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Exponential State Estimation for Stochastic Complex Dynamical Networks with Multi-Delayed Base on Adaptive Control

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Abstract: This paper discusses the exponential state estimation problem for stochastic complex dynamical networks involving multi-delayed and adaptive control. A new approach, very different to the linear matrix inequality (LMI) method, has been developed to solve the above problem. Meanwhile, some sufficient conditions are derived to ensure the exponential stability in pth moment for the dynamics of state estimator error. The feedback gain update law is found by the adaptive control technique. An illustrative example is provided to show the usefulness and effectiveness of the proposed design method.

Keywords: Adaptive control, complex dynamical networks, exponential state estimation, multi-delayed, stochastic noise.

1. INTRODUCTION

Complex networks can be found everywhere in our daily life, such as social networks, electrical power, World Wide Web, disease transmission, and so on. These networks are multi-links, which mean that there are more than one link between two nodes and each of these links has its own property. At the same time, the dynamic behavior of stochastic complex networks contain inherent time delays, which may cause instability or oscillation. Those may lead to the complex networks with multi-delayed. Moreover, "stochastic complex networks" means that inputs and outputs of complex networks evolve and change over time. This kind of complex networks is widely studied by many researchers [1,2].

On the other hand, in practical complex networks, some state variables are unknown and must be estimated. The aim of state estimation is to find an estimation of system quantities (the state) via a set of measured system quantities. Moreover, the adaptive control can help to deliver both stability and good response for systems with variability in parameters that can either be predicted or are uncertain. The goal of adaptive control is to adjust the unknown or changing plant parameters. The adaptive estimation problem has been extensively investigated over the last decade due to their successful applications in many areas (see e.g., [3-10]), such as missile defense system, the Kalman filter, nonlinear systems, etc. In [3], based on the H_∞ performance analysis of this unified model using the LMIs approach, novel state feedback controllers are established not only to guarantee exponentially stable synchronization between two unified models with different initial conditions but also to reduce the effect of external disturbance on the synchronization error to a minimal H_∞ norm constraint. In [9], by employing a Lyapunov-Krasovskii functional, sufficient delay-distribution-dependent conditions are established in terms of LMIs that guarantee the existence of the state estimator which can be checked readily by the Matlab toolbox.

It should be pointed out that, up to now, the problem of adaptive exponential state estimation for stochastic complex dynamical networks with multi-delayed has received very little research attention, which is the motivation of this paper.

The main novelty of our contribution lies in three folds: 1) A new adaptive exponential estimation for stochastic complex dynamical networks with multi-delayed is addressed; 2) Using the adaptive feedback control techniques, several suitable parameters update laws are found; 3) A M-matrix algorithm of the adaptive estimator is given by employing a new nonnegative function.

2. PROBLEM FORMULATION AND PRELIMINARIES

The multi-delayed coupled complex networks [1] can be called drive system and described as follows:

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$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + \sum_{j=1}^{N} (b_{ij})_1 \Gamma x_j(t-\tau_1(t)) + \ldots + \sum_{j=1}^{N} (b_{ij})_m \Gamma x_j(t-\tau_m(t)), \quad \mathrm{i} = 1,2,\cdots,N, \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)]^T \in \mathbb{R}^n$ is the state vector of the ith node, $f(x_i(t)) \in \mathbb{R}^n$ is a nonlinear vector-valued function. $\Gamma = I_n = \mathrm{diag}\{1,1,\cdots,1\} \in \mathbb{R}^{n\times n}$ is a inner-coupling matrix, $A = (a_{ij})^{N\times N} \in \mathbb{R}^{N\times N}$ and $B_l = ((b_{ij})_l)^{N\times N} \in \mathbb{R}^{N\times N}$ are the connection weight and the delayed connection weight matrices, and a_{ij} and $(b_{ij})_l$ are the weight or coupling strength. If there exists a link from node i to j $(i \neq j)$, then $a_{ij} \neq 0$ and $(b_{ij})_l \neq 0$. Otherwise, $a_{ij} = 0$ and $(b_{ij})_l = 0$. $\tau_l(t)$ is the time-varying delay satisfying that $0 < \tau_l(t) \le \overline{\tau}_l$ and $\dot{\tau}_l(t) \le \hat{\tau}_l < 1$, where $\overline{\tau}_l$, $\hat{\tau}_l$ are constants, $l = 1,2,\cdots,m$.

