International Journal of Control, Automation, and Systems (2014) 12(4):749-758
DOI 10.1007/s12555-013-0377-1
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

A Game Theoretic Approach for the Distributed Control of Multi-Agent Systems under Directed and Time-Varying Topology

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Abstract: This paper presents a new game theory based method to control multi-agent systems under directed and time varying interaction topology. First, the sensing/communication matrix is introduced to cope with information sharing among agents, and to provide the minimal information requirement which ensures the system level objective is desirable. Second, different from traditional methods of controlling multi-agent systems, non-cooperative games are investigated to enforce agents to make rational decisions. And a new game model, termed stochastic weakly acyclic game, is developed to capture the optimal solution to the distributed optimization problem for multi-agent systems with directed topology. It is worth noting that the system level objective can be achieved at the points of the corresponding equilibriums of the new game model. The proposed method is illustrated with an example in smart grid where multiple distributed generators are controlled to reach the fair power utilization profile in the game formulation to ensure the aggregated power output are optimal.

Keywords: Distributed control, distributed optimization, game theory, multi-agent system.

1. INTRODUCTION

Recent technological advances have spurred a broad interest in distributed coordination of dynamic agents in the presence of uncertainty and limited information exchange [1-10]. The study forms an active area of research, giving rise to new control paradigms such as the fields of formation control [6], autonomous underwater vehicles [7], smart grid control [8,9] and so on. Although each of these areas poses its own unique challenges, several common threads can be found. In most cases, the central problem for multi-agent system is to design appropriate local control laws for each agent such that system level objective can be achieved. And the local control laws should possess several desirable attributes such as the real-time adaption and robustness to dynamic uncertainties [22]. However, realizing these benefits comes with several underlying challenges, such as dealing with overlapping and distributed information, as well as making decisions for a potentially large number of interacting and self-interested agents. Interestingly, these paradigms of making distributed decisions with such challenges perfectly fit into the framework of non-cooperative game theory [10-16].

Recently, the appeal of applying game theoretic methodology to multi-agent systems is receiving significant attention [10,13-16]. In order to utilize game theory, system designer must set up game model for the individual agent and specify the distributed learning algorithm that enables all the self-interested agents to reach desirable global behaviors. The most advantage by using game theoretic approach is that it provides a hierarchical decomposition between the game model design and the distributed learning algorithm design [10,13-16]. As is nature, by using the language of learning in game theory, the problem of designing local control laws for multi-agent system becomes the problem of designing game model for each agents [10,13,14, 21,22]. There is a large and growing literature that focuses on this topic. In [10], by designing potential game [11] models for agents, the relationship between potential game and multi-agent cooperative control is established. And the class of potential games captures many application domains and is beginning to receive great interest in multi-agent systems. However, the framework of potential game is not broad enough or even impossible to meet the diverse set of challenges such as locality of the designed agent objective functions as well as efficiency guarantees for the resulting equilibriums [13,14,21]. Inspired by [19], where the moods (or states) are introduced into game theory and a payoff based learning algorithm is developed to implement desired equilibriums, in [14,21,22], an additional state space is introduced into potential game model to cope with the design challenges mentioned above.

Generally, the approach in [14,21,22] is based on the framework of state based potential game [23] and it requires the information exchange among agents should be undirected. However, there are a variety of practical situations where information among agents may be directed and time-varying, such as the leader-following scenarios [3,4]. As a result, there is a need to extend the

Manuscript received August 6, 2013; revised December 19, 2013; accepted January 24, 2014. Recommended by Associate Editor Juhoon Back under the direction of Editor Zengqi Sun.

This work was supported by the Natural Science Foundation of China under Grant 61371095, 51177146, and the Fundamental Research Funds for the Central Universities under Grant 2014 QNA4011.

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results in [14,21-23] to conditions where information exchange can be directed and the interaction topology may be not connected at some time instants. This requires a deep inspection into a more general game theoretical framework beyond potential games.

Compared with existing results, the main contribution of this paper is twofold. First is to develop a novel game framework termed stochastic weakly acyclic games to deal with the directed and time-varying information flow among agents. The new game model introduces an underlying state space into the weakly acyclic games [16,19,20], which are a class of games particularly relevant for multi-agent cooperative control problems. The new game model possesses several desired attributes. First, different from potential games [10,11], the inherent structure of new game allows the interaction topology to be directed. It is noticed that the games in [14,21-23] are a special case of the game model in this paper. Second, much like potential games, the new game model possesses an underlying structure that can be exploited by using the existing distributed learning algorithms for state based games [14-16,19]. Third, the state can be considered as the coordinating entity which decouples the system level objective of the multi-agent system into agent objectives of virtually any degree of locality. It provides much more degree of freedom for us to design local control laws for multi-agent systems, especially with time varying topologies or certain physical meanings. Besides, the state can be used to improve group behaviors of agents and provide system designer the mechanism to choose the desirable equilibriums for distributed engineering systems.

By using the time varying and binary-valued matrix to capture the changes of sensing/communicating among agents, the second contribution of this paper is to extend both cases of connected and undirected communication requirements in [14,22] to practical conditions such as directed, time-variant and not always connected topologies. Different from and complementary to graph theory [2,4], matrix theory not only can be used to deal with agents with high order dynamics, but also provides a new more intuitive concept to explore the convergence of the system dynamics by analyzing the property of matrix sequence. Specifically, the matrix theory based analysis admits the best results obtained by using graph theory [7], and all the existing graph theory results have their counterparts in algebraic matrix theory. Besides, the minimal requirement of the communication topology under which the equilibriums are efficient with regards to the system level objective is also provided in the language of matrix theory.

The rest of this paper is organized as follows. In Section 2, we present the background and problems to be solved in the distributed optimization formation of multi-agent systems. The matrix theory based communication description and minimal information requirement on the matrix sequences are introduced in Section 3. In Section 4, the stochastic weakly acyclic games are introduced to design local control laws for agents under directed information network. And the analytical properties of the game model are given to show the efficiency of the resulting equilibriums. After that the simulation results are shown in Section 5 and finally conclusions are given in Section 6.

