Disturbance Observer based Backstepping for Position Control of Electro-hydraulic Systems

Daehee Won and Wonhee Kim*

Abstract: We propose a disturbance observer (DOB) based backstepping control which improves the position tracking performance in the presence of both friction and load force in an electro-hydraulic systems. The DOB is designed to estimate the disturbance including friction and load force, while avoiding amplification of the measurement noise. We use an auxiliary state variable to avoid the use of the derivative of the measured signal. This results in the avoidance of the amplification of the measurement noise. For position tracking with compensation of disturbances, a backstepping controller is design. The backstepping controller guarantees the ultimate boundedness of the tracking error in the presence of both friction and load force. The closed-loop stability is proven using Lyapunov's theory.

Keywords: Backstepping control, disturbance observer, electro-hydraulic systems, position control.

1. INTRODUCTION

Various feedback control methods have been developed to improve the tracking performance of the position or force of the Electro-hydraulic systems (EHSs). In one study [1], the local linearization of the nonlinear dynamics was investigated, but the global stability of that method was not proven. Variable structure control (VSC) methods were proposed for the control of the EHS in other works [2,3]. Chattering in the VSC control signal can result in excitement of high-frequency modes. In order to compensate for the global nonlinearities of EHSs, input-output linearization has been proposed [4], [5,6]. However, its control input signal can often have a high amplitude due to cancellation of the nonlinear terms. Because the EHS dynamics is in the form of the strict feedback, a backstepping control method can easily be applied to the EHS [7,8]. Thus, various backsteppingbased control designs using the dynamics properties of an EHS have been proposed [9-11]. All these methods improve the position tracking performance, however, they do not consider their disturbances, i.e., the friction and/or the load torque. Although the output feedback control was obtained in one study [13], zero friction and zero load torque were assumed. However, when the disturbance significantly affects the position tracking performance, the position tracking performance can

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degrade. Therefore, a means of compensating for disturbances is needed in order to improve EHS performance levels.

Generally, because it is difficult directly to measure this type of a disturbance, adaptation or estimation algorithms are required to estimate them. In order to estimate a disturbance, several disturbance estimation methods for an EHSs have been proposed [6,12,13]. However, because previous observers have used the derivative of the measurement signal, which contains measurement noise, the system can become unstable due to the amplification of the noise.

In this paper, we proposed a DOB based backstepping control method to improve the position tracking performance in the presence of both friction and load force. The proposed method consists of a DOB and a backstepping controller. The DOB is designed to estimate the disturbances, which include friction and load force, while avoiding amplification of the measurement noise. We use an auxiliary state variable to avoid the use of the derivative of the measured signal. Doing so results in the avoidance of the amplification of the measurement noise. For position tracking with compensation of disturbances, the backstepping controller is designed. The backstepping controller guarantees the ultimate boundedness of the tracking error in the presence of both friction and load force. The performance of the proposed method is validated via simulations and experiments.

2. MATHEMATICAL MODEL OF AN ELECTRO-HYDRAULIC SYSTEM

In many EHS applications, for simplicity the valve dynamics can be approximated [4-6] as

$$
x_{\nu} = k_{\nu} i,\tag{1}
$$

where x_v is the spool position of the servo-valve [m], *i* is the input current of the torque motor [mA], and k_v is the torque motor gain [m/mA]. The control flow equation of

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the hydraulic valve for the load flow rate can be written as

$$
Q_L = C_d w x_v \sqrt{\frac{1}{\rho} (P_s - \text{sgn}(x_v) P_L)},
$$
\n(2)

where Q_L is the load flow rate $[m^3/s]$, C_d is the discharge coefficient, w is the area gradient of the servo-valve spool [m], P_s is the supply pressure of the pump [N/m²], P_L is the differential pressure between P_A and P_B [N/m²], *a μ* is the density of the hydraulic oil [kg/m³]. By applying the law of continuity to each actuator chamber, the load flow rate continuity equation is given by (3) [1], $Q_t = A_x \dot{x}_t + C_d P_t + \frac{V_t}{r} \dot{P}_t$. (3) applying the law of continuity to each actuator chamber, the load flow rate continuity equation is given by (3) [1],

