International Journal of Control, Automation, and Systems (2014) 12(5):930-938
DOI 10.1007/s12555-013-0309-0
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Consensus for Double-Integrator Dynamics with Velocity Constraints

Tales A. Jesus*, Luciano C. A. Pimenta, Leonardo A. B. Tôrres, and Eduardo M. A. M. Mendes

Abstract: The problem of consensus for double-integrator dynamics with velocity constraints and a constant group reference velocity is addressed such that: (i) the control law of an agent does not depend on the local neighbors' velocities or accelerations, but only on the neighbors' positions and on the own agent velocity; (ii) the constraints are non-symmetric; (iii) the class of nonlinear functions used to account for the velocity constraints is more general than the ones that are normally considered in the literature. We propose a decentralized control strategy with the neighboring topology described by an undirected interaction graph that is connected. Mathematical guarantees of convergence without violating the constraints are given. A numerical experiment is provided to illustrate the effectiveness of our approach.

Keywords: Consensus, multi-agent systems, second-order dynamics, velocity constraints.

1. INTRODUCTION

1.1. Literature review

Coordination of multiple agents has received great attention in the last years due to its potential to solve problems in different applications. In the context of mobile robotics, some examples are surveillance and boundary coverage [1], forest fire monitoring [2], convoy protection for unmanned ground vehicles [3], and target localization and encirclement with a multi-robot system [4]. Despite the progress in the area, many of these and other theoretical works systematically ignored that the majority of physical system exhibits constraints in the inputs and in the state variables. Take, for instance, two important papers that dealt with the application of consensus algorithms [5] and passivity [6] in group coordination. They do not consider the presence of constraints in the inputs or in the state variables of the agents. published recently dealing with double integrator dynamics.

Similarly to what was proposed in [5], several consensus algorithms to attain multi-agent coordination were developed (see [7] and references therein). The basic idea of consensus is to make a team of agents reach an agreement on a common value by exchanging information with neighbors. Such a problem is normally addressed without considering constraints in the inputs or in the state variables [8-12]. It was shown in [13] that a consensus protocol designed without considering saturation constraints still works when the saturation is present, however such an approach is restricted to first order multi-agent systems with symmetrical saturation constraints.

Concerning second-order consensus of multi-agent systems, many works have been published recently. Some of them address the consensus problem with switching communication topologies [14-18]. Some other works focus on strategies that take into account communication constraints between agents, such as time-delays [19-24]. Finite-time consensus algorithms have also received attention in the scientific literature [25-27], and in order to deal with disturbances, some robust strategies were also developed by many research groups [16,27-29]. All of these aspects are extremely important and deserve to be taken into consideration in the development of consensus algorithms. However, none of the above publications addressed the effects of constraints in the agent input/state variables, which is also a relevant point that must not be neglected, since almost all physical systems in the real world exhibit constraints in the inputs or in the state variables.

Hu *et al.* [30] proposed a second-order consensus algorithm that takes into account the presence of unknown but bounded disturbance in the velocity measurements and the presence of symmetric constraints in the inputs (accelerations). However, they did not consider asymmetric constraints in the state variable velocity as we do in the present work.

Although the so-called *constrained consensus problem* was addressed in the literature, the authors of [31-33] have studied only the case of discrete dynamical systems.

Manuscript received June 27, 2013; revised December 21, 2013; accepted March 27, 2014. Recommended by Associate Editor Tae-Hyoung Kim under the direction of Editor Hyungbo Shim.

This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG) and Financiadora de Estudos e Projetos (FINEP) - Brazil.

Tales A. Jesus is with the Computation Department, Centro Federal de Educação Tecnológica de Minas Gerais, Av. Amazonas, 7675 - Gameleira, 30510-000, Belo Horizonte (MG), Brazil (e-mail: talesargolo@decom.cefetmg.br).

Luciano C. A. Pimenta, Leonardo A. B. Tôrres, and Eduardo M. A. M. Mendes are with the School of Engineering, Universidade Federal de Minas Gerais, Av. Antônio Carlos, 6627, Pampulha, 31270-901, Belo Horizonte (MG), Brazil (e-mails: {lucpim, torres}@cpdee.ufmg.br, emammendes@gmail.com).

* Corresponding author.

© ICROS, KIEE and Springer 2014

An interesting work in which the continuous case was discussed is [34], however the authors have considered only first-order dynamics and have not taken into account the possibility of defining a group reference velocity, since the final state reached by the agents depends on the initial conditions.

There are some works in which symmetric acceleration input saturations have been investigated [35-40]. In [35] the author proposes some consensus algorithms for double integrator-dynamics considering limits in the acceleration control inputs and a possibly time-varying group reference velocity, among other aspects. Most part of the results in consensus usually implies that, after convergence, all agents reach the same position and the same velocity. However, there are some extensions based on such algorithms that have as a goal to obtain collective motions including rendezvous and circular patterns [41-45]. Note, however, that these extensions did not consider the presence of input saturations or bounds in the values of the state variables.

1.2. Statement of contributions

In this paper we propose a methodology for consensus of multiple agents that are described as double integrators. Differently from the above-cited papers, particularly [35,37,39,40], that consider the presence of symmetric constraints in the acceleration input, we consider non-symmetric constraints in the velocity state variable. In [37] the authors proposed a consensus strategy for double integrator dynamics with input saturation (constrained acceleration) and demonstrated that the velocity of each agent is bounded. However, they do not consider explicit limits for the velocities as we do in the present work. In addition, our approach allows for a different class of nonlinear functions to be used to account for the velocity saturations (see Section 2), since they do not have to be odd functions (as in [35,37,39]), and they do not have to be differentiable (as in [40]).

The non-symmetric velocity constraints are important when dealing, for example, with the problem of coordinating fixed-wing unmanned aerial vehicles (UAVs) that have minimum and maximum positive limits for their forward velocities. Therefore, the simple model used in this work could be directly used as a reference model to be followed in the context of designing UAVs guidance and navigation strategies aiming to the coordination of such vehicles [46]. For example, our approach could be applied to the problem of making n UAVs, each one flying in a straight line and distributed in parallel and adjacent lanes, span a rectangular area with constant velocity and paired side by side, without violating the velocity constraints. The accomplishment of this task is schematically illustrated in Fig. 1.

Besides, we consider that the control law of each agent does not depend on its local neighbor's velocities or accelerations, differently from the consensus algorithm with input constraints and with group reference position, velocity and acceleration presented in [35], for example.

Finally, we provide mathematical guarantees that the proposed strategy will make the multi-agent system reach consensus asymptotically with a desired group velocity without violating the constraints.