Let $x(t) = [x_1^T(t), x_2^T(t), \cdots, x_N^T(t)]^T \in \mathbb{R}^{n\times N}$, $f(x(t)) = [f^T(x_1(t)), f^T(x_2(t)), \cdots, f^T(x_N(t))]^T$, the drive system (1) can be rewritten as

$$dx(t) = [f(x(t)) + A \otimes I_n x(t) + B_1 \otimes I_n x(t-\tau_1(t)) + \cdots + B_m \otimes I_n x(t-\tau_m(t))]dt. \tag{2}$$

The network measurements are assumed to satisfy

$$y(t) = x(t) + g(x(t)), \tag{3}$$

where $y(t)$ is the measurement output, $g(x(t))$ is the nonlinear disturbances on the complex dynamical networks output.

Based on the drive system (2), we construct the following response system

$$\begin{aligned} d\hat{x}(t) = {} & [f(\hat{x}(t)) + A \otimes I_n \hat{x}(t) + B_1 \otimes I_n \hat{x}(t-\tau_1(t)) \\ & + \cdots + B_m \otimes I_n \hat{x}(t-\tau_m(t)) \\ & + K(\hat{x}(t) + g(\hat{x}(t)) - y(t))]dt \\ & + \sigma(t, \hat{x}(t) - x(t), \hat{x}(t-\tau_1(t)) - x(t-\tau_1(t)), \\ & \cdots, \hat{x}(t-\tau_m(t)) - x(t-\tau_m(t)))dw(t), \end{aligned} \tag{4}$$

where $\hat{x}(t)$ is the state vector of the state estimator (4), $K = \mathrm{diag}\{k_1, k_2, \cdots, k_N\}$ is the estimator gain matrix to be designed. $w(t) = [w_1(t), w_2(t), \cdots, w_n(t)]^T$ is an n-di-men-sional Brown moment defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\ge 0}$, and $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the noise intensity matrix and can be regarded as a result from the occurrence of eternal random fluctuation and other probabilistic causes.

Let $e_i(t) = \hat{x}_i(t) - x_i(t)$, $e(t) = [e_1^T(t), e_2^T(t), \cdots, e_N^T(t)]^T \in \mathbb{R}^{n\times N}$. As a matter of convenience, we mark $e(t-\tau_l(t)) = e_{\tau l}(t)$, $\phi(e(t)) = f(\hat{x}(t)) - f(x(t))$, $\varphi(e(t)) = g(\hat{x}(t)) - g(x(t))$. From the drive system (2) and the state estimator (4), the error system is arranged as

$$\begin{aligned} de(t) = {} & [\phi(e(t)) + A \otimes I_n e(t) + B_1 \otimes I_n e_{\tau_1}(t) \\ & + \cdots + B_m \otimes I_n e_{\tau_m}(t) + K(e(t) + \varphi(e(t)))]dt \\ & + \sigma(t, e(t), e_{\tau_1}(t), \cdots, e_{\tau_m}(t))dw(t). \end{aligned} \tag{5}$$

Next, we firstly introduce some concepts and lemmas which will be used in the proofs of main results.

Assumption 1: The activation functions $f(x(t))$ in (2) and $g(x(t))$ in (3) satisfy the Lipschitz condition. That is to say, there exist constants $L_1 > 0$ and $L_2 > 0$ such that $|f(u) - f(v)| \le L_1 |u - v|$ and $|g(u) - g(v)| \le L_2 |u - v|$, $\forall u, v \in R^n$, respectively.