$$
Q_L = A_p \dot{x}_p + C_{tl} P_L + \frac{V_t}{4\beta_e} \dot{P}_L,
$$
\n(3)

where x_p is the piston position [m], A_p is the pressure where x_p is the piston position [m], A_p is the pressure area of the piston [m²], $C_{il} = C_{il} + C_{el}/2$ is the total leakage coefficient $\left[\text{m}^{5}/\text{Ns}\right]$, C_{il} is the internal leakage coefficient $[m^5/Ns]$, C_{el} is the external leakage coefficient [m⁵/Ns], V_t is the total actuator volume [m³], and β_e is the effective bulk modulus of the system $[N/m^2]$. Combining the control flow rate equation (2) and the load flow rate continuity equation (3), the fluid dynamics of the actuator is given by control flow ray
tinuity equation
tator is given by
 $\dot{\phi} = \frac{4\beta_e A_p}{r}$

$$
\dot{P}_L = -\frac{4\beta_e A_p}{V_t} \dot{x}_p - \frac{4\beta_e C_{tl}}{V_t} P_L
$$
\n
$$
+ \frac{4\beta_e C_d w}{V_t \sqrt{\rho}} \sqrt{(P_s - \text{sgn}(x_v) P_L)} x_v.
$$
\n(4)

Finally, by applying Newton's second law, the actuator's force balance equation is given by
 $m\ddot{x} = -k\dot{x} - b\dot{x} + A P_x - d$ $V_t \sqrt{\rho}$ V^{(\cdot s</sub> \sim example, $V_t \sqrt{\rho}$ and $V_t \sqrt{\rho}$ and V_t and V_t and V_t is a position is given by $m\ddot{x}_p = -kx_p - b\dot{x}_p + A_p P_L - d$, (5)} -
-
.t

$$
m\ddot{x}_p = -kx_p - b\dot{x}_p + A_p P_L - d,\t\t(5)
$$

where *m* is the mass of the piston [kg], k is the load spring constant $[N/m]$, *b* is the viscous damping coefficient $[N/(m/s)]$, and where the disturbance d includes both the unknown load force F_L and the friction F_F [N]. The goal is to make the piston position track the desired position with disturbance compensation.

Combining $(1)-(5)$ the dynamics of the EHS can be reformulated as the following state space representation:

$$
\dot{x}_1 = x_2,\n\dot{x}_2 = -\frac{kx_1 + bx_2}{m} + \frac{A_p}{m} x_3 - \frac{d}{m},\n\dot{x}_3 = -\frac{4\beta_e A_p}{V_t} x_2 - \frac{4\beta_e C_{tl}}{V_t} x_3\n+ \frac{4\beta_e C_d w k_v}{V_t \sqrt{\rho}} \sqrt{P_s - \text{sgn}(u) x_3} u,\ny = x_1.
$$
\n(6)

Here, $x = [x_1, x_2, x_3]^T$ represents the states, x_1 is the position of the piston $[m]$, x_2 is the velocity of the piston

[m/s], x_3 is the pressure difference between chambers A and B [N/m], $u = i$ is the current input i [mA], and y is the output. We assume that all of the states are [m/s], x_3 is the pressure difference between chambers A and B [N/m], $u = i$ is the current input *i* [mA], and *y* is the output. We assume that all of the states are measurable. In practice, $\sqrt{P_s - \text{sgn}(u)x_3}$ is sel as $|x_3|$ is seldom close to P_s . In the rare case that d B [N/m], $u = i$ is the current input i [mA], and
is the output. We assume that all of the states are
easurable. In practice, $\sqrt{P_s - \text{sgn}(u)x_3}$ is seldom zero,
 $|x_3|$ is seldom close to P_s . In the rare case that
 P positive number [11].

