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H_∞ Synchronization of Two Different Discrete-Time Chaotic Systems via a Unified Model

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Abstract: This paper presents some novel synchronization methods for two discrete-time chaotic systems with different time delays, which are transformed into two unified models. First, the H_∞ performance of the synchronization error dynamical system between the drive unified model and the response one is analyzed using the linear matrix inequality (LMI) approach. Second, the novel state feedback controllers are established to guarantee H_∞ performance for the overall system. The parameters of these controllers are determined by solving the eigenvalue problem (EVP). Most discrete-time chaotic systems with or without time delays can be converted into this unified model, and H_∞ synchronization controllers are designed in a unified way. The effectiveness of the proposed design methods are demonstrated by three numerical examples.

Keywords: H_∞ synchronization, chaotic systems, different time delays, discrete-time system, drive-response conception.

1. INTRODUCTION

In recent years, synchronization problems in chaotic systems have attracted much attention, and many possible applications such as secure communication, have been discussed by computer simulations and even realized under laboratory conditions [1-3]. Since Pecora and Carroll [4] firstly proposed the drive-response (master-slave) concept for achieving the synchronization of coupled chaotic systems, many researchers have also proposed a variety of alternative schemes for the control and synchronization of chaotic systems with or without delays, which include linear and nonlinear feedback control, impulsive control method, sliding mode control, adaptive design control, and invariant manifold method, among many others (see [1-10] and references cited therein).

In real physical systems, some noises or disturbances always exist that may cause the instability and poor performance. Therefore, how to reduce the influence of the noises or disturbances on the synchronization process of chaotic systems becomes an important issue. Suykens *et al.* [11] firstly adopted the H_∞ control concept to reduce the effect of the disturbance for synchronization problem of chaotic Lur'e systems. Based on the work of Suykens *et al.*, authors in [12] and [13] designed H_∞ synchronization controllers for a general class of chaotic systems with external disturbances. On the other hand, there has been increasing interest in time-delayed chaotic systems since chaos phenomenon in time-delayed systems was first found by Mackey and Glass [14]. The H_∞ synchronization problem for time-delayed chaotic systems is also investigated by some researchers [15-18].

Here one thing should be pointed out. The underlying assumption in the aforementioned methods is that the drive and the response systems have identical dynamic structures and the same parameters. And external disturbances and different time delays both exist in engineering practice while both of them have seldom been considered in synchronization problems between chaotic systems. Therefore, to our best knowledge, synchronization problems between systems of different time delays with external disturbances haven't been (not much if any) touched so far, which is just the main job in this paper, thus it makes sense for a deep investigation.

In this paper, we first put forward a unified model to describe discrete-time chaotic systems [19] and continuous-time chaotic systems [20]. This unified model is the interconnection of a linear dynamic system and a bounded static nonlinear operator. Most chaotic systems with or without time delays, such as chaotic neural networks, Chua's circuits, and Hénon map, etc, can be transformed into this unified model with the H_∞

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synchronization controller designed in a unified way. A kind of state feedback controller for the synchronization between two unified models with different time delays is proposed. By the state feedback control scheme, the closed-loop synchronization error system is asymptotically stable and the H_∞-norm from the disturbance to controlled output is reduced to the lowest level.

Notation: The superscript "T" stands for matrix transposition. $l_2[0, \infty)$ is the space of square integrable vectors. $\Re^n$ denotes n dimensional Euclidean space, and $\Re^{n\times m}$ is the set of all $n\times m$ real matrices. I denotes identity matrix of appropriate orders. $*$ denotes the symmetric parts. diag{…} stands for a block-diagonal matrix. The notations $X > Y$ and $X \geq Y$, where X and Y are matrices of the same dimensions, mean that the matrix $X - Y$ is positive definite and positive semi-definite, respectively. If $X\in\Re^p$ and $Y\in\Re^q$, $C(X; Y)$ denotes the space of all continuous functions mapping $\Re^p \to \Re^q$.

2. PROBLEM FORMULATION

The unified model we suggested consists of a linear dynamic system and a bounded static nonlinear operator [19]:

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-\tau_x) + B_p\phi(\xi(k)), \\ \xi(k) = C_q x(k) + C_{qd} x(k-\tau_x) + D_p\phi(\xi(k)), \\ z_x(k) = Cx(k), \end{cases} \tag{1}$$

with the initial condition function $x(k) = \varpi(k)$, $\forall k\in[-\tau_x, 0]$, where $x(k)\in\Re^n$ is the system state, $A\in\Re^{n\times n}$, $A_d\in\Re^{n\times n}$, $B_p\in\Re^{n\times L}$, $C_q\in\Re^{L\times n}$, $C_{qd}\in\Re^{L\times n}$, $D_p\in\Re^{L\times L}$, and $C\in\Re^{l\times n}$, are the corresponding state-space matrices, $\xi\in\Re^L$ is the input of nonlinear function ϕ, $\phi\in C(\Re^L; \Re^L)$ is nonlinear function satisfying $\phi(0)=0$, $z_x(k)\in\Re^l$ is the output vector, $L\in N$ is the number of nonlinear functions, $\tau_x \geq 0$ is the time delay, $\varpi(k)$ is the given function on $[-\tau_x, 0]$.

