International Journal of Control, Automation, and Systems (2014) 12(6):1187-1196
DOI 10.1007/s12555-013-0461-6
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Over Networks H_∞ Filtering for Discrete Singular Jump Systems with Interval Time-varying Delay

Yongbo Lai* and Guoping Lu

Abstract: This paper deals with the robust H_∞ filtering for discrete singular systems with jump parameters and interval time-varying delay, whose system mode is transmitted through an unreliable network. The class of systems under consideration is more general and covers the singular jump systems with mode-dependent and mode-independent as two special cases, over networks a novel delay-dependent and partially mode-dependent filter is established via using a mode-dependent Lyapunov function and a finite sum inequality based on quadratic terms. The corresponding filter parameters can be obtained by solving a set of linear matrix inequalities without decomposing the original system matrix. The proposed linear robust filter ensuring that the filtering error singular jump system is to be regular, causal, stochastically stable and satisfies H_∞ performance. In addition, two numerical examples are given to illustrate the effectiveness of the proposed approach.

Keywords: Delay-dependent, discrete singular jump systems, H_∞ filtering, interval time-varying delay, linear matrix inequality (LMI), partially mode-dependent.

1. INTRODUCTION

As a special class of hybrid systems, Markovian jump systems (MJSs) have been attracting extensive research attention over the past decades due to their widely practical applications in manufacturing systems, power systems, aerospace systems and networked control systems. The control and filtering problems related to MJSs with or without time-delay have been fully investigated, see e.g., [1-8] and the references therein. Recently, many notions and results in state-space systems have been successfully extended to singular Markovian jump systems (SMJSs), such as stability and stabilization [9,10], H_∞ control [11-13], filtering problem researching [14-16] and so on. When Markovian jump parameters appear, it should be pointed out that the problem for SMJSs is much more complicated than that of state-space jump systems, because it requires to consider not only stability and modes switching, but also regularity and impulse elimination (for continuous-time singular systems) or causality (for discrete-time singular systems) simultaneously, while the latter two do not appear in regular ones.

Filtering is a class of important approaches to estimate the state information when the system plant is disturbed. Currently there are many approaches proposed for filter design, such as Kalman filtering [17,18], H_∞ filtering [19-21], $l_2 - l_\infty$ filtering [22-24] etc. In the H_∞ filtering, the input is supposed to be an energy signal and the corresponding energy-to-energy gain can be minimized. Recently, many results on the H_∞ mode-dependent and mode-independent filtering for MJSs have been presented in [25-35] and the references therein. However, these results on the filtering for the MJSs require critical assumption on the accessibility of the jumping mode and main classified to three types of stochastic filters, which the first type is mode-dependent filtering with completely known transition modes of systems [25,26]; the second type is mode-independent filtering design ignoring mode information in the filter construction [27,28]; the last one is with partially unknown transition probabilities of jump mode, such as researched in [30-32].

In practical applications, the aforementioned assumptions may sometimes be impossible to satisfy, such as the networked control systems (NCSs) [33,34,40-42], the introduction of communication networks in feedback control loops complicates the system analysis and synthesis, which also introduces new interesting and challenging problems. For the cases that the underlying NCSs is an MJS, both system state and mode are transmitted, when the system mode transmitted through networks suffers being lost and observed simultaneously, it is said that mode-dependent method is too ideal, whereas mode-independent algorithm is too absolute. Thus, both of the two extreme filter design methods are not suitable to the case where the system mode is available to a filter with some probabilities and time-varying delay through unreliable networks. Although in [32], a new filtering method was established for a class of MJSs, it should be pointed out that discrete-time SMJSs with time-varying delay are much more important

Manuscript received October 10, 2013; revised March 21, 2014; accepted April 20, 2014. Recommended by Associate Editor Shengyuan Xu under the direction of Editor Myotaeg Lim.

This paper was supported by the National Natural Science Foundation of China and Grant No.61174066, also supported by the Jiangsu Overseas Research & Training Program for University Prominent Young and Middle aged teachers & Presidents.

Yongbo Lai and Guoping Lu are with the Department of Electrical and Engineering, Jiangsu College of Information Technology, Wuxi 214153, China (e-mails: yongbo100@sina.com, lgp@jsit.edu.cn).
* Corresponding author.

than their continuous-time counterparts in our digital world. The problem of delay-dependent robust H_∞ filtering for discrete-time SMJSs with system mode available to a filter through unreliable networks has not been fully investigated. It is, therefore, the main purpose of the present research to shorten such a gap by making the first attempt to deal with the delay-dependent and partially mode-dependent H_∞ filtering problem of a class of discrete-time SMJSs over networks.

This paper is concerned with the delay-dependent and partially mode-dependent robust H_∞ filtering problem for a class of discrete-time SMJSs over networks. The considered systems are more general and cover the singular Markovian systems with completely known or complete- ly unknown transition modes as two special cases. First in contrast with the traditionally mode-dependent or mode-independent filtering method, the accessible probability of mode available to a filter is taken into consideration for the partially mode-dependent filter design with interval time-varying delay. Based on this, a new delay-dependent and partially mode dependent filter is established via using a mode-dependent Lyapunov function and a finite sum inequality based on quadratic terms. The suitable filter parameters which is solved by employing the LMIs technique and without decomposing the original system matrix. The desired filter which guarantees the admissibility and the H_∞ performance of the corresponding filtering error system. Last, two numerical examples are provided to illustrate the effectiveness of the developed theoretical results.

Notations: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is semi-positive definite(respectively, positive definite); $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all n×m real matrices, respectively; I is the identity matrix with appropriate dimension; the superscript T represents the transpose of a matrix; $\| X \|$ refers to Euclidean norm of the vector X, Z denotes the set of non-negative integer numbers; $\varepsilon\{\bullet\}$ denotes the mathematical expectation, $diag(\bullet)$ means block diagonal matrix, $He(M)$ stands for $M+M^T$, and * denotes the symmetric term in a symmetric matrix.

2. PROBLEM STATEMENTS AND PRELIMINARIES

In this paper, fix a probability space (Ω, F, P) and consider the following discrete SMJSs with interval time -varying delay

$$\begin{cases} Ex(k+1) = A(\theta(k))x(k) + A_d(\theta(k))x(k-\tau(k)) \\ \qquad\qquad +B(\theta(k))\omega(k), \\ y(k) = C(\theta(k))x(k) + C_d(\theta(k))x(k-\tau(k)) \\ \qquad\quad + D(\theta(k))\omega(k), \\ z(k) = L(\theta(k))x(k) + L_d(\theta(k))x(k-\tau(k)) \\ \qquad\quad + G(\theta(k))\omega(k), \\ x(k) = \phi(k), \qquad k = -\overline{\tau}, -\overline{\tau}+1, \ldots, 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is the system state, the matrix $E \in \mathbf{R}^{n\times n}$ may be singular, we shall assume that $rank(E) = r \leq n$, $y(k) \in \mathbf{R}^m$ is the measurement vector, $\omega(k) \in \mathbf{R}^q$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(k) \in \mathbf{R}^p$ is the signal to be estimated, $\phi(k)$ is a known initial condition, where $\{\theta(k), k \in Z\}$ is discrete Markov chains that takes values in $l = \{1, 2, \ldots, N\}$. Its transition probability matrix is $\Lambda = \{\lambda_{ij}\}$, which is defined $\lambda_{ij} = P(\theta(k+1) = j | \theta(k) = i)$ with $\lambda_{ij} \geq 0$, $\forall i, j \in l$ and $\sum_{j=1}^{N} \lambda_{ij} = 1$ for all $i \in l$. For each possible value of $\theta(k) = i \in l$, A_i, A_{di}, B_i, C_i, C_{di}, D_i, L_i, L_{di} and G_i are known constant matrices with appropriate dimensions. $\tau(k)$ is time-varying delay and satisfies

$$0 < \underline{\tau} \leq \tau(k) \leq \overline{\tau} < \infty, \tag{2}$$

where $\underline{\tau}$ and $\overline{\tau}$ are known positive integers.