2. PROBLEM SETUP

2.1. Background

For multi-agent problems, we are interested in optimization algorithms that can be distributed across individual agents. Suppose the set of agents is denoted by set $N=\{1,2,\cdots,n\}$, and each agent $i\in N$ is endowed with a set of possible actions (or decisions) denoted by V_i, which is a nonempty and convex subset of R. Any specific joint action profile is denoted by the vector $v\triangleq(v_1,v_2,\cdots,v_n)$, where $v\in V=\prod_{i\in N}V_i$ and V is the closed, convex and nonempty set consisting of all possible joint decisions. Suppose the global objective of the multi-agent system can be captured by a differentiable and convex function $\phi: V\rightarrow R$. More specifically, the distributed optimization formulation of the multi-agent cooperative control problem takes on the general form:

$$\begin{aligned}&\min_{v\in V}\quad \phi(v_1,v_2,\cdots,v_n).\\ &s.t.\quad v_i\in V_i, i\in N.\end{aligned}\tag{1}$$

The well known distributed optimization algorithms, such as the gradient or gradient related methods [17,18], always have limited applicability because of local or overlapping information between agents and the inherent structure constraints of the system. As we know, game theory is powerful to analyze interactions between agents and to react to limited or overlapping information. In addition, it provides a way for agents to make individual and rational decisions to capture the system level objective through optimizing their local objectives. In this paper, we are interested in solving the distributed optimization problem by designing game models for agents in the framework of non-cooperative game [12]. In order to use game theory for agents to make rational decisions $\{v_i(t)\}_{i\in N}$ to solve the optimization problem (1), there are some problems to be solved as follows.

2.2. Problems to be solved

Problem 1: Design game models for agents to produce desirable decisions $\{v_i(t)\}_{i\in N}$. Specifically, the new decision of agent i at time t, denoted by $v_i(t)$, can be formulated by the local objective function based on the available information at time $t-1$:

$$v_i(t)=U_i(v_1(t-1),v_2(t-1),\cdots,v_n(t-1)),\tag{2}$$

where $i=1,2,\cdots,n$, $U_i(\bullet)$ is the local objective function of agent i at time t that we want to design. In order to make sure the decision $v_i(t)$ is feasible, the local control law $U_i(t)$ must change according to the available information at time t. Generally, the more information the agents share between each other, the faster the system converges. As a result, this will bring out the problem of determining minimum information sharing among agents. This is the problem as follows.

Problem 2: Design the interaction topology to ensure the global objective is desirable and the local control laws are robust with respect to possible variations and limitations of communication networks among agents, as well as minimizing information flowing between agents.

After the game models for agents are established through such a design of local communication network, a sequence of decisions of agents are produced by the designed local objective functions. However, whether the decisions will converge to one equilibrium and whether the equilibrium will be the desired result become our main concern. In other words, there may exist equilibriums that are suboptimal and fail to solve the distributed optimization problem for multi-agent systems. This will be the problem as follows.

Problem 3: When distributed optimization problem is formulated in game model, whether the equilibriums solve the optimization problem should be verified further.

For the multi-agent systems, operation at the point of equilibrium may reflect certain degree of optimization for the global objective. The game model developed in [10,13] asymptotically guarantee that the actions of agents will constitute at least one equilibrium. However, the equilibriums may fail to optimize the global objective due to the inherent limitations of non-cooperative games [13,21], where each agent aims to optimize its own performance without regarding to the costs it imposes on others. To that end, we aim to develop a novel game model to suppress the aggressive competition among agents as well as taking into account the individual agent's effect on the performance of equilibriums.

3. COMMUNICATION TOPOLOGY DESIGN

In order to solve Problem 2, first the matrix based topology model is introduced in Section 3.1. Based on the model, in Section 3.2, the rule of the topology design is introduced to ensure the desirable system dynamics with minimal information requirements.

3.1. Matrix based topology model

Different from the representation of information flow among agents in the fashion of graph theory, a matrix theory based model is proposed to analyze and design the information sensing and communication for a group of individual agents [7]. Generally, the sensing and communication is described mathematically by a time-varying and piecewise-constant matrix whose dimension is equal to the number of dynamical agents and whose elements assume binary values. And the sensing/ communication matrix can be defined without loss of any generality [7]:

$$S(t)=\begin{pmatrix} s_{11}(t) & s_{12}(t) & \cdots & s_{1n}(t) \\ s_{21}(t) & s_{22}(t) & \cdots & s_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}(t) & s_{n2}(t) & \cdots & s_{nn}(t) \end{pmatrix},$$

where $s_{ii}(t)=1$ because agent i can always acquire its own information, whether other agent's information can be acquired or not by agent i is completely determined by the binary entry in the i th row of the matrix. In general, for any $i \neq j$, at time t, $s_{ij}(t)=1$ if the agent i can get the information of agent j, and $s_{ij}(t)=0$ if otherwise. Over time, binary changes of matrix $S(t)$ occur at an infinite sequence of time instants denoted by $\{t_k : k \in \Omega\}$, where $\Omega \triangleq \{0,1,\cdots,\infty\}$ and $S(t)$ is piecewise constant as $S(t)=S(t_k)$ for all $t \in [t_k, t_{k+1})$.

Remark 1: By utilizing the piecewise-constant and binary-valued matrix, dynamical changes of sensing and communication among agents can be captured. In addition, based on augmentation of irreducible and reducible matrix theory, not only identical agents but also heterogeneous dynamical systems of arbitrary but finite degree can be dealt with. Furthermore, through adopting lower triangulation of reducible matrix as a tool for analysis, all the existing results based on the graph theory, such as a strongly connected graph, a spanning tree, have their counterparts in the matrix theory [7,24].

3.2. Rule of the communication topology design

As we know, in order to ensure individual agents collectively accomplish the system level objective, local information needs to be shared among agents. Heuristically, the more information channels there are, the faster the system converges. However, this quickly becomes an uneconomical solution to the problem. Is there a minimal information requirement which will guarantee the validity of the proposed control strategy as well as the efficiency of the resulting behavior? The rule in [7] gives the answer to this question.