In this paper, we assume that the nonlinear functions in (1) are monotonically non-decreasing and globally Lipschitz. That is, there exists a positive scalar h_i such that

$$0 \leq \frac{\phi_i(\alpha)-\phi_i(\beta)}{\alpha-\beta} \leq h_i, \qquad i=1, \ldots, L, \tag{2}$$

for all arbitrary $\alpha \neq \beta$ and $\alpha, \beta\in\Re$.

According to the drive-response concept [4], if the system (1) is regarded as the drive system, a suitable response system with control input should be constructed to synchronize the drive system. The response unified model can be described by the following equations:

$$\begin{cases} y(k+1) = Ay(k) + A_d y(k-\tau_y) + B_p\phi(\zeta(k)) \\ \qquad\qquad + u_1(k) + Dw(k), \\ \zeta(k) = C_q y(k) + C_{qd} y(k-\tau_y) + D_p\phi(\zeta(k)) \\ \qquad\qquad + u_2(k), \\ z_y(k) = Cy(k), \end{cases} \tag{3}$$

with the initial condition function $y(k) = \sigma(k)$, $\forall k\in[-\tau_y, 0]$, where $y(k)\in\Re^n$ is the state vector of response system, $D\in\Re^{n\times s}$ is a constant matrix, $\tau_y \in \Re$ is the time delay, which is generally assumed to satisfy $\tau_y \geq \tau_x \geq 0$, $\sigma(\cdot)$ is the given continuous function on $[-\tau_y, 0]$, $w(k)\in\Re^s$ is the external disturbance which belongs to $l_2[0, \infty)$, $z_y(k)\in\Re^l$ is the output of the response system, $u_1(k)\in\Re^n$ and $u_2(k)\in\Re^L$ are the control inputs and will be appropriately designed such that the specific control objective is achieved.

Defining the synchronization error $e(k)=y(k)-x(k)$, we have the following error dynamical system between (1) and (3):

$$\begin{cases} e(k+1) = Ae(k) + A_d(y(k-\tau_y) - x(k-\tau_x)) \\ \qquad\qquad + B_p f(\eta(k)) + u_1(k) + Dw(k), \\ \eta(k) = C_q e(k) + C_{qd}(y(k-\tau_y) - x(k-\tau_x)) \\ \qquad\qquad + D_p f(\eta(k)) + u_2(k), \\ z_e(k) = Ce(k), \end{cases} \tag{4}$$

where $e(k)\in\Re^n$, $z_e(k) = z_y(k) - z_x(k)$, $\eta(k) = \zeta(k) - \xi(k)$, and $f(\eta(k)) = \phi(\zeta(k)) - \phi(\xi(k)) = \phi(\eta(k) + \xi(k)) - \phi(\xi(k))$, therefore $f(0) = 0$. From (2), we derived that $f_i(\cdot)$ satisfy the sector conditions, i.e. , for each $i = 1, \ldots, L$,

$$0 \leq f_i(\eta_i(k))/\eta_i(k) \leq h_i \text{ or}$$
$$f_i(\eta_i(k))\cdot[f_i(\eta_i(k)) - h_i\eta_i(k)] \leq 0. \tag{5}$$

In order to synchronize drive system (1) with response one (3) in the sense of H_∞ control [21], we consider the following state feedback controller:

$$\begin{cases} u_1(k) = K_1 e(k) + A_d(y(k-\tau_x) - x(k-\tau_y)), \\ u_2(k) = K_2 e(k) + C_{qd}(y(k-\tau_x) - x(k-\tau_y)), \end{cases} \tag{6}$$

where $x(k) = x(-\tau_x)$, $\forall k\in[-\tau_y, -\tau_x]$, $K_1\in\Re^{n\times n}$ and $K_2\in\Re^{L\times n}$ are feedback gains. With the control law (6), the error dynamic system (4) can be rewritten as the follows:

$$\begin{cases} e(k+1) = \bar{A}e(k) + A_d(e(k-\tau_y) + e(k-\tau_x)) \\ \qquad\qquad + B_p f(\eta(k)) + Dw(k), \\ \eta(k) = \bar{C}_q e(k) + C_{qd}(e(k-\tau_y) + e(k-\tau_x)) \\ \qquad\qquad + D_p f(\eta(k)), \\ z_e(k) = Ce(k), \end{cases} \tag{7}$$

where $\bar{A} = A + K_1$ and $\bar{C}_q = C_q + K_2$. Since $f(0) = 0$, system (7) has a trivial solution $e(k) \equiv 0$ while $w(k) = 0$.

Definition 1 (H_∞ synchronization): The drive system (1) and the response system (3) are said to be H_∞ synchronized if the following two conditions are satisfied:

(i) With zero disturbances, the synchronization error system (7) is asymptotically stable.