Assumption 2: The noise intensity matrix $\sigma(\cdot,\cdot,\cdots,\cdot)$ satisfies the linear growth condition. That is to say, there exist positives η, $\lambda_1, \cdots \lambda_m$, such that

$$\mathrm{trace}(\sigma(t, e, e_{\tau_1}, \cdots, e_{\tau_m}))^T (\sigma(t, e, e_{\tau_1}, \cdots, e_{\tau_m})) \le (\eta |e|^2 + \lambda_1 |e_{\tau_1}|^2 + \cdots + \lambda_m |e_{\tau_m}|^2).$$

Definition 1: The trivial solution $e(t, \xi(s))$ of the error system (5) is said to be exponential stability in pth moment if $\limsup_{t\to\infty} \frac{1}{t} \log(\mathcal{E} |e(t, \xi(s))|^p) < 0$, for any $\xi(s) \in L^p_{\mathcal{L}_0}([-\overline{\tau}, 0]; R^n)$, where $p \ge 2$, $p \in Z$. When $p = 2$, it is said to be exponential stability in mean square.

Lemma 1 [11]**:** Consider an n-dimensional stochastic delay differential equation (SDDE, for short)

$$dx(t) = f(t, x(t), x_\tau(t))dt + g(t, x(t), x_\tau(t))d\omega(t) \tag{6}$$

on $t \in [0, \infty)$ with the initial data given by

$$\{x(\theta) : -\overline{\tau} \le \theta \le 0\} = \xi \in L^p_{\mathcal{L}_0}([-\overline{\tau}, 0]; \mathbb{R}^n).$$

If $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$, define an operator $\mathcal{L}$ from $\mathbb{R}_+ \times \mathbb{R}^n$ to $\mathbb{R}$ by

$$\mathcal{L}V(t,x) = V_t(t,x) + V_x(t,x) f(t,x,x_\tau) + (1/2)\,\mathrm{trace}(\mathrm{g}^T(t,x,t_\tau) V_{xx}(t,x) g(t,x,t_\tau)),$$

where

$$V_t(t,x) = \frac{\partial V(t,x)}{\partial t}, \quad V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_j \partial x_k} \right)_{n\times n},$$

$$V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \cdots, \frac{\partial V(t,x)}{\partial x_n} \right).$$

Let $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$ and τ_1, τ_2 be bounded stopping times such that $0 \le \tau_1 \le \tau_2$ a.s. If $V(t, x(t))$ and $\mathcal{L}V(t, x(t))$ are bounded on $t \in [\tau_1, \tau_2]$ with probability 1, then

$$\mathcal{E}V(\tau_2, x(\tau_2)) = \mathcal{E}V(\tau_1, x(\tau_1)) + \mathcal{E}\int_{\tau_1}^{\tau_2} \mathcal{L}V(s, x(s))ds.$$

Lemma 2 [11]**:** Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$. Then $x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y$ for any $\varepsilon > 0$.

Lemma 3 [11]**:** Let $a, b \in \mathbb{R}$ and $\beta \in [0,1]$. Then

$$|a|^\beta |b|^{(1-\beta)} \le \beta |a| + (1-\beta)|b|.$$

Lemma 4 [11]**:** Let $T > 0$ and $u(\cdot)$ be a Borel measurable bounded non-negative function on $[0, T]$. If

$$u(t) \le c + v\int_0^t u(s)ds, \quad \forall 0 \le t \le T,$$

for some constants c and v, then $u(t) \le c\exp(vt)$, $\forall 0 \le t \le T$.

3. MAIN RESULTS

In this section, the criterion of adaptive exponential estimation in pth moment will be obtained for the system (2)-(5).