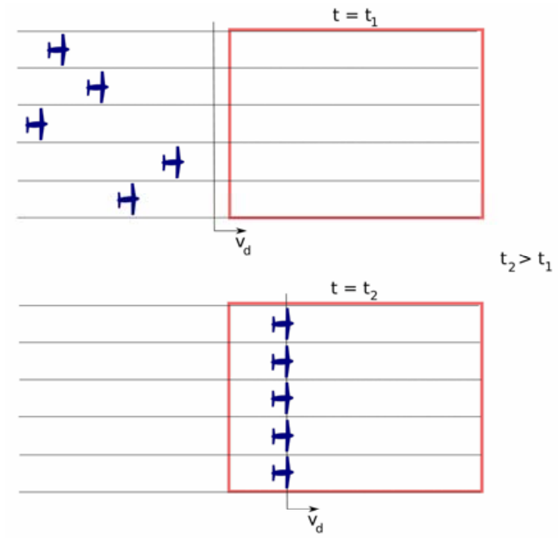

Fig. 1. Coordination of n UAVs in order to span a rectangular area.

1.3. Organization

This paper is organized as follows: In Section 2 some background on graph theory is presented and the consensus problem to be solved is stated. Section 3 shows the consensus strategy itself. A numerical experiment is shown and analysed in Section 4. Finally, in Section 5 the conclusions are presented.

2. PROBLEM FORMULATION

2.1. Background

It is common to model the information exchange topology in a multi-agent system by using graphs. In this paper we consider that if agent i can obtain information from agent j, then agent j can also obtain information from agent i. Such a communication scheme can be described by a graph that contains both edges (i, j) and (j, i), i.e., by an undirected graph [47]. So it is important to establish some notions on undirected graphs. This section is strongly based on [35].

A graph consists of a vertex set or node set $\mathcal{V} = \{1,\ldots,n\}$, an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and an adjacency matrix $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with zeros in the diagonal ($a_{ii} = 0, i \in \{1,...,n\}$). If (i, j) is an edge of an undirected graph, i.e., $(i, j) \in \mathcal{E} \Rightarrow (j,i) \in \mathcal{E}$, it means that agents i and j communicate with each other, and that $a_{ij} = a_{ji} = 1$. Otherwise, $a_{ij} = a_{ji} = 0$. According to that, the adjacency matrix $\mathbf{A}$ of an undirected graph is always symmetric.

A path is a sequence of edges on a graph of the form $(i_1, i_2), (i_2, i_3), \ldots,$ where $i_j \in \mathcal{V}$. We say that an undirected graph is *connected* if there is a path between every pair of different nodes.

The Laplacian matrix $\mathbf{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathbf{A}$ is defined as

$$l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}, \quad l_{ij} = -a_{ij}, \quad i \neq j. \tag{1}$$

The Laplacian matrix $\mathbf{L}$ of an undirected graph is symmetric positive semidefinite. It is always true that 0 is an eigenvalue of $\mathbf{L}$ associated with the eigenvector $\mathbf{1}_{n\times 1}$. Another fact concerning undirected graphs is that 0 is a simple eigenvalue of $\mathbf{L}$ and all of the other eigenvalues are positive if and only if the undirected graph is connected.

2.2. Problem Statement

In this work, we consider a group of n agents with dynamic behavior described by the following model:

$$\begin{aligned}\dot{\mathbf{r}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= \mathbf{u}_i,\end{aligned} \tag{2}$$

where $\mathbf{r}_i \in \mathbb{R}^m$ and $\mathbf{v}_i \in \mathbb{R}^m$ are, respectively, the position and the velocity of the ith agent, and $\mathbf{u}_i \in \mathbb{R}^m$ is the acceleration control input. The agents are subjected to the following constraint:

$$\mathbf{v}_{min} \le \mathbf{v}_i \le \mathbf{v}_{max}, \tag{3}$$

where $\mathbf{v}_{min} \in \mathbb{R}^m$ and $\mathbf{v}_{max} \in \mathbb{R}^m$ are, respectively, the minimum and the maximum constant limits in the velocity, and the operation represented by '$\le$' is defined component-wise, that is, $[x_1 \cdots x_m]^T \le [y_1 \cdots y_m]^T \to x_1 \le y_1, \cdots, x_m \le y_m$, where $x_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$. Differently from previous work [13], it should be emphasized that (3) is a non-symmetric constraint, i.e., $\mathbf{v}_{min} \neq -\mathbf{v}_{max}$. We will define a control law of the form

$$\mathbf{u}_i = -\mathbf{K}(\mathbf{v}_i - \mathbf{v}_{ci}), \tag{4}$$

where $\mathbf{K} \in \mathbb{R}^{m\times m}$ is a positive-definite diagonal matrix and $\mathbf{v}_{ci}$ is a virtual control input that is, essentially, a velocity command. The great motivation for considering such a control law is the presence of the velocity-dependent term $-\mathbf{K}\mathbf{v}_i$ that implies a damping behavior in the velocity dynamics. Therefore, the dynamical behavior of an agent can be described by

$$\begin{aligned}\dot{\mathbf{r}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= -\mathbf{K}(\mathbf{v}_i - \mathbf{v}_{ci}),\end{aligned} \tag{5}$$

and now there is a first-order differential equation that relates $\mathbf{v}_i$ to the virtual input $\mathbf{v}_{ci}$. The following theorem can now be stated:

Theorem 1: Consider the dynamical system

$$\dot{\mathbf{v}}_i = -\mathbf{K}(\mathbf{v}_i - \mathbf{v}_{ci}). \tag{6}$$

If the constraints

$$\begin{cases}\mathbf{v}_{min} \le \mathbf{v}_i(0) \le \mathbf{v}_{max} \\ \mathbf{v}_{min} \le \mathbf{v}_{ci} \le \mathbf{v}_{max}\end{cases} \tag{7}$$

are always satisfied, then the constraint

$$\mathbf{v}_{min} \le \mathbf{v}_i \le \mathbf{v}_{max}$$

will always be satisfied.

Proof: The solution of (6) is

$$\mathbf{v}_i(t) = e^{-\mathbf{K}t}\mathbf{v}_i(0) + \int_0^t e^{-\mathbf{K}(t-\tau)}\mathbf{K}\mathbf{v}_{ci}(\tau)d\tau,$$

which implies that

$$\begin{cases}\mathbf{v}_i(t) \le e^{-\mathbf{K}t}\mathbf{v}_{max} + \int_0^t e^{-\mathbf{K}(t-\tau)}d\tau\,\mathbf{K}\max\{\mathbf{v}_{ci}\}, \\ \mathbf{v}_i(t) \ge e^{-\mathbf{K}t}\mathbf{v}_{min} + \int_0^t e^{-\mathbf{K}(t-\tau)}d\tau\,\mathbf{K}\min\{\mathbf{v}_{ci}\}.\end{cases} \tag{8}$$

Replacing

$$\int_0^t e^{-\mathbf{K}(t-\tau)}d\tau = \left(\mathbf{I} - e^{-\mathbf{K}t}\right)\mathbf{K}^{-1},$$

$$\max\{\mathbf{v}_{ci}\} = \mathbf{v}_{max}, \quad \text{and} \quad \min\{\mathbf{v}_{ci}\} = \mathbf{v}_{min},$$

into (8), we have that

$$\mathbf{v}_{min} \le \mathbf{v}_i(t) \le \mathbf{v}_{max}.$$

It should be noted from (3), (4), and (7) that $\|\mathbf{u}_i\|_\infty \le \|\mathbf{K}\|_\infty \|\mathbf{v}_{max} - \mathbf{v}_{min}\|_\infty$. It means that we can also impose an acceleration constraint to the control strategy by choosing any matrix $\mathbf{K}$ such that $\|\mathbf{K}\|_\infty < \infty$.

