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Consensus of Leader-Following Multi-Agent Systems in Time-Varying Networks via Intermittent Control

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Abstract: This paper is concerned with the issue of consensus for leader-following multi-agent systems, wherein the agents acting as followers update states based on the information received from the time-varying neighbors and the virtual leader. Moreover, the neighbors of an agent are divided into three types according to their relative position, which may also be changed with time. Consensus protocol is derived mainly by using intermittent control, and based on the Lyapunov stability theory, sufficient conditions for consensus are presented and proved theoretically. Finally, some numerical examples are given to demonstrate the effectiveness of the results.

Keywords: Consensus, intermittent control, Lyapunov function, multi-agent systems, time-varying networks.

1. INTRODUCTION

Over the past decade, great interest has been shown to the study of the consensus problem for multi-agent systems, due to its broad applications in control of unmanned aerial vehicles, formation control of mobile robots, and so on [1-3]. Generally speaking, leaderless consensus means that each agent updates its state based on local information of its neighbors such that all agents eventually reach an agreement on a common value, while leader-following consensus means that there exists a virtual leader which specifies an objective for all agents to follow [4,5]. In the existing works, consensus issue has been investigated from many perspectives and a great deal of results have been proposed. For instances, consensus of heterogeneous multi-agent systems was investigated in [6,7], multi-agent systems with noises or time-delays have also been considered in consensus problem, and so on [8-13].

It is well known that multiple agents in a system can be taken as nodes in a network, and communication channels among the agents can be viewed as edges. Consequently, network topology plays an important role in determining consensus of the agents. So far networks with fixed topologies [14,15], switching or time-varying topologies [16-18] have been researched, wherein switching or time-varying topologies may be more realistic since there might exist link failures or creations in a network of mobile agents.

Recently, [19-21] investigated intermittent consensus algorithms for multi-agent systems in networks with fixed or switching topologies. Intermittent control [22, 23] is an effective strategy in comparison to continuous control for consideration of the cost. On the other hand, communication among agents may be interrupted due to the external disturbances or limitations of technology, therefore, information transmission among nodes may occur intermittently rather than continuously in many real-world networks.

Motivated by the above discussion, in this paper we further consider consensus of multiple agents in time-varying networks via intermittent control. The network consists of continuous dynamic agents acting as the followers and the virtual leader, and the follower updates the state based on the information received from its neighbors and the leader. However, differs from most of the current literature [19-21], we introduce the sign function in the consensus protocol. Since the corresponding relationship between nodes can be competitive besides cooperative [24-26], we divide the neighbors of a follower into different types by the sign function. Accordingly, for an agent, the neighbors in front of it can be regarded as cooperative neighbors, and the ones behind it are competitive. Moreover, if a neighbor's state is the same as this agent's, then this neighbor can be viewed as an ineffective neighbor. Obviously, coopera-

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tive and competitive dynamic systems are more realistic in practice, while in most of the existing works [19-21], only cooperative relationship has been considered. Based on the Lyapunov stability theory, this paper will demonstrate some sufficient conditions for consensus of the proposed multi-agent system. The main contributions of this work can be summarized as: The first one is that the condition of bounded time intervals for intermittent consensus protocol can be easily satisfied even though the topologies of the network are time-varying; The second one is that the neighbors of an agent can be cooperative or competitive, which may be more consistent with the real-world networks.

The rest of the paper is organized as follows. In Section 2, some concepts in graph theory are described, and the problem to be investigated is formulated. Theoretical results for consensus are derived in Section 3. In Section 4, some numerical examples are shown to illustrate the analysis. Finally, concluding remarks are presented and discussed.

The following notations are given which will be used throughout this paper: Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^m$ the m-dimensional Euclidean space and $\mathbb{R}^{m\times n}$ the set of $m\times n$ real matrices. $\mathbf{1}_m$ denotes the m-dimensional vector of ones. $\mathbf{0}_m$ denotes the m-dimensional vector of zeros. $\mathbf{0}_{m\times n}$ denotes the $m\times n$ zero matrix. $\mathbf{I}_m$ denotes the $m\times m$ identity matrix. $\mathbf{diag}\{a_1,a_2,\cdots,a_m\}$ denotes the $m\times m$ diagonal matrix with elements $a_1,a_2,\cdots,a_m\in\mathbb{R}$ on the diagonal. $\mathbf{X}^T$ indicates the transpose of matrix or vector $\mathbf{X}$. $\mathbf{X}^{-1}$ indicates the inverse of matrix $\mathbf{X}$. $\|\cdot\|$ indicates the Euclidean norm. $|\cdot|$ stands for the absolute value. $\lfloor x\rfloor$ stands for the largest integer which is less than or equal to $x\in\mathbb{R}$.