3. DISTURBANCE OBSERVER BASED BACKSTEPPING CONTROL

3.1. Disturbance observer design

In this subsection, we design the disturbance observer to estimate the disturbance d in (6). The dynamics (6) can be written as follows: 21. Disturbance observer designt

21. Disturbance disturbance diversions the disturbance diversion is

21. Disturbance diversion in the written as follows:

21. $d = -m\dot{x}_2 - kx_1 - bx_2 + A_p x_3$

$$
d = -m\dot{x}_2 - kx_1 - bx_2 + A_p x_3. \tag{7}
$$

We define the estimations of the disturbance \hat{d} . The estimation error is defined as $d = -m\dot{x}_2$

∴ define t

imation er
 $\tilde{d} = d - \hat{d}$.

timation error is defined as
\n
$$
\tilde{d} = d - \hat{d}.
$$
\n(8)
\nThe dynamics of \hat{d} is designed as

$$
\tilde{d} = d - \hat{d}.
$$
\n(8)
\nThe dynamics of \hat{d} is designed as
\n
$$
\dot{\hat{d}} = -l_o \left(m\dot{x}_2 + kx_1 + bx_2 - A_p x_3 + \hat{d} \right),
$$
\n(9)
\nere l_o is the observer gain.
\n**Assumption 1:** The disturbance, *d* is bounded, and its

where l_o is the observer gain.

Assumption 1: The disturbance, d is bounded, and its $d = -l_o (m\dot{x}_2 + kx_1 + bx_2 - A_px_3 + d),$
where l_o is the observer gain.
Assumption 1: The disturbance, *d* is bounded derivative is also bounded such that $|\dot{d}| \le \dot{d}_{\text{max}}$.

The disturbance d consists of the load force and the friction. The friction is not actually differentiable at zero velocity due to the Coulomb friction effect. Fortunately, the actual friction cannot have infinite jumps. Thus, the Coulomb friction of the friction model can be approximated as a smooth function (for example $tanh(x_2)$) instead of a discontinuous function (for example sgn(x_2)). Thus, Assumption 1 is physically reasonable [14].

To suppress the bounded derivatives of the disturbance, high gain is required. Essentially, measurement noise Thus, Assumption 1 is physically reasonable [14].
To suppress the bounded derivatives of the disturbance,
high gain is required. Essentially, measurement noise
appears in the sensors. The dynamics of \hat{d} (9) uses the derivative of the state. If high observer gain is used, the noise is amplified by the high gain. Thus, the observer is not practical to implement. To avoid this problem, we use the auxiliary variables, as represented by ξ [15]. We can then ensure the uniformly ultimate boundedness for ξ through the following Theorem 1.

Theorem 1: Given the auxiliary variables, ξ such that

$$
\xi = -\hat{d} - l_o m x_2,\tag{10}
$$

the dynamics of the auxiliary variable is

Theorem 1: Given the auxiliary variables,
$$
\zeta
$$
 such that
\n
$$
\xi = -\hat{d} - l_o m x_2,
$$
\n(10)
\nthe dynamics of the auxiliary variable is
\n
$$
\dot{\xi} = -l_o \left(\xi + l_o m x_2 \right) + l_o \left(k x_1 + b x_2 - A_p x_3 \right).
$$
\n(11)
\nThen, $|\tilde{d}| \le e^{-l_o t} \cdot |\tilde{d}(0)| + 1/l_o \rho(t)$ for envelope function

the dynamics of the auxiliar
 $\dot{\xi} = -l_o(\xi + l_o m x_2) + l_o(i)$

Then, $|\tilde{d}| \leq e^{-l_o t} \cdot |\tilde{d}(0)| + 1/\rho(t)$ such that $\rho(t) \geq |\dot{d}|$, $\rho(t)$ such that $\rho(t) \ge |\dot{d}|$, $\forall t \ge 0$.

