bounded time-varying function $\tau(t)$ satisfying $0 < \dot{\tau}(t) \le d < 1$.

4. EXTENSIONS

In this section, we consider an extended nonlinear system with delay in the input

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + B_0 u(t-\tau_2) \\ &\quad + \phi(t, x, u(t), x(t-\tau_1), u(t-\tau_1)), \\ y(t) &= C_0 x(t), \end{aligned} \tag{25}$$

where A_0, B_0, C_0, u and $\phi(\cdot)$ are defined in the system (1). $0 \le \tau_i \le d$, $i = 1, 2$, are unknown constant time delays.

Lemma 2 [28]**:** For any constant $\tau > 0$ and continuous vector $\eta(t) \in R^n$, the following inequality holds

$$\int_{t-\tau}^{t} \eta^T(s)\mathrm{d}s \int_{t-\tau}^{t} \eta(s)\mathrm{d}s \le \tau \int_{t-\tau}^{t} \|\eta(s)\|^2 \,\mathrm{d}s. \tag{26}$$

Under Assumption 1, we can obtain a result similar to Theorem 1, which is described in the following theorem.

Theorem 2: Suppose that Assumption 1 holds, There exist two appropriate vectors $a = (a_1, \ldots, a_n)^T$, $b = (b_1, \ldots, b_n)^T$ such that the output feedback controller (4) globally regulates the uncertain nonlinear system (25).

Proof: We give the outline of the proof. Let

$$[\tilde{x}_1, \ldots, \tilde{x}_{n-1}, \tilde{x}_n]^T = \left[x_1, \ldots, x_{n-1}, x_n + \int_{t-\tau_2}^{t} u(s)\mathrm{d}s \right], \tag{27}$$

then the system (25) becomes

$$\begin{cases} \dot{\tilde{x}}(t) = A_0 \tilde{x}(t) + B_0 u(t) - B_1 \int_{t-\tau_2}^{t} u(s)\mathrm{d}s + \phi(\cdot), \\ y(t) = C_0 \tilde{x}(t), \end{cases} \tag{28}$$

where $\tilde{x} = [\tilde{x}_1, \ldots, \tilde{x}_{n-1}, \tilde{x}_n]^T$, $B_1 = [0, \ldots, 1, 0]^T \in \mathbb{R}^n$. Define

$$\varepsilon_i = \frac{\tilde{x}_i - \hat{x}_i}{L^{n-i+1}}, \quad z_i = \frac{\hat{x}_i}{L^{n-i+1}}, \quad i = 1, 2, \ldots, n. \tag{29}$$

From (28) and (4), a simple calculation gives

$$\begin{cases} \dot{\varepsilon} = \frac{1}{L} A\varepsilon - \frac{1}{L^2} B_1 \int_{t-\tau_2}^{t} u(s)\mathrm{d}s + \Phi(\cdot) - \frac{\dot{L}}{L} D\varepsilon, \\ \dot{z} = \frac{1}{L} Bz + \frac{1}{L} a\varepsilon_1 - \frac{\dot{L}}{L} Dz. \end{cases} \tag{30}$$

Let $V_\varepsilon = \varepsilon^T P \varepsilon$, $V_z = z^T Q z$. Note that $u = -b^T z$ and Lemma 2, then

$$\begin{aligned} &-2\varepsilon^T \frac{PB_1}{L^2} \int_{t-\tau_2}^{t} u(s)\mathrm{d}s \\ &\le 2\|\varepsilon\| \frac{\|P\|\|b\|}{L^2} \int_{t-\tau_2}^{t} \|z(s)\|\,\mathrm{d}s \\ &\le \frac{\mu_1}{L^2}\|\varepsilon\|^2 + \frac{\mu_1}{L^2}\left(\int_{t-\tau_2}^{t} \|z(s)\|\,\mathrm{d}s \right)^2 \\ &\le \frac{\mu_1}{L^2}\|\varepsilon\|^2 + \mu_1 \tau_2 \int_{t-\tau_2}^{t} \frac{\|z(s)\|^2}{L^2(s)}\mathrm{d}s, \end{aligned} \tag{31}$$