(ii) With zero initial conditions and a given constant $\gamma > 0$, the following condition holds:

$$J=\sum_{k=0}^{\infty}[z_e^T(k)z_e(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)]<0$$
$$\text{(i.e.,}\quad \sup_{w(k)\neq 0,w(k)\in l_2[0,\infty)}\frac{\|z_e(k)\|}{\|w(k)\|}<\gamma). \tag{8}$$

Then, the controller (6) is said to be an H_∞ synchronization controller with the disturbance attenuation γ. The parameter γ is called the H_∞-norm bound of the controller. If we find a minimal positive γ to satisfy the above conditions, then the controller (6) is an optimal H_∞ synchronizer.

3. H_∞ PERFORMANCE ANALYSIS

Theorem 1: If there exist symmetric positive definite matrices P, R and Q, diagonal positive semi-definite matrices Λ and Σ, matrices K_1 and K_2, and a positive scalar γ that satisfy

$$M=\begin{bmatrix} \bar{A}^{\mathrm{T}}P\bar{A}+\Pi_1+C^{\mathrm{T}}C & \bar{A}^{\mathrm{T}}PA_d & \bar{A}^{\mathrm{T}}PA_d & \bar{A}^TPB_p+\bar{C}_q^{\mathrm{T}}(\Sigma H+\Lambda) & \bar{A}^TPD \\ * & A_d^{\mathrm{T}}PA_d-Q & A_d^{\mathrm{T}}PA_d & A_d^TPB_p+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & A_d^TPD \\ * & * & A_d^{\mathrm{T}}PA_d-R & A_d^TPB_p+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & A_d^TPD \\ * & * & * & B_p^TPB_p+\Pi_2 & B_p^TPD \\ * & * & * & * & D^TPD-\gamma^2 I \end{bmatrix}<0, \tag{9}$$

where $\Pi_1=-P+Q+R$, $\Pi_2=D_p^T(\Sigma H+\Lambda)+(\Sigma H+\Lambda)D_p-2\Sigma$, $H=\mathrm{diag}\{h_1, h_2, \ldots, h_L\}$, then system (7) with $w(k)=0$ is globally asymptotically stable and the L_2 gain of the system (7) is less than or equal to γ. The minimum of γ can be obtained by solving the following eigenvalue problem (EVP):

$$\begin{aligned}&\text{minimize} \quad \gamma,\\ &\text{subject to} \quad (9), P>0, Q>0, R>0, \Lambda\geq 0, \Sigma\geq 0.\end{aligned} \tag{10}$$

Proof: First, consider system (7) with $w(k)=0$, that is

$$\begin{cases} e(k+1)=\bar{A}e(k)+A_d(e(k-\tau_y)\\ \qquad\qquad +e(k-\tau_x))+B_pf(\eta(k)),\\ \eta(k)=\bar{C}_qe(k)+C_{qd}(e(k-\tau_y)\\ \qquad\qquad +e(k-\tau_x))+D_pf(\eta(k)),\\ z_e(k)=Ce(k). \end{cases} \tag{11}$$

Since $e(k)=0$ and $\eta(k)=0$ are solutions to (11), there exists at least one equilibrium located at the origin, i.e., $e_{eq}=0$, $\eta_{eq}=0$. For system (11), we adopt the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(e(k),\eta(k))&=e^T(k)Pe(k)+\sum_{i=-\tau_x}^{-1}e^{\mathrm{T}}(i+k)Qe(i+k)\\ &+\sum_{i=-\tau_y}^{-1}e^{\mathrm{T}}(i+k)Re(i+k)\\ &+2\sum_{j=1}^{L}\lambda_j\int_0^{\eta_j(k-1)}f_j(\sigma)d\sigma, \end{aligned} \tag{12}$$

where $P>0$, $Q>0$, $R>0$ and $\lambda_j\geq 0$ ($j=1,2,\ldots,L$). Thus, $\forall e(k)\neq 0$, $\forall\eta(k)\neq 0$, $V(e(k), \eta(k))>0$ and $V(e(k), \eta(k))=0$ iff $e(k)=0$ and $\eta(k)=0$. We first give an estimation of the term of the integral $\int_0^{\eta_j(k)}f_j(\sigma)d\sigma$ by the sectors condition (5) and integral mean-value theorem. While $\eta_j(k)\geq 0$, we have

$$\int_0^{\eta_j(k)}f_j(\sigma)d\sigma=\eta_j(k)f_j(\beta)\leq\eta_j(k)f_j(\eta_j(k)), \tag{13}$$

where $0\leq\beta\leq\eta_j(k)$, $0\leq f_j(\beta)\leq f_j(\eta_j(k))$. While $\eta_j(k)\leq 0$, the inequality (13) also holds, where $\eta_j(k)\leq\beta\leq 0$, $f_j(\eta_j(k))\leq f_j(\beta)\leq 0$.