2. PROBLEM FORMULATION

2.1. Graph theory

We shall present the graph theory [27] in this subsection, which is fundamental to the later development. For a network of N nodes, its topology can be modeled as a graph $\mathbf{G}=(\mathbf{W},\mathbf{E},\mathbf{A})$, where $\mathbf{W}=\{w_1,w_2,\cdots,w_N\}$ is the set of nodes, and $\mathbf{E}\subset\mathbf{W}\times\mathbf{W}$ is the set of edges. The set of neighbors of a node w_i is denoted by $\mathcal{N}_i=\{w_j\in\mathbf{W}:(w_j,w_i)\in\mathbf{E}\}$. A path on $\mathbf{G}$ from node w_{i1} to node w_{ik} is a sequence of ordered edges in the form $(w_{i1},w_{i2}),(w_{i2},w_{i3}),\cdots,(w_{i(k-1)},w_{ik})$. If there exists a special vertex that has a directed path to all the other nodes, then $\mathbf{G}$ is said to have a spanning tree. Moreover, the graph is said to be strongly connected if there exists a path between every pair of distinct nodes. $\mathbf{A}=(a_{ij})\in\mathbb{R}^{N\times N}$ denotes the weighted adjacent matrix of $\mathbf{G}$, when $i\neq j$, if $(w_j,w_i)\in\mathbf{E}$, then $a_{ij}>0$; otherwise $a_{ij}=0$. The diagonal elements a_{ii} equal zero. Additionally, in an undirected graph $\mathbf{G}$, $a_{ji}=a_{ij}$, then $\mathbf{A}$ is a symmetric matrix.

2.2. Consensus protocol

Consider a leader-following multi-agent system with N agents acting as followers, which are moving with the first-order dynamics

$$\dot{x}_i(t)=u_i(t),\ \ i=1,2,\cdots,N, \tag{1}$$

where $x_i(t)\in\mathbb{R}$ is the state or position of the agent i, $u_i(t)\in\mathbb{R}$ is the so called consensus protocol, which represents for the velocity of the agent i, and $t\geq 0$.

It is supposed that the state of the virtual leader in the multi-agent system is denoted as $x_0(t)\in\mathbb{R}$, which satisfies the following second-order system:

$$\begin{cases}\dot{x}_0(t)=v_0(t),\\ \dot{v}_0(t)=a_0,\end{cases} \tag{2}$$

where $v_0(t)\in\mathbb{R}$ denotes the velocity of the leader at time t, and $a_0\in\mathbb{R}$ denotes the acceleration of the leader and is assumed to be known. The leader can also be viewed as a target for consensus tracking.

The intermittent method will be utilized in the consensus protocol $u_i(t)$ here, which is described as:

$$u_i(t)=pK_i(t)+v_i(t), \tag{3}$$
$$\dot{v}_i(t)=qpK_i(t)+a_0, \tag{4}$$

where $i=1,2,\cdots,N$. $p>0$ and $0<q<1$ denote the control parameters. $v_i(t)$ is the estimate of $v_0(t)$ by the agent i due to that $v_0(t)$ cannot be measured easily. $K_i(t)$ is an intermittent control.

Suppose that the time $t\in[0,+\infty)$ is divided into $[t_k,t_{k+1})$, where $k=0,1,\cdots$, $t_0=0$. The sequence of $[t_k,t_{k+1})$ is uniformly bounded and non-overlapping. We assume that when $t\in[t_{2k},t_{2k+1})$, the control $K_i(t)$ will "work"; when $t\in[t_{2k+1},t_{2k+2})$, the control $K_i(t)$ will "rest". On the other hand, in general the state of each follower in the multi-agent system is considered to be updated according to the information from the neighbors and the leader, then the intermittent control $K_i(t)$ in this paper will be supposed as follows in order to achieve consensus:

When $t\in[t_{2k},t_{2k+1})$,

$$\begin{aligned}K_i(t)=&\sum_{j=1}^{N}a_{ij}(t)sign[x_j(t)-x_i(t)][x_j(t)-x_i(t)]\\&+b_i(t)[x_0(t)-x_i(t)],\end{aligned} \tag{5}$$

when $t\in[t_{2k+1},t_{2k+2})$,

$$K_i(t)=0, \tag{6}$$

where $k=0,1,\cdots$, and $i=1,2,\cdots,N$. $a_{ij}(t)$ is described as $a_{ij}(t)=a_{ji}(t)=1$, if $j\in\mathcal{N}_i(t)$; $a_{ij}(t)=a_{ji}(t)=0$, if $j\notin\mathcal{N}_i(t)$; and $a_{ii}(t)=0$. $\mathcal{N}_i(t)$ stands for the neighbors of agent i at time t. It can be seen that the network consisting of all agents can be time-varying according to $\mathcal{N}_i(t)$. $b_i(t)\geq 0$ denotes the weight of the edge from the leader to the agent i, which will be discussed later. $sign(\cdot)$ is the sign function defined by

$$sign(z) = \begin{cases} 1, & z > 0, \\ -1, & z < 0, \\ 0, & z = 0. \end{cases}$$