Theorem 1: Under Assumptions 1-2, the full-order estimator (4) can be adaptive exponential estimated with the original system (2), if the following condition holds

$$\theta > \max_{1\le l\le m} \left\{\frac{1}{1-\hat{\tau}_l}\right\}\sum_{i=1}^n \mu_i, \tag{7}$$

where

$$\theta = p\left(\frac{1}{2}(1+L_1^2)+\alpha+\sum_{i=1}^n \beta_l - \frac{1}{2}(1-L_2^2)H\right) + \frac{1}{2}(p-2)+\frac{1}{2}p(p-1)\eta+\frac{1}{2}(p-1)(p-2)\sum_{i=1}^n \lambda_i,$$

$$\alpha = \lambda_{\max}(A\otimes I_n), \quad \mu_i = (p-1)\lambda_i + 1,$$

$$\beta_l = \lambda_{\max}\left(\frac{1}{2}(B_l\otimes I_n)(B_l\otimes I_n)^T\right).$$

And the feedback gain $K(t)$ with the update law is chosen as

$$\dot{k}_j = -\frac{1}{2}v_j p\,|e|^{p-2}\,[e_j^2 + e_j\varphi(e_j)]. \tag{8}$$

Proof: Choose a non-negative function candidate as

$$V(t,e) = |e|^p + \sum_{j=1}^n \frac{1}{v_j}(k_j+H)^2, \tag{9}$$

where H is a sufficiently large positive constant.

The compute of $\mathcal{L}V(t,e)$ along the solution of error system (5), and using (8) is

$$\begin{aligned}
\mathcal{L}V(t,e) &= V_t(t,e)+V_e(t,e)[\phi(e(t))+A\otimes I_n e(t)\\
&\quad + B_1\otimes I_n e_{\tau 1}(t)+\cdots+B_m\otimes I_n e_{\tau_m}(t)\\
&\quad + K(e(t)+\varphi(e(t)))]\\
&\quad + \frac{1}{2}\mathrm{trace}(\sigma^T(t,e,e_{\tau_1},\cdots,e_{\tau_m})\\
&\quad \cdot V_{ee}(t,e)\sigma(t,e,e_{\tau_1},\cdots,e_{\tau_m}))\\
&= 2\sum_{j=1}^n \frac{1}{v_j}(k_j+H)\dot{k}_j + p\,|e|^{p-2}\,e^T[\phi(e(t))\\
&\quad + A\otimes I_n e(t)+B_1\otimes I_n e_{\tau_1}(t)+\cdots\\
&\quad + B_m\otimes I_n e_{\tau_m}(t)+K(e(t)+\varphi(e(t)))]\\
&\quad + \frac{1}{2}\mathrm{trace}(\sigma^T(t,e,e_{\tau_1},\cdots,e_{\tau_m})\\
&\quad \cdot V_{ee}(t,e)\sigma(t,e,e_{\tau_1},\cdots,e_{\tau_m}))
\end{aligned}$$

$$\begin{aligned}
&= p\,|e|^{p-2}\,e^T[\phi(e(t))+A\otimes I_n e(t)\\
&\quad + B_1\otimes I_n e_{\tau_1}(t)+\cdots\\
&\quad + B_m\otimes I_n e_{\tau_m}(t)-H(e+\varphi(e))]\\
&\quad + \frac{1}{2}\mathrm{trace}(\sigma^T(t,e,e_{\tau_1},\cdots,e_{\tau_m})\\
&\quad \cdot p(p-1)\,|e|^{p-2}\,\sigma(t,e,e_{\tau_1},\cdots,e_{\tau_m})).
\end{aligned} \tag{10}$$