The objective of this paper is to develop a novel partially mode-dependent filter over networks with state-space realization of the following form:

$$\begin{cases} x_f(k+1) = A_f x_f(k) + B_f y(k) + \alpha(k)(A_f(\theta(k))x_f(k) \\ \qquad\qquad + B_f(\theta(k))y(k)), \\ z_f(k) = C_f x_f(k) + D_f y(k) + \alpha(k)(C_f(\theta(k))x_f(k) \\ \qquad\quad + D_f(\theta(k))y(k)), \end{cases} \tag{3}$$

where $x_f(k) \in \mathbf{R}^n$ filter state, and $z_f \in \mathbf{R}^p$ is the estimation single, $A_f \in \mathbf{R}^{n\times n}$, $B_f \in \mathbf{R}^{n\times m}$, $C_f \in \mathbf{R}^{p\times n}$, $D_f \in \mathbf{R}^{p\times m}$, $A_f(\theta(k)) \in \mathbf{R}^{n\times n}$, $B_f(\theta(k)) \in \mathbf{R}^{n\times m}$, $C_f(\theta(k)) \in \mathbf{R}^{p\times n}$ and $D_f(\theta(k)) \in \mathbf{R}^{p\times m}$ are filter parameters to be determined.

Notice that filter (3) and system (1) have the same mode, $\theta(k)$ is assumed to be known in this paper. $\alpha(k)$ is an indicator function described as: if $\theta(k)$ transmitted successfully, then $\alpha(k)=1$, otherwise $\alpha(k)=0$. Assume $\alpha(k)$ is a Bernoulli distributed sequence with

$$\Pr\{\alpha(k)=1\} = \varepsilon\{\alpha(k)\} = \alpha,\ \Pr\{\alpha(k)=0\} = 1-\alpha, \tag{4}$$

where α is a constant $0 \leq \alpha \leq 1$. Furthermore, we have

$$\varepsilon\{\alpha(k)-\alpha\} = 0,\ \beta^2 := \varepsilon\{(\alpha(k)-\alpha)^2\} = \alpha(1-\alpha). \tag{5}$$

In addition, the two random processes $\alpha(k)$ and $\theta(k)$ are assumed to be independent. Define the augmented state vector $\overline{x}(k) = [x(k)^T \quad x_f(k)^T]^T$, and the estimation error $\overline{z}(k) = z(k) - z_f(k)$, for simplicity, let the mode at time k is i, that is $\theta(k) = i$, then the filtering error singular jump system is obtained as follows:

$$\begin{cases} \overline{E}\overline{x}(k+1) = \overline{A}_i\overline{x}(k) + \overline{A}_{di}K\overline{x}(k-\tau(k)) + \overline{B}_i\omega(k) \\ \qquad\qquad +\alpha(k)(\tilde{A}_i\overline{x}(k) + \tilde{A}_{di}K\overline{x}(k-\tau(k)) + \tilde{B}_i\omega(k)), \\ \overline{z}(k) = \overline{L}_i\overline{x}(k) + \overline{L}_{di}K\overline{x}(k-\tau(k)) + \overline{D}_i\omega(k) \\ \qquad\quad + \alpha(k)(\tilde{L}_i\overline{x}(k) + \tilde{L}_{di}K\overline{x}(k-\tau(k)) + \tilde{D}_i\omega(k)), \end{cases} \tag{6a}$$

$$\overline{x}(k) = [\phi(k)^T \quad 0]^T, \qquad k = -\overline{\tau}, -\overline{\tau}+1, \ldots, 0, \tag{6b}$$

where

$$\bar{E}=\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix},\quad \bar{A}_i=\begin{bmatrix} A_i & 0 \\ B_f C_i & A_f \end{bmatrix},\quad \bar{A}_{di}=\begin{bmatrix} A_{di} \\ B_f C_{di} \end{bmatrix},$$

$$\bar{B}_i=\begin{bmatrix} B_i \\ B_f D_i \end{bmatrix},\quad \tilde{A}_i=\begin{bmatrix} 0 & 0 \\ B_{fi} C_i & A_{fi} \end{bmatrix},\quad \tilde{A}_{di}=\begin{bmatrix} 0 \\ B_{fi} C_{di} \end{bmatrix},$$

$$\tilde{B}_i=\begin{bmatrix} 0 \\ B_{fi} D_i \end{bmatrix},\quad \bar{L}_i=\begin{bmatrix} L_i - D_f C_i & -C_f \end{bmatrix},$$

$$\bar{L}_{di}=\begin{bmatrix} L_{di} - D_f C_{di} & 0 \end{bmatrix},\quad \bar{D}_i = G_i - D_f D_i,$$

$$\tilde{D}_i=-D_{fi}D_i,\quad \tilde{L}_i=\begin{bmatrix} -D_{fi}C_i & -C_{fi} \end{bmatrix},$$

$$\tilde{L}_{di}=-D_{fi}C_{di},\quad K=[I \quad 0].$$

By rewriting (6), it is equivalent to

$$\begin{cases} \bar{E}\bar{x}(k+1)=\hat{A}_i\bar{x}(k)+\hat{A}_{di}K\bar{x}(k-\tau(k))+\hat{B}_i\omega(k) \\ \qquad +\hat{\beta}(\tilde{A}_i\bar{x}(k)+\tilde{A}_{di}K\bar{x}(k-\tau(k))+\tilde{B}_i\omega(k)), \\ \bar{z}(k)=\hat{L}_i\bar{x}(k)+\hat{L}_{di}K\bar{x}(k-\tau(k))+\hat{D}_i\omega(k) \\ \qquad +\hat{\beta}(\tilde{L}_i\bar{x}(k)+\tilde{L}_{di}K\bar{x}(k-\tau(k))+\tilde{D}_i\omega(k)), \end{cases} \tag{7}$$

where

$$\hat{A}_i=\bar{A}_i+\alpha\tilde{A}_i,\quad \hat{A}_{di}=\bar{A}_{di}+\alpha\tilde{A}_{di},\quad \hat{B}_i=\bar{B}_i+\alpha\tilde{B}_i,$$

$$\hat{L}_i=\bar{L}_i+\alpha\tilde{L}_i,\quad \hat{L}_{di}=\bar{L}_{di}+\alpha\tilde{L}_{di},\quad \hat{D}_i=\bar{D}_i+\alpha\tilde{D}_i,$$

$$\hat{\beta}=\alpha(k)-\alpha.$$

Remark 1: In system (7) a Bernoulli variable $\alpha(k)$ reflects the jam degree of network and denotes whether the current mode is accessible or not. In contrast to traditional filtering methods, the filter (3) is more advantageous in terms of the following two aspects. Firstly, compared with fully mode-dependent filter needing system mode obtained exactly all the time, filter (3) can bear the system mode lost in terms of some probabilities. Thus, we can measure or drop the mode signal with some probability, which could reduce the burden of data transmission. Secondly, different from fully mode-independent filter completely ignoring mode information in the filter construction, the probability of mode accessible to a filter is considered here. Because mode-independent filter design method is to find a common filter for all modes, the solvable solution set is smaller than the one generated by (3). That is, when the mode is accessible with some probability and there is no solution to a mode-independent filter, we may still get an effective filter of form (3) and minimum H_∞ performance, which can be provided from the Example 1.

We first introduce the following definitions for the filtering error system (7):

Definition 1 [16]**:** For all $i \in l$, the discrete singular system $\bar{E}\bar{x}(k+1)=\hat{A}_i\bar{x}(k)+\hat{A}_{di}K\bar{x}(k-\tau(k))$ is said to be

i) Regular if $\det(z\bar{E}-\hat{A}_i)$ is not identically zero, causal if $\deg(\det(z\bar{E}-\hat{A}_i))=rank(\bar{E})$.

ii) Stochastically admissible if it is regular, causal and stochastically stable.

Definition 2 [26]**:** The discrete singular filtering error system (6) with $\omega(k)=0$ is said to be stochastically stable, if for any $x_0\in\mathbf{R}^n$, there exists $\delta(x_0,\theta_0)>0$ and a scalar ρ, such that

$$\lim_{N\to\infty}\varepsilon\left\{\sum_{k=0}^{N}\|x\|^2\Big|x_0,\theta_0\right\}\le\rho\delta(x_0,\theta_0). \tag{8}$$

Definition 3 [26]**:** System (7) is said to be robustly mean square quadratic stability, if there exists a scalar $\gamma>0$, such that $\|\bar{z}(k)\|_2\le\gamma\|\omega(k)\|_2$, for any nonzero disturbance $\omega(k)\in L_2[0,\infty)$, where

$$\begin{aligned} \|\bar{z}(k)\|_2&=\sum_{k=0}^{\infty}\varepsilon\left\{\bar{z}^T(k)\bar{z}(k)\right\}, \\ \|\omega(k)\|_2&=\sum_{k=0}^{\infty}\varepsilon\left\{\omega^T(k)\omega(k)\right\}. \end{aligned} \tag{9}$$

The objective of this paper is to design a filter (3) such that the filtering error system (7) is stochastically stable and satisfies H_∞ performance.