To account for the velocity saturations, the command $\mathbf{v}_{ci}$ should be designed based on a continuous function $\sigma : \mathbb{R} \mapsto (-1,1)$ that satisfies the following properties:

P1. $\lim_{x\to -\infty} \sigma(x) = -1$;
P2. $\lim_{x\to \infty} \sigma(x) = 1$;
P3. $\sigma(0) = 0$;
P4. $\sigma(x)$ is a strictly increasing function.

It is immediate from property P4 that $\sigma(x)$ is an invertible function. In addition, if we define the function

$$\gamma(x) = \int_0^x \sigma(\xi)d\xi, \tag{9}$$

we have that $\gamma(x) \ge 0, \ \forall x \in \mathbb{R}$.

Finally, the problem to be solved can be stated as:

Given a team of n agents with individual dynamical behavior described by (2), each one under the actuation of the control input $\mathbf{u}_i$ *given by (4), find a distributed virtual control law* $\mathbf{v}_{ci}$ *for each agent, relying only on the neighbor's positions, that makes this multi-agent system, without violating the constraints (7), reach consensus with a group reference velocity* $\mathbf{v}_d$ *that satisfies* $\mathbf{v}_{min} \le \mathbf{v}_d \le \mathbf{v}_{max}$, *that is, for all* $\mathbf{r}_i(0)$ *and admissible* $\mathbf{v}_i(0)$, $\mathbf{r}_i(t) \to \mathbf{r}_j(t)$ *and* $\mathbf{v}_i(t) \to \mathbf{v}_d$ *asymptotically as* $t \to \infty$.

3. CONSENSUS STRATEGY

Consider the following error variable associated with the i-th agent:

$$\mathbf{e}_i = \sum_{j=1}^{n} a_{ij}\mathbf{r}_{ij}, \qquad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j. \tag{10}$$

Writing the above definition for all n agents, we obtain the expression

$$\mathbf{e} = (\mathbf{L} \otimes \mathbf{I}_m)\mathbf{r}, \tag{11}$$

where $\mathbf{e} = [\mathbf{e}_1^T, \cdots, \mathbf{e}_n^T]^T$, $\mathbf{r} = [\mathbf{r}_1^T, \cdots, \mathbf{r}_n^T]^T$, and $\otimes$ denotes the Kronecker product operator. In this case:

$$\begin{aligned}\sum_{i=1}^{n} \mathbf{e}_i &= (\mathbf{1}_{1\times n} \otimes \mathbf{I}_m)\mathbf{e} \\ &= (\mathbf{1}_{1\times n} \otimes \mathbf{I}_m)(\mathbf{L} \otimes \mathbf{I}_m)\mathbf{r} \\ &= (\mathbf{1}_{1\times n}\mathbf{L} \otimes \mathbf{I}_m)\mathbf{r},\end{aligned} \tag{12}$$

where $\mathbf{1}_{p_1\times q_1}$ is a matrix of ones with p_1 rows and q_1 columns.

Since $\mathbf{1}_{1\times n}\mathbf{L} = \mathbf{0}_{1\times m}$, where $\mathbf{0}_{p_0\times q_0}$ is a matrix of zeros with p_0 rows and q_0 columns, we have that

$$\sum_{i=1}^{n} \mathbf{e}_i = \mathbf{0}_{m\times 1}. \tag{13}$$

Note that in the unidimensional case $(m = 1)$, (11) would reduce to $\mathbf{e} = \mathbf{L}\mathbf{r}$, and (12) and (13) would reduce to $\sum_{i=1}^{n} e_i = \mathbf{1}_{1\times n}\mathbf{e} = \mathbf{1}_{1\times n}\mathbf{L}\mathbf{r} = 0$, where $\mathbf{e} = [e_1, \cdots, e_n]^T \in \mathbb{R}^n$.

Finally, let the neighboring topology be described by an undirected interaction graph $G(\mathscr{V}, \mathscr{E})$ that is connected. We will also need the definition of $\mathscr{N}_i$, which is the set of the i-th agent's neighbors. The cardinality of $\mathscr{N}_i$ is represented by $|\mathscr{N}_i|$.

Based on the above setting, the following lemmas can be stated:

Lemma 1: Let $\mathbf{w}_i \in \mathbb{R}^{m\times 1}$, $i = 1, 2, \ldots, n$, and define $\mathbf{w} = [\mathbf{w}_1^T\ \mathbf{w}_2^T \cdots \mathbf{w}_n^T]^T$. If $\mathbf{L}$ is the Laplacian matrix associated with a connected interaction graph, then

$$\left(\mathbf{L} \otimes \mathbf{I}_m\right)\mathbf{w} = \mathbf{0}_{mn\times 1} \quad \Leftrightarrow \quad \mathbf{w}_1 = \mathbf{w}_2 = \cdots = \mathbf{w}_n.$$

Proof: First notice that

$$\left(\mathbf{L} \otimes \mathbf{I}_m\right)\mathbf{w} = \left(\mathbf{L} \otimes \mathbf{I}_m\right)\mathrm{vec}\left(\mathbf{W}\right),$$

where the operator $\mathrm{vec}(\mathbf{W})$ represents the vertical stacking of the columns of the matrix $\mathbf{W}$, with $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_n] \in \mathbb{R}^{m\times n}$. Using the fact [48], valid for any matrices $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times l}$ and $\mathbf{C} \in \mathbb{R}^{l\times k}$, that

$$(\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B}) = \mathrm{vec}(\mathbf{ABC}),$$

it follows that

$$\begin{aligned}&(\mathbf{L} \otimes \mathbf{I}_m)\mathrm{vec}(\mathbf{W}) = \mathbf{0}_{mn\times 1} \Leftrightarrow \mathrm{vec}(\mathbf{I}_m\mathbf{W}\mathbf{L}^T) = \mathbf{0}_{mn\times 1}, \\ &\Leftrightarrow \mathbf{W}\mathbf{L}^T = \mathbf{0}_{m\times n} \Leftrightarrow \mathbf{L}\mathbf{W}^T = \mathbf{0}_{n\times m}, \\ &\Leftrightarrow \mathbf{L}\begin{bmatrix}\mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \vdots \\ \mathbf{w}_n^T\end{bmatrix} = \mathbf{0}_{n\times m}, \\ &\Leftrightarrow \mathbf{L}\begin{bmatrix}w_{j1} \\ w_{j2} \\ \vdots \\ w_{jn}\end{bmatrix} = \mathbf{0}_{n\times 1}; \quad j = 1, 2, \ldots, m.\end{aligned}$$