Remark 1: Notice that the sign function is introduced in the intermittent consensus algorithm (5), then for the i th follower, the weight of the edge from its neighboring agent j can be viewed as $\tilde{a}_{ij}(t) = a_{ij}(t)sign[x_j(t) - x_i(t)]$, which can be negative. As discussed in [24,25], two nodes in a network are cooperative (or competitive) if the coupling strength between them is positive (or negative). Therefore, the role of the agent j playing on the agent i at time t can be described by $\tilde{a}_{ij}(t)$, i.e., the neighboring agent j is cooperative (or competitive) with the agent i if $\tilde{a}_{ij}(t) = 1$ (or $\tilde{a}_{ij}(t) = -1$). In addition, if the state of the agent j equals that of the agent i, then $\tilde{a}_{ij}(t) = 0$, which means that the agent j is an ineffective neighbor.

Remark 2: Since the neighbors of the i th follower can change according to $\mathscr{N}_i(t)$, the topologies of the network consisting of all the followers will be time-varying. Meanwhile, the network is a cooperative and competitive system. Therefore, the multi-agent system considered here may be consistent with the real-world network. However, it should be noticed that the multi-agent system discussed in this paper is a little rigorous, since the neighbors are classified based on the relative position and all the agents are supposed to move in one-dimensional space. Our future work will try to extend the system to the more general case.

In the following, we will derive sufficient conditions for consensus of the above multi-agent system, which can guarantee that each follower's state will finally converge to the leader's as time going on, and the estimations of $v_0(t)$ by the followers will converge to the leader's velocity, i.e.,

$$\begin{cases} \lim_{t\to\infty} x_i(t) = x_0(t), \\ \lim_{t\to\infty} v_i(t) = v_0(t) \end{cases}$$

for $i = 1, 2, \cdots, N$.

3. CONSENSUS ANALYSIS

Let $\mathbf{x}(t) = (x_1(t), x_2(t), \cdots, x_N(t))^T \in \mathbb{R}^N$, $\mathbf{v}(t) = (v_1(t), v_2(t), \cdots, v_N(t))^T \in \mathbb{R}^N$, $\bar{\mathbf{x}}(t) = \mathbf{x}(t) - 1_N x_0(t)$, $\bar{\mathbf{v}}(t) = \mathbf{v}(t) - 1_N v_0(t)$, $\mathbf{e}(t) = \begin{bmatrix} \bar{\mathbf{x}}(t) \\ \bar{\mathbf{v}}(t) \end{bmatrix} \in \mathbb{R}^{2N}$. Based on the problem formulation of the above section, we can get the following:
When $t \in [t_{2k}, t_{2k+1})$,

$$\begin{cases} \dot{\mathbf{x}}(t) = p[\tilde{\mathbf{A}}(t) - \mathbf{D}(t) - \mathbf{B}(t)]\mathbf{x}(t) \\ \qquad + p\mathbf{B}(t)1_N x_0(t) + \mathbf{v}(t), \\ \dot{\mathbf{v}}(t) = qp[\tilde{\mathbf{A}}(t) - \mathbf{D}(t) - \mathbf{B}(t)]\mathbf{x}(t) \\ \qquad + qp\mathbf{B}(t)1_N x_0(t) + 1_N a_0, \end{cases} \tag{7}$$

when $t \in [t_{2k+1}, t_{2k+2})$,

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{v}(t), \\ \dot{\mathbf{v}}(t) = 1_N a_0, \end{cases} \tag{8}$$

where $k = 0, 1, \cdots$; $\tilde{\mathbf{A}}(t) = (\tilde{a}_{ij}(t))_{N\times N} \in \mathbb{R}^{N\times N}$, $\tilde{a}_{ij}(t) = a_{ij}(t)sign[x_j(t) - x_i(t)]$. It can be seen that $\tilde{a}_{ij}(t) = -\tilde{a}_{ji}(t)$ and $\tilde{a}_{ii}(t) = 0$ due to the definition of $a_{ij}(t)$ in Section 2.2, then $\tilde{\mathbf{A}}(t)$ is an anti-symmetric matrix at all times. $\mathbf{D}(t) = \mathbf{diag}\{d_1(t), d_2(t), \cdots, d_N(t)\} \in \mathbb{R}^{N\times N}$ with $d_i(t) = \sum_{j=1}^{N} \tilde{a}_{ij}(t)$. $\mathbf{B}(t) = \mathbf{diag}\{b_1(t), b_2(t), \cdots, b_N(t)\} \in \mathbb{R}^{N\times N}$.