From the sector conditions (5), we have

$$f_i(\eta_i(k))\cdot\varepsilon_i\cdot[f_i(\eta_i(k))-u_i\eta_i(k)]\leq 0, \tag{14}$$

where $\varepsilon_i\geq 0$ ($i=1, \ldots, L$). From the inequality (14), we have

$$2\varepsilon_i f_i^2(\eta_i(k))-2\varepsilon_i u_i f_i(\eta_i(k))\eta_i(k)\leq 0. \tag{15}$$

The difference of $V(e(k), \eta(k))$ along the solution to (11) is

$$\begin{aligned} &\Delta V(e(k),\eta(k))\\ &=V(e(k+1),\eta(k+1))-V(e(k),\eta(k))\\ &\leq e^T(k+1)Pe(k+1)-e^T(k)Pe(k)+e^T(k)(Q+R)e(k)\\ &\quad -e^{\mathrm{T}}(k-\tau_x)Qe(k-\tau_x)-e^{\mathrm{T}}(k-\tau_y)Re(k-\tau_y)\\ &\quad +2\sum_{j=1}^{L}\lambda_j\int_0^{\eta_j(k)}f_j(\sigma)d\sigma-2\sum_{i=1}^{L}\varepsilon_i f_i^2(\eta_i(k))\\ &\quad +2\sum_{i=1}^{L}\varepsilon_i u_i f_i(\eta_i(k))\eta_i(k)\\ &\leq[\bar{A}e(k)+A_de(k-\tau_x)+A_de(k-\tau_y)+B_pf(\eta(k))]^TP\\ &\quad\times[\bar{A}e(k)+A_de(k-\tau_x)+A_de(k-\tau_y)+B_pf(\eta(k))]\\ &\quad -e^T(k)Pe(k)+e^T(k)(Q+R)e(k)\\ &\quad -e^{\mathrm{T}}(k-\tau_x)Qe(k-\tau_x)\\ &\quad -e^{\mathrm{T}}(k-\tau_y)Re(k-\tau_y)-2\sum_{i=1}^{L}\varepsilon_i f_i^2(\eta_i(k))\\ &\quad +2\sum_{i=1}^{L}(\varepsilon_i h_i+\lambda_i)f_i(\eta_i(k))\eta_i(k)\\ &=\left[e^{\mathrm{T}}(k)\quad e^{\mathrm{T}}(k-\tau_x)\quad e^{\mathrm{T}}(k-\tau_y)\quad f^{\mathrm{T}}(\eta(k))\right]G\\ &\quad\times\left[e^{\mathrm{T}}(k)\quad e^{\mathrm{T}}(k-\tau_x)\quad e^{\mathrm{T}}(k-\tau_y)\quad f^{\mathrm{T}}(\eta(k))\right]^{\mathrm{T}}, \end{aligned} \tag{16}$$

where

$$G=\begin{bmatrix}\begin{pmatrix}\bar{A}^T P\bar{A}\\+\Pi_1\end{pmatrix} & \bar{A}^T PA_d & \bar{A}^T PA_d & \begin{pmatrix}\bar{A}^T PB_p\\+\bar{C}_q^{\mathrm{T}}(\Sigma H+\Lambda)\end{pmatrix}\\ * & \begin{pmatrix}A_d^T PA_d\\-Q\end{pmatrix} & A_d^T PA_d & \begin{pmatrix}A_d^T PB_p\\+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda)\end{pmatrix}\\ * & * & \begin{pmatrix}A_d^T PA_d\\-R\end{pmatrix} & \begin{pmatrix}A_d^T PB_p\\+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda)\end{pmatrix}\\ * & * & * & B_p^T PB_p+\Pi_2\end{bmatrix},$$

$\Sigma=\mathrm{diag}\{\varepsilon_1,\ \varepsilon_2,\ \ldots,\ \varepsilon_L\}\geq 0$, $\Lambda=\mathrm{diag}\{\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_L\}\geq 0$. With the Schur complement [22], $M<0$ is equivalent to

$$\begin{bmatrix}\bar{A}^T P\bar{A}+\Pi_1 & \bar{A}^T PA_d & \bar{A}^T PA_d & \bar{A}^T PB_p+\bar{C}_q^{\mathrm{T}}(\Sigma H+\Lambda) & \bar{A}^T PD & C^T\\ * & A_d^T PA_d-Q & A_d^T PA_d & A_d^T PB_p+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & A_d^T PD & 0\\ * & * & A_d^T PA_d-R & A_d^T PB_p+C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & A_d^T PD & 0\\ * & * & * & B_p^T PB_p+\Pi_2 & B_p^T PD & 0\\ * & * & * & * & D^T PD-\gamma^2 I & 0\\ * & * & * & * & * & -I\end{bmatrix}<0. \quad (17)$$

Since G in (16) is the principal minor of the left-hand side of inequality (17), we have $G<0$. So system (7) with $w(k)=0$, i.e. system (11), is globally asymptotically stable.