Since the Laplacian matrix $\mathbf{L}$ is associated with a connected interaction graph, the last equation leads to the conclusion that

$$\begin{bmatrix}w_{j1} \\ w_{j2} \\ \vdots \\ w_{jn}\end{bmatrix} = \alpha_j \mathbf{1}_{n\times 1}; \quad j = 1, 2, \ldots, m;$$

for constants $\alpha_j \in \mathbb{R}$, and therefore

$$w_{j1} = \ldots = w_{jn}; \quad j = 1, 2, \ldots, m \quad \Leftrightarrow \quad \mathbf{w}_1 = \cdots = \mathbf{w}_n.$$

Note that in the unidimensional case $(m = 1)$, Lemma 1 would reduce to $\mathbf{L}\mathbf{w} = \mathbf{0}_{n\times 1} \Leftrightarrow w_1 = w_2 = \cdots = w_n$, where $\mathbf{w} = [w_1, \cdots, w_n]^T \in \mathbb{R}^n$, and this is a result that comes directly from the fact that the null space of the Laplacian associated with a connected graph is $\beta\mathbf{1}_{n\times 1}$, $\beta \in \mathbb{R}$.

Lemma 2: Let the set of vector functions $\mathbf{g}_{ij} : \mathbb{R}^{m(|\mathscr{N}_i| + |\mathscr{N}_j|)} \mapsto \mathbb{R}^m$ be

$$\mathbf{g}_{ij} = a_{ij}\left[\sigma\left(\mathbf{K}_e \sum_{k=1}^{n} a_{jk}\mathbf{r}_{jk} + \mathbf{c}\right) - \sigma\left(\mathbf{K}_e \sum_{l=1}^{n} a_{il}\mathbf{r}_{il} + \mathbf{c}\right)\right]$$

for $(i, j) \in \mathscr{E}$, where $\mathbf{K}_e$ is a positive-definite diagonal matrix, $\mathbf{c} \in \mathbb{R}^m$, and $\sigma(\cdot)$ is an invertible function defined component-wise according to Properties P1-P4. Then,

$$\mathbf{g}_{ij} = \mathbf{0}_{m\times 1} \quad \Leftrightarrow \quad \mathbf{r}_{ij} = \mathbf{0}_{m\times 1}; \quad (i, j) \in \mathscr{E}.$$

Proof: Notice that

$$\mathbf{g}_{ij} = a_{ij}(\mathbf{w}_j - \mathbf{w}_i)$$

with

$$\mathbf{w}_j = \sigma\left(\mathbf{K}_e \sum_{k=1}^{n} a_{jk}\mathbf{r}_{jk} + \mathbf{c}\right), \text{ and } \mathbf{w}_i = \sigma\left(\mathbf{K}_e \sum_{l=1}^{n} a_{il}\mathbf{r}_{il} + \mathbf{c}\right).$$

If $\mathbf{g}_{ij} = \mathbf{0}_{m\times 1}$, $\forall (i, j) \in \mathscr{E}$, and using the fact that $a_{ij} = 0$, $\forall (i, j) \notin \mathscr{E}$, one has that

$$\sum_{i=1}^{n} \mathbf{g}_{ij} = \sum_{i=1}^{n} a_{ij}\left(\mathbf{w}_j - \mathbf{w}_i\right) = \mathbf{0}_{m\times 1} \Leftrightarrow \left(\mathbf{L} \otimes \mathbf{I}_m\right)\mathbf{w} = \mathbf{0}_{mn\times 1};$$

with $\mathbf{w} = [\mathbf{w}_1^T\ \mathbf{w}_2^T \cdots \mathbf{w}_n^T]^T$. In this case, from Lemma 1 it follows that $\mathbf{w}_1 = \mathbf{w}_2 = \cdots = \mathbf{w}_n$. Since $\sigma(\cdot)$ is an invertible function, and $\mathbf{K}_e$ is an invertible matrix, one has that

$$\begin{aligned}\mathbf{w}_j - \mathbf{w}_i = \mathbf{0}_{m\times 1} \quad &\Leftrightarrow \quad \sum_{k=1}^{n} a_{jk}\mathbf{r}_{jk} = \sum_{l=1}^{n} a_{il}\mathbf{r}_{il}, \\ &\Leftrightarrow \quad \mathbf{e}_j = \mathbf{e}_i,\end{aligned}$$

for $i=1,2,\ldots,n$ and $j=1,2,\ldots,n$. Using the fact that $\sum_{i=1}^{n}\mathbf{e}_i=\mathbf{0}_{m\times 1}$, the above result implies that $\mathbf{e}_i=\mathbf{0}_{m\times 1}$, $i=1,2,\ldots,n$. Since, in this case, $(\mathbf{L}\otimes\mathbf{I}_m)\mathbf{r}=\mathbf{e}=\mathbf{0}_{mn\times 1}$, and using again the result in Lemma 1, one has that

$$\mathbf{r}_1=\mathbf{r}_2=\cdots=\mathbf{r}_n \quad\Leftrightarrow\quad \mathbf{r}_{ij}=0.$$

On the other hand, it is clear that if $\mathbf{r}_{ij}=\mathbf{0}_{mn\times 1}\Rightarrow\mathbf{w}_j=\mathbf{w}_i\Rightarrow\mathbf{g}_{ij}=\mathbf{0}_{mn\times 1}$.

It is important to emphasize that the velocity command $\mathbf{v}_{ci}$ in (5) should be designed taking into consideration two major aspects: (i) $\mathbf{v}_{ci}$ must converge to the desired group velocity $\mathbf{v}_d$ when the multiagent system is close to consensus; (ii) $\mathbf{v}_{ci}$ must satisfy constraint (7) throughout the execution of the entire task. Based on these specifications, the solution to the proposed consensus problem can now be given.