Proof: Differentiating the auxiliary variable with respect to time gives

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\n
$$
\dot{\xi} = -\dot{\hat{d}} - l_o m \dot{x}_2.
$$
\n(12)

Substituting (9) into (12), the disturbance observer comes (11). From (7), (8), (11), and (12), the llowing estimation error dynamics is obtained as $\dot{\tilde{d}} = -l_o \tilde{d} + \dot{d}$. (13) becomes (11) . From (7) , (8) , (11) , and (12) , the following estimation error dynamics is obtained as $\frac{1}{2}$

$$
\tilde{d} = -l_o \tilde{d} + \dot{d}.\tag{13}
$$

Therefore, $|\tilde{d}| \leq e^{-l_0 t} \cdot |\tilde{d}(0)| + 1/l_o \rho(t)$.

Note that the DOB gain l_0 determines the convergence rate and the boundedness of the estimation error.

Remark 1: Observer (11) with auxiliary variable (10) does not require the derivatives of x to obtain \hat{d} . If (10) and (11) are used to estimate the unknown disturbance instead of (9), amplification of the measurement noise by the high gain can be reduced so that it is negligible in practice. Consequently, the large observer gain can be used to reduce the estimation error without the amplification of the measurement noise when \tilde{d} is large.

3.2. Controller design

In this subsection, the backstepping controller is In this subsection, the backstepping controller is
designed to compensate for an unknown disturbance and track the desired output y_d . We define the position/output tracking error z_1 as $z_1 = x_1 - y_d$. The states of the closed-loop system z_{cl} are defined as $z_{cl} = [z_1, z_2, z_3,$ \tilde{d}]^T where z_2 and z_3 are defined in the following theorem.

Theorem 2: Consider the EHS (6) and the disturbance observer (10) and (11). Suppose that the control law is

given by
 $z_1 = x_1 - y_d$, $\alpha_1 = -k_1 z_1$, $z_2 = x_2 - y_d - \alpha_1$,
 $kx_1 + kx_2 - (A_1 - m)\ddot{v}_1 + m(-k_2 z_2 + \dot{\alpha}_1 - z_1) + \hat{d}$ given by

$$
z_{1} = x_{1} - y_{d}, \quad \alpha_{1} = -k_{1}z_{1}, \quad z_{2} = x_{2} - \dot{y}_{d} - \alpha_{1},
$$
\n
$$
\alpha_{2} = \frac{kx_{1} + bx_{2} - (A_{p} - m)\ddot{y}_{d} + m(-k_{2}z_{2} + \dot{\alpha}_{1} - z_{1}) + \hat{d}}{A_{p}},
$$
\n
$$
z_{3} = x_{3} - \ddot{y}_{d} - \alpha_{2},
$$
\n
$$
u = \frac{\varphi(x, z, y_{d})}{\frac{4\beta_{e}C_{d}wk_{v}}{V_{t}\sqrt{\rho}}\sqrt{P_{s} - \text{sgn}(\varphi(x, z, y_{d}))x_{3}}},
$$
\n(14)

where k_1 , k_2 , and k_3 are positive controller gains, and

$$
\varphi(x, z, y_d) = \frac{4\beta_e A_p}{V_t} x_2 + \frac{4\beta_e C_{tl}}{V_t} x_3
$$

+ $\ddot{y}_d + \dot{\alpha}_2 - k_3 z_3 - \frac{A_p}{m} z_2$.

If we take $\frac{1}{l_o} < 4m^2 k_2$, then $z_{cl}(t)$ is globally uniformly
bounded.
Proof: Step 1: The derivative of z_1 with respect to
time gives us
 $\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d$. (15) bounded.

Proof: Step 1: The derivative of z_1 with respect to time gives us $\frac{2}{5}$

$$
\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d.
$$
 (15)

Let us define the control Lyapunov function (CLF) candidate, V_1 as

$$
V_1 = \frac{1}{2}z_1^2.
$$
 (16)

The derivative of V_1 with respect to time is given by

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\nThe derivative of
$$
V_1
$$
 with respect to time is given by
\n $\dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 + \alpha_1).$ (17)
\nSubstituting α_1 in (14) into (19) results in
\n $\dot{V}_1 = -k_1 z_1^2 + z_1 z_2.$ (18)

Substituting α_1 in (14) into (19) results in , .
.