Now, according to Assumptions 1 and 2 together with Lemma 2, one obtains that

$$e^T\phi(e(t)) \le \frac{1}{2}e^Te+\frac{1}{2}\phi^T(e)\phi(e) \le \frac{1}{2}(1+L_1^2)|e|^2, \tag{11}$$

$$e^TA\otimes I_n e \le \alpha|e|^2, \tag{12}$$

$$\begin{aligned}
e^TB_l\otimes I_n e_{\tau_l} &\le \frac{1}{2}e^T(B_l\otimes I_n)(B_l\otimes I_n)^Te+\frac{1}{2}e_{\tau_l}^Te_{\tau_l}\\
&\le \beta_l|e|^2+\frac{1}{2}|e_{\tau_l}|^2,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\frac{1}{2}\mathrm{trace}\Big((\sigma^T(t,e,e_{\tau 1},\cdots,e_{\tau_m})\\
&\quad \cdot p(p-1)\,|e|^{p-2}\,\sigma(t,e,e_{\tau 1},\cdots,e_{\tau_m})\Big)\\
&\le \frac{1}{2}p(p-1)\,|e|^{p-2}\,(\eta|e|^2\\
&\quad + \lambda_1|e_{\tau_1}|^2+\cdots+\lambda_n|e_{\tau_m}|^2),
\end{aligned} \tag{14}$$

$$\begin{aligned}
-e^TH\varphi(e(t)) &\le \frac{1}{2}He^Te+\frac{1}{2}H\varphi^T(e)\varphi(e)\\
&\le \frac{1}{2}H(1+L_2^2)|e|^2.
\end{aligned} \tag{15}$$

Also, Lemma 3 yields

$$|e|^{p-2}|e_{\tau_l}|^2 \le \frac{p-2}{p}|e|^p+\frac{2}{p}|e_{\tau_l}|^p. \tag{16}$$

Substituting (11)-(16) into (10), one gets

$$\begin{aligned}
\mathcal{L}V(t,e) &\le p\,|e|^{p-2}\Big[\frac{1}{2}(1+L_1^2)|e|^2+\alpha|e|^2\\
&\quad +(\beta_1|e|^2+\frac{1}{2}|e_{\tau_1}|^2)+\cdots\\
&\quad +(\beta_m|e|^2+\frac{1}{2}|e_{\tau_m}|^2)\\
&\quad -H|e|^2+\frac{1}{2}H(1+L_2^2)|e|^2\Big]\\
&\quad +\frac{1}{2}p(p-1)\,|e|^{p-2}\,(\eta|e|^2\\
&\quad +\lambda_1|e_{\tau_1}|^2+\cdots+\lambda_n|e_{\tau_m}|^2)\\
&\le \Big[p\Big((\frac{1}{2}(1+L_1^2)+\alpha+\sum_{i=1}^n\beta_l-\frac{1}{2}(1-L_2^2)H\Big)\\
&\quad +\frac{1}{2}(p-2)+\frac{1}{2}p(p-1)\eta
\end{aligned}$$

$$+\frac{1}{2}(p-1)(p-2)\sum_{i=1}^{n}\lambda_i\Bigg]|e|^p$$
$$+((p-1)\lambda_1+1)|e_{\tau_1}|^p$$
$$+\cdots+((p-1)\lambda_n+1)|e_{\tau_m}|^p$$
$$=-\theta|e|^p+\mu_1|e_{\tau_1}|^p+\cdots+\mu_n|e_{\tau_m}|^p. \quad (17)$$

For the function $V(t, e)$, making use of the Lemma 1 and the inequality (7), one can obtain

$$\mathcal{E}|e(t)|^p \le \mathcal{E}V(0,\xi(0))$$
$$+\mathcal{E}\int_0^t \mathcal{L}V(s,e(s),e_{\tau_1}(s),\cdots,e_{\tau_m}(s))ds$$
$$\le \mathcal{E}V(0,\xi(0))$$
$$+\mathcal{E}\int_0^t(-\theta|e|^p+\mu_1|e_{\tau_1}|^p+\cdots+\mu_n|e_{\tau_m}|^p)ds.$$