Theorem 2: Given n agents with individual dynamical behavior described by (5) with freely chosen positive-definite diagonal matrices $\mathbf{K}$ and $\mathbf{K}_e$, and a group reference velocity $\mathbf{v}_d$ satisfying $\mathbf{v}_{min}\le\mathbf{v}_d\le\mathbf{v}_{max}$, if the interaction graph $G(\mathscr{V},\mathscr{E})$ is connected, then the distributed control law

$$\mathbf{v}_{ci}=\mathbf{a}-\mathbf{B}\sigma\left(\mathbf{K}_e\sum_{j=1}^{n}a_{ij}\,\mathbf{r}_{ij}+\mathbf{c}\right),\quad i\in\{1,\ldots,n\} \tag{14}$$

with

$$\mathbf{a}=\frac{1}{2}(\mathbf{v}_{min}+\mathbf{v}_{max}), \tag{15}$$

$$\mathbf{B}=\frac{1}{2}\operatorname{diag}\{v_{\max}^1-v_{\min}^1,\ldots,v_{\max}^m-v_{\min}^m\}, \tag{16}$$

$$\mathbf{c}=\sigma^{-1}\left(\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_d)\right), \tag{17}$$

where $\sigma(\cdot)$ and $\sigma^{-1}(\cdot)$ are defined component-wise according to Properties P1-P4, ensures that, without violating the constraints (7), the multi-agent system will reach consensus with the group reference velocity $\mathbf{v}_d$, that is, for all $\mathbf{r}_i(0)$ and $\mathbf{v}_i(0)$ satisfying the first constraint in (7), $\mathbf{r}_i(t)\to\mathbf{r}_j(t)$ and $\mathbf{v}_i(t)\to\mathbf{v}_d$ asymptotically as $t\to\infty$.

Proof: The lowest value of $\mathbf{v}_{ci}$ occurs when $\sigma(\cdot)=\mathbf{1}_{m\times 1}$, and the highest value occurs when $\sigma(\cdot)=-\mathbf{1}_{m\times 1}$, i.e., at $\mathbf{v}_{min}$ and $\mathbf{v}_{max}$, respectively. Therefore, if $\mathbf{v}_{min}\le\mathbf{v}_i(0)\le\mathbf{v}_{max}$, the constraints in (7) will be automatically satisfied for $t\ge 0$.

We need to show that the argument $\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_d)$ of the function $\sigma(\cdot)^{-1}$ in (17) obeys the inequality $-\mathbf{1}_{m\times 1}\le\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_d)\le\mathbf{1}_{m\times 1}$. Considering that $\sigma(\cdot)^{-1}$ is an strictly increasing function, it is enough to evaluate it in the extremal points corresponding to $\mathbf{v}_d=\mathbf{v}_{min}$ and $\mathbf{v}_d=\mathbf{v}_{max}$, since $\mathbf{v}_{min}\le\mathbf{v}_d\le\mathbf{v}_{max}$. If $\mathbf{v}_d=\mathbf{v}_{min}$, then $\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_{min})=\mathbf{1}_{m\times 1}$; if $\mathbf{v}_d=\mathbf{v}_{max}$, then $\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_{max})=-\mathbf{1}_{m\times 1}$. Therefore $\mathbf{v}_{min}\le\mathbf{v}_d\le\mathbf{v}_{max}\Rightarrow-\mathbf{1}_{m\times 1}\le\mathbf{B}^{-1}(\mathbf{a}-\mathbf{v}_d)\le\mathbf{1}_{m\times 1}$.

From (5), (10), and (14), and considering only the variables $\mathbf{r}_{ij}$ that have corresponding edges in the interaction graph $G(\mathscr{V},\mathscr{E})$, we have that:

$$\ddot{\mathbf{r}}_{ij}=-\mathbf{K}(\dot{\mathbf{r}}_{ij}-\mathbf{B}\mathbf{g}_{ij}),\quad (i,j)\in\mathscr{E}. \tag{18}$$

To determine the equilibrium points of this error system, we simply set $\ddot{\mathbf{r}}_{ij}=\dot{\mathbf{r}}_{ij}=\mathbf{0}_{m\times 1}$, resulting in $\mathbf{g}_{ij}=\mathbf{0}_{m\times 1}$, for $(i,j)\in\mathscr{E}$, since $\mathbf{KB}$ is invertible. Therefore, according to Lemma 2, the system with dynamical behavior described by (18) that satisfies (13) has the unique fixed point, or equilibrium configuration,

$$\begin{bmatrix}\mathbf{r}_{ij}\\ \dot{\mathbf{r}}_{ij}\end{bmatrix}=\mathbf{0}_{2m\times 1},\quad (i,j)\in\mathscr{E}. \tag{19}$$

Now consider the function

$$H=\frac{1}{4}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\dot{\mathbf{r}}_{ij}+\phi, \tag{20}$$

where

$$\phi=\mathbf{1}_{1\times m}\mathbf{KBK}_e^{-1}\sum_{i=1}^{n}\gamma\left(\mathbf{K_e}\sum_{j\in\mathscr{N}_i}\mathbf{r}_{ij}+\mathbf{c}\right). \tag{21}$$

It must be emphasized that H is a function of $\mathbf{r}_{ij}$ and $\dot{\mathbf{r}}_{ij}$, for $(i,j)\in\mathscr{E}$.

The function $\gamma(\cdot)$ in (21) is defined component-wise. Since $\gamma(x)\ge 0$, for $x\in\mathbb{R}$, H is a positive semidefinite function. Therefore it is possible to verify that H is radially unbounded with respect to $\mathbf{r}_{ij}$ and $\dot{\mathbf{r}}_{ij}$, for $(i,j)\in\mathscr{E}$. Note that $\frac{\partial\phi}{\partial\gamma}\frac{\partial\gamma}{\partial\mathbf{r}_{ij}}=\mathbf{KB}\sigma\left(\mathbf{K_e}\sum_{l\in\mathscr{N}_i}\mathbf{r}_{il}+\mathbf{c}\right)$, and $a_{ij}=1$, for $(i,j)\in\mathscr{E}$. Thus, the time derivative of H is given by

$$\begin{aligned}\dot{H}&=\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\ddot{\mathbf{r}}_{ij}+\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\left(\frac{\partial\phi}{\partial\gamma}\frac{\partial\gamma}{\partial\mathbf{r}_{ij}}\right)^T\dot{\mathbf{r}}_{ij}\\&=\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\left[-\mathbf{K}(\dot{\mathbf{r}}_{ij}-\mathbf{B}\mathbf{g}_{ij})\right]\\&\quad+\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\mathbf{KB}\sigma\left(\mathbf{K_e}\sum_{l\in\mathscr{N}_i}\mathbf{r}_{il}+\mathbf{c}\right)\\&=-\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\mathbf{K}\dot{\mathbf{r}}_{ij}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\mathbf{KB}\mathbf{g}_{ij}\\&\quad+\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\mathbf{KB}\sigma\left(\mathbf{K_e}\sum_{l\in\mathscr{N}_i}\mathbf{r}_{il}+\mathbf{c}\right)\\&=-\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathscr{N}_i}\dot{\mathbf{r}}_{ij}^T\mathbf{K}\dot{\mathbf{r}}_{ij}\\&\le 0.\end{aligned} \tag{22}$$

In the third equality of (22) we have performed some algebra using the property $\dot{\mathbf{r}}_{ij}=-\dot{\mathbf{r}}_{ji}$ and the equation

$$\mathbf{g}_{ij} = \left[\sigma\left(\mathbf{K}_e \sum_{k\in\mathcal{N}_j} \mathbf{r}_{jk} + \mathbf{c}\right) - \sigma\left(\mathbf{K}_e \sum_{l\in\mathcal{N}_i} \mathbf{r}_{il} + \mathbf{c}\right)\right], \ (i,j)\in\mathcal{E},$$

to show that the sum of the second and the third term is null.