$$
\dot{V}_1 = -k_1 z_1^2 + z_1 z_2.
$$
\n(18)
\nStep 2: The derivative of z_2 with respect to time is
\n
$$
\dot{z}_2 = \dot{x}_2 - \ddot{v}_1 - \dot{\alpha}.
$$

Step 2: The derivative of z_2 with respect to time is

Step 2: The derivative of
$$
z_2
$$
 with respect to time is
\n
$$
\dot{z}_2 = \dot{x}_2 - \ddot{y}_d - \dot{\alpha}_1
$$
\n
$$
= -\frac{1}{m} (kx_1 + bx_2) + \frac{A_p}{m} x_3 - \frac{d}{m} - \ddot{y}_d - \dot{\alpha}_1.
$$
\nLet us define V_2 as
\n
$$
V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{d}^2.
$$
\n(20)

Let us define V_2 as

$$
V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{d}^2,
$$
\n
$$
m,
$$
\n
$$
\vec{V} = \vec{V} + \vec{z} \cdot \vec{z} + \tilde{d}\dot{\vec{d}}.
$$
\n(20)

then,

$$
\vec{v}_2 = \vec{V}_1 + z_2 \vec{z}_2 + \tilde{d}\dot{\vec{d}}
$$
\n
$$
= \vec{V}_1 + z_2 \left(-\frac{1}{m} (kx_1 + bx_2) + \frac{A_p}{m} x_3 - \frac{d}{m} - \ddot{y}_d - \dot{\alpha}_1 \right)
$$
\n
$$
+ \tilde{d}\dot{\vec{d}}.
$$
\nSubstituting (13) and (14) into (21) results in

\n
$$
\vec{v}_1 = \begin{pmatrix} 1 & \dots & A_n \end{pmatrix}
$$

Substituting (13) and (14) into (21) results in

$$
+dd.
$$
\n(21)
\nSubstituting (13) and (14) into (21) results in
\n
$$
\dot{V}_2 = \dot{V}_1 + z_2 \left(-\frac{1}{m} (kx_1 + bx_2) + \frac{A_p}{m} (z_3 + \dot{y}_d + \alpha_2) - \frac{d}{m} - \ddot{y}_d - \dot{\alpha}_1 \right) + \tilde{d} (\dot{d} - l_o \tilde{d})
$$
\n(22)
\n
$$
+ \frac{d}{m} - \ddot{y}_d - \dot{\alpha}_1 + \tilde{d} (\dot{d} - l_o \tilde{d})
$$
\n
$$
= \frac{d}{m} - \ddot{y}_d - \dot{\alpha}_1 + \tilde{d} (\dot{d} - l_o \tilde{d}) + \frac{A_p}{m} \tilde{d} \tilde{d} + \frac{A_p}{m} \tilde{d} \tilde{d} \tilde{d} + \frac{A_p}{m} \tilde{d} \tilde{d} \tilde{d}
$$

with α_2 , the time derivative of V_2 becomes $\frac{1}{2}$

th *a*₂, the time derivative of *V*₂ becomes
\n
$$
\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 - \frac{1}{m} z_2 \tilde{d} + \tilde{d} (\dot{d} - l_o \tilde{d}) + \frac{A_p}{m} z_2 z_3
$$
\n
$$
\leq -k_1 z_1^2 - k_2 \left(z_2^2 + \frac{1}{k_2 m} z_2 \tilde{d} \right) - l_o \tilde{d}^2 + |\tilde{d}| |\dot{d}|
$$
\n
$$
+ \frac{A_p}{m} z_2 z_3
$$
\n
$$
= -k_1 z_1^2 - k_2 \left(z_2 + \frac{1}{2mk_2} \tilde{d} \right)^2 - \gamma \left(|\tilde{d}| - \frac{1}{2\gamma} |\dot{d}| \right)^2
$$
\n
$$
+ \frac{1}{4\gamma} |\dot{d}|^2 + \frac{A_p}{m} z_2 z_3,
$$
\n(23)
\nhere $\gamma = \frac{4m^2 l_o k_2 - 1}{4m^2 k_o}$. From Assumption 1, because $|\dot{d}|$