where μ_1 is defined by (10). Using Assumption 1, (10) and (27), we get

$$\begin{aligned} &|2\varepsilon^T P \Phi(\cdot)| \\ &\le 2C\|\varepsilon\|\|P\| \frac{1}{L^2} [\sum_{i=1}^{n} (|\frac{x_i}{L^{n-i+1}}| \\ &\quad + |\frac{x_i(t-\tau_1)}{L^{n-i+1}}|) + |u| + |u(t-\tau_1)|] \\ &\le 2C\|\varepsilon\|\|P\| \frac{1}{L^2} [\sum_{i=1}^{n} (|\varepsilon_i| + |z_i| + |\varepsilon_i(t-\tau_1)| \\ &\quad + |z_i(t-\tau_1)|) + |u| + |u(t-\tau_1)|] \\ &\quad + 2C\|\varepsilon\|\|P\| \frac{1}{L^3} [| \int_{t-\tau_2}^{t} u(s)\mathrm{d}s | \\ &\quad + | \int_{t-\tau_1-\tau_2}^{t-\tau_1} u(s)\mathrm{d}s |] \\ &\le \frac{C\mu_1}{L^2(t-\tau_1)} (\|\varepsilon(t-\tau_1)\|^2 + \|z(t-\tau_1)\|^2) \\ &\quad + \frac{C\mu_1}{L^2} (\|\varepsilon\|^2 + \|z\|^2) \\ &\quad + 2\frac{C\mu_1}{L^2} \|\varepsilon\| \int_{t-\tau_2}^{t} [\|z(s)\| + \|z(s-\tau_1)\|]\mathrm{d}s \\ &\le \frac{C\mu_1}{L^2(t-\tau_1)} (\|\varepsilon(t-\tau_1)\|^2 + \|z(t-\tau_1)\|^2) \\ &\quad + \frac{3C\mu_1}{L^2} (\|\varepsilon\|^2 + \|z\|^2) \\ &\quad + \frac{C\mu_1\tau_2}{L^2} \int_{t-\tau_2}^{t} [\|z(s)\|^2 + \|z(s-\tau_1)\|^2]\mathrm{d}s \\ &\le \frac{C\mu_1}{L^2(t-\tau_1)} (\|\varepsilon(t-\tau_1)\|^2 + \|z(t-\tau_1)\|^2) \\ &\quad + \frac{3C\mu_1}{L^2} (\|\varepsilon\|^2 + \|z\|^2) \\ &\quad + C\mu_1\tau_2 \int_{t-\tau_2}^{t} [\frac{\|z(s)\|^2}{L^2(s)} + \frac{\|z(s-\tau_1)\|^2}{L^2(s-\tau_1)}]\mathrm{d}s. \end{aligned} \tag{32}$$

Consider the Lyapunov-Krasovskii functional

$$\begin{aligned} V_1(t) &= C\mu_1\tau_2 \int_{-\tau_2}^{0} \int_{t+\theta}^{t} [\frac{\|z(s)\|^2}{L^2(s)} + \frac{\|z(s-\tau_1)\|^2}{L^2(s-\tau_1)}]\mathrm{d}s\mathrm{d}\theta \\ &\quad + V_\varepsilon + \mu_1\tau_2 \int_{-\tau_2}^{0} \int_{t+\theta}^{t} \frac{\|z(s)\|^2}{L^2(s)}\mathrm{d}s\mathrm{d}\theta, \end{aligned} \tag{33}$$

we have

$$\begin{aligned}\dot{V}_1(t) \le & -\frac{1}{L}\|\varepsilon\|^2 + \frac{3C\mu_1}{L^2}(\|\varepsilon\|^2 + \|z\|^2) \\ & + \frac{C\mu_1}{L^2(t-\tau_1)}(\|\varepsilon(t-\tau_1)\|^2 + \|z(t-\tau_1)\|^2) \\ & + C\mu_1\tau_2^2\left(\frac{\|z\|^2}{L^2} + \frac{\|z(t-\tau_1)\|^2}{L^2(t-\tau_1)}\right) \\ & + \frac{\mu_1}{L^2}\|\varepsilon\|^2 + \mu_1\tau_2^2\frac{\|z\|^2}{L^2} \\ \le & -\frac{1}{L}\|\varepsilon\|^2 + \frac{(3C+1)\mu_1(\tau_2^2+1)}{L^2}(\|\varepsilon\|^2 + \|z\|^2) \\ & + \frac{(3C+1)\mu_1(\tau_2^2+1)}{L^2(t-\tau_1)}(\|\varepsilon(t-\tau_1)\|^2 + \|z(t-\tau_1)\|^2),\end{aligned} \tag{34}$$

which is similar to (13). Then, the reminder of the proof is very similar to that of Theorem 1. We can conclude that $\lim_{t\to+\infty}\tilde{x}(t) = \lim_{t\to+\infty}\hat{x}(t) = \lim_{t\to+\infty}u(t) = 0$. Therefore, using (27), we have $\lim_{t\to+\infty}x(t) = \lim_{t\to+\infty}\hat{x}(t) = 0$.