For $\int_0^t|e_{\tau_l}|^p\,ds$, let $u=s-\tau_l(s)$, then $du=(1-\dot{\tau}_l(s))ds$ and

$$\int_0^t|e_{\tau_l}(s)|^p\,ds=\int_{-\tau_l(0)}^{t-\tau_l(t)}\frac{1}{1-\dot{\tau}(s)}|e(s)|^p\,ds$$
$$\le\frac{1}{1-\hat{\tau}_l}\int_{-\bar{\tau}_l}^{t}|e(s)|^p\,ds$$
$$=\frac{1}{1-\hat{\tau}_l}\int_{-\bar{\tau}_l}^{0}|e(s)|^p\,ds+\frac{1}{1-\hat{\tau}_l}\int_0^t|e(s)|^p\,ds$$
$$\le\frac{\bar{\tau}_l}{1-\hat{\tau}_l}\max_{\bar{\tau}_l\le s\le 0}|\xi(s)|^p+\frac{1}{1-\hat{\tau}_l}\int_0^t|e(s)|^p\,ds$$
$$\le\max_{1\le l\le m}\left(\frac{\bar{\tau}_l}{1-\hat{\tau}_l}\right)\max_{\bar{\tau}_l\le s\le 0}|\xi(s)|^p$$
$$+\max_{1\le l\le m}\left(\frac{1}{1-\hat{\tau}_l}\right)\int_0^t|e(s)|^p\,ds.$$

Therefore

$$\mathcal{E}|e|^p\le\mathcal{E}V(0,\xi(0))+\max_{1\le l\le m}\left(\frac{\bar{\tau}_l}{1-\hat{\tau}_l}\right)\sum_{i=1}^{n}\mu_i\max_{\bar{\tau}_l\le s\le 0}|\xi(s)|^p$$
$$+\int_0^t\left[-\delta+\max_{1\le l\le m}\left(\frac{1}{1-\hat{\tau}_l}\right)\sum_{i=1}^{n}\mu_i\right]\mathcal{E}|e(s)|^p\,ds,$$

that is to say

$$\mathcal{E}|x|^p\le c+\int_0^t v\mathcal{E}|x|^p\,ds,$$

where

$$c=\mathcal{E}V(0,\xi(0))+\max_{1\le l\le m}\left(\frac{\bar{\tau}_l}{1-\hat{\tau}_l}\right)\sum_{i=1}^{n}\mu_i\max_{\bar{\tau}_l\le s\le 0}|\xi(s)|^p,$$
$$v=-\delta+\max_{1\le l\le m}\left(\frac{1}{1-\hat{\tau}_l}\right)\sum_{i=1}^{n}\mu_i.$$

It can be seen that c, v are constants, where $c>0$ and $v<0$. By Lemma 4, one gets $\mathcal{E}|x|^p\le c\exp(vt)$.

So $\limsup_{t\to\infty}\frac{1}{t}\log(\mathcal{E}|e(t,\xi)|^p)\le c<0$. Thereby, the error system (5) is exponential stability in pth moment. The proof is completed.

Remark 1: In Theorem 1, the condition (7) of the adaptive exponential state estimation for stochastic complex dynamical networks with multi-delayed obtained by using new method is delay-dependent and very different to those, such as linear matrix inequality method. And the condition can be checked if the drive system and the response system are given.

4. ILLUSTRATIVE EXAMPLE

In this section, an illustrative example will be given to demonstrate the effectiveness of the proposed methods.

Example 1: The Rössler system is described by

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}-x_2(t)-x_3(t)\\ x_1(t)+ax_2(t)\\ b+(x_1(t)-c)x_3(t)\end{bmatrix},$$

where $a=0.2$, $b=0.2$, $c=5.7$.