Next, for system (7) under zero initial conditions, J in (8) is equivalent to

$$\begin{aligned}
&J(w(k))\\
&=\sum_{k=0}^{\infty}\left[z_e^{\mathrm{T}}(k)z_e(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)\right]\\
&=\sum_{k=0}^{\infty}\left[z_e^{\mathrm{T}}(k)z_e(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)+\Delta V(e(k),\eta(k))\right]\\
&\quad-\{V(e(\infty),\eta(\infty))-V(e(0),\eta(0))\}\\
&\leq\sum_{k=0}^{\infty}\left[z_e^{\mathrm{T}}(k)z_e(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)+\Delta V(e(k),\eta(k))\right]\\
&\leq\sum_{k=0}^{\infty}\Big\{e^{\mathrm{T}}(k)C^{\mathrm{T}}Ce(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)\\
&\quad+\left[e^{\mathrm{T}}(k)\quad e^{\mathrm{T}}(k-\tau_x)\quad e^{\mathrm{T}}(k-\tau_y)\quad f^{\mathrm{T}}(\eta(k))\right]G\\
&\quad\times\left[e^{\mathrm{T}}(k)\quad e^{\mathrm{T}}(k-\tau_x)\quad e^{\mathrm{T}}(k-\tau_y)\quad f^{\mathrm{T}}(\eta(k))\right]^{\mathrm{T}}\\
&\quad+[Dw(k)]^{\mathrm{T}}P[Dw(k)]+[Dw(k)]^{\mathrm{T}}P\\
&\quad\times P\left[\bar{A}e(k)+A_d e(k-\tau_x)+A_d e(k-\tau_y)+B_p f(\eta(k))\right]\\
&\quad+\left[\bar{A}e(k)+A_d e(k-\tau_x)+A_d e(k-\tau_y)\right.\\
&\quad\left.+B_p f(\eta(k))\right]^T P[Dw(k)]\Big\}\\
&=\sum_{k=0}^{\infty}\Big\{\left[e^{\mathrm{T}}(k)\ e^{\mathrm{T}}(k-\tau_x)\ e^{\mathrm{T}}(k-\tau_y)\ f^{\mathrm{T}}(\eta(k))\ w^{\mathrm{T}}(k)\right]\\
&\quad\times M\left[e^{\mathrm{T}}(k)\ e^{\mathrm{T}}(k-\tau_x)\ e^{\mathrm{T}}(k-\tau_y)\ f^{\mathrm{T}}(\eta(k))\ w^{\mathrm{T}}(k)\right]^{\mathrm{T}}\Big\}.
\end{aligned} \quad (18)$$

Since $M<0$ in inequality (17) , $J(w(k))<0$ holds for any $[e^{\mathrm{T}}(k)\ e^{\mathrm{T}}(k-\tau_x)\ e^{\mathrm{T}}(k-\tau_y) f^{\mathrm{T}}(\eta(k))\ w^{\mathrm{T}}(k)]^{\mathrm{T}}\neq 0$, $w(k)\in l_2[0,\infty)$. From Definition 1, it can be concluded that the drive system (1) and the response system (3) are H_∞ synchronized. This completes the proof.

While γ reaches its minimum, system (7) has the optimal perturbation resistance performance. It requires the solution of the eigenvalue problem (EVP) in inequality (10), which is a convex optimization problem that can be solved by using the MATLAB LMI Control Toolbox [23].

Remark 1: It should be noted here that the disturbance attenuate rate γ actually needs to be chosen as an appropriate value according to the real system to achieve the optimal performance, which will be illustrated in detail in Remark 3 and Corollary 2.

4. H_∞ SYNCHRONIZATION CONTROLLER DESIGN

Based on Theorem 1, we can obtain the following theorem to design the synchronization controller (6) for the drive system (1) and the response system (3).

Theorem 2: If there exist symmetric positive definite matrices P, Q, and R, diagonal positive semi-definite matrices Λ and Σ, matrices S_1 and S_2, and a positive scalar γ that satisfy the following EVP:

$$\text{minimize } \gamma, \quad (19)$$

subject to

$$\begin{bmatrix}-P & \begin{pmatrix}PA\\+S_1\end{pmatrix} & PA_d & PA_d & PB_p & PD\\ * & \begin{pmatrix}\Pi_1\\+C^T C\end{pmatrix} & 0 & 0 & \begin{pmatrix}C_q^{\mathrm{T}}(\Sigma H+\Lambda)\\+S_2^T\end{pmatrix} & 0\\ * & * & -Q & 0 & C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & 0\\ * & * & * & -R & C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & 0\\ * & * & * & * & \Pi_2 & 0\\ * & * & * & * & * & -\gamma^2 I\end{bmatrix}<0, \quad (20)$$

then the drive system (1) and the response system (3) can be synchronized by the H_∞ controller (6), and the H_∞-norm bound of the error system (7) does not exceed γ. Moreover, the feedback gains of optimal H_∞ controller

(6) are obtained by $K_1 = P^{-1}S_1$ and $K_2 = (H\Sigma+\Lambda)^{-1}S_2$.