Therefore, $\dot{H}$ is a negative semidefinite function. To apply LaSalle's invariant set theorem [49], notice that the region where $\dot{H}$ vanishes is

$$\Gamma_1 = \left\{\left[\mathbf{r}_{ij}^T \ \ \dot{\mathbf{r}}_{ij}^T\right]^T \in \mathbb{R}^{2m} \,|\, \dot{\mathbf{r}}_{ij} = 0_{m\times 1}\right\}, \quad (i,j)\in\mathcal{E}.$$

Within Γ_1, system (18) is governed by equations

$$\ddot{\mathbf{r}}_{ij} = \mathbf{KBg}_{ij}, \quad (i,j)\in\mathcal{E}. \tag{23}$$

If the system begins its trajectory in Γ_1, with $\mathbf{r}_{ij} \neq \mathbf{0}_{m\times 1}$, for any pair $(i,j)\in\mathcal{E}$, we have that $\ddot{\mathbf{r}}_{ij} \neq \mathbf{0}_{m\times 1}$, and the system will leave the region Γ_1. For this reason the largest invariant set in Γ_1 is given by

$$\Gamma_2 = \left\{\left[\mathbf{r}_{ij}^T \ \ \dot{\mathbf{r}}_{ij}^T\right]^T \in \mathbb{R}^{2m} \,|\, \dot{\mathbf{r}}_{ij} = \mathbf{r}_{ij} = \mathbf{0}_{m\times 1}\right\}, \quad (i,j)\in\mathcal{E},$$

which is the fixed point of system (18). Hence the equilibrium point (19) is asymptotically stable, and it means that $\mathbf{r}_i(t) \to \mathbf{r}_j(t)$ asymptotically as $t\to\infty$. In addition, with the proposed control law in (15), the velocity of the ith agent described by (5) will behave according to

$$\dot{\mathbf{v}}_i = -\mathbf{K}(\mathbf{v}_i - \mathbf{v}_d) + \mathbf{s}, \tag{24}$$

where $\mathbf{s} = -\mathbf{K}\left[\mathbf{v}_d - \mathbf{a} + \mathbf{B}\sigma\left(\mathbf{K}_e \sum_{j=1}^{n} a_{ij}\,\mathbf{r}_{ij} + \mathbf{c}\right)\right]$. Since $\mathbf{r}_{ij} \to \mathbf{0}_{m\times 1}$ as $t\to\infty$, for $(i,j)\in\mathcal{E}$, we have that $\lim_{t\to\infty} \mathbf{s} = \mathbf{0}_{m\times 1}$, which implies that $\mathbf{v}_i(t) \to \mathbf{v}_d$ asymptotically as $t\to\infty$. Therefore, consensus with a group reference velocity $\mathbf{v}_d$ will be reached, that is, for all $\mathbf{r}_i(0)$ and $\mathbf{v}_i(0)$ satisfying the first constraint in (7), $\mathbf{r}_i(t)\to\mathbf{r}_j(t)$ and $\mathbf{v}_i(t)\to\mathbf{v}_d$ asymptotically as $t\to\infty$.

Two important remarks should be made at this point:

1) One could think that a trivial consensus (i.e., all states of the multiagent system converging to the origin) would be a possible scenario. However, Theorem 2 guarantees that, for all $\mathbf{r}_i(0)$ and $\mathbf{v}_i(0)$ satisfying the first constraint in (7), $\mathbf{r}_i(t)\to\mathbf{r}_j(t)$ and $\mathbf{v}_i(t)\to\mathbf{v}_d$ asymptotically as $t\to\infty$. So, if $\mathbf{v}_d \neq 0$, then trivial consensus will never be achieved. If $\mathbf{v}_d = 0$, then the trivial consensus becomes a remote possibility that will depend on the initial conditions of the multiagent system;
2) The choice of the expression (14) for $\mathbf{v}_{ci}$ was made taking into account the function H given in (20) and the constraints on the agents velocities given in (3). The H function was inspired by the Lyapunov function proposed in [35], that is composed of a kinetic-energy-like term and a potential-energy-like term.

4. SIMULATION EXAMPLE

System (2) was simulated with $m=1$, $n=5$, $\mathbf{v}_{min}$ = 1 m/s, $\mathbf{v}_{max} = 5$ m/s, $\mathbf{v}_d = 3$ m/s, $\mathbf{K}=2$, and $\mathbf{K}_e = 2$. In the velocity command $\mathbf{v}_{ci}$ we considered the function $\sigma(x)=\tanh(x)$, so $\gamma(x)=\ln(\cosh(x))$. The interaction graph that represents the communication topology is presented in Fig. 2. Note, from Fig. 3, that the consensus with a group reference velocity was achieved and that the velocity constraints were satisfied all the time. The time evolution of the Lyapunov function H is presented in Fig. 4, which confirms the convergence of the coordinated control algorithm.

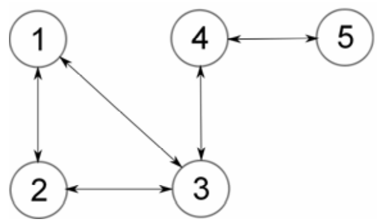

Fig. 2. Interaction graph for the simulation example.

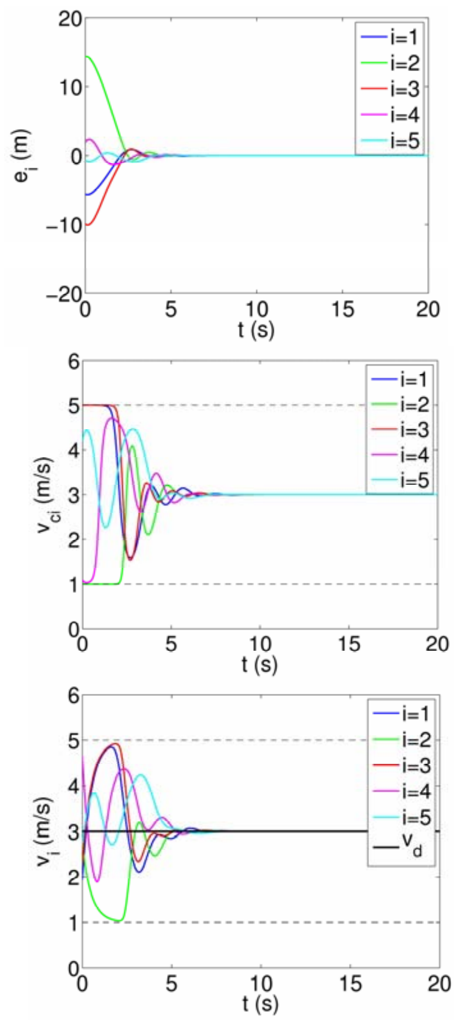

Fig. 3. Simulation results for the proposed consensus strategy.