5. SIMULATION EXAMPLES

Example 1: In [30], Jo *et al*. have shown that the nonlinear LLC resonant circuit system, through appropriate transformation, can be changed into the following system

$$\begin{cases}\dot{x}_1(t) = x_2(t) + c_0x_3(t), \\ \dot{x}_2(t) = x_3(t), \\ \dot{x}_3(t) = u(t), \\ y(t) = x_1(t).\end{cases} \tag{35}$$

Since time delay and the uncertainty are frequently encountered in a variety of practical systems, we consider the following three-order time-delay system

$$\begin{cases}\dot{x}_1(t) = x_2(t) + \dfrac{c_1x_1(t-\tau_1)[x_3(t-\tau_1)+u(t)]}{1+x_1^2(t-\tau_1)} + c_2x_3(t), \\ \dot{x}_2(t) = x_3(t) + \ln[1+c_3^2u^2(t-\tau_1)], \\ \dot{x}_3(t) = u(t), \\ y(t) = x_1(t),\end{cases} \tag{36}$$

where $c_i, i = 1,2,3,$ are unknown parameters. The unknown time-delay constants τ_1 satisfies $0 < \tau_1 \le 1$.

It is not difficult to prove that the uncertain time-delay system (36) satisfies Assumption 1. In fact, we have

$$\begin{aligned}|\phi_1(\cdot)| &= \left|\frac{c_1x_1(t-\tau_1)[x_3(t-\tau_1)+u(t)]}{1+x_1^2(t-\tau_1)} + c_2x_3(t)\right| \\ &\le c[|x_3(t)| + |x_3(t-\tau_1)| + |u(t)|], \\ |\phi_2(\cdot)| &= |\ln[1+c_3^2u^2(t-\tau_1)]| \le c|u(t-\tau_1)|,\end{aligned}$$

where $c = \max\{\frac{|c_1|}{2}, |c_2|, |c_3|\}$. Then, by Remark 1, we get that there exists an unknown constant $C > 0$ such that Assumption 1 holds.

According to Theorem 1, we construct the observer dynamics and the output feedback controller for (36)

$$\begin{cases}\dot{\hat{x}}_1 = \hat{x}_2 + \dfrac{3}{L}(y-\hat{x}_1), \\ \dot{\hat{x}}_2 = \hat{x}_3 + \dfrac{3}{L^2}(y-\hat{x}_1), \\ \dot{\hat{x}}_3 = u + \dfrac{1}{L^3}(y-\hat{x}_1), \\ u = -\left(\dfrac{\hat{x}_1}{L^3} + 3\dfrac{\hat{x}_2}{L^2} + 3\dfrac{\hat{x}_3}{L}\right), \\ \dot{L} = \dfrac{1}{L^2}\left(\dfrac{y-\hat{x}_1}{L^3}\right)^2, \text{ with } L(t) = 1, \text{ for } t\in[-1,0].\end{cases} \tag{37}$$

Picking $c_1 = c_2 = 1$, $c_3 = 2$ and $\tau_1 = 1$, the simulation results are shown in Fig. 1 for the closed-loop system consisting of (36) and (37). The initial condition is chosen as for $t\in[-1,0]$, $[x_1(t), x_2(t), x_3(t), \hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t), L(t)] = [7,2,-3,5,3,-1,1]$.

Example 2: Consider the following time-delay system

$$\begin{cases}\dot{x}_1(t) = x_2(t) + c_1\sqrt{\ln[1+x_2^4(t-\tau_1)]\ln[1+u^4(t)]}, \\ \dot{x}_2(t) = x_3(t) + c_2u(t-\tau_1), \\ \dot{x}_3(t) = u(t-\tau_2), \\ y(t) = x_1(t),\end{cases} \tag{38}$$

where $c_i \ne 0$, $i = 1,2,$ are unknown parameters. The unknown time-delay constants τ_j satisfy $0 < \tau_j \le 1$, $j = 1,2.$ Then, for any $s > 0$