Proof: With the Schur complement [22], the equality (9) in Theorem 1 is equivalent to

$$\begin{bmatrix} -P & P\bar{A} & PA_d & PA_d & PB_p & PD \\ * & \Pi_1 + C^T C & 0 & 0 & \bar{C}_q^{\mathrm{T}}(\Sigma H+\Lambda) & 0 \\ * & * & -Q & 0 & C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & 0 \\ * & * & * & -R & C_{qd}^{\mathrm{T}}(\Sigma H+\Lambda) & 0 \\ * & * & * & * & \Pi_2 & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0. \quad (21)$$

Defining $S_1 = PK_1$ and $S_2 = (H\Sigma+\Lambda)K_2$ in (21), we can obtain Theorem 2.

Remark 2: When the response system is constructed, all available information is taken use of such as A, A_d, B_p, C_q, C_{qd}, D_p, and C. And there are only two parameters K_1 and K_2 required to be determined.

If $A_d = 0$ and $C_{qd} = 0$ or $\tau_x = \tau_y = 0$, the system (1) is a chaotic system without time delays, which is represented as:

$$\begin{cases} x(k+1) = Ax(k) + B_p\phi(\xi(k)), \\ \xi(k) = C_q x(k) + D_p\phi(\xi(k)), \\ z_x(k) = Cx(k). \end{cases} \quad (22)$$

The response system corresponding to the drive system (22) is given by the following equations:

$$\begin{cases} y(k+1) = Ay(k) + B_p\phi(\zeta(k)) + u_1(k) + Dw(k), \\ \zeta(k) = C_q y(k) + D_p\phi(\zeta(k)) + u_2(k), \\ z_y(k) = Cy(k). \end{cases} \quad (23)$$

The H_∞ synchronization controller is of the following form:

$$\begin{cases} u_1(k) = K_1 e(k), \\ u_2(k) = K_2 e(k). \end{cases} \quad (24)$$

With the control law (24), the error dynamical system between (22) and (23) can be expressed by the following form:

$$\begin{cases} e(k+1) = (A+K_1)e(k) + B_p f(\eta(k)) + Dw(k), \\ \eta(k) = (C_q + K_2)e(k) + D_p f(\eta(k)), \\ z_e(k) = Ce(k). \end{cases} \quad (25)$$

Since $f(0) = 0$, the system (25) has a trivial solution $e(k) \equiv 0$. For the drive system (22) and the response system (23), we can use the following corollary to design the optimal H_∞ synchronization controller (24).

Corollary 1: If there exist a symmetric positive definite matrix P, diagonal positive semi-definite matrices Λ and Σ, matrices S_1 and S_2, and a positive scalar γ that satisfy the following EVP:

$$\text{minimize } \gamma, \quad (26)$$

subject to

$$\begin{bmatrix} -P & PA+S_1 & PB_p & PD \\ * & -P+C^T C & C_q^{\mathrm{T}}(\Sigma H+\Lambda)+S_2^T & 0 \\ * & * & \Pi_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (27)$$

then the drive system (22) and the response system (23) can be synchronized by the H_∞ controller (24), and the H_∞-norm bound of the error system (25) does not exceed γ. Moreover, the feedback gains of optimal H_∞ controller (24) are obtained as $K_1 = P^{-1}S_1$ and $K_2 = (H\Sigma+\Lambda)^{-1}S_2$.

The proof of Corollary 1 follows the same ideas as those in the proofs of Theorems 1 and 2, which is thus omitted here. For Corollary 1, the following Lyapunov functional is chosen:

$$V(e(k),\eta(k)) = e^T(k)Pe(k) + 2\sum_{j=1}^{L}\lambda_j \int_0^{\eta_j(k-1)} f_j(\sigma)d\sigma. \quad (28)$$

Remark 3: When the disturbance attenuation rate is given beforehand, the synchronization performance can still be achieved and the feedback gains can be determined by the following corollary.

Corollary 2: For a given γ, if there exist symmetric positive definite matrices P, Q, and R (P only for non-delayed systems), diagonal positive semi-definite matrices Λ and Σ, matrices S_1 and S_2 that satisfy (20) (or (27) for non-delayed systems), the feedback gains of controller (6) can be determined by $K_1 = P^{-1}S_1$ and $K_2 = (H\Sigma+\Lambda)^{-1}S_2$.

5. ILLUSTRATIVE EXAMPLES

In order to apply Theorem 2 (or Corollary 1) to solve the synchronization problems for the chaotic systems, we need to transform them into the unified model (1) (or (22)). The following three examples, i.e., synchronization of two chaotic Hopfield neural networks with different time delays, synchronization of two chaotic recurrent multilayer perceptrons (RMLPs) without and with time delays, and synchronization of two hyperchaotic Hénon maps, illustrate that the unified model can be widely applied to synchronization problems of a large class of chaotic systems.