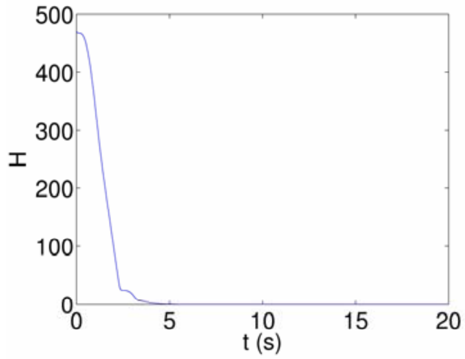

Fig. 4. Time evolution of the Lyapunov function H.

5. CONCLUSION AND FUTURE WORKS

In this paper a distributed control law has been proposed for a team of n vehicles with double-integrator dynamics in search for consensus. Differently from previous works, we have considered asymmetric velocity constraints, which are an interesting aspect when dealing, for example, with the problem of coordinating multiple fixed-wing unmanned aerial vehicles. In addition, we have considered that the control law of each agent does not depend on its local neighbors' velocities or accelerations. The strategy is decentralized because each vehicle only needs to access information from their neighbors. Mathematical guarantees have been derived within the proposed approach and a numerical simulation has been given to illustrate the effectiveness of the theoretical results. Future works will consider switching neighboring topologies and formation control with the inclusion of relative reference positions.

REFERENCES

[1] M. A. Hsieh, V. Kumar, and L. Chaimowicz, "Decentralized controllers for shape generation with robotic systems," *Robotica*, vol. 26, no. 5, pp. 691-701, September 2008.

[2] D. W. Casbeer, D. B. Kingston, R. W. Beard, T. W. McLain, S.-M. Li, and R. Mehra, "Cooperative forest fire surveillance using a team of small unmanned air vehicles," *International Journal of Systems Science*, vol. 37, no. 6, pp. 351-360, May 2006.

[3] X. C. Ding, A. R. Rahmani, and M. Egerstedt, "Multi-UAV convoy protection: an optimal approach to path planning and coordination," *IEEE Trans. on Robotics*, vol. 26, no. 2, pp. 256-268, April 2010.

[4] A. Franchi, P. Stegagno, M. D. Rocco, and G. Oriolo, "Distributed target localization and encircling with a multi-robot system," *Proc. of the 7th IFAC Symposium on Intelligent Autonomous Vehicles*, pp. 1-6, 2010.

[5] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. on Automatic Control*, vol. 49, no. 9, pp. 1520-1533, September 2004.

[6] M. Arcak, "Passivity as a design tool for group coordination," *IEEE Trans. on Automatic Control*, vol. 52, no. 9, pp. 1380-1390, August 2007.

[7] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination," *Proc. of the American Control Conference*, pp. 1859-1864, 2005.

[8] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. on Automatic Control*, vol. 50, no. 5, pp. 655-661, May 2005.

[9] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Trans. on Automatic Control*, vol. 51, no. 3, pp. 401-420, March 2006.

[10] G. Xie and L. Wang, "Consensus control for a class of networks of dynamic agents," *International Journal of Robust and Nonlinear Control*, vol. 17, pp. 941-949, November 2007.

[11] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," *International Journal of Robust and Nonlinear Control*, vol. 17, pp. 1002-1033, November 2007.

[12] F. Xiao, L. Wang, J. Chen, and Y. Gao, "Finite-time formation control for multi-agent systems," *Automatica*, vol. 45, no. 11, pp. 2605-2611, November 2009.

[13] Y. Li, J. Xiang, and W. Wei, "Consensus problems for linear time-invariant multi-agent systems with saturation constraints," *IET Control Theory & Applications*, vol. 5, no. 6, pp. 823-829, April 2011.

[14] L. Gao, J. Zhang, and W. Chen, "Second-order consensus for multiagent systems under directed and switching topologies," *Mathematical Problems in Engineering*, pp. 1-21, 2012.

[15] Z. Wang and J. Cao, "Quasi-consensus of second-order leader-following multi-agent systems," *IET Control Theory & Applications*, vol. 6, no. 4, pp. 545-551, March 2012.

[16] H. Zhao, S. Xu, and D. Yuan, "An LMI approach to consensus in second-order multi-agent systems," *International Journal of Control, Automation, and Systems*, vol. 9, no. 6, pp. 1111-1115, December 2011.

[17] L. Gao, X. Zhu, and W. Chen, "Leader-following consensus problem with an accelerated motion leader," *International Journal of Control, Automation, and Systems*, vol. 10, no. 5, pp. 931-939, October 2012.

[18] X. Luo, D. Liu, X. Guan, and S. Li, "Flocking in target pursuit for multi-agent systems with partial informed agents," *IET Control Theory & Applications*, vol. 6, no. 4, pp. 560-569, March 2012.

[19] G. Wen, Z. Duan, W. Yu, and G. Chen, "Consensus in multi-agent systems with communication constraints," *International Journal of Robust and Nonlinear Control*, vol. 22, pp. 170-182, January 2012.

[20] G. Wen, Z. Duan, Z. Lu, and G. Chen, "Consensus and its L_2 gain performance of multi-agent systems