Since the sum of each row of the matrix $\tilde{\mathbf{A}}(t) - \mathbf{D}(t)$ is equal to zero, then $[\tilde{\mathbf{A}}(t) - \mathbf{D}(t)]\mathbf{1}_N x_0(t) = \mathbf{0}_N$. Therefore, the consensus error system can be described as:
When $t \in [t_{2k}, t_{2k+1})$,

$$\dot{\mathbf{e}}(t) = \begin{bmatrix} \dot{\bar{\mathbf{x}}}(t) \\ \dot{\bar{\mathbf{v}}}(t) \end{bmatrix} = \begin{bmatrix} p[\tilde{\mathbf{A}}(t) - \mathbf{D}(t) - \mathbf{B}(t)] & \mathbf{I}_N \\ qp[\tilde{\mathbf{A}}(t) - \mathbf{D}(t) - \mathbf{B}(t)] & 0_{N\times N} \end{bmatrix} \mathbf{e}(t), \tag{9}$$

when $t \in [t_{2k+1}, t_{2k+2})$,

$$\dot{\mathbf{e}}(t) = \begin{bmatrix} \dot{\bar{\mathbf{x}}}(t) \\ \dot{\bar{\mathbf{v}}}(t) \end{bmatrix} = \begin{bmatrix} 0_{N\times N} & \mathbf{I}_N \\ 0_{N\times N} & 0_{N\times N} \end{bmatrix} \mathbf{e}(t), \tag{10}$$

where $k = 0, 1, \cdots$.

Now, we give the sufficient conditions as the Theorem 1 which guarantee the consensus of the multi-agent system, i.e., $\lim_{t\to\infty} \| \mathbf{e}(t) \| = 0$. Firstly, a useful lemma [28] is given as follows:

Lemma 1: Suppose that a symmetric matrix is described as

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 \\ \mathbf{L}_2^T & \mathbf{L}_3 \end{bmatrix},$$

where $\mathbf{L}_1$ and $\mathbf{L}_3$ are square matrices. $\mathbf{L}$ is positive definite if and only if both $\mathbf{L}_1$ and $\mathbf{L}_3 - \mathbf{L}_2^T \mathbf{L}_1^{-1} \mathbf{L}_2$ are positive definite.

Theorem 1: Multiple agents in the system described as (7) and (8) will achieve consensus under the following conditions:

(i) $b_i(t)$ in (5) satisfies:

$$b_i(t) = \begin{cases} 0, & \text{if} \quad d_i(t) > 0, \\ 1 - d_i(t), & \text{otherwise} \end{cases} \tag{11}$$

for $i = 1, 2, \cdots, N$.

(ii) Control parameters p and q satisfy:

$$p > \frac{1}{4q(1 - q^2)}, \tag{12}$$

where $0 < q < 1$.

(iii) Let $\Delta_k = t_{k+1} - t_k$, $k = 0, 1, \cdots$, suppose that there exist $T_w, T_r > 0$, which satisfy (13) and (14):

$$\begin{cases} \Delta_{2k} \geq T_w, \\ \Delta_{2k+1} \leq T_r, \end{cases} \tag{13}$$

$$\frac{T_w}{T_r} > \frac{\tilde{s}^*(1+q)}{r^*(1-q)}, \tag{14}$$

where

$$\tilde{s}^* = \sqrt{q^2+1} - q > 0,$$
$$r^* = (1-q^2)p + q - \sqrt{[(1-q^2)p-q]^2+1} > 0.$$

Proof: Construct the following Lyapunov function:

$$V(t) = \mathbf{e}^T(t)\mathbf{M}\mathbf{e}(t), \tag{15}$$

where $\mathbf{M} = \begin{bmatrix} \mathbf{I}_N & -q\mathbf{I}_N \\ -q\mathbf{I}_N & \mathbf{I}_N \end{bmatrix} \in \mathbb{R}^{2N\times 2N}$, we can get the minimum eigenvalue of $\mathbf{M}$ is $\lambda_{min}(\mathbf{M}) = 1-q > 0$, while the maximum eigenvalue of $\mathbf{M}$ is $\lambda_{max}(\mathbf{M}) = 1+q > 0$, and $\mathbf{M}$ is a positive definite matrix.
When $t \in [t_{2k}, t_{2k+1})$, $k = 0,1,\cdots,$

$$\begin{aligned}\dot{V}(t) &= \dot{\mathbf{e}}^T(t)\mathbf{M}\mathbf{e}(t) + \mathbf{e}^T(t)\mathbf{M}\dot{\mathbf{e}}(t) \\ &= -\mathbf{e}^T(t)\mathbf{R}(t)\mathbf{e}(t),\end{aligned} \tag{16}$$

where $\mathbf{R}(t) = \begin{bmatrix} 2(1-q^2)p[\mathbf{D}(t)+\mathbf{B}(t)] & -\mathbf{I}_N \\ -\mathbf{I}_N & 2q\mathbf{I}_N \end{bmatrix} \in \mathbb{R}^{2N\times 2N}$;
When $t \in [t_{2k+1}, t_{2k+2})$, $k = 0,1,\cdots,$