Rule [7]**:** The sensing/communication matrices sequence $S_{\infty:0}=\{S(t_0), S(t_1), \cdots\}$ is sequentially complete.

The completeness condition is a very precise method to design desired communication networks and to schedule local communications among agents. It provides the minimal information that needs to be shared among agents for distributed control. Especially, it gives the cumulated effects of information in a time interval and shows that there exists connection between a pair of agents in the cumulated communication network even if at some time instants the pair of agents can not communication with each other [7]. In addition, it provides system designer much more freedom to develop distributed control laws for a broad class of multi-agent system.

In order to illustrate the application of sequentially complete condition in time varying communication network, we assume that for any t_0, there exist a constant $T>0$ such that for any $t_1 \in [t_0, t_0+T]$, consider the binary product of matrixes as $S_{t_1:t_0}=S(t_0)\wedge S(t_0+1) \wedge\cdots\wedge S(t_1-1)\wedge S(t_1)$, if the matrix $S_{t_1:t_0}$ is lower triangularly complete [7], and is uniformly bounded, then sequence is sequentially complete. Intuitively, it means that in the corresponding graph which is constructed by linking the agents according to nonzero entries in $S_{t_1:t_0}$, there exists at least one agent, denoted as agent i, such that by following the directed branches, every other agent can be reached from the agent *i*. It should be also noted that the convergence rate of the system relates to the connectivity of the communication network, so it is important to design a reasonably local communication network within certain physical and economic considerations.

4. STOCHASTIC WEAKLY ACYCLIC GAMES DESIGN

In this section, the notion of a new game named stochastic weakly acyclic game is introduced in Section 4.1. In Section 4.2, the new game model is developed and the local objective functions for agents are designed to solve the Problem 1. In Section 4.3, under the rule of sequentially complete for the directed communication among agents, the analytical properties of the designed game are verified to show the efficiency of equilibriums, which gives the answer to the Problem 3.

4.1. Stochastic weakly acyclic games

Recently, applying potential games to the control of multi-agent systems has been addressed in [10,13,14, 21,22]. However, the inherent structure of potential game models requires the information topology to be undirected [10]. In many situations, information may be exchanged via directed sensing or communication. The class of weakly acyclic games [16,19,20] forms a generalization of the class of potential games and has an natural appeal of coping with directed information flowing among agents.

However, the weakly acyclic game still falls under the framework of non-cooperative games. Because of the aggressive competition among agents, it is still not capable of developing agent's local objective functions which meet objectives such as locality of information and efficiency of resulting equilibriums. In addition, the inherent structure of weakly acyclic games can not handle the interaction of agents under time-varying topologies. These limitations lead us move beyond weakly acyclic games to games with a new and broader structure. Inspired by the work of [13,14,19-23], a new game model, named stochastic weakly acyclic games, is developed by introducing an underlying state space into the framework of weakly acyclic games. This new game model has some advantages, such as ensuring agents collectively accomplish global objective under directed, time varying and local information, as well as providing system designer much more additional degree of freedom to coordinate group behaviors.

A stochastic weakly acyclic game, denoted by $G = \{N, \{A_i\}_{i\in N}, \{U_i\}_{i\in N}, X, f, \varphi\}$, consists of a finite player set N and an underlying finite state space X. Each agent i has a state dependent action set $A_i(x)$, and a state dependent payoff function $U_i: X\times A \to R$. The state transition is defined according to the function $f: X\times A\times\Gamma \to X$ which depends on state X, joint action A and system noise Γ.

Repeated play of stochastic weakly acyclic games produces a sequence of actions and a sequence of states, which are generated according to the following process: at time t, given state $x(t)$, all the agents take turns, in some arbitrary order, with other players to select actions $a(t) = \{a_1(t), \cdots, a_n(t)\}$ according to some specified decision rule, such as the myopic Cournot process. The state $x(t)$ and joint action profile $a(t)$ together determine the agent's local objective function at time t, i.e., $U_i(x(t), a(t))$. After all agents select their respective actions, the ensuring state $x(t+1)$ is determined according to the state transition function $x(t+1) = f(x(t), a(t), \Gamma(t+1))$. The process is repeated until no agents want to change their actions anymore. Accordingly, the final results are defined by the notion of state equilibrium for stochastic weakly acyclic game in the same fashion of Nash equilibrium for general strategic form game [11,12].

Definition 1 (State Equilibrium)**:** A state action pair $p^* \triangleq (x^*, a^*)$ is called state equilibrium if for any $i\in N$, given state x^*, $U_i(p^*) = \min_{a_i\in A_i} U_i(x^*, a_i, a_{-i}^*)$.

In order to introduce the definition of stochastic weakly acyclic games, we will also define the notions of state based disagreement function and best response improvement path as follows.

Definition 2 (State based Disagreement Function)**:** For any differential and convex function $g: X\times A\to R$, given some specific action profile $a = (a_i, a_{-i})\in A$ and state $x\in X$, where a_{-i} is the joint action profiles other than i, there exists an agent $i\in N$ with action $a_i^*\in A_i$ while others keep their actions unchanged. Then the disagreement function $D(g, a_i^*, a_{-i}, x)$ for the function g and some specific action a and state x can be defined as follows:

$$D(g, a_i^*, a_{-i}, x) \triangleq g(a_i, a_{-i}, x) - g(a_i^*, a_{-i}, x), \tag{3}$$

where $a_i^* \in \arg\min_{a_i'\in A}(g(a_i', a_{-i}, x))$.

Definition 3 (Best Response Improvement Path)**:** A best response improvement path is a sequence of state action pair $p_0, p_1, \cdots p_L$, where for any time t_l, $0\le l\le L$, $L\in N$, we have $p_l \triangleq (x^l, a^l) \triangleq (x(t_l), a(t_l))$, such that for any $0\le m\le L-1$, there exists one agent i with action $a_i^m \triangleq a_i(t_m)$ at time t_m and a new action $a_i^{m+1}\triangleq a_i(t_{m+1})$ at time t_{m+1} such that i) $a_i^m \ne a_i^{m+1}$; ii) $a_{-i}^m = a_{-i}^{m+1}$; iii) $D(U_i, a_i^{m+1}, a_{-i}^m, x(t_{m+1})) > 0$.