where $\gamma = \frac{4m^2 l_0 k_2 - 1}{2}$ $\sqrt{4m^2k_2}$ where $\leq d_{\text{max}}$, \dot{d}_{max} , \dot{V}_2 becomes ere $\gamma = \frac{4m^2 \omega^2}{4m^2 k_2}$. From Assumption 1, because V_{max} , \vec{V}_2 becomes -
-
-

$$
\dot{V}_2 \le -k_1 z_1^2 - k_2 \left(z_2 + \frac{1}{2mk_2} \tilde{d} \right)^2 - \gamma \left(|\tilde{d}| - \frac{1}{2\gamma} \dot{d}_{\text{max}} \right)^2 + \frac{1}{4\gamma} \dot{d}_{\text{max}}^2 + \frac{A_p}{m} z_2 z_3.
$$
\n(24)

Final step: The dynamics of z_3 is

$$
\dot{z}_{3} = -\frac{4\beta_{e}A_{p}}{V_{t}}x_{2} - \frac{4\beta_{e}C_{tl}}{V_{t}}x_{3}
$$
\n
$$
+\frac{4\beta_{e}C_{d}wk_{v}}{V_{t}\sqrt{\rho}}\sqrt{P_{s}-sgn(u)x_{3}}u - \ddot{y}_{d} - \dot{\alpha}_{2}.
$$
\n(25)

We defined the overall Lyapunov function candidate V_3 as

$$
V_3 = V_2 + \frac{1}{2}z_3^2.
$$
 (26)
nen we obtain \dot{V}_3 as
 $\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3.$ (27)

Then we obtain \dot{V}_3 as

$$
\dot{V}_3 = \dot{V}_2 + z_3 \dot{z}_3.
$$
\nsubstituting (25) into (27) yields

\n
$$
\begin{pmatrix}\n4\beta A_2 & 4\beta C_1\n\end{pmatrix}
$$

Substituting (25) into (27) yields

$$
\dot{V}_3 = \dot{V}_2 + z_3 \left(-\frac{4\beta_e A_p}{V_t} x_2 - \frac{4\beta_e C_{tl}}{V_t} x_3 + \frac{4\beta_e C_d w k_v}{V_t \sqrt{\rho}} \sqrt{P_s - \text{sgn}(u) x_3} u - \ddot{y}_d - \dot{\alpha}_2 \right).
$$
\n
$$
\text{with the control input, } \dot{V}_3 \text{ can be written as}
$$

With the control input, \dot{V}_3 can be written as

10.11. The equation is given by:

\n
$$
\vec{V}_3 = \vec{V}_2 - k_3 z_3^2 - \frac{A_p}{m} z_3 z_3
$$
\n
$$
\leq -k_1 z_1^2 - k_2 \left(z_2 + \frac{1}{2mk_2} \tilde{d} \right)^2 - \gamma \left(|\tilde{d}| - \frac{1}{2\gamma} \dot{d}_{\text{max}} \right)^2
$$
\n
$$
-k_3 z_3^2 + \frac{1}{4\gamma} \dot{d}_{\text{max}}^2
$$
\n
$$
\leq -(1-\theta)\dot{V}_{3_0} - \theta \dot{V}_{3_0} + \frac{1}{4\gamma} \dot{d}_{\text{max}}^2,
$$
\nwhere

\n
$$
\vec{V}_1 = k_1 z_1^2 + k_2 \left(z_1 + \frac{1}{4} \dot{d}_{\text{max}}^2 \right)^2 + \gamma \left(|\tilde{d}| - \frac{1}{4} \dot{d} \right)^2
$$

where

$$
\dot{V}_{3_0} = k_1 z_1^2 + k_2 \left(z_2 + \frac{1}{2mk_2} \tilde{d} \right)^2 + \gamma \left(\|\tilde{d}\| - \frac{1}{2\gamma} \dot{d}_{\text{max}} \right)^2
$$