$$\begin{aligned}\dot{V}(t) &= \dot{\mathbf{e}}^T(t)\mathbf{M}\mathbf{e}(t) + \mathbf{e}^T(t)\mathbf{M}\dot{\mathbf{e}}(t) \\ &= -\mathbf{e}^T(t)\mathbf{S}(t)\mathbf{e}(t),\end{aligned} \tag{17}$$

where $\mathbf{S}(t) = \begin{bmatrix} 0_{N\times N} & -\mathbf{I}_N \\ -\mathbf{I}_N & 2q\mathbf{I}_N \end{bmatrix} \in \mathbb{R}^{2N\times 2N}$.

The above equation (16) holds since $\tilde{\mathbf{A}}(t)$ always is an anti-symmetric matrix. Now, we prove that the matrix $\mathbf{R}(t)$ is positive definite. Since $\mathbf{D}(t)+\mathbf{B}(t) = \mathbf{diag}\{d_1(t)+b_1(t), d_2(t)+b_2(t), \cdots, d_N(t)+b_N(t)\}$, then the eigenvalues of $\mathbf{D}(t)+\mathbf{B}(t)$ are $\lambda_i(\mathbf{D}(t)+\mathbf{B}(t)) = d_i(t)+b_i(t)$, $i = 1,2,\cdots,N$, where $1-N \le d_i(t) = \sum_{j=1}^N \tilde{a}_{ij}(t) \le N-1$. If the condition (i) in the Theorem 1 is satisfied, then $\mathbf{D}(t)+\mathbf{B}(t)$ is positive definite with the minimum eigenvalue being $\lambda_{min}(\mathbf{D}(t)+\mathbf{B}(t)) = 1$. Accordingly, based on the Lemma 1, if the condition (ii) in the Theorem 1 is satisfied, then $\mathbf{R}(t)$ is positive definite. Furthermore, we can get all eigenvalues of $\mathbf{R}(t)$ as follows:

$$\begin{cases} \lambda_{i,1}(\mathbf{R}(t)) = (1-q^2)p(d_i(t)+b_i(t)) + q \\ \qquad\qquad + \sqrt{[(1-q^2)p(d_i(t)+b_i(t))-q]^2+1}, \\ \lambda_{i,2}(\mathbf{R}(t)) = (1-q^2)p(d_i(t)+b_i(t)) + q \\ \qquad\qquad - \sqrt{[(1-q^2)p(d_i(t)+b_i(t))-q]^2+1}, \end{cases} \tag{18}$$

when $i = 1,2,\cdots,N$. Now, we search for the minimum eigenvalue of $\mathbf{R}(t)$, which will exist in the $\lambda_{i,2}(\mathbf{R}(t))$. It is easy to find out that the derivative of $\lambda_{i,2}(\mathbf{R}(t))$ with respect to $d_i(t)+b_i(t)$ is greater than zero, then $\lambda_{i,2}(\mathbf{R}(t))$ is an increasing function with $d_i(t)+b_i(t)$. Accordingly, when $d_i(t)+b_i(t) = 1$, $\lambda_{i,2}(\mathbf{R}(t))$ is minimum, which equals $r^* = (1-q^2)p + q - \sqrt{[(1-q^2)p-q]^2+1} > 0$.

Similarly, the eigenvalues of $\mathbf{S}(t)$ are

$$\begin{cases} \lambda_{i,1}(\mathbf{S}(t)) = q + \sqrt{q^2+1}, \\ \lambda_{i,2}(\mathbf{S}(t)) = q - \sqrt{q^2+1}, \end{cases} \tag{19}$$

when $i = 1,2,\cdots,N$. It is easy to see that the minimum eigenvalue of $\mathbf{S}(t)$ will be $-\tilde{s}^*$, where $\tilde{s}^* = \sqrt{q^2+1} - q > 0$. Therefore, according to (16) and (17), we obtain the following:
When $t \in [t_{2k}, t_{2k+1})$, $k = 0,1,\cdots,$

$$\dot{V}(t) \le -r^*\mathbf{e}^T(t)\mathbf{e}(t). \tag{20}$$

When $t \in [t_{2k+1}, t_{2k+2})$, $k = 0,1,\cdots,$

$$\dot{V}(t) \le \tilde{s}^*\mathbf{e}^T(t)\mathbf{e}(t). \tag{21}$$

Then, for that $\lambda_{max}(\mathbf{M}) = 1+q$, $\lambda_{min}(\mathbf{M}) = 1-q$, we can get:
When $t \in [t_{2k}, t_{2k+1})$, $k = 0,1,\cdots,$

$$\dot{V}(t) \le -\frac{r^*}{1+q}V(t).$$

When $t \in [t_{2k+1}, t_{2k+2})$, $k = 0,1,\cdots,$

$$\dot{V}(t) \le \frac{\tilde{s}^*}{1-q}V(t).$$

Accordingly, when $t \in [t_0, t_1)$,

$$V(t) \le V(t_0)\exp\left\{-\frac{r^*}{1+q}(t-t_0)\right\}. \tag{22}$$

When $t \in [t_1, t_2)$,

$$V(t) \le V(t_0)\exp\left\{-\frac{r^*}{1+q}\Delta_0 + \frac{\tilde{s}^*}{1-q}(t-t_1)\right\}. \tag{23}$$