3. MAIN RESULTS

In this section, we first present a performance analysis result for the filtering error system (7) and then give a representation of filter gains in terms of the feasible solutions to a set of LMIs.

For simplicity in the sequel, let the following

$$\xi(k):=\begin{bmatrix}\bar{x}(k)^T & \bar{x}(k-\tau(k))^T K^T & \omega(k)^T\end{bmatrix}^T,$$

$$\bar{y}(k):=\bar{x}(k+1)-\bar{x}(k),$$

then

$$\begin{cases} \bar{E}\bar{x}(k+1)=\begin{bmatrix}\hat{A}_i,\hat{A}_{di},\hat{B}_i\end{bmatrix}\xi(k)+\hat{\beta}\begin{bmatrix}\tilde{A}_i,\tilde{A}_{di},\tilde{B}_i\end{bmatrix}\xi(k) \\ \qquad :=(\Pi_{1i}+\hat{\beta}\bar{\Pi}_{1i})\xi(k), \\ K\bar{E}\bar{y}(k)=\begin{bmatrix}(A_i-E)K,A_{di},B_i\end{bmatrix}\xi(k):=\Pi_{2i}\xi(k), \\ \bar{z}(k)=\begin{bmatrix}\bar{L}_i,\bar{L}_{di},\bar{D}_i\end{bmatrix}\xi(k)+\hat{\beta}\begin{bmatrix}\tilde{L}_i,\tilde{L}_{di},\tilde{D}_i\end{bmatrix}\xi(k) \\ \qquad :=(\Pi_{3i}+\hat{\beta}\bar{\Pi}_{3i})\xi(k). \end{cases} \tag{10}$$

Before presenting our propose, we introduce the following lemma:

Lemma 1: For any $i\in l$, a positive definite matrix $S_i\in\mathbf{R}^{n\times n}$ and constant matrices $M_i=[M_{1i}\quad M_{2i}]\in\mathbf{R}^{n\times 2n}$, $M_{3i}\in\mathbf{R}^{n\times n}$, $W_i\in\mathbf{R}^{n\times q}$, and a positive integer time-varying delay $\tau(k)$ then

$$\begin{aligned} &-\sum_{j=k-\tau(k)}^{k-1}\bar{y}(j)^T\bar{E}^T K^T S_i K\bar{E}\bar{y}(j) \\ &\le\xi(k)^T\left\{\Pi_i+\bar{\tau}Y_i^T S_i^{-1}Y_i\right\}\xi(k), \end{aligned} \tag{11}$$

where

$$Y_i=[M_i \quad M_{3i} \quad W_i],$$

$$\Pi_i=\begin{bmatrix} He(M_i^T EK) & K^T E^T M_{3i}-M_i^T E & K^T E^T W_i \\ * & -M_{3i}^T E-E^T M_{3i} & -E^T W_i \\ * & * & 0 \end{bmatrix}.$$

Proof: When $i=1$, similar to the proof of Lemma 1 in [36], let $C=\begin{bmatrix} S^{1/2} & S^{1/2}Y \\ 0 & 0 \end{bmatrix}$, $C^TC=\begin{bmatrix} S & Y \\ Y^T & Y^TS^{-1}Y \end{bmatrix}\geq 0$, it follows that

$$\sum_{j=k-\tau(k)}^{k-1}\begin{bmatrix} K\bar{E}\bar{y}(j) \\ \xi(k) \end{bmatrix}^T\begin{bmatrix} S & Y \\ Y^T & Y^TS^{-1}Y \end{bmatrix}\begin{bmatrix} K\bar{E}\bar{y}(j) \\ \xi(k) \end{bmatrix}\geq 0. \quad (12)$$

Notice that

$$-\sum_{j=k-\tau(k)}^{k-1} 2\xi(k)^TY^TK\bar{E}\bar{y}(j)=2\xi(k)^TY^T[EK,-E,0]\xi(k) \quad (13)$$

rearranging (12) yields (11).

Obviously, when $i>1$, it is also true of (11).

Theorem 1: Given scalar $\gamma>0$, for each $i\in l$, if and only if there exist positive definite matrices P_i, S_i, Q and M_i, M_{3i}, W_i, Z_i such that

$$\Xi_i=\begin{bmatrix} \Xi_{1i} & \Xi_{2i} \\ * & \Xi_{3i} \end{bmatrix}<0, \quad (14)$$

then the discrete singular filtering error system (7) is stochastically stable, moreover, satisfies H_∞ performance γ norm, where

$$\Xi_{1i}=\begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} & \Gamma_{3i} \\ & \Gamma_{4i} & -E^TW_i \\ & & -\gamma^2I \end{bmatrix},\quad \hat{A}^T=K^T(A_i-E)^T,$$

$$\Xi_{2i}=\begin{bmatrix} \bar{\tau}M_i^T & \bar{\tau}\hat{A}^T & \tilde{L}_i^T & \beta\tilde{L}_i^T & \hat{A}_i^T & \beta\tilde{A}_i^T \\ \bar{\tau}M_{3i}^T & \bar{\tau}A_{di}^T & \tilde{L}_{di}^T & \beta\tilde{L}_{di}^T & \hat{A}_{di}^T & \beta\tilde{A}_{di}^T \\ \bar{\tau}W_i^T & \bar{\tau}B_i^T & \hat{G}_i^T & \beta\tilde{G}_i^T & \hat{B}_i^T & \beta\tilde{B}_i^T \end{bmatrix},$$

$$\Xi_{3i}=diag[-\bar{\tau}S_i,-\bar{\tau}S_i^{-1},-I,-I,-X_i^{-1},-X_i^{-1}],$$

$$\Gamma_{1i}=He(Z_i\bar{R}^T\hat{A}_i)+He(M_i^TEK)-\bar{E}P_i\bar{E}+\tau^*Q,$$

$$\Gamma_{2i}=Z_i\bar{R}^T\hat{A}_{di}-M_i^TE+K^TE^TM_{3i},$$

$$\Gamma_{3i}=Z_i\bar{R}^T\hat{B}_i+K^TE^TW_i,\quad \Gamma_{4i}=-He(M_{3i}^TE)-\bar{Q},$$

$$X_i=\sum_{j=1}^N\lambda_{ij}P_j,\quad \tau^*=\bar{\tau}-\underline{\tau}+1,\quad Q=diag(\bar{Q},\bar{Q}),$$

$\bar{R}\in\mathbf{R}^{(n+n)\times(n+n)}$ is any matrix with full column satisfying $\bar{E}^T\bar{R}=0$ with $rank(\bar{R})=2n-r$.

Proof: Under the given condition, we first show that system (7) with $\omega(k)=0$ is regular and causal. Since rank $\bar{E}=n+r$, we choose two nonsingular matrices $\hat{M},\hat{N}\in\mathbf{R}^{2n\times 2n}$ such that

$$\bar{E}=\hat{M}\begin{bmatrix} I_{n+r} & 0 \\ 0 & 0 \end{bmatrix}\hat{N},\quad \hat{A}_i=\hat{M}\begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix}\hat{N}, \quad (15)$$

then $\bar{R}$ can be given as

$$\bar{R}=\hat{M}^{-T}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\hat{H}, \quad (16)$$

where $\hat{H}\in\mathbf{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix. Write

$$\hat{N}^{-T}Z_i\hat{H}^T=\begin{bmatrix} Z_{1i} \\ Z_{2i} \end{bmatrix},\quad \hat{M}^TP_i\hat{M}=\begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}. \quad (17)$$

From inequality (14), it can be obtained that

$$\begin{bmatrix} \Gamma_{1i} & \hat{A}_i \\ * & -X_i^{-1} \end{bmatrix}<0 \quad (18)$$

by Schur complement (18), and $\tau^*Q>0$, then

$$\hat{A}_i^TX_i\hat{A}_i+He(Z_i\bar{R}^T\hat{A}_i)-\bar{E}P_i\bar{E}<0, \quad (19)$$

where the partition is compatible with that of $\hat{A}_i$ in (15). Now, substituting (15), (16) and (17) into (19) gives

$$\hat{N}^T\begin{bmatrix} \# & \# \\ \# & \hat{W} \end{bmatrix}\hat{N}<0, \quad (20)$$

where # represents matrices that are not relevant in the following discussion, and

$$\begin{aligned}\hat{W}=&A_{2i}^T\hat{P}_{1i}A_{2i}+A_{4i}^T\hat{P}_{2i}^TA_{2i}+A_{2i}^T\hat{P}_{2i}A_{4i}+A_{4i}^T\hat{P}_{3i}A_{4i}\\&+Z_{2i}A_{4i}+A_{4i}^TZ_{2i}^T,\end{aligned} \quad (21)$$

where $\hat{P}_{li}=\sum_{j=1}^N\lambda_{ij}P_{lj}$, $(l=1,2,3)$.