Much like weakly acyclic games [19,20], roughly speaking, the stochastic weakly acyclic games is a kind of game which satisfies the following conditions: from any pair of state action profile $p_0 \triangleq (x^0, a^0)$, there exist some best response improvement path, which is a sequence of agent's state and action pairs $p_0, p_1, \cdots p_L$, where $p_l \triangleq (x^l, a^l)$, $0\le l\le L$, leading from state action pair p_0 to a state equilibrium p_L. However, the above definition is too rough to model and solve the control problems for multi-agent systems. To that end, through utilizing potential functions in the same fashion of weakly acyclic games [10,19,20], a precise and equivalent definition of stochastic weakly acyclic games is driven in the Proposition 1.

Proposition 1: A game is called a stochastic weakly acyclic game if and only if the following conditions are satisfied: for a finite player set N and an underlying finite state space X, at any time t_0, given any state action pair as $p_0 \triangleq (x^0, a^0) = (x(t_0), a(t_0))$ that is not a state equilibrium, there exists a differential and convex potential function $\varphi: X \times A \to R$ and a time $t \in [t_0, t_0 + T]$, where $T > 0$ is a time constant, there exist one agent $i \in N$ with an action $a_i(t) \in A_i$, such that $D(U_i, a_i(t), a_{-i}(t_0), x(t)) > 0$, $D(\varphi, a_i(t), a_{-i}(t_0), x(t)) > 0$ provided that $a(t_0) = a(t_0+1) = \cdots = a(t-1)$, and $x(t_0) = x(t_0+1) = \cdots = x(t-1)$.

Proof: ($\Leftarrow$) At any time t_0, select any state action pair $p_0 = (x^0, a^0) \triangleq (x(t_0), a(t_0))$. If p_0 is not a state equilibrium, then there exists an agent i with an action $a_i' \in A_i$ and a time $t \in [t_0, t_0+T]$ such that $D(U_i, a_i', a_{-i}^0, x(t)) > 0$ and $D(\varphi, a_i', a_{-i}^0, x(t)) > 0$. Then we get a new state action pair as $p^1 \triangleq (x^1, a^1)$, where $a^1 \triangleq a(t) = (a_i', a_{-i}^0)$, $x^1 \triangleq x(t) = f(x^0, a^0, \Gamma(t))$.

Repeat this process and construct a best response improvement path $p_0, p_1, \cdots, p_L$. It is noticed that such a path can not enter inescapable oscillations because the potential function φ is strictly decreasing along the path. Moreover, the path can not be extended indefinitely because the Cartesian product of state and action set $P = X \times A$ is finite. Hence, the last state action pair in the path is obviously guaranteed to exist and must be the state equilibrium.

($\Rightarrow$) At time t_0, select any state action pair $p_0 \triangleq (x^0, a^0)$ from finite set $P = \{(x,a)\}_{x\in X, a\in A}$. Because the game is a stochastic weakly acyclic game, there must exist a best response improvement path $p_0, p_1, \cdots, p_{L_0}$, where $p_l \triangleq (x^l, a^l) \triangleq (x(t_l), a(t_l))$, $t_l \in [t_{l-1}, t_{l-1}+T_l]$, $T_l > 0$, $0 \le l \le L_0$, leading from the state action pair p_0 to a state equilibrium p_{L_0}. Suppose the best response improvement path can be denoted as the set $P^0 \triangleq \{p_0, p_1, \cdots, p_{L_0}\}$, then we can define potential function $\varphi: X \times A \to R$ over the set P^0 to satisfy the condition as follows:

$$\varphi(p_0) > \varphi(p_1) > \cdots > \varphi(p_{L_0}). \tag{4}$$

Obviously, for any $0 \le l \le L_0 - 1$, there exists an agent i with a new action a_i^{l+1} and state x^{l+1} such that $D(U_i, a_i^{l+1}, a_{-i}^l, x^{l+1}) > 0$ and $D(\varphi, a_i^{l+1}, a_{-i}^l, x^{l+1}) > 0$.

Now, select any state action pair $\tilde{p}_0 \in P \setminus P^0$, where $P = X \times A$. Since the game is stochastic weakly acyclic game, then there must exist a best response improvement path $P^1 \triangleq \{\tilde{p}_0, \tilde{p}_1, \cdots, \tilde{p}_{L_1}\}$ leading from that state action pair $\tilde{p}_0$ to a state equilibrium $\tilde{p}_{L_1}$. If $P^1 \cap P^0 = \varnothing$, then we can define potential function φ over the set P^1 to satisfy the condition as follows:

$$\varphi(\tilde{p}_0) > \varphi(\tilde{p}_1) > \cdots > \varphi(\tilde{p}_{L_1}). \tag{5}$$

Obviously, for any $0 \le l \le L_1 - 1$, there exists an agent i with new action $\tilde{a}_i^{l+1}$ and new state $\tilde{x}^{l+1}$, such that $D(U_i, \tilde{a}_i^{l+1}, \tilde{a}_{-i}^l, \tilde{x}^{l+1}) > 0$ and $D(\varphi, \tilde{a}_i^{l+1}, \tilde{a}_{-i}^l, \tilde{x}^{l+1}) > 0$.

Otherwise if $P^1 \cap P^0 \ne \varnothing$, we will choose the index $k' = \min\{k \in \{1, 2, \cdots, L_1\} : \tilde{p}_k \in P^0\}$. Define the potential function φ over the set $P^1 \triangleq \{\tilde{p}_0, \tilde{p}_1, \cdots, \tilde{p}_{k'-1}\}$ to satisfy the condition as follows:

$$\varphi(\tilde{p}_0) > \varphi(\tilde{p}_1) > \cdots > \varphi(\tilde{p}_{k'-1}). \tag{6}$$

Obviously, for any $0 \le l \le k' - 2$, there exists an agent i with new action $\tilde{a}_i^{l+1}$ and new state $\tilde{x}^{l+1}$, such that $D(U_i, \tilde{a}_i^{l+1}, \tilde{a}_{-i}^l, \tilde{x}^{l+1}) > 0$ and $D(\varphi, \tilde{a}_i^{l+1}, \tilde{a}_{-i}^l, \tilde{x}^{l+1}) > 0$.