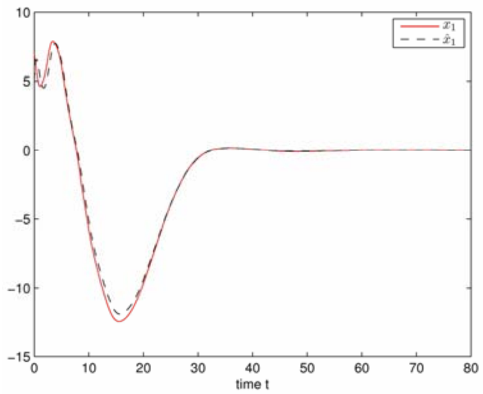
$$\begin{aligned}|\phi_1(\cdot)| &= |c_1\sqrt{\ln[1+x_2^4(t-\tau)]\ln[1+u^4(t)]}| \\ &\le 9|c_1|\sqrt{s|x_2(t-\tau)|\frac{|u|}{s}} \\ &\le \frac{9|c_1|}{2}\left[s|x_2(t-\tau)| + \frac{|u|}{s}\right].\end{aligned}$$

Accordingly, for $\forall s > 0$

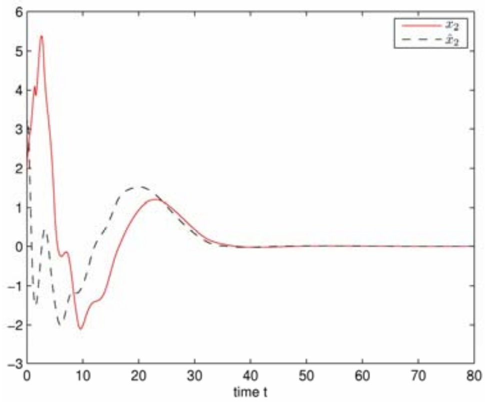
$$\begin{aligned}& s^3|\phi_1(\cdot)| + s^2|\phi_2(\cdot)| \\ & \le \frac{9|c_1|}{2}s^2[s^2|x_2(t-\tau)| + |u|] + |c_2|s^2|u(t-\tau)| \\ & \le Cs^2[s^2|x_2(t-\tau)| + |u| + |u(t-\tau)|] \\ & \le Cs^2\left[\sum_{i=1}^{3}s^{4-i}(|x_i| + |x_i(t-\tau)|)|u| + |u(t-\tau)|\right],\end{aligned}$$

where $C \ge \max\{\frac{9|c_1|}{2}, |c_2|\}$ is an unknown positive constant. Therefore, the system (38) satisfies Assumption 1. It is easy to see that (3) does not hold.

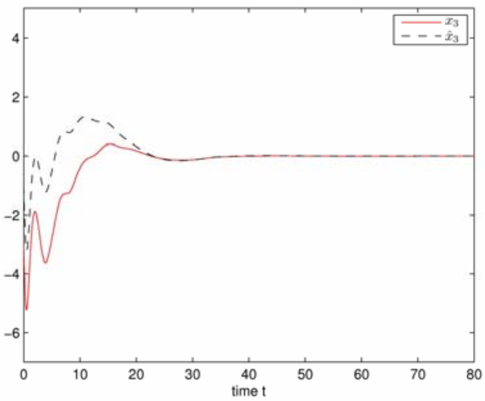
Based on Theorem 2, the output feedback controller is designed as



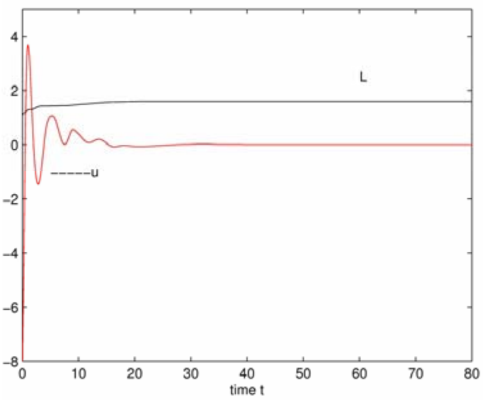
(a) Trajectories of x_1 and $\hat{x}_1$.



(b) Trajectories of x_2 and $\hat{x}_2$.

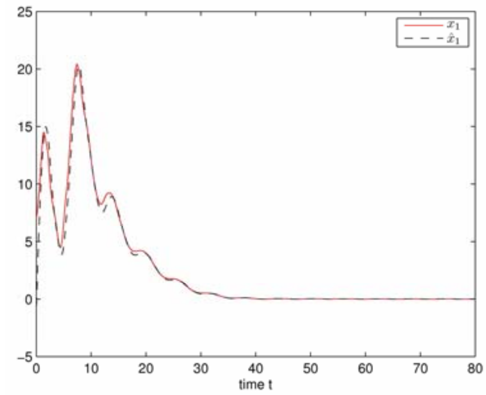


(c) Trajectories of x_3 and $\hat{x}_3$.