From (20), it is easy to see $\hat{W}<0$, which implies A_{4i} is nonsingular for any $i\in l$, thus the pair $(\bar{E},\hat{A}_i)$ is regular and causal.

According to Definition 1, filtering error system (7) with $\omega(k)=0$ is regular and causal.

Next we will show that system (7) is stochastically stable, construct the following Lyapunov function as

$$V(k,\theta(k))=V_1(k)+V_2(k)+V_3(k)+V_4(k), \quad (22)$$

where

$$V_1(k)=\bar{x}(k)^T\bar{E}P_i\bar{E}\bar{x}(k),$$

$$V_2(k)=\sum_{\theta=-\bar{\tau}+1}^{0}\sum_{j=k-1+\theta}^{k-1}\bar{y}(j)^T\bar{E}^TK^TS_iK\bar{E}\bar{y}(j),$$

$$V_3(k)=\sum_{i=k-\tau(k)}^{k-1}\bar{x}(i)^TQ\bar{x}(i),$$

$$V_4(k)=\sum_{j=-\bar{\tau}+2}^{-\underline{\tau}+1}\sum_{l=k+j-1}^{k-1}\bar{x}(l)^TQ\bar{x}(l),$$

taking the forward difference of $\Delta V(k)$ along the trajectory of system (7) and taking the mathematical expectation, yields

$$\begin{aligned}\varepsilon\Delta V_1(k)&=\varepsilon\{V(\bar{x}(k+1),\theta(k+1))\}-V(\bar{x}(k),\theta(k))\\&=\varepsilon\left\{\xi^T(k+1)\bar{E}^TX_i\bar{E}\xi(k+1)\right\}\\&\quad-\xi^T(k)\bar{E}^TP_i\bar{E}\xi(k)\\&=\xi^T(k)\{(\Pi_{1i}+\beta\bar{\Pi}_{1i})^TX_i(\Pi_{1i}+\beta\bar{\Pi}_{1i}),\end{aligned} \quad (23)$$

$$\begin{aligned}\varepsilon\Delta V_2(k)&=\bar{\tau}\bar{y}(k)^T\bar{E}^TK^TS_iK\bar{E}\bar{y}(k)\\&\quad-\sum_{j=k-\bar{\tau}}^{k-1}\bar{y}(j)^T\bar{E}^TK^TS_iK\bar{E}\bar{y}(j)\\&\leq\bar{\tau}\xi(k)^T\Pi_{2i}^TS_i\Pi_{2i}\xi(k)\\&\quad-\sum_{j=k-\tau(k)}^{k-1}\bar{y}(j)^T\bar{E}^TK^TS_iK\bar{E}\bar{y}(j),\end{aligned}$$

by Lemma 1 to obtain

$$\varepsilon\Delta V_2(k) \le \xi(k)^T\left\{(\Pi_i + \bar{\tau}Y_i^T S_i^{-1} Y_i) + \Pi_{2i}^T S_i \Pi_{2i}\right\}\xi(k), \tag{24}$$

$$\begin{aligned}\varepsilon\Delta V_3(k) &\le \bar{x}(i)^T Q\bar{x}(i) - \bar{x}(k-\tau(k))^T \bar{Q}\bar{x}(k-\tau(k)) \\ &\quad + \sum\nolimits_{i=k-\bar{\tau}+1}^{k-\underline{\tau}} \bar{x}(i)^T Q\bar{x}(i), \\ \varepsilon\Delta V_4(k) &\le \tau^* \bar{x}(i)^T Q\bar{x}(i) - \sum\nolimits_{i=k-\bar{\tau}+1}^{k-\underline{\tau}} \bar{x}(i)^T Q\bar{x}(i), \\ \varepsilon\Delta V_3(k) + \varepsilon\Delta V_4(k) &\le \tau^* \bar{x}(i)^T Q\bar{x}(i) \\ &\quad - \bar{x}(k-\tau(k))^T K^T \bar{Q} K\bar{x}(k-\tau(k)) \\ &= \xi(k)^T\left\{diag(\tau^* Q, \bar{Q}, 0)\right\}\xi(k).\end{aligned} \tag{25}$$

Noting $\bar{E}^T\bar{R} = 0$ and $\bar{y}(k) := \bar{x}(k+1) - \bar{x}(k)$, the following equation holds

$$\begin{cases} 2\bar{x}(k+1)^T \bar{E}^T \bar{R} Z_i^T \bar{x}(k) = 0, \\ 0 = \bar{x}(k+1)^T \bar{E}^T \bar{R}^T P_i \bar{R}\bar{E}\bar{x}(k+1) = -(\Pi_{1i} \\ \quad + \beta\bar{\Pi}_{1i})^T \bar{R}^T X_i \bar{R}(\Pi_{1i} + \beta\bar{\Pi}_{1i}). \end{cases} \tag{26}$$

Combining (23)-(26) one obtains

$$\begin{aligned}&\varepsilon\Delta V(k) - \gamma^2\omega(k)^T\omega(k) \\ &\le \xi(k)^T\left\{(\Pi_{1i} + \beta\bar{\Pi}_{1i})^T X_i (\Pi_{1i} + \beta\bar{\Pi}_{1i})\right. \\ &\quad - diag(\bar{E}^T P_i \bar{E}, 0, 0) + (\Pi_i + \bar{\tau}Y_i^T S_i^{-1} Y_i) + \Pi_{2i}^T S_i \Pi_{2i} \\ &\quad \left. - diag(\tau^* Q, \bar{Q}, 0) - diag(0, 0, \lambda^2 I)\right\}\xi(k) \\ &= \xi^T(k)\Xi_i\xi(k),\end{aligned} \tag{27}$$

where

$$\Xi_i = \begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} & \bar{\tau}M_i^T & \bar{\tau}\hat{A}^T & \hat{A}_i^T & \beta\tilde{A}_i^T \\ & \Gamma_{4i} & \bar{\tau}M_{3i}^T & \bar{\tau}A_{di}^T & \hat{A}_{di}^T & \beta\tilde{A}_{di}^T \\ & & -\bar{\tau}S_i & 0 & 0 & 0 \\ & & & -\bar{\tau}S_i^{-1} & 0 & 0 \\ & & & & -X_i^{-1} & 0 \\ & & & & & -X_i^{-1} \end{bmatrix}. \tag{28}$$

By Schur complement (14), it can be obtained $\Xi_i < 0$, considering (27) with $\omega(k) = 0$, yields

$$\begin{aligned}\varepsilon\Delta V(k) &= \varepsilon\left\{V((k+1), \theta(k+1))\right\} - \varepsilon\left\{V(0, \theta(0))\right\} \\ &\le -\lambda_{\min}(-\Xi_i)\sum\nolimits_{k=0}^{v}\varepsilon\left\{\xi^T(k)\xi(k)\right\} < 0.\end{aligned} \tag{29}$$

Let $\rho = \lambda_{\min}(-\Xi_i)$, then $\rho > 0$, so

$$\sum\nolimits_{k=0}^{v}\varepsilon\left\{\xi^{\mathrm{T}}(k)\xi(k)\right\} \le \frac{1}{\rho}\varepsilon\left\{V(\xi(0), \theta(0))\right\},$$

let $v \to \infty$, then

$$\begin{aligned}&\sum\nolimits_{k=0}^{\infty}\varepsilon\left\{x^{\mathrm{T}}(k)x(k)\middle|\phi(0), \theta(0)\right\} \\ &\le \frac{1}{\rho}\varepsilon\left\{V(\xi(0), \theta(0))\right\} < \infty.\end{aligned} \tag{30}$$

By Definition 2, one obtains the system (7) is stochastically stable with $\omega(k) = 0$.