Now selecting any state action pair $\hat{p}_0 \in P \setminus (P^0 \cup P^1)$ and repeat above process until no such state action pair exists.

The construction of potential function φ guarantees that for any state action pair $p_0 \triangleq (x^0, a^0)$ at time t_0 that is not a state equilibrium, there exists a differential and convex potential function $\varphi: X \times A \to R$ and a time constant T such that there exists a player $i \in N$ with an action $a_i \in A_i$, $D(U_i, a_i, a_{-i}^0, x(t)) > 0$ and $D(\varphi, a_i, a_{-i}^0, x(t)) > 0$. This completes the proof.

Remark 2: Obviously, the game models proposed in [14,22] require all the agents satisfy the necessary condition in Proposition 1. Therefore, these game models could be regarded as a special case of stochastic weakly acyclic game, which only requires that there exists one agent satisfying the necessary condition.

Remark 3: Generally, there are some advantages to extend the game models in [14,22] into the framework of stochastic weakly acyclic game. First, the potential game formulation in [14,22] requires the interaction topology among agents should be undirected. But this requirement is no longer necessary in the structure of stochastic weakly acyclic games. On the other hand, it is noticed that potential games requires that all the agent's local objective function should be appropriately aligned with the potential function. However, the stochastic weakly acyclic game relaxes this alignment requirement by requiring at least one agent's local objective functions to be somewhat aligned with the potential of the game. As a result, this gives system designer much more flexibility in designing agent's objective functions and acquiring the desirable results for multi-agent problems.

Remark 4: The stochastic weakly acyclic games are also known as a special case of the stochastic games [25]. Intuitively, the agents in the stochastic games seek to make decisions to optimize a discounted sum of their historical payoffs. However, in stochastic weakly acyclic games, agents are with finite rationality and make decisions only to optimize their present payoffs.

4.2. Design of stochastic weakly acyclic games

In order to use the framework of stochastic weakly acyclic games in the control of multi-agent systems, local objective functions should be specified for individual agents in the game environment. Accordingly, this will arise the problem of designing game model for agents, which includes specifying the state space, the action space, the state transition function and the local objective for each agent. The details of design process are based on the work of [14,22] and the difference from [14,22] will be introduced as follows.

First, the state transition function is designed as follows. Generally, the state evolves as a function of the sequence of action profile $a(t_0), a(t_1), \cdots$ and sequence of system noise $\Gamma(t_0), \Gamma(t_1), \cdots$. Then the state at time $t+1$ can be defined as $x(t+1) = f(x(t), a(t), \Gamma(t+1))$.

It is noted that the state $x(t+1)$ at time $t+1$ is constructed by using only observations from the state $x(t)$ and action $a(t)$ at time t and also the system noise $\Gamma(t+1)$ at time $t+1$. Actually, there are many alternative possibilities for the state selection. For example, we will use the information from last two time periods, such as time periods t and $t-1$, to formulate the state at time $t+1$ as follows.

$$x(t+1) = f(x(t), x(t-1), a(t), a(t-1), \Gamma(t+1)). \tag{7}$$

However, in this paper we will omit the influence from system noise for ease of exposition.

Second, the local objective function is developed as follows. Different from [14,22], the equally shared utility [13], which is with truly physical meaning, is introduced into the design of local objective function. Given $v = (v_1, v_2, \cdots, v_n) \in R$ as the tuple of value profile for n agents' decisions, the average of local objective functions of agent i's neighboring agents can be chosen to act as the new local objective function for agent i, termed equally shared utility in [13] as follows:

$$U_i(v) = \sum\nolimits_{j=1}^{n} s_{ij} \cdot U_j(v) / \sum\nolimits_{j=1}^{n} s_{ij}. \tag{8}$$

However, in practical applications with directed and intermittent communication or time delays, agents may not have complete knowledge about true value of local agents' actions as soon as possible. To this end, the estimation term in the state space is used to estimate the true value of actions. Accordingly, the equally shared utility above can be rewritten as

$$U_i(e_j|_{s_{ij}=1}) = \sum\nolimits_{j=1}^{n} s_{ij} \cdot \phi(e_j^1, \cdots, e_j^n) / \sum\nolimits_{j=1}^{n} s_{ij}. \tag{9}$$

Meanwhile, considering the error between estimation items and true value, a penalty term is introduced in the local objective function to minimize the errors and ensure the desirable system dynamics. Accordingly, the local objective function can be defined as follows:

$$U_i(x,a) = U_i^{\phi}(x,a) + \alpha \bullet U_i^{e}(x,a), \tag{10}$$

where α is a positive tradeoff parameter. The first term $U_i^{\phi}(x,a) = U_i(e_j|_{s_{ij}=1})$ is inspired from the notion of equally shared utility which distributes the global objective across individual agents. The second penalty term is defined as

$$U_i^{e}(x,a) = (\sum\nolimits_{j=1}^{n} \sum\nolimits_{k} s_{ij} (e_j^k)^2 - n v_i^2) / \sum\nolimits_{j=1}^{n} s_{ij}, \tag{11}$$

which is used to minimize the error caused by introducing estimate items into game model.

4.3. Analytical properties of stochastic weakly acyclic games

After establishing the game model for the distributed optimization problem, next the properties of the model need to be analyzed and verified to see whether the model meets the desired goals or not. Naturally, whether the model results in the framework of stochastic weakly acyclic game or not need to be verified first. This is stated in Theorem 1 as follows.

Theorem 1: Model the optimization problem in (1) as the game model in Section 4.2 with any positive constant α. Given the differential and convex potential function $\varphi: X \times A \to R$ as

$$\varphi(x,a) = \varphi^{\phi}(x,a) + \alpha \bullet \varphi^{e}(x,a),$$

where

$$\varphi^{\phi}(x,a) = \sum\nolimits_{j=1}^{n} \phi(e_j^1, \cdots, e_j^n) / n,$$

$$\varphi^{e}(x,a) = \sum\nolimits_{j=1}^{n} \sum\nolimits_{k} (e_j^k)^2 / n - \sum\nolimits_{i=1}^{n} v_i^2.$$

Then game model is a stochastic weakly acyclic game with potential function φ.