+ $k_3 z_3^2$
d 0 < \theta < 1. If we take $\frac{1}{l_0} < 4m^2 k_2$ and $||z_{cl}(t)||$
where $B_r = \left\{ z_{cl} | \theta V_{3_0} = \frac{1}{4\gamma} \dot{d}_{\text{max}}^2 \right\}$, then

and $0 < \theta < 1$. If we take $\frac{1}{l_0} < 4m^2 k_2$ and $||z_{cl}(t)|| \ge$ B_r where $B_r = \{ z_{cl} | \theta V_{3_0} = \frac{z_{01}}{4 \gamma} d_{\max}^2 \}$ 2 $\dot{V}_3 \leq -(1-\theta)\dot{V}_{3_0}$. + $k_3 z_3^2$

d 0 < θ < 1. If

where $B_r = \begin{cases} z \\ z \end{cases}$
 $\dot{V}_3 \le -(1-\theta)\dot{V}_{3_0}$. -
-
-
-(30)

Thus $z_{cl}(t)$ is globally uniformly bounded.

In Theorem 2, the size of the ball B_r mainly depends on the control and observer gains as well as $\frac{1}{4\gamma} \dot{d}_{\text{max}}^2$. The controller gain uses the derivatives of the measured signals so that it is difficult to use the high control gains. On the other hand, the high observer gain can be used since the proposed DOB does not use the derivatives of the measured signals. Thus, the high observer gain can shrink size of B_r .

Remark 2: In actuality, the term $\sqrt{P_s - \text{sgn}(u)x_3}$ should be used to calculate u in (14) instead of $\sqrt{P_s - \text{sgn}(\varphi)x_3}$. Unfortunately, it cannot be solved "as

is," because it contains the control variable u on both sides of the equation. However, u on the right side is used for only the sign function. Thus, the sign of u is is," because it contains the control variable *u* on both sides of the equation. However, *u* on the right side is used for only the sign function. Thus, the sign of *u* is determined by φ . Consequently, $\sqrt{P_s - \text{sgn}($ is," because it contains the control varsides of the equation. However, u on t
used for only the sign function. Thus, if
determined by φ . Consequently, $\sqrt{P_s}$ – s
substituted for $\sqrt{P_s}$ – sgn(φ)x₃ in (14).

4. SIMULATIONS AND EXPERIMENTS

In the simulations and experiments, the system, disturbance observer, and controller parameters are used as follows: $m = 10$, $k = 50$, $b = 1000$, $A_p = 4.812 \times 10^{-4}$, $\beta_e = 1.8 \times 10^9$, $V_t = 6.2 \times 10^{-5}$, $C_{il} = 2.48815 \times 10^{-14}$, C_{el} as follows: $m=10$, $k = 50$, $b=1000$, $A_p = 4.812 \times 10^{-4}$,
 $\beta_e = 1.8 \times 10^9$, $V_t = 6.2 \times 10^{-5}$, $C_{il} = 2.48815 \times 10^{-14}$, $C_{el} = 1.666 \times 10^{-14}$, $w = 5.2 \times 10^{-3}$, $\rho = 840$, $C_d = 0.6$, $k_v =$ $\beta_e = 1.8 \times 10^9$, $V_t = 6.2 \times 10^{-5}$, $C_{il} = 2.48815 \times 10^{-14}$, C_{el}
= 1.666×10⁻¹⁴, $w = 5.2 \times 10^{-3}$, $\rho = 840$, $C_d = 0.6$, $k_v =$
1.33×10⁻⁵, $P_s = 12.0 \times 10^6$, $k_1 = 350$, $k_2 = 1700$, $k_3 = 130$, $I_0 = 1.49 \times 10^3$. The proposed method, as defined in (14), $I_0 = 1.49 \times 10^3$. The proposed method, as defined in (14), was compared with the backstepping control without a disturbance observer.