Last, to prove that the system (7) satisfies H_∞ performance. For any nonzero $\omega(k) \in L_2[0, \infty)$ and zero initial condition, define

$$J = \sum\nolimits_{k=0}^{v}\varepsilon\left\{z^T(k)z(k) - \gamma^2\omega^T(k)\omega(k)\right\},\ v \in Z, \tag{31}$$

as $\bar{z}(k)^T\bar{z}(k) = \xi(k)^T(\Pi_{3i} + \beta\bar{\Pi}_{3i})^T(\Pi_{3i} + \beta\bar{\Pi}_{3i})\xi(k)$, then one obtains

$$\begin{aligned}J &= \varepsilon\sum\nolimits_{k=0}^{v}\left\{\Delta V(\xi(k), \theta(k)) + z^T(k)z(k)\right. \\ &\quad \left.-\gamma^2\omega^T(k)\omega(k)\right\} - \varepsilon\left\{\sum\nolimits_{k=0}^{v}\Delta V(\xi(k), \theta(k))\right\} \\ &\le \xi(k)^T\left\{\Xi_i + \left[(\Pi_{3i}, 0, 0, 0) + \beta(\bar{\Pi}_{3i}, 0, 0, 0)\right]^T\right. \\ &\quad \left.\times\left[(\Pi_{3i}, 0, 0, 0) + \beta(\bar{\Pi}_{3i}, 0, 0, 0)\right]\right\}\xi(k) \\ &\le \varepsilon\sum\nolimits_{k=0}^{v}\left\{\xi^T(k)\hat{\Xi}_i\xi(k)\right\},\end{aligned} \tag{32}$$

where

$$\begin{aligned}\hat{\Xi}_i &= \begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} & \Gamma_{3i} \\ & \Gamma_{4i} & -E^T W_i \\ & & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \hat{A}_i^T \\ \hat{A}_{di}^T \\ \hat{B}_i^T \end{bmatrix} X_i \begin{bmatrix} \hat{A}_i^T \\ \hat{A}_{di}^T \\ \hat{B}_i^T \end{bmatrix}^T \\ &\quad + \beta^2\begin{bmatrix} \tilde{A}_i^T \\ \tilde{A}_{di}^T \\ \tilde{B}_i^T \end{bmatrix} X_i \begin{bmatrix} \tilde{A}_i^T \\ \tilde{A}_{di}^T \\ \tilde{B}_i^T \end{bmatrix}^T \\ &\quad + \begin{bmatrix} \hat{L}_i^T \\ \hat{L}_{di}^T \\ \hat{G}_i^T \end{bmatrix}\begin{bmatrix} \hat{L}_i^T \\ \hat{L}_{di}^T \\ \hat{G}_i^T \end{bmatrix}^T + \beta^2\begin{bmatrix} \tilde{L}_i^T \\ \tilde{L}_{di}^T \\ \tilde{G}_i^T \end{bmatrix}\begin{bmatrix} \tilde{L}_i^T \\ \tilde{L}_{di}^T \\ \tilde{G}_i^T \end{bmatrix}^T.\end{aligned}$$

By Schur complement (14), it is easy to show that $\hat{\Xi}_i < 0$, so

$$\lim_{v\to\infty}\mathcal{E}\sum\nolimits_{k=0}^{v}\left\{\xi^T(k)\hat{\Xi}_i\xi(k)\right\} < 0, \quad \forall i \in l. \tag{33}$$

That is, $\sum\nolimits_{k=0}^{\infty}\varepsilon\{z^T(k)z(k)\} < \gamma^2\sum\nolimits_{k=0}^{\infty}\varepsilon\{\omega^T(k)\omega(k)\}$ is true, which completes the proof.

Remark 2: It should be pointed out that Theorem 1 established a sufficient existent condition of delay-dependent and partially mode-dependent filter (3) for system (1). Compared with fully mode-dependent filter, such as those of [25,26,41], where the system modes need to be known exactly, and is thus invalid when the jump mode $\theta(k)$ is not available to the filter.

Based on Theorem1, we are now ready to deal with the H_∞ filter design problem for discrete SMJSs with interval time-varying delay (1), and a sufficient condition for the existence of a suitable filter is presented as follows.

Theorem 2: For a prescribed scalar $\gamma > 0$, the filtering error system (7) is stochastically stable with H_∞ performance γ, for each $i \in l$, if there exist matrices M_{1i}, $\bar{M}_{2i}$, M_{3i}, W_i, S_i, $\bar{Z}_i$, Q, H_1, F_1, F_2, X_1, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{D}_f$, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{C}_{fi}$, $\bar{D}_{fi}$ and $\begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} > 0$, such that the inequalities (34) are feasible

$$\tilde{\Xi}_i = \begin{bmatrix} \Omega_{1i} & \Omega_{2i} & \Omega_{3i} & \Omega_{4i} & \Omega_{5i} \\ * & \Omega_{6i} & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & \Omega_{7i} & 0 \\ * & * & * & * & \Omega_{8i} \end{bmatrix} < 0, \tag{34}$$

where

$$\Omega_{1i} = \begin{bmatrix} \bar{\Omega}_{1i} & E^T \bar{M}_{2i} - E^T P_{2i} & \bar{\Omega}_{2i} & \bar{\Omega}_{3i} \\ * & -P_{3i} + \tau^* Q & -\bar{M}_{2i}^T & * \\ * & * & \bar{\Omega}_{4i} & -E^T W_i \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Omega_{2i} = \begin{bmatrix} \bar{\tau} M_{1i}^T & \bar{\tau} \hat{A}^T H_1^T & L_i^T - C_i^T \bar{D}_f^T - \alpha C_i^T \bar{D}_{fi}^T \\ \bar{\tau} M_{2i}^T & 0 & -\bar{C}_f^T - \alpha \bar{C}_{fi}^T \\ \bar{\tau} M_{3i}^T & \bar{\tau} A_{di}^T H_1^T & L_{di}^T - C_{di}^T \bar{D}_f^T - \alpha C_{di}^T \bar{D}_{fi}^T \\ \bar{\tau} W_i^T & \bar{\tau} B_i^T H_1^T & G_i^T - D_i^T \bar{D}_f^T - \alpha D_i^T \bar{D}_{fi}^T \end{bmatrix},$$

$$\Omega_{4i} = \begin{bmatrix} A_i^T X_1^T + C_i^T \bar{B}_f^T & A_i^T F_1^T + C_i^T \bar{B}_f^T \\ \bar{A}_f^T & \bar{A}_f^T \\ A_{di}^T X_1^T + C_{di}^T \bar{B}_f^T & A_{di}^T F_1^T + C_{di}^T \bar{B}_f^T \\ B_i^T X_1^T + D_i^T \bar{B}_f^T & B_i^T F_1^T + D_i^T \bar{B}_f^T \end{bmatrix} + \alpha \begin{bmatrix} A_i^T X_1^T + C_i^T \bar{B}_{fi}^T & A_i^T F_1^T + C_i^T \bar{B}_{fi}^T \\ \bar{A}_{fi}^T & \bar{A}_{fi}^T \\ A_{di}^T X_1^T + C_{di}^T \bar{B}_{fi}^T & A_{di}^T F_1^T + C_{di}^T \bar{B}_{fi}^T \\ B_i^T X_1^T + D_i^T \bar{B}_{fi}^T & B_i^T F_1^T + D_i^T \bar{B}_{fi}^T \end{bmatrix},$$

$$\Omega_{5i} = \beta \begin{bmatrix} 0 & C_i^T \bar{B}_{fi}^T \\ 0 & \bar{A}_{fi}^T \\ 0 & C_{di}^T \bar{B}_{fi}^T \\ 0 & D_i^T \bar{B}_{fi}^T \end{bmatrix},$$

$$\Omega_{6i} = diag\left\{-\bar{\tau} S_i, \bar{\tau}(S_i - H_1 - H_1^T), -I\right\},$$

$$\Omega_{7i} = \Omega_{8i} = \begin{pmatrix} X_{1i} - X_1 - X_1^T & X_{2i} - F_1^T - F_2 \\ * & X_{3i} - F_2 - F_2^T \end{pmatrix},$$

$$\bar{\Omega}_{1i} = He(A_i^T R Z_i^T) + He(M_{1i}^T E) - E^T P_{1i} E + \tau^* \bar{Q},$$

$$\bar{\Omega}_{2i} = Z_i R^T A_{di} - M_{1i}^T E - E^T M_{3i},$$

$$\bar{\Omega}_{3i} = Z_i R^T B_i + E^T W_i, \quad \bar{\Omega}_{4i} = -He(M_{3i}^T E) - \bar{Q},$$

$$X_{li} = \sum_{j=1}^N \lambda_{ij} P_{lj} (l = 1,2,3),$$

$R \in \mathbf{R}^{n\times(n-r)}$ is any matrix with full column satisfying $E^T R = 0$.