Proof: It is obvious to verify the game model in Section 4.2 is a stochastic weakly acyclic game.

Remark 5: It is noticed that the local objective function $U_i(x,a)$ and potential function $\varphi(x,a)$ are independent of communication topology. So the designed stochastic weakly acyclic games possess an underlying structure that can be exploited in time varying communication topologies besides directed topologies.

After establishing the stochastic weakly acyclic game, then whether there exists an equilibrium becomes our main concern, which is stated in the theorem as follows.

Theorem 2: A stochastic weakly acyclic game possesses at least one state equilibrium.

Proof: The best response improvement path consists of a sequence of state action pairs $p \triangleq (x,a)$ where $p \in P$ and $P = X \times A$ is the set of state action pairs. Obviously the set P is finite because it is the Cartesian product of the finite state set X and finite action set A. Therefore, the last state action pair in the best response improvement path is guaranteed to exist, and the potential function achieves the minimal value at the point of the last state action pair. In addition, both the $D(U_i, a(t), x(t)) > 0$ and $D(\varphi, a(t), x(t)) > 0$ in Proposition 1 imply that the equilibrium set of stochastic weakly acyclic games coincides with the equilibrium set of the games by replacing the local objective function U_i with potential function φ. Consequently, if potential function φ admits a minimal value in the set P, then the stochastic weakly acyclic game possesses a state equilibrium. This completes the proof.

The potential function captures the global objective of distributed optimization problem for multi-agent system. And the game model guarantees at least an equilibrium.

However, whether the equilibriums of the game model are the solution to the optimization problem in (1) is the next problem, which will be stated in Theorem 3.

Theorem 3: Suppose the interaction topology is directed, time-varying, and for any time $t_0 > 0$, there exists a constant $T > 0$ and $t_1 \in [t_0, t_0 + T]$, such that the sequence of topology matrixes $\{S(t_0), S(t_0+1), \cdots, S(t_1)\}$ is sequentially complete, then at the point of state equilibrium, $\forall i, k \in N$, we have $e_i^k = v_k$.

Proof: Suppose the state equilibrium of the designed game is denoted by the state action pair $p \triangleq (x, a) = ((v, e), (\hat{v}, \hat{e}))$. Then $\forall i \in N$, for any joint action profile $a' = (a_i', a_{-i}) = ((\hat{v}_i', \hat{v}_{-i}), (\hat{e}_i', \hat{e}_{-i}))$, we have $U_i(x, a) \le U_i(x, a')$.

Since the corresponding sequence of topology matrixes $\{S(t_0), S(t_0+1), \cdots, S(t_1)\}$ is sequentially complete, then it is easy to find an agent, denoted by i with an action $a_i \in A_i$ and any two agent $j_1, j_2 \in L_i$ where $L_i = \{l : s_{li}(t) = 1\}$ is the set of adjacent agents of agent i. $\forall \delta \in R$, the new action for the agent i, which is denoted by $a_i' = (\hat{v}_i', \hat{e}_i')$, can be defined as $\hat{v}_i' = \hat{v}_i$ and

$$\hat{e}_{i\to j}^{k}{}' = \begin{cases} \hat{e}_{i\to j}^k + \delta & j = j_1 \\ \hat{e}_{i\to j}^k - \delta & j = j_2 \\ \hat{e}_{i\to j}^k & j \in L_i \setminus \{j_1, j_2\}. \end{cases}$$

Accordingly, the change in the local objective function of agent i can be expressed as follows:

$$\sum_{j=1}^n s_{ij} \Delta U_i = \sum_{j=1}^n s_{ij} U_i(x, a') - \sum_{j=1}^n s_{ij} U_i(x, a). \tag{12}$$

When $\delta \to 0$, equation (12) can be expressed as

$$\sum_{j=1}^n s_{ij} \Delta U_i = \left(\frac{\partial \phi}{\partial e_{j_1}^k} - \frac{\partial \phi}{\partial e_{j_2}^k} + 2\alpha \sum_{k \in N} e_{j_1}^k - e_{j_2}^k \right) \delta + o(\delta^2). \tag{13}$$

As we know, $\forall \delta \in R$, $\Delta U_i \ge 0$, therefore, $\forall i, k \in N$, (13) can be translated to

$$\frac{\partial \phi}{\partial e_{j_1}^k} - \frac{\partial \phi}{\partial e_{j_2}^k} + 2\alpha \sum_{k \in N} (e_{j_1}^k - e_{j_2}^k) = 0. \tag{14}$$

As the global objective function $\phi(\cdot)$ is convex and differentiable, then we have

$$\frac{\partial \phi}{\partial e_{j_1}^k} - \frac{\partial \phi}{\partial e_{j_2}^k} = H(\phi)|_{\xi e_{j_1}^k + (1-\xi) e_{j_2}^k} (e_{j_1}^k - e_{j_2}^k), \tag{15}$$

where $\xi \in (0,1)$, $H(\phi)$ is the Hessian matrix of function ϕ. Accordingly, we have

$$\begin{aligned} 0 &\ge -2\alpha \sum_{k \in N} (e_{j_1}^k - e_{j_2}^k)^2 \\ &= H(\phi)|_{\xi e_{j_1}^k + (1-\xi) e_{j_2}^k} (e_{j_1}^k - e_{j_2}^k)^2. \end{aligned} \tag{16}$$

As the Hessian matrix of convex function $\phi(\cdot)$ will be positive semi-definite, then we have

$$0 \ge -\alpha \sum_{k \in N} (2e_{j_1}^k - 2e_{j_2}^k)^2 \ge 0. \tag{17}$$

The inequality (17) implies that $\forall i, k \in N$, $\forall j_1, j_2 \in L_i$,

$$e_{j_1}^k = e_{j_2}^k. \tag{18}$$

In the state space design process [14,22], it is noticed that the sum of the estimation from all the agents regarding any specific agent k's value is equal to n times the agent k's value, that is

$$\sum_{i \in N} e_i^k(t) = n v_k(t). \tag{19}$$

Substituting (19) with (18), we have $\forall i, k \in N$, $e_i^k = v_k$. This completes the proof.