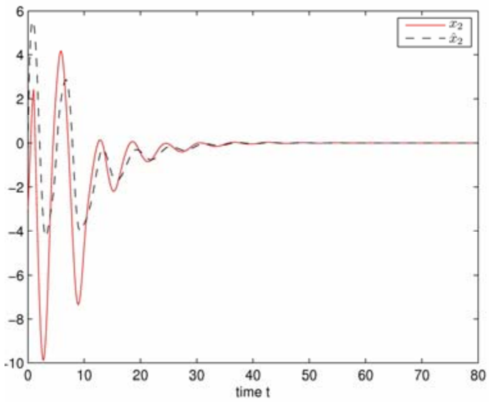


(d) The control input u and the observer's gain L.

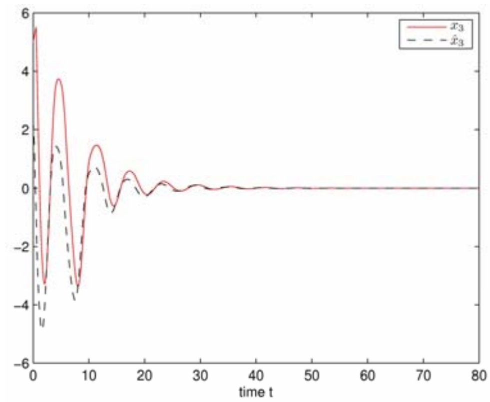
Fig. 1. Transient response of the closed-loop system consisting of (36) and (37).



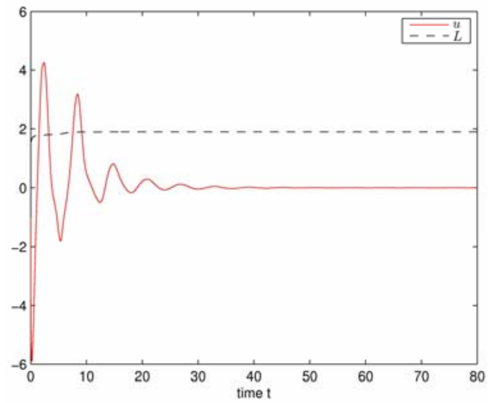
(a) Trajectories of x_1 and $\hat{x}_1$.



(b) Trajectories of x_2 and $\hat{x}_2$.



(c) Trajectories of x_3 and $\hat{x}_3$.



(d) The control input u and the observer's gain L.

Fig. 2. Transient response of the closed-loop system consisting of (38) and (39).

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \dfrac{4}{L}(y - \hat{x}_1), \\ \dot{\hat{x}}_2 = \hat{x}_3 + \dfrac{5}{L^2}(y - \hat{x}_1), \\ \dot{\hat{x}}_3 = u + \dfrac{2}{L^3}(y - \hat{x}_1), \\ u = -\left(2\dfrac{\hat{x}_1}{L^3} + 5\dfrac{\hat{x}_2}{L^2} + 4\dfrac{\hat{x}_3}{L}\right), \\ \dot{L} = \dfrac{1}{L^2}\left(\dfrac{y - \hat{x}_1}{L^3}\right)^2, \text{ with } L(t) = 1, \text{ for } t \in [-1, 0]. \end{cases} \tag{39}$$

Let $c_1 = c_2 = 1$ and $\tau_1 = 1$, $\tau_2 = 0.5$, the simulation results are shown in Fig. 2 for the closed-loop system consisting of (38) and (39). The initial condition is chosen as, for $t \in [-1, 0]$, $[x_1(t), x_2(t), x_3(t), \hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t), L(t)] = [7, -3, 5, -2, -1, 2, 1]$.

6. CONCLUSIONS

In this paper, we have investigated the problem of global states regulation by output feedback for a class of nonlinear time-delay systems whose nonlinearity satisfies certain growth condition. The uncertainty of unknown time delays has been compensated by the use of appropriate Lyapunov-Krasovskii functionals. By designing the dynamic gain observer and using the rescaling transformation of coordinates, a dynamic output feedback controller which has a linear-like structure can be constructed to achieve a global adaptive regulation of system. Simulation results have been provided to show the effectiveness of the proposed approach.

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