The network (drive system $\dot{x}_{i1}$, $\dot{x}_{i2}$, $\dot{x}_{i3}$ and state estimator $\dot{\hat{x}}_{i1}$, $\dot{\hat{x}}_{i2}$, $\dot{\hat{x}}_{i3}$) with four nodes described as follows

$$\dot{x}_{i1}=-x_{i2}(t)-x_{i3}(t)+\sum_{j=1}^{4}a_{ij}x_{j1}(t)$$
$$+\sum_{j=1}^{4}(b_{ij})_1x_{j1}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2x_{j1}(t-\tau_2),$$
$$\dot{x}_{i2}=-x_{i1}(t)+ax_{i2}(t)+\sum_{j=1}^{4}a_{ij}x_{j2}(t)$$
$$+\sum_{j=1}^{4}(b_{ij})_1x_{j2}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2x_{j2}(t-\tau_2),$$
$$\dot{x}_{i3}=b+(x_{i1}(t)-c)x_{i3}(t)+\sum_{j=1}^{4}a_{ij}x_{j3}(t)$$
$$+\sum_{j=1}^{4}(b_{ij})_1x_{j3}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2x_{j3}(t-\tau_2).$$
$$\dot{\hat{x}}_{i1}=-\hat{x}_{i2}(t)-\hat{x}_{i3}(t)+\sum_{j=1}^{4}a_{ij}\hat{x}_{j1}(t)$$
$$+\sum_{j=1}^{4}(b_{ij})_1\hat{x}_{j1}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2\hat{x}_{j1}(t-\tau_2)$$
$$+k^i[\hat{x}_{i1}-x_{i1}+\tanh(\hat{x}_{i1})-\tanh(x_{i1})]$$
$$+0.4[\hat{x}_{i1}(t)-x_{i1}(t)]$$
$$+0.3[\hat{x}_{i1}(t-\tau_1)-x_{i1}(t-\tau_1)],$$
$$\dot{\hat{x}}_{i2}=-\hat{x}_{i1}(t)+a\hat{x}_{i2}(t)+\sum_{j=1}^{4}a_{ij}\hat{x}_{j2}(t)$$
$$+\sum_{j=1}^{4}(b_{ij})_1\hat{x}_{j2}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2\hat{x}_{j2}(t-\tau_2)$$

$$
\begin{aligned}
&+k^i[\hat{x}_{i2}-x_{i2}+\tanh(\hat{x}_{i2})-\tanh(x_{i2})]\\
&+0.3[\hat{x}_{i2}(t)-x_{i2}(t)]\\
&+0.4[\hat{x}_{i2}(t-\tau_2)-x_{i1}(t-\tau_2)],
\end{aligned}
$$

$$
\begin{aligned}
\dot{\hat{x}}_{i3} &= b+(\hat{x}_{i1}(t)-c)\hat{x}_{i3}(t)+\sum_{j=1}^{4}a_{ij}\hat{x}_{j3}(t)\\
&+\sum_{j=1}^{4}(b_{ij})_1\hat{x}_{j3}(t-\tau_1)+\sum_{j=1}^{4}(b_{ij})_2\hat{x}_{j3}(t-\tau_2)\\
&+k^i[\hat{x}_{i3}-x_{i3}+\tanh(\hat{x}_{i3})-\tanh(x_{i3})]\\
&+0.3[\hat{x}_{i3}(t-\tau_1)-x_{i3}(t-\tau_1)]\\
&+0.5[\hat{x}_{i3}(t-\tau_2)-x_{i3}(t-\tau_2)].
\end{aligned}
$$

In the simulation, let

$$
A=\begin{bmatrix}-6 & 2 & 1 & 3\\ 2 & -5 & 2 & 1\\ 0 & 1 & -1 & 0\\ 3 & 1 & 0 & -4\end{bmatrix},\quad B_1=\begin{bmatrix}-8 & 3 & 1 & 4\\ 3 & -5 & 0 & 2\\ 1 & 0 & -3 & 2\\ 1 & 3 & 2 & -6\end{bmatrix},
$$

$$
B_2=\begin{bmatrix}-2 & 1 & 1 & 0\\ 2 & -3 & 0 & 1\\ 1 & 0 & -2 & 1\\ 0 & 2 & 2 & -4\end{bmatrix},\quad \tau_1=0.1,\quad \tau_2=0.2,\quad p=2.
$$

These parameters fully satisfy Assumptions 1 and 2, the condition (7). Therefore, the full-order estimator (4) can be adaptive exponential estimated with the original system (2) by Theorem 1.