Next, whether the equilibrium is the optimal solution to the optimization problem becomes our main concern, which is stated in the next theorem.

Theorem 4: Model the optimization problem in (1) as stochastic weakly acyclic game with any positive constant α. Then all the state equilibriums are optimal solutions to the distributed optimization problem.

Proof: Suppose $(x, a) = ((v, e), (\hat{v}, \hat{e}))$ is the state equilibrium of the game. Considering a new action profile $a' = (a_i', a_{-i}) = ((\hat{v}_i', \hat{v}_{-i}), (\hat{e}_i', \hat{e}_{-i}))$, which causes the value of the agent i, denoted by v_i, to change to a new value $\hat{v}_i'$, where $\hat{v}_i' = \hat{v}_i + \delta$, $\forall \delta \in R$.

Accordingly, the change in local objective function for agent i can be expressed as follows:

$$\sum_{j=1}^n s_{ij} \cdot \Delta U_i = U_i(x', a') - U_i(x, a). \tag{20}$$

When $\delta \to 0$, we can express (20) as

$$\sum_{j=1}^n \Delta U_i = n \frac{\partial \phi}{\partial v_i} \delta. \tag{21}$$

As we know, $\forall \delta \in R$, $\Delta U_i \ge 0$. Therefore, $\forall i \in N$, (21) can be translated to

$$\frac{\partial \phi}{\partial v_i} = 0. \tag{22}$$

Since the global objective function $\phi(\cdot)$ is convex and differentiable, then the minimum of the distributed optimization problem (1) can be achieved at the point where its derivative equals zero. Meanwhile, the point is also the point of equilibrium of the game. This completes the proof.

5. SIMULATION STUDY

In this section we will illustrate applicability of the theoretical results on the problem of controlling multiple distributed generators (DGs) in distribution networks. Suppose there are n DGs which form a virtual power plant(VPP) [9]. The objective is to ensure the aggregated power output of all the DGs can be controlled through coordinating the power output of each DGs. A simple solution is to specify the fair power utilization profile, where the same ratio of power output versus maximum available capability is imposed on all DGs [8]. The desired utilization profile, denoted by γ^*, is determined by the high level control in grid, such as the transmission and distribution control center [8,9]. Then the objective is to establish a set of local control laws for DGs such that the power output ratios of all DGs seek to γ^* and further it ensures the desirable aggregated power output of VPP, even though the output capabilities of some individual DGs may have large swings. Consequently, the above DGs control problem can be formalized as the optimization problem as follows:

$$\begin{aligned} &\min_{\gamma}\ \phi(\gamma_1,\cdots,\gamma_n)=\sum\nolimits_{i=1}^{n}(\gamma_i-\gamma^*)^2, \\ &s.t.\quad \gamma_i\in(0,1), \gamma_i\in R. \end{aligned} \tag{23}$$

The dynamics of the problem is described as follows. Suppose at time t_0, the high level control command DG i to increase its output power ratio to the desired utilization profile γ^*. Then DG i distributes the profile γ^* to its connected DGs according to the sensing/communication networks and commands them to increase their ratios. Finally, all the DGs asymptotically reach the same utilization profile γ^*.

First, the communication topology between DGs should be designed. The optimization problem will be simulated under two different communication topologies. The first topology is defined as follows. At any time t, suppose there is only one different DG, denoted by j, which can get the information of DG 1, that is, $s_{j1}(t)=1$. And the second topology is defined by adding more connections than in first topology at each time step. It is noticed that information flow among DGs is directed and some entries in matrix may switch from 1 to 0 intermittently as the abnormal operation of a single communication channel. Therefore, the communication matrix is asymmetric and time-varying. However, in order to ensure the output of VPP converges to the expected operational point and the system is robust to time varying and intermittent conditions, the matrix sequence $\{S(t_0),\cdots,S(t_m)\}$ corresponding to the topologies should be sequentially complete. That is to say, in certain time interval $[t_0,\cdots,t_m]$, the resulting graph has at least one globally reachable DG [7]. It is obvious that if the time interval is large enough, the sequentially complete condition is satisfied and the resulting graph has at least one globally reachable DG, i.e., the DG 1 in the VPP.

Then the game model for the optimization (23) is described as follows. For DG i, its state is defined as $x_i=(\gamma_i,e_i)$, where $e_i=(e_i^1,\cdots,e_i^n)$ and e_i^k is the estimation for DG k's ratio γ_k from DG i. Accordingly, the action for DG i is defined as $a_i=(\hat{\gamma}_i,\hat{e}_i)$ where $\hat{\gamma}_i$ is the change of the ratio of DG i and the $\hat{e}_i=(\hat{e}_i^1,\cdots,\hat{e}_i^n)$ is the changes of the estimation term of DG i. The local objective function for DG i, denoted by $U_i(x,a)$, is defined as follows:

$$U_i(x,a)=U_i^{\phi}(x,a)+\alpha\bullet U_i^{e}(x,a), \tag{24}$$

where

$$U_i^{\phi}(x,a)=\sum\nolimits_{j=1}^{n}s_{ij}\cdot\phi(e_j^1,\cdots,e_j^n)/\sum\nolimits_{j=1}^{n}s_{ij},$$

$$U_i^{e}(x,a)=\left(\sum\nolimits_{j=1}^{n}\sum\nolimits_{k}s_{ij}(e_j^k)^2-n\gamma_i^2\right)/\sum\nolimits_{j=1}^{n}s_{ij}.$$

Accordingly, the game model is a stochastic weakly acyclic game with potential function $\varphi(x,a)$ which is defined as follows:

$$\varphi(x,a)=\varphi^{\phi}(x,a)+\alpha\bullet\varphi^{e}(x,a), \tag{25}$$

where

$$\varphi^{\phi}(x,a)=\sum\nolimits_{j=1}^{n}\phi(e_j^1,\cdots,e_j^n)/n,$$

$$\varphi^{e}(x,a)=\sum\nolimits_{j=1}^{n}\sum\nolimits_{k}(e_j^k)^2/n-\sum\nolimits_{i=1}^{n}\gamma_i^2,$$

Under the two different topology designs, Fig. 1 and Fig. 3 illustrate the evolution of the ratios of DGs, while Fig. 2 and Fig. 4 shows the evolution of the optimization function (23) by using the stochastic weakly acyclic game design in (24) and (25) and the gradient play learning algorithm. We set the number of DGs $n=10$, the desired utilization profile $\gamma^*=0.5$ and the tradeoff parameter $\alpha=0.01$.