Therefore, when $t \in [t_{2k}, t_{2k+1})$, $k = 1,2,\cdots,$

$$\begin{aligned}V(t) \le V(0)\exp\Bigg\{&\frac{-r^*}{1+q}(\Delta_0+\Delta_2+\cdots+\Delta_{2k-2}+t-t_{2k}) \\ &+\frac{\tilde{s}^*}{1-q}(\Delta_1+\Delta_3+\cdots+\Delta_{2k-1})\Bigg\}.\end{aligned} \tag{24}$$

When $t \in [t_{2k+1}, t_{2k+2})$, $k = 1,2,\cdots,$

$$\begin{aligned}V(t) \le V(0)\exp\Bigg\{&\frac{-r^*}{1+q}(\Delta_0+\Delta_2+\cdots+\Delta_{2k}) \\ &+\frac{\tilde{s}^*}{1-q}(\Delta_1+\Delta_3+\cdots+\Delta_{2k-1}+t-t_{2k+1})\Bigg\}.\end{aligned} \tag{25}$$

If the condition (iii) in Theorem 1 is satisfied, then when $t \in [t_k, t_{k+1})$, $k = 0,1,\cdots,$

$$V(t) \le V(0)\exp\left\{\frac{-r^*}{1+q}\left\lfloor\frac{k}{2}\right\rfloor T_w + \frac{\tilde{s}^*}{1-q}\left(\left\lfloor\frac{k}{2}\right\rfloor+1\right)T_r\right\}.$$

Consequently, when $k \to +\infty$, $V(t) \to 0$, i.e., $\lim_{t\to\infty} x_i(t) = x_0(t)$, $\lim_{t\to\infty} v_i(t) = v_0(t)$ for $i = 1,2,\cdots,N$. It means that the followers will realize consensus as time going on. The proof of the Theorem 1 is thus completed.

Remark 3: The condition (i) demonstrates that at time t, for the agent i, if the number of cooperative neighbors is more than that of competitive neighbors, i.e., $d_i(t) > 0$, then communication from the leader to the i th follower is not needed $(b_i(t) = 0)$; otherwise, an edge with weight being $b_i(t) = 1 - d_i(t)$ exists from the leader to the i th follower.

Remark 4: In the condition (iii), T_w indicates the shortest continuous "working" time needed for the control $K_i(t)$, while T_r indicates the longest continuous "resting" time. T_w can also be viewed as the shortest dwell time for $K_i(t)$. T_w and T_r just only depend on the control parameters p and q.

4. NUMERICAL EXAMPLES

In order to illustrate the aforementioned theoretical analysis clearly, in this section, we take a model of multi-agent system consisting of 5 followers and one leader, i.e., $N = 5$. The initial states of followers are selected as $\mathbf{x}(0) = (-3.5, 1, 2.8, 0, -2)^T$, The acceleration of the leader a_0 equals 0.2, the initial state and velocity of the leader are chosen as $x_0(0) = 10$, $v_0(0) = 1$. The neighbors of the agent i at time t here are described as $\mathcal{N}_i(t) = \{j : |x_j(t) - x_i(t)| < 2\}$, accordingly, the network consisting of the followers is time-varying, and $\tilde{\mathbf{A}}(0)$, $\mathbf{D}(0)$ can be represented as

$$\tilde{\mathbf{A}}(0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{D}(0) = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

We can see that $\tilde{\mathbf{A}}(0)$ is an anti-symmetric matrix. The initial value of $\mathbf{v}(t)$ are selected as $\mathbf{v}(0) = (0.5, 0.2, 0.8, 1.3, 0.7)^T$. Furthermore, let $\Delta_{2k} = T_w$ and $\Delta_{2k+1} = T_r$ for all $k = 0,1,\cdots$.