5.1. Synchronization of two chaotic Hopfield neural networks with different time delays

We consider the following discrete-time chaotic delayed Hopfield neural network [24]:

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} 0.2 & -0.01 \\ -0.5 & 0.45 \end{bmatrix} \tanh(x(k)) \\ \qquad + \begin{bmatrix} -0.15 & -0.01 \\ -0.04 & -0.8 \end{bmatrix} \tanh(x(k-10)), \\ z_x(k) = x_2(k), \end{cases} \quad (29)$$

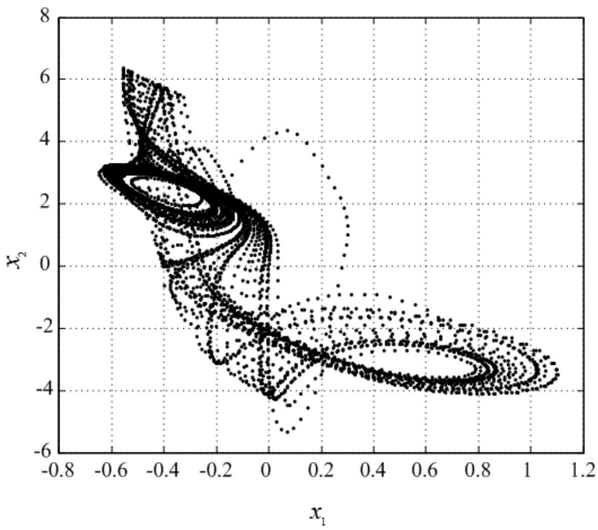

Fig. 1. The phase curve of the delayed Hopfield neural network (29) with the initial condition $[x_1(k)\ x_2(k)]^T = [-0.4\ -0.6]^T$ for $-10 \le k \le 0$ (5000 iterations have been plotted).

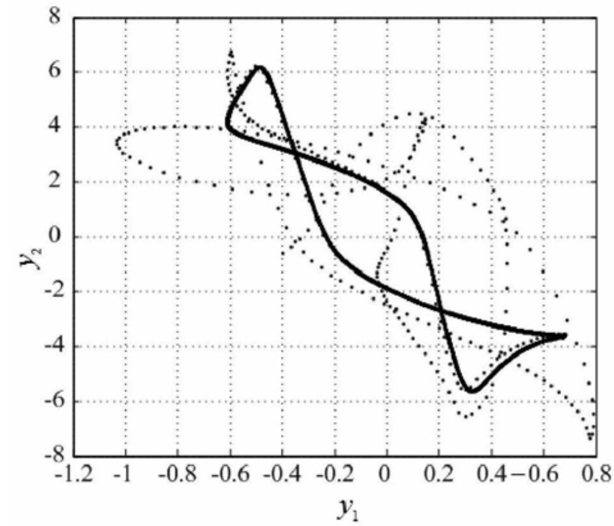

Fig. 2. The phase curve of the response chaotic delayed Hopfield neural network with the initial condition $[x_1(k)\ x_2(k)]^T = [-0.4\ -0.6]^T$ for $-11 \le k \le 0$ (5000 iterations have been plotted).

where $x(k) = [x_1(k)\ \ x_2(k)]^T$, with the initial condition $[x_1(k)\ x_2(k)]^T = [-0.4\ -0.6]^T$ for $-10 \le k \le 0$. Fig. 1 shows the chaotic behavior of the system (29). We convert the delayed Hopfield neural network (29) into the system (1), where

$$\tau_x = 10,$$

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad A_d = 0_{2\times 2},$$

$$B_p = \begin{bmatrix} 0.2 & -0.01 & -0.15 & -0.01 \\ -0.5 & 0.45 & -0.02 & -0.8 \end{bmatrix}, \quad C_q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_{qd} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_p = 0_{4\times 4}, \quad C = [0 \quad 1], \ H = I_{4\times 4},$$

$\phi_i(\xi_i(k)) = \tanh(x_i(k))$, $i = 1, 2$, $\phi_3(\xi_3(k)) = \tanh(x_1(k-10))$, $\phi_4(\xi_4(k)) = \tanh(x_2(k-10))$. The response chaotic delayed Hopfield neural network with external disturbances is described in the form of (3), where $\tau_y = 11$, $D = [1\ 1]^T$.

In the absence of disturbance $w(k)$ and controller inputs $u_1(k)$ and $u_2(k)$, the behavior of the delayed Hopfield neural network described by (29) is shown in Fig. 2. The controller (6) is employed to synchronize two delayed Hopfield neural networks with different time delays. The external disturbance $w(k) \in l_2[0, \infty)$ is defined as

$$w(k) = r\sin(k)\exp(-0.05k), \tag{30}$$

where r is a random number taken from a uniform distribution over [0, 1]. By solving the EVP (19)-(20) given in Theorem 2, we obtain the solutions of EVP and the controller parameters as follows:

$$\gamma_{\min} = 9.7361, \quad P = \begin{bmatrix} 34.3652 & -5.8129 \\ -5.8129 & 10.2218 \end{bmatrix},$$

$$Q = \begin{bmatrix} 13.9846 & -2.3090 \\ -2.3090 & 4.4991 \end{bmatrix}, \ R = \begin{bmatrix} 13.9846 & -2.3090 \\ -2.3090 & 4.4991 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} -30.5469 & 5.1670 \\ 5.1670 & -9.0860 \end{bmatrix},$$

$$S_2 = 10^8 \times \begin{bmatrix} -4.5663 & -0.0000 \\ -0.0000 & -4.5437 \\ -0.0000 & 0.0000 \\ 0.0000 & -0.0000 \end{bmatrix},$$

$$\Sigma = 10^8 \times \mathrm{diag}\{2.4704, 2.4448, 0.0000, 0.0000\},$$

$$K_1 = \begin{bmatrix} -0.8889 & 0 \\ 0 & -0.8889 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -1.0000 & -0.0000 \\ -0.0000 & -1.0000 \\ -0.0000 & 0.0000 \\ 0.0000 & -0.0000 \end{bmatrix}.$$