4.1. Simulation results

Figs. 1 and 2 show the position tracking errors of the two methods and the estimation performances of the unknown disturbance, i.e., $d = -600 - 50\sin(\pi t) + \text{sgn}(x_2)$. known disturbance, i.e., $a = -600 - 50 \sin(\pi t) + \text{sgn}(x_2)$.
In these simulations, $x(0) = [0 \ 0 \ 0]^T$, $d(0) = -600$ In these simulations, $x(0) = [0 \ 0 \ 0]$, $a(0) = -000$
and $\hat{d}(0) = 0$. Due to the disturbance, the position tracking performance of the backstepping control without a disturbance observer in Fig. 1 was relatively poor compared to that of the proposed method shown in Fig. 2. Fig. 2(c) shows that the unknown disturbance was well estimated such that the proposed method demonstrates the best position tracking performance between the two methods. Furthermore, due to the disturbance, offset tracking errors appeared in the backstepping control method without a disturbance observer. On the other hand, the proposed method had almost zero position tracking error because it compensates for the unknown disturbance.

Fig. 1. Backstepping control.

To evaluate the performance of the proposed DOB in the presence of a measurement noise, the disturbance estimation performance of proposed DOB (10) and (11) was compared with that of conventional DOB (9). Fig. 3 shows that the results of the disturbance estimation performance of the unknown disturbance with the measurement noises, i.e., $|x_{1n}| < 200 \text{ µm}$, where x_{1n} is

Fig. 2. DOB based backstepping control.

Fig. 3. Disturbance estimation performance of the CLF with the conventional DOB (9) and CLF with the proposed DOB (10) and (11) with measurement noises.

the position measurement noise. Because the conventional Wonhee Kim
the position measurement noise. Becau
DOB (9) used the derivative of \dot{x}_2 to obtain \hat{d} , the estimated disturbance of the DOB (9) had large ripples due the amplification of the measurement noises. On the other hand, since the proposed DOB did not use the derivative of x_2 , the measurement noises were not amplified. Thus, the proposed DOB had the small ripples.

4.2. Experimental results

The position and force feedback were measured by a linear variable differential transformer (LVDT) and a load cell, respectively. The velocity was numerically approximated by differentiating the position. The derivatives were calculated using the forward Euler method. The sampling rate was set to 1 kHz. In these method. The sampling
experiments, $\hat{d}(0) = 0$.

Fig. 4 shows the position tracking errors of the two methods and the estimation performance with the unknown disturbance. In order to evaluate the validity of the proposed method, the desired reference position profile shown in Fig. 4 was used. Due to the unknown

Fig. 4. Position tracking performances.

disturbance, the backstepping control performance without the disturbance observer was poor relatively to that of the proposed method due to the unknown disturbance. Furthermore, the unknown disturbance resulted in offset position tracking errors in the backstepping control method without the disturbance observer. On the other hand, the proposed method had the best position tracking performance between the two methods, as the proposed method compensated for the unknown disturbance. Fig. 4(d) shows the unknown disturbance estimation result. Thus, in order to verify the estimation performance, the actual disturbance is shown in Fig. 4(e). Ripples in the disturbance appeared due to structural vibration and sensor noise. Actually, the estimated disturbance was similar to the load force because the required load force to move the actuator was much smaller than that required to overcome the disturbance, but the disturbance was not equal to the load force [6]. Thus, we evaluated the estimation performance by means of a comparison between the estimated disturbance and the load force. Given that the estimated force [6]. Thus, we evaluated the estimation performance
by means of a comparison between the estimated
disturbance and the load force. Given that the estimated
disturbance \hat{d} was similar to the actual load force F_L we can conclude that the unknown disturbance was well estimated despite the structural vibration. Also, as discussed in Remark 1, because we did not use the derivatives of x_i to estimate the disturbance, the sensor noise was suppressed significantly. Due to the estimations and the compensation of the disturbance, the position tracking error was satisfactory and the position tracking performance was improved using the proposed method.

5. CONCLUSION

The disturbance observer based backstepping control scheme for EHSs was proposed in order to improve the position tracking performance in the presence of an unknown disturbance. The proposed method consists of a disturbance observer and a backstepping controller. The position tracking performance of the proposed method was validated via simulations and experiments. The position tracking performance was improved by the compensation of the unknown load force using a disturbance observer.

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