Example 1: The control parameters p and q are taken as 10 and 0.5, respectively, which satisfy the condition (ii) in Theorem 1. Since $r^* = 0.5376$ and $\tilde{s}^* = 0.6180$

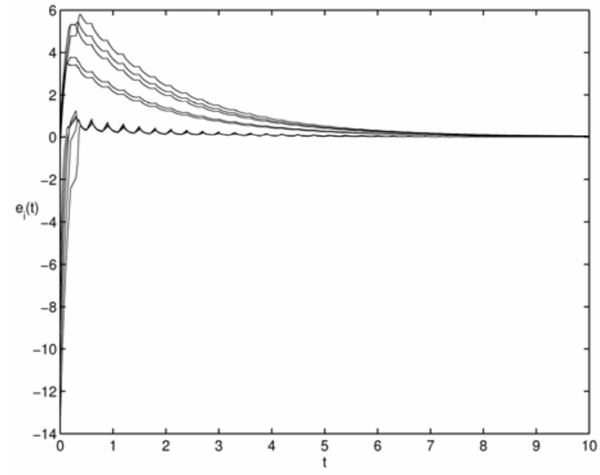

Fig. 1. Graphical representations of the consensus errors $e_i(t)$ $(i = 1,2,\cdots,10)$ in Example 1.

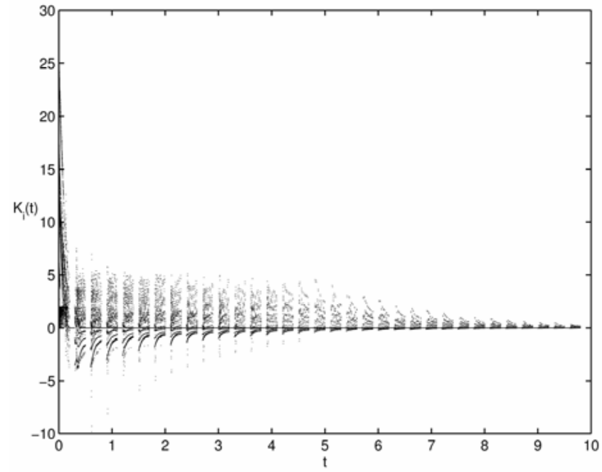

Fig. 2. Graphical representations of the intermittent control $K_i(t)$ $(i = 1,2,\cdots,5)$ in Example 1.

here, then T_w and T_r are chosen as 0.2 second and 0.1 second, which satisfy the condition (iii) in Theorem 1. $b_i(t)$ in the consensus protocol (5) is described as (11), i.e.,

$$b_i(t) = \begin{cases} 0, & \text{if } \ d_i(t) > 0, \\ 1 - d_i(t), & \text{otherwise} \end{cases} \quad \text{for } \ i = 1,2,\cdots,5.$$

Then, the numerical results are demonstrated in Fig. 1 to Fig. 3. Fig. 1 indicates that all followers will asymptotically reach the leader, since $e_i(t) \to 0$, where $e_i(t) = x_i(t) - x_0(t)$, when $i = 1,2,\cdots,5$; $e_i(t) = v_{i-5}(t) - v_0(t)$, when $i = 6,7,\cdots,10$. Fig. 2 demonstrates that $K_i(t)$ is an intermittent control. In Fig. 3, we give "+" sign denoting the $g(t)$ $(t \in [0, 1.2))$, where $g(t)$ stands for the number of followers are not connected to the leader at time t, we can see that in the "resting" time, the leader do not communicate with any follower, i.e., $g(t) = 5$; while in the "working" time, the leader also do not need to communicate all followers, since $g(t) > 0$ in most of the time.

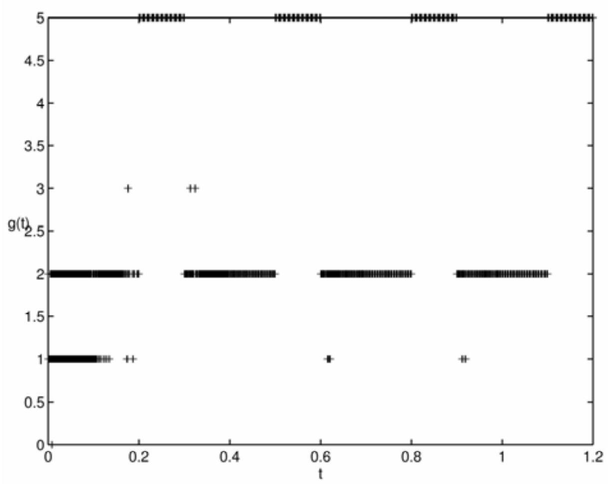

Fig. 3. Graphical representation of the $g(t)$ in Example 1.

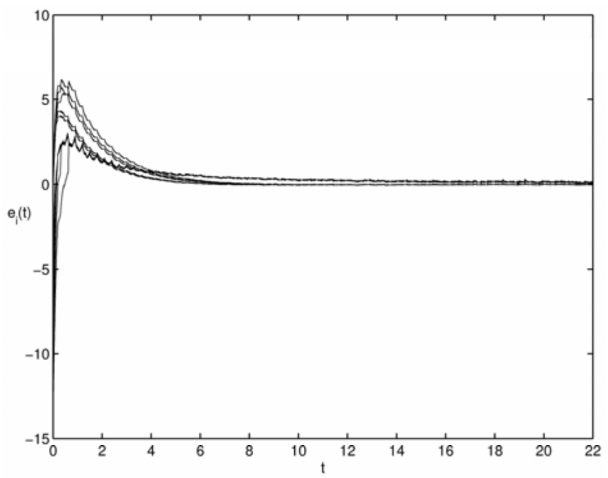

Fig. 4. Graphical representations of the consensus errors $e_i(t)$ $(i=1,2,\cdots,10)$ infected by noises in Example 1.