with intermittent information transmissions," *International Journal Control*, vol. 85, no. 4, pp. 384-396, January 2012.
[21] P. Lin and Y. Jia, "Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies," *Automatica*, vol. 45, no. 9, pp. 2154-2158, September 2009.
[22] Q. Zhang, Y. Niu, L. Wang, L. Shen, and H. Zhu, "Average consensus seeking of high-order continuous-time multi-agent systems with multiple time-varying communication delays," *International Journal of Control, Automation, and Systems*, vol. 9, no. 6, pp. 1209-1218, December 2011.
[23] W. Yu, G. Chen, M. Cao, and J. Kurths, "Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics," *IEEE Trans. on Systems, Man, and Cybernetics - Part B: Cybernetics*, vol. 40, no. 3, pp. 881-891, June 2010.
[24] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, no. 6, pp. 1089-1095, June 2010.
[25] S. Li, H. Du, and X. Lin, "Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics," *Automatica*, vol. 47, no. 8, pp. 1706-1712, August 2011.
[26] H. Du, S. Li, and C. Qian, "Finite-time attitude tracking control of spacecraft with application to attitude synchronization," *IEEE Trans. on Automatic Control*, vol. 56, no. 11, pp. 2711-2717, November 2011.
[27] H. Li, X. Liao, and G. Chen, "Leader-following finite-time consensus in second-order multi-agent networks with nonlinear dynamics," *International Journal of Control, Automation, and Systems*, vol. 11, no. 2, pp. 422-426, April 2013.
[28] D. Zhao, T. Zou, S. Li, and Q. Zhu, "Adaptive backstepping sliding mode control for leader-follower multi-agent systems," *IET Control Theory & Applications*, vol. 6, no. 8, pp. 1109-1117, May 2012.
[29] H. Du, S. Li, and P. Shi, "Robust consensus algorithm for second-order multi-agent systems with external disturbances," *International Journal of Control*, vol. 85, no. 12, pp. 1913-1928, December 2012.
[30] H. Hu, G. Chen, and G. Xie, "Second-order consensus of multi-agent systems with unknown but bounded disturbance," *International Journal of Control, Automation, and Systems*, vol. 1, no. 2, pp. 258-267, April 2013.
[31] G. Ferrari-Trecate, L. Galbusera, M. P. E. Marciandi, and R. Scattolini, "Contractive distributed MPC for consensus in networks of single- and double-integrators," *Proc. of the 17th World Congress of the International Federation of Automatic Control*, pp. 9033-9038, 2008.
[32] A. Nedić, A. Ozdaglar, and P. A. Parrilo, "Constrained consensus and optimization in multi-agent networks," *IEEE Trans. on Automatic Control*, vol. 55, no. 4, pp. 922-938, April 2010.
[33] J. Lee, J.-S. Kim, H. Song, and H. Shim, "A constrained consensus problem using MPC," *International Journal of Control, Automation and Systems*, vol. 9, no. 5, pp. 952-957, October 2011.
[34] U. Lee and M. Mesbahi, "Constrained consensus via logarithmic barrier functions," *Proc. of the 5th Conf. on Decision and Control*, pp. 3608-3613, 2011.
[35] W. Ren, "On consensus algorithms for double-integrator dynamics," *IEEE Trans. on Automatic Control*, vol. 53, no. 6, pp. 1503-1509, July 2008.
[36] K. Peng, H.-S. Su, and Y.-P. Yang, "Coordinated control of multi-agent systems with a varying-velocity leader and input saturation," *Communications in Theoretical Physics*, vol. 52, no. 3, pp. 449-456, September 2009.
[37] A. Abdessameud and A. Tayebi, "On consensus algorithms for double-integrator dynamics without velocity measurements and with input constraints," *Systems & Control Letters*, vol. 59, no. 12, pp. 812-821, December 2010.
[38] W. Ren, "Distributed leaderless consensus algorithms for networked Euler-Lagrange systems," *Systems & Control Letters*, vol. 82, no. 11, pp. 2137-2149, November 2009.
[39] J. R. T. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Trans. on Robotics and Automation*, vol. 19, no. 6, pp. 933-941, December 2003.
[40] A. Abdessameud and A. Tayebi, "Synchronization of networked Lagrangian systems with input constraints," *Proc. of the 18h IFAC World Congress*, pp. 2382-2387, 2011.
[41] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Trans. on Automatic Control*, vol. 52, no. 5, pp. 916-922, May 2007.
[42] W. Ren, "Collective motion from consensus with Cartesian coordinate coupling," *IEEE Trans. on Automatic Control*, vol. 54, no. 6, pp. 1330-1335, June 2009.
[43] P. Lin, K. Qin, Z. Lin, and W. Ren, "Collective rotating motions of second-order multi-agent systems in three-dimensional space," *Systems & Control Letters*, vol. 60, no. 6, pp. 365-372, June 2011.
[44] P. Lin and Y. Jia, "Distributed rotating formation control of multi-agent systems," *Systems & Control Letters*, vol. 59, no. 10, pp. 587-595, October 2010.
[45] X.-P Chen, H.-B. Xu, and Y.-X. Ban, "Rotating consensus of multi-agent systems without relative velocity measurement," *Chinese Physics B*, vol. 20, no. 9, 090515, September 2011.
[46] S. Bayraktar, J. E. Fainekos, and G. J. Pappas, "Experimental cooperative control of fixed-wing unmanned aerial vehicles," *Proc. of the 43rd Conf. Decision and Control*, pp. 4292-4298, 2004.
[47] F. Bullo, J. Cortés, and S. Martínez, *Distributed Control of Robotic Networks*, Princeton University

Press, New Jersey, 2009.

[48] D. S. Bernstein, *Matrix Mathematics: Theory, Facts and Formulas*, Princeton University Press, New Jersey, 2009.

[49] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, Englewood Cliffs, NJ, 1991.

Tales A. Jesus received his B.S. degree in Control and Automation Engineering, and his M.Sc. and Ph.D. degrees in Electrical Engineering from the Federal University of Minas Gerais (UFMG), Belo Horizonte, Brazil, in 2006, 2008, and 2013, respectively. He is currently an Assistant Professor with the Computation Department at Centro Federal de Educação Tecnológica de Minas Gerais (CEFET-MG), Belo Horizonte, Brazil. His research interests include mobile robotics, multiagent coordination, and nonlinear control theory.

Luciano C. A. Pimenta received his B.S., M.Sc., and Ph.D. degrees in Electrical Engineering from the Universidade Federal de Minas Gerais (UFMG), Belo Horizonte, Brazil, in 2003, 2005, and 2009, respectively. From April 2007 to June 2008, he was a visiting Ph.D. student at the General Robotics, Automation, Sensing and Perception (GRASP) Laboratory at the University of Pennsylvania, Philadelphia, USA. He is currently an Assistant Professor with the Department of Electronic Engineering at UFMG. His research interests include robotics, multi-robot systems, and control theory.

Leonardo A. B. Tôrres received his B.Eng. in 1997, and his PhD degree in 2001, both in Electrical Engineering, from the Federal University of Minas Gerais (UFMG), Brazil, and in 2010-2011 he was a visiting scholar at University of California Santa Barbara. In 2002, he joined the Department of Electronic Engineering, at UFMG. As Associate Professor, he has been working with control and synchronization of nonlinear dynamical systems, with applications to information transmission, uninterruptible power supplies, and robotic coordination.

Eduardo M. A. M. Mendes received his Ph.D. degree in Automatic Control and Systems Engineering from the University of Sheffield, Sheffield, U.K. in 1995. He was appointed to his current position of Associate Professor, Department of Electronic Engineering, UFMG, Brazil in 2002. His research interests include system identification for nonlinear systems, NARMAX methods, model validation, prediction, spectral analysis, chaos, control of nonlinear systems, signal processing and analysis of biomedical signals and images. He has authored over 100 research and scientific publications in various fields.