Then the discrete SMJSs (1) has a desired filter (3) can be chosen with parameters as

$$\begin{aligned} &A_f = \bar{A}_f F_2^{-1}, \ B_f = \bar{B}_f, \ C_f = \bar{C}_f F_2^{-1}, \ D_f = \bar{D}_f, \\ &A_{fi} = \bar{A}_{fi} F_2^{-1}, \ B_{fi} = \bar{B}_{fi}, \ C_{fi} = \bar{C}_{fi} F_2^{-1}, \ D_{fi} = \bar{D}_{fi}. \end{aligned} \tag{35}$$

Proof: Similar to the method in [36], by introducing the slack variable $H_i = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}$, an equivalent form of (14) is given in the following

$$\hat{\Xi}_i = \begin{bmatrix} \hat{\Xi}_{1i} & \hat{\Xi}_{2i} \\ * & \hat{\Xi}_{3i} \end{bmatrix} < 0, \tag{36}$$

where

$$\hat{\Xi}_{1i} = \Xi_{1i}, \quad \hat{\Xi}_{2i} = \Xi_{2i}[I, H_1, I, I, H_2, H_2]^T,$$

$$\hat{\Xi}_{3i} = diag[-\bar{\tau} S_i, \bar{\tau}(S_i - H_1^T - H_1), -I, -I, \bar{X}_i, \bar{X}_i],$$

$$\bar{X}_i = X_i - H_2^T - H_2.$$

Inequality (36) shows that $H_1^T + H_1 > 0$, then $X_4 + X_4^T > 0$, which implies that X_4 is invertible, define

$$J = \begin{bmatrix} I & 0 \\ 0 & X_2 X_4^{-1} \end{bmatrix}, \ \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} = J P_i J^T, \ \bar{R} = \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{Z}_i = \begin{bmatrix} Z_i & I \\ 0 & I \end{bmatrix}, \ F_1 = X_2 X_4^{-1} X_3, \ F_2 = X_2 X_4^{-T} X_2^T,$$

$$T = diag\{J, I, I, I, I, I, J, J\}.$$

Pre- and post-multiplying the left hand side of (36) by T and T^T, respectively, and let

$$\bar{A}_f = X_2 A_f X_4^{-T} X_2^T, \ \bar{B}_f = X_2 B_f, \ \bar{C}_f = C_f X_4^{-T} X_2^T,$$

$$\bar{D}_f = D_f, \ \bar{A}_{fi} = X_2 A_{fi} X_4^{-T} X_2^T, \ \bar{B}_{fi} = X_2 B_{fi},$$

$$\bar{C}_{fi} = C_{fi} X_4^{-T} X_2^T, \ \bar{D}_{fi} = D_{fi}.$$

We can find (36) is equivalent to (34). Therefore, the partially mode-dependent filtering error singular jump system (7) is regular, causal, and stochastically stable with H_∞ norm bound γ. The desired filter can be obtained from (34). This completes the proof.

Remark 3: It is worth noting that by Theorem 2, a delay-dependent and partially mode-dependent H_∞ filter design method for singular jump systems with time-varying delay is proposed, all solutions including filter variables can be directly obtained from the LMI condition (33) without decomposing the original system matrix, and thus making the design procedure reliable. The filter existing condition depends on the upper bound as well as the lower bound of the time varying delay. Moreover, it can be seen that the mode accessible probability α is involved, compared with the results in [26,28,29] for singular jump systems, our delay-dependent and partially mode-dependent result is much more desirable and applicable than that of those.

Remark 4: To get the minimum H_∞ norm bound γ, LMI condition (34) is changed to the optimization problem as follows:

Minimize γ,
subjects to LMI (34) and $\gamma_{\min} = \sqrt{\gamma}$. (37)

Remark 5: In (3), if $\alpha(k)=0$, it reduces to a general delay-dependent and mode-independent filter designing for discrete-time singular jump system with time-varying delay, we have the following corollary based on the Theorem 2.

Corollary 1: For the discrete SMJSs (1), the filtering error system

$$\begin{cases}\bar{E}\bar{x}(k+1) = \bar{A}_i\bar{x}(k) + \bar{A}_{di}K\bar{x}(k-\tau(k)) + \bar{B}_i\omega(k)\\ \bar{z}(k) = \bar{L}_i\bar{x}(k) + \bar{L}_{di}K\bar{x}(k-\tau(k)) + \bar{D}_i\omega(k),\end{cases} \tag{38}$$

is stochastically admissible, moreover, satisfies H_∞ performance γ norm. For each $i \in l$, if there exist matrices M_{1i}, $\bar{M}_{2i}$, M_{3i}, W_i, S_i, $\bar{Z}_i$, Q, H_1, F_1, F_2, X_1, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{D}_f$, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{C}_{fi}$, $\bar{D}_{fi}$ and $\begin{bmatrix} P_{1i} & P_{2i}\\ * & P_{3i}\end{bmatrix} > 0$, such that

$$\hat{\Xi}_i = \begin{bmatrix}\Gamma_{11i} & \Gamma_{12i} & \Gamma_{13i}\\ & \Gamma_{14i} & 0\\ & & \Gamma_{15i}\end{bmatrix} < 0, \tag{39}$$

where

$$\Gamma_{11i} = \Omega_{1i},\quad \Gamma_{15i} = \Omega_{7i} = \Omega_{8i},$$

$$\Gamma_{14i} = diag\left\{-\bar{\tau}S_i, \bar{\tau}(S_i - H_1 - H_1^T), -I\right\},$$

$$\Gamma_{12i} = \begin{bmatrix}\bar{\tau}M_{1i}^T & \bar{\tau}K^T(A_i - E)^T H_1^T & L_i^T - C_i^T\bar{D}_f^T\\ \bar{\tau}M_{2i}^T & 0 & -\bar{C}_f^T\\ \bar{\tau}M_{3i}^T & \bar{\tau}A_{di}^T H_1^T & L_{di}^T - C_{di}^T\bar{D}_f^T\\ \bar{\tau}W_i^T & \bar{\tau}B_i^T H_1^T & G_i^T - D_i^T\bar{D}_f^T\end{bmatrix},$$

$$\Gamma_{13i} = \begin{bmatrix}A_i^T X_1^T + C_i^T\bar{B}_f^T & A_i^T F_1^T + C_i^T\bar{B}_f^T\\ \bar{A}_f^T & \bar{A}_f^T\\ A_{di}^T X_1^T + C_{di}^T\bar{B}_f^T & A_{di}^T F_1^T + C_{di}^T\bar{B}_f^T\\ B_i^T X_1^T + D_i^T\bar{B}_f^T & B_i^T F_1^T + D_i^T\bar{B}_f^T\end{bmatrix}.$$

Then a desired filter

$$\begin{cases}x_f(k+1) = A_f x_f(k) + B_f y(k),\\ z_f(k) = C_f x_f(k) + D_f y(k),\end{cases} \tag{40}$$

can be chosen with parameters as defined in (35), which was researched in [16]. Furthermore, when $E = I$ and $I = 1$, it reduces to the results of [36] which is a special case of this corollary.

Remark 6: If $A_f = B_f = C_f = D_f = 0$ in (3), the output of measurement and the estimated signal fully transmitted under without data loss, that is $\alpha(k)=1$, then a desired delay-dependent and mode-dependent filter designing in Corollary 1 becomes

$$\begin{cases}x_f(k+1) = A_f(\theta(k))x_f(k) + B_f(\theta(k))y(k),\\ z_f(k) = C_f(\theta(k))x_f(k) + D_f(\theta(k))y(k),\end{cases} \tag{41}$$

which was researched in [25,29].

4. NUMERICAL EXAMPLES

Example 1: For system (1) parameters borrowed from the Example 2 in [30], let $R = diag\{0,5\}$, given time varying delay $\tau(k) \in [1,4]$, $[1,5]$ and $[2,6]$ respectively, and by solving (36) the minimum allowed γ are obtained for the transition probabilities completely known case in Table 4 of [30]. The corresponding computation results are listed in Table 1 for different α and time-varying delay.