It is noticed that convergence rates of both the ratios and the optimization function in the second topology are faster than the ones in the first topology. It is because the convergence rate is mainly determined by the Fiedler value, which reflects how well connected the topology is.

For a fixed number of DGs, the Fiedler value in the second topology is bigger than the one in the first topology. Intuitively, there are more information channels in second topology, therefore, the system converges to the desired behaviors faster. Also, it is noticed in both Fig. 2 and Fig. 4 the value of optimization function will be substantially high at the initial time since the difference between the ratios of DGs and the desired utilization profile is large at the

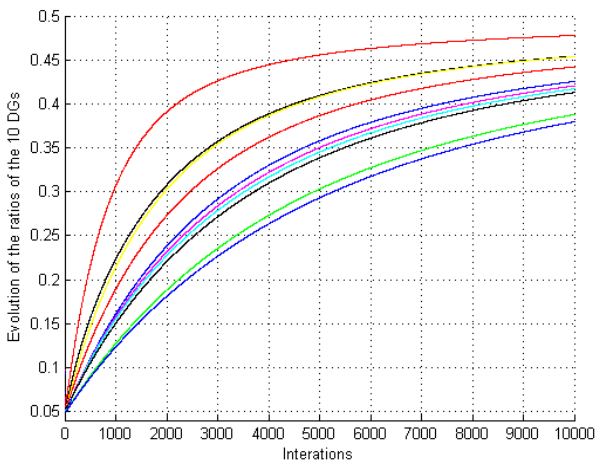

Fig. 1. Evolution of the ratios for 10 DGs in VPP under first topology.

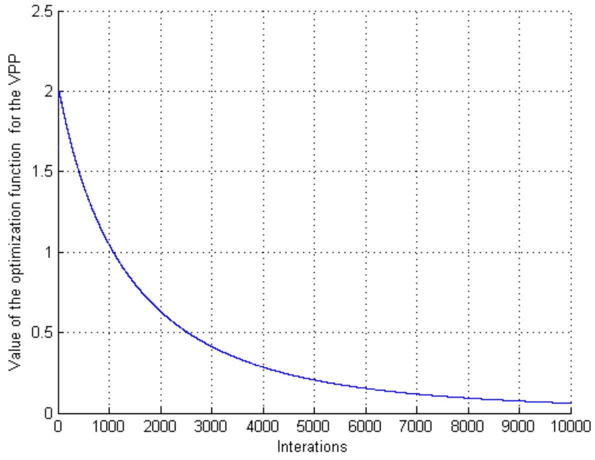

Fig. 2. Value of the optimization function for the VPP under first topology.

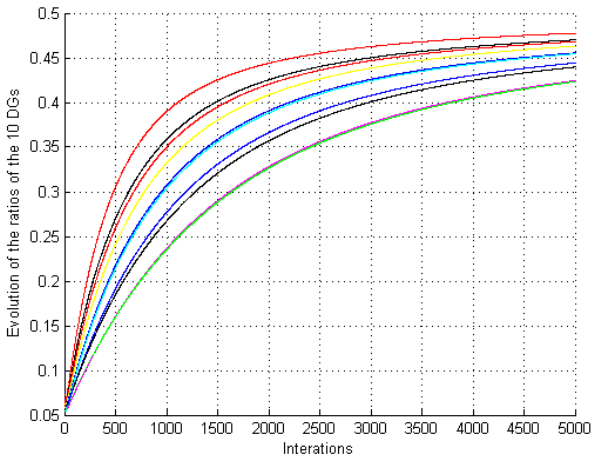

Fig. 3. Evolution of the ratios for 10 DGs in VPP under second topology.

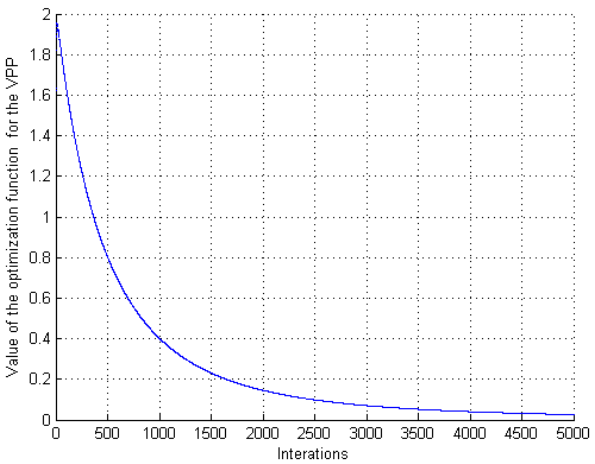

Fig. 4. Value of the optimization function for the VPP under second topology.

beginning. However, it will asymptotically converge to zero, which shows that the designed local objective function for DGs in our game model will ensure the VPP converges to an expected operational point effectively and efficiently.

6. CONCLUSIONS

In this paper, a new theoretical framework for controlling multi-agent system is developed based on the non-cooperative game theory. In order to suppress the aggressive competition among agents and ensure the desired results, a state space is introduced into the existing non-cooperative game model and a novel game named stochastic weakly acyclic game is proposed. It is noticed that the system level objective can be achieved at the points of the corresponding equilibriums of the new game model. Further research direction includes two problems: (i) exploring the influence of parameter α on the system behaviors. (ii) developing a broad class of learning algorithms for stochastic weakly acyclic games to meet the challenges inherent in the multi-agent systems.

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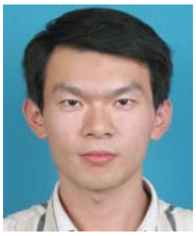

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