In addition, the consensus protocol is robust for some noises actually existing in the transmission channels. If $x_j(t)-x_i(t)$ in the $u_i(t)$ are changed into $x_j(t)+\eta_{ij}(t)-x_i(t)$ for $j=0,1,\cdots,5;$ $i=1,2,\cdots,5,$ where $\eta_{ij}(t)$ denote the noises, then the consensus may also be reached. For example, $\eta_{ij}(t)$ are assumed to be the different white noises with intensities being $\frac{2}{t+1}$, the consensus result will be shown in Fig. 4.

Example 2: If we take $b_i(t)=0$ for all followers in the "working" time, and the other parameters are selected the same as in Example 1, then all followers cannot achieve consensus with the leader, the representations of the state errors and velocity estimation errors will be demonstrated in Fig. 5.

Example 3: If we take $T_w=0.2,$ $T_r=5,$ and the other parameters are selected the same as in Example 1, then the consensus cannot be reached, the representations of the state errors and velocity estimation errors are demonstrated in Fig. 6.

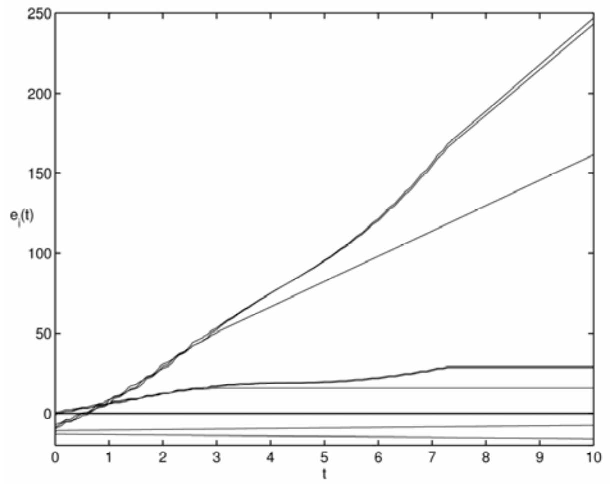

Fig. 5. Graphical representations of the consensus errors $e_i(t)$ $(i=1,2,\cdots,10)$ in Example 2.

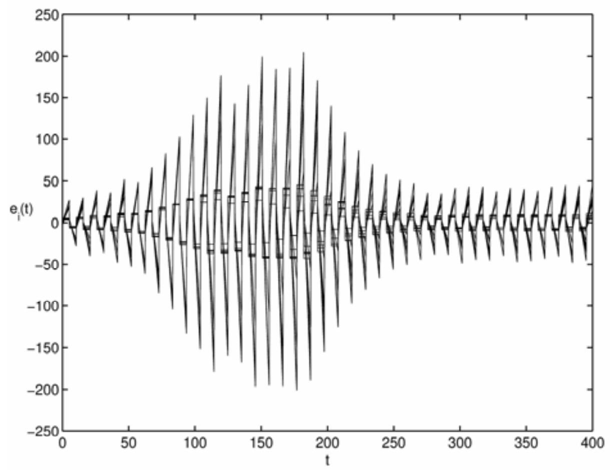

Fig. 6. Graphical representations of the consensus errors $e_i(t)$ $(i=1,2,\cdots,10)$ in Example 3.

5. CONCLUSIONS AND DISCUSSION

In this paper, we consider the consensus problem of a leader-following multi-agent system with an active leader. The intermittent control approach has been applied in the consensus protocol. The theoretical results have been derived mainly based on the Lyapunov stability. In the model of the multi-agent system, neighbors of a follower can be time-varying on account of agents' motions, especially, for the agent i, its neighbors can be divided into cooperative and competitive types. It is found out that the sufficient conditions for consensus of the proposed model include three aspects: The first one is the condition about the weight of the edge from the leader to the follower; The second one is the condition of the control parameters p and q; And the last one is about the continuous "working" time and "resting" time. All the conditions are easily satisfied, and we do not need to solve the linear matrix inequalities (LMIs) [29], which is usually needed in the stability analysis. It should be pointed out these are sufficient but not necessary conditions for consensus. The results proposed here may be practical since study on multiple agents moving in one-dimensional space has some significance [30]. The future work regarding this topic will focus on exploring the better consensus algorithms for general multi-agent systems.

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