It shows that the lower value of α results the larger value of $\gamma_{\min}$ under the same transition matrix and time varying delay.

Remark 7: A key feature of this paper is that singular jump, time-varying delay and noise perturbations are all considered, moreover, with the probability of mode accessible to a filter over networks, we may still get effective minimum H_∞ performance. The filter design methods in [7,31,32,35,39] cannot be applied to this example, as there exists singular system matrices.

Example 2: Considering the system (1) with the parameters borrowed from the Example in [29], the same selecting $R = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$, the transition probability matrix $\Pi = \begin{bmatrix}0.7 & 0.3\\ 0.2 & 0.8\end{bmatrix}$, and time-varying delay is assumed to be $1 \le \tau(k) \le 3$.

Assume $\gamma = 2.6$, with $\alpha = 1$, 0.5 and 0.1 respectively, using Matlab LMI control Tool box to solve the LMIs in (20), we have the filter of form (3) with parameters as:

Case one: $\alpha = 0$

$$A_f = \begin{bmatrix}-0.1945 & 0.0393\\ -0.6114 & 0.1225\end{bmatrix},\quad B_f = \begin{bmatrix}-1.0214\\ -0.5311\end{bmatrix},$$

$$C_f = \begin{bmatrix}0.1389 & -0.0270\end{bmatrix},\quad D_f = 1.3991,$$

$$A_{f1} = \begin{bmatrix}0.8375 & -0.1697\\ -0.2025 & 0.0389\end{bmatrix},\quad B_{f1} = \begin{bmatrix}0.2238\\ 0.1126\end{bmatrix},$$

$$C_{f1} = \begin{bmatrix}-0.1095 & 0.0218\end{bmatrix},\quad D_{f1} = 0.8476.$$

Case two: $\alpha = 1$

$$A_f = \begin{bmatrix}-0.2716 & 0.0450\\ -0.8558 & 0.1754\end{bmatrix},\quad B_f = \begin{bmatrix}-1.4297\\ -0.7434\end{bmatrix},$$

Table 1. The minimun H_∞ performance $\gamma_{\min}$ with different α and time-varying delay.

$\gamma_{\min}$ \ α	0	0.5	1
$\tau(k) \in [1,4]$	1.985	1.223	0.697
$\tau(k) \in [1,5]$	2.212	1.402	0.785
$\tau(k) \in [2,6]$	2.736	1.779	0.839

$$C_f = \begin{bmatrix} 0.1931 & -0.0376 \end{bmatrix}, \quad D_f = 1.958,$$

$$A_{f1} = \begin{bmatrix} 0.2173 & -0.0360 \\ -0.6836 & 0.1403 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 1.1436 \\ 0.5947 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} -0.1545 & 0.0301 \end{bmatrix}, \quad D_{f1} = 1.5664,$$

$$A_{f2} = \begin{bmatrix} 0.1738 & -0.0288 \\ -0.5469 & 0.1122 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.9149 \\ 0.4758 \end{bmatrix},$$

$$C_{f2} = \begin{bmatrix} -0.1236 & 0.0241 \end{bmatrix}, \quad D_{f2} = 1.2531.$$

Case three: $\alpha = 0.5$

$$A_f = \begin{bmatrix} -0.1358 & 0.0275 \\ -0.4279 & 0.0877 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.7150 \\ -0.3717 \end{bmatrix},$$

$$C_f = \begin{bmatrix} 0.0966 & -0.0189 \end{bmatrix}, \quad D_f = 0.9793,$$

$$A_{f1} = \begin{bmatrix} 0.2513 & -0.0509 \\ -0.0608 & 0.0116 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.2145 \\ 0.0111 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} -0.0288 & 0.0056 \end{bmatrix}, \quad D_{f1} = 0.2937,$$

$$A_{f2} = \begin{bmatrix} 0.2010 & -0.0407 \\ -0.0486 & 0.0093 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.1716 \\ 0.0890 \end{bmatrix},$$

$$C_{f2} = \begin{bmatrix} -0.023 & 0.0045 \end{bmatrix}, \quad D_{f2} = 0.2350.$$

the same initial condition is $\phi(k) = [1 \quad -1/6]^T$ and the noise signal is $\omega(k) = 0.5e^{-0.3k}$ with different α the trajectory of the filtering error responses are shown in Fig. 1.

Remark 8: As discussed in Remarks 1-6, the criteria obtained in [7,16,25-29,31,32,35,39,41] fails in Examples 1 and 2. when there is a jam in network that the system mode cannot be observed totally, accessed with some probabilities, a partially mode-dependent filter is less conservative due to consideration of the distributed property of system mode, compared with the previous results, which are more realistic in the application.

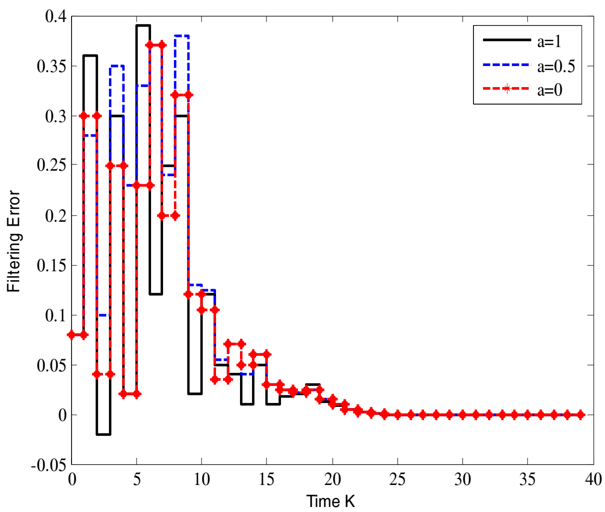

Fig. 1. The trajectory of filtering error.

5. CONCLUSION

In this paper, the problem of robust H_∞ filter design has been investigated for discrete-time singular Markovian jump systems over networks. Attention has been focused on using switched Lyapunov function to characterize sufficient conditions for the solvability of the switched singular filtering problem in terms of LMIs under the mode accessible probability α is involved. Based on the obtained analysis result, a novel H_∞ filter has been designed, which guarantees the filtering error system to be regular, causal, and stochastically stable with a given H_∞ performance γ, the delay-dependent and partially mode-dependent result is much more desirable and applicable than that of past results. Lastly, numerical examples are used to illustrate the benefit and applicability of the developed results.

REFERENCES

[1] M. S. Mahmoud and P. Shi, "Robust control for Markovian jump linear discrete-time systems with unknown nonlinearities," *IEEE Trans. on Automatic Control*, vol. 49, no. 4, pp. 538-542, 2002.

[2] J. Xiong, J. Lam, H. J. Gao, and D. W. C. Ho, "On robust stabilization of Markovian jump systems with uncertain switching probabilities," *Automatica*, vol. 41, pp. 897-903, 2005.

[3] E. K. Boukas and Z. K. Liu, "Robust stability and H_∞ control of discrete-time Jump linear systems with time delay: an LMI approach," *Proc. of the 39th IEEE conf. on decision and control*, USA, pp. 1527-1532, 2000.

[4] H. P. Liu, D. W. C. Ho, and F. C. Sun, "Design of H_∞ filtering for Markovian jump systems with non-accessible mode information," *Automatica*, vol. 44, no. 10, pp. 2655-2660, 2008.

[5] L. Zhang, Y. Shi, C. Wang, and H. Gao, "Robust H_∞ filtering for switched linear discrete-time systems with polytopic uncertainties," *International Journal of Adaptive Control and Signal Processing* vol. 20, pp. 291-304, 2006.

[6] P. Shi, M. Mahmoud, and S. Nguang, "Robust filtering for jumping systems with mode-dependent delays," *Signal Processing*, vol. 86, pp. 140-152, 2006.

[7] L. Zhang and E. K. Boukas, "Mode-dependent H_∞ filtering for discrete time Markovian jump linear systems with partly unknown transition probabilities," *Automatic*, vol. 45, no. 7, pp. 1462-1467, 2009.

[8] D. Du, B. Jiang, P. Shi, and S. Zhou, "H_∞ filtering of discrete-time switched systems with state-delays via switched Lyapunov function approach," *IEEE Trans. Automatica Control*, vol. 52, no. 8, pp. 1520-1525, 2007.