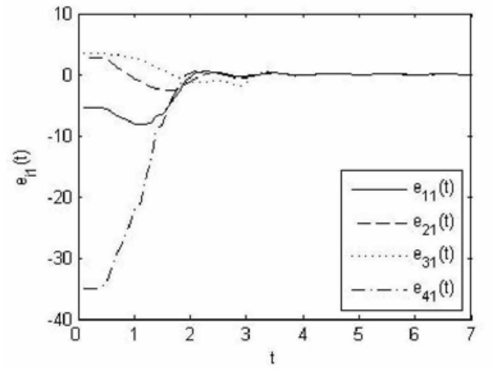

Fig. 1. The error states of complex network $e_{i1}(t)$.

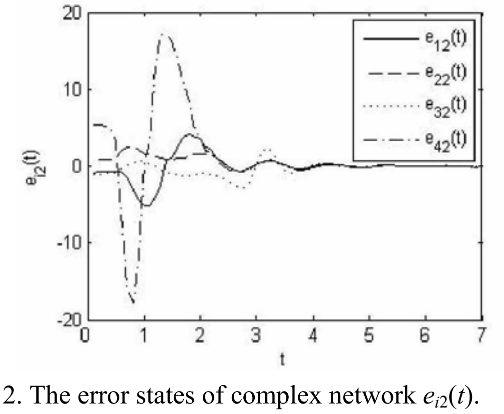

Fig. 2. The error states of complex network $e_{i2}(t)$.

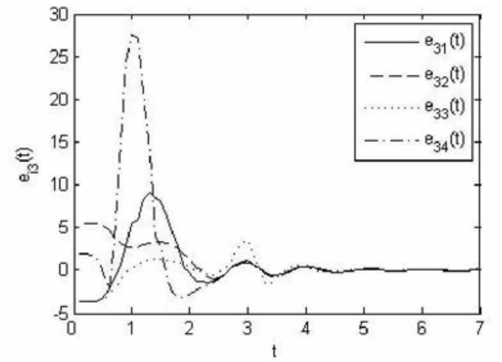

Fig. 3. The error states of complex network $e_{i3}(t)$.

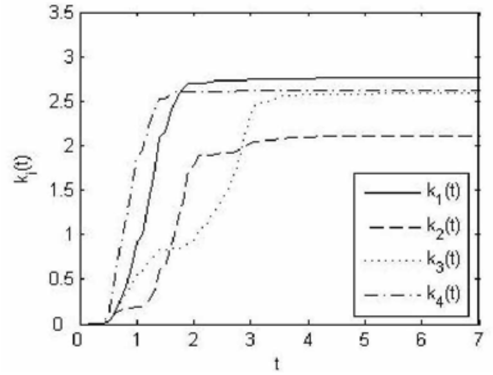

Fig. 4. The feedback gain.

To illustrate the effectiveness of the developed theory, we employ the non-negative function to solve the solutions for delayed neural networks and to simulate the dynamics of error system and the adaptive feedback gain. The simulation figures are shown in Fig. 1-4. Among them, Figs. 1-3 plot the error states of complex network $e_{i1}(t)$, $e_{i2}(t)$ and $e_{i3}(t)$. Fig. 4 depicts the adaptive feedback gain. All these figures show us that the stochastic complex dynamical networks with multi-delayed is an estimation.

5. CONCLUSIONS

In this paper, we have dealt with the problem of the exponential state estimation for stochastic complex dynamical networks with multi-delayed base on adaptive control. The traditional monotonicity and smoothness assumptions on the activation function have been removed. A new approach has been developed to solve this problem. The conditions for the adaptive exponential state estimation have been derived in terms of some algebraical inequalities. These state estimation conditions are much different to those of linear matrix inequality.

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