[9] Y. Q. Xia, J. H. Zhang, and E. K. Boukas, "Control for discrete singular hybrid systems," *Automatic*, vol. 44, no. 10, pp. 2635-2641, 2008.

[10] S. Ma, E. K. Boukas, and Y. Chinniah, "Stability and stabilization of discrete-time singular Markov

jump systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 12, pp. 531-543, 2010.

[11] I. Masubuchi, Y. Kamitane, A. Ohara, and N. Suda, "H_∞ control for descriptor systems: a matrix inequalities approach," *Automatica*, vol. 33, no. 4, pp. 669-673, 1997.

[12] X. Ji, H. Su, and J. Chu, "Robust state feedback H_∞ control for uncertain linear discrete singular systems," *IET Control Theory and Applications* vol. 1, no. 6, pp. 195-200, 2007.

[13] J. Lam, Z. Shu, S. Y. Xu, and E. K. Boukas, "Robust H_∞ control of descriptor discrete-time Markovain jump systems," *International Journal of Control*, vol. 83, no. 3, pp. 374-385, 2007.

[14] A. Germani, C. Manes, and P. Palumbo, "Filtering of switching systems via a singular minimax approach," *Proc. of the 41st IEEE Conf. on Decision and Control*, Las Vegas, Nevada USA, December, pp. 1223-1228, 2002.

[15] C. E. de Souza, K. A. Barbosaa, and M. Y. Fu, "Robust filtering for uncertain linear discrete-time descriptor systems," *Automatica*, vol. 44, no. 9, pp. 792-798, 2008.

[16] S. Y. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer-Verlag, Berlin, Germany, 2006.

[17] M. S. Mahmoud, P. Shi, and A. Ismail, "Robust Kalman filtering for discrete-time Markovian jump systems with parameter uncertainty," *J. Comput. Appl. Math.*, vol. 169, no. 1, pp. 53-69, 2004.

[18] P. Shi, E. K. Boukas, and R. K. Agarwal, "Kalman filtering for continuous-time uncertain systems with Markovian jumping parameters," *IEEE Trans. on Automatic Control,* vol. 44, no. 8, pp. 1592-1597, 2009.

[19] S. Xu and J. Lam, "Reduced-order H_∞ filtering for singular systems," *Systems and Control Letters*, vol. 56, no. 1, pp. 48-57, 2007.

[20] J. H. Kim, "Delay-dependent robust H_∞ filtering for uncertain discrete-time singular systems with interval time-varying delay," *Automatica*, vol. 46, no. 5, pp. 591-597, 2010.

[21] D. Yue and Q. L. Han, "Robust H_∞ filter design of uncertain descriptor systems with discrete and distributed delays," *IEEE Trans. on signal processing*, vol. 52, no. 7, pp. 3200-3212, 2014.

[22] Y. Zhou and J. Li, "Reduced-order l_2-l_∞ filtering for singular systems: a linear matrix inequality approach," *IET Control Theory and Applications*, vol. 2, no. 8, pp. 228-238, 2007.

[23] Z. G. Wu, P. Shi, H. Y. Su, and J. Chu, "l_2-l_∞ filter design for discrete-time singular Markovian jump systems with time-varying delays," *Information Sciences*, vol. 181, no. 9, pp. 5534-5547, 2011.

[24] M. S. Mahmoud and Y. Xia, "Design of reduced-order l_2-l_∞ filter design for singular discrete-time systems using strict linear matrix inequalities," *IET Control Theory and Applications*, vol. 4, no. 3, pp. 509-519, 2010.

[25] S. Ma and E. K. Boukas, "Robust H_∞ filtering for uncertain discrete Markov jump singular systems with mode-dependent time delay," *IET Control Theory and Applications*, vol. 3, no. 3, pp. 351-361, 2009.

[26] E. K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Springer-Verlag, Berlin, 2008.

[27] C. E. DeSouza, A. Trofino, and K. A. Barbosa, "Mode-independent H_∞ filters for Markovian jump linear systems," *IEEE Trans. Automatic Control*, vol. 51, no. 6, pp. 1837-1841, 2006.

[28] Z. G. Wu, H. Y. Su, and J. Chu, "H_∞ filtering for singular Markovian jump systems with time delay," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 8, pp. 939-957, 2009.

[29] D. Zhang, L. Yu, Q. G. Wang, and C. J. Wong, "Exponential H_∞ filtering for discrete-time jump singular systems with time-varying delays," *Journal of Franklin Institute*, vol. 349, no. 12, pp. 2323-2342, 2012.

[30] J. X. Lin, S. M. Fei, and J. Shen, "Delay-dependent H_∞ filtering for discrete-time singular Markovian jump systems with time-varying delay and partially unknown transition probabilities," *Signal Processing* vol. 91, pp. 277-289, 2011.

[31] Y. L. Wei, J. B. Qiu, H. R. Karimi, and M. Wang, "A new design of H_∞ filtering for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information," *Signal Processing*, vol. 93, pp. 2392-2407, 2013.

[32] G. L. Wang, Q. L. Zhang, and V. Sreeram, "Partially mode-dependent H_∞ filtering for discrete-time Markovian jump systems with partly unknown transition probabilities," *Signal Processing*, vol. 90, pp. 548-556, 2012.

[33] K. Mathiyalagana, J. H. Park, R. Sakthivel, and S. Marshal Anthoni, "Robust mixed H_∞ and passive filtering for networked Markovian jump systems with impulses," *Signal Processing*, vol. 101, pp. 162-173, 2014.

[34] M. X. Liu, X. T. Liu, Y. Shi, and S. Q. Wang, "T-S fuzzy mode-based H_2 and H_∞ filtering for networked control systems with two-channel Markovian random delays," *Digital Signal Processing*, vol. 27, no. 4, pp. 167-174, 2014.

[35] Y. Q. Zhang, G. F. Cheng, and C. X. Liu, "Finite-time unbiased H_∞ filtering for discrete jump time-delay systems," *Applied Mathematical Modelling*, vol. 38, no. 13, pp. 3339-3349, 2014.

[36] X. M. Zhang and Q. L. Han, "Delay-dependent Robust H_∞ filtering for uncertain discrete-time systems with time-varying delay based on a finite sum inequality," *IEEE Trans. on Circuits and Systems—II: Express Briefs*, vol. 53, no. 12, pp. 1466-1470, 2006.

[37] J. L. Liang, F. B. Sun, and X. H. Liu, "Finite-horizon H_∞ filtering for time-varying delay systems with randomly varying nonlinearities and sensor saturations," *System Science & Control Engineer-*

ing: An Open Access Journal, vol. 2, no. 5, pp. 108-118, 2014.

[38] Z. Gu, S. M. Fei, D. Yue, and E. G. Tian, "H_∞ filtering for discrete-time systems subject to stochastic missing measurements: a decomposition approach," *International Journal of Systems Science*, DOI: 10.1080/00207721.2013.871372, 2014.

[39] L. Y. Chung, C. H. Lien, K. W. Yu, and J. D. Chen, "Robust H_∞ filtering for discrete switched systems with interval time-varying delay," *Signal Processing*, vol. 94, pp. 661-669, 2014.

[40] D. Zhang, W. J Cai, and Q. G Wang, "Energy-efficient H_∞ filtering for networked systems with stochastic signal transmissions," *Signal Processing*, vol. 101, pp. 134-141, 2014.

[41] L. Li, F. Li, and Z. X. Zhang, "On mode-dependent H_∞ filtering for network-based discrete time systems," *Signal Processing*, vol. 93, pp. 634-640, 2013.

[42] Z. D. Wang, H. L. Dong, and B. Shen, "Finite-horizon H_∞ filtering with missing measurements and quantization effects," *IEEE Trans. on Automatic Control*, vol. 58, no. 7, pp. 1707-1718, 2013.

Yongbo Lai received his M.S. degree from the School of Internet of Things Engineering, Jiangnan University, China, in 2007. He is currently a lecturer at the Jiangsu College of Information Technology, Jiangsu, China. His current research interests include robust control and networked control.

Guoping Lu received his B.S. degree from the Department of Applied Mathematics, Chengdu University of Science and Technology, China in 1984, and his M.S. and Ph.D. degrees from the Department of Mathematics, East China Normal University, China, in 1989 and 1998, respectively. He is currently a Professor at the Jiangsu College of Information Technology, Jiangsu, China. His current research interests include nonlinear control, robust control and networked control.