In order to verify control performance of synchronization between two chaotic delayed Hopfield neural networks with different time delays in the numerical simulation, the controller (6) with the above K_1 and K_2 is applied. First, without disturbance signals, the synchronization error between drive and response systems is given in Fig. 3, which shows that the synchronization errors converge to zero asymptotically. To observe the H_∞ performance with disturbance attenuation performances, the response of the controlled output error $z_e(k)$ is depicted in Fig. 4, which shows that the state feedback H_∞ controller (6) reduces the effect of the disturbance input $w(k)$ on the controlled output error $z_e(k)$ to the lowest level.

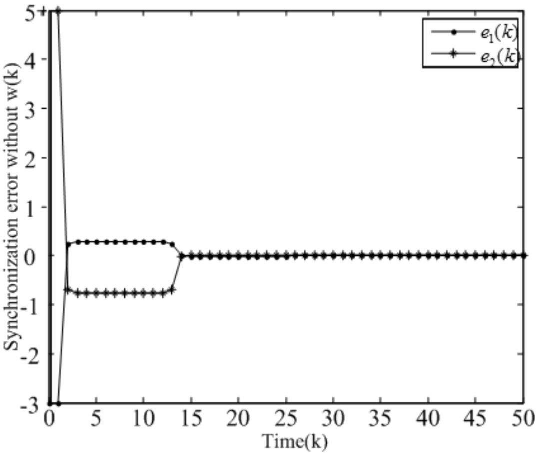

Fig. 3. The synchronization error of discrete-time Hopfield neural networks without disturbance signal $w(k)$.

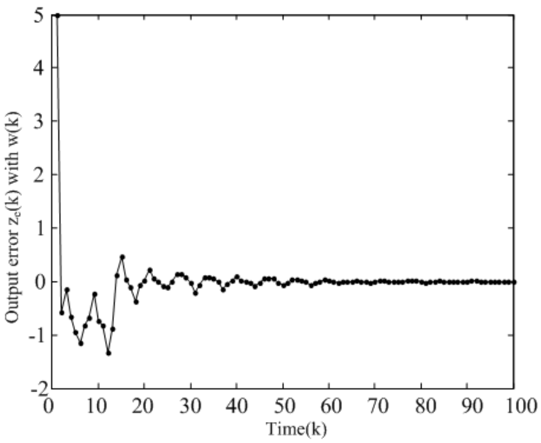

Fig. 4. The controlled output error $z_e(k)$ of discrete-time Hopfield neural networks with disturbance signal $w(k)$ defined as (30).

5.2. Synchronization of two chaotic recurrent multilayer perceptrons (RMLPs) without and with time delays

We consider the following discrete-time chaotic RMLPs [25]:

$$\begin{cases} x(k+1) = W\tanh(Vx(k)), \\ z_x(k) = Cx(k), \end{cases} \tag{31}$$

where

$$W = \begin{bmatrix} 0.9690 & 0.6967 & 0.2985 \\ -0.7473 & 3.2069 & 0.2840 \\ -2.7960 & 0.5360 & 0.9597 \end{bmatrix},$$

$$V = \begin{bmatrix} 2.0876 & 0.0173 & 1.1578 \\ 1.5247 & 0.2463 & 0.1619 \\ -0.1953 & -0.8545 & 1.5571 \end{bmatrix}, \quad C = [1 \quad 1 \quad 1].$$

The behavior of the chaotic neural network with the initial value $x_1(0) = 0$, $x_2(0) = -1$ and $x_3(0) = 0$ is shown in Fig. 5. Since some states are delayed, the response chaotic RMLPs are designed as follows:

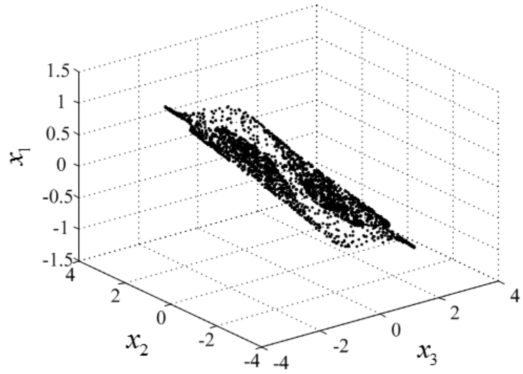

Fig. 5. Chaotic trajectory of the RMLPs (31) with the initial condition $[x_1(0)\ x_2(0)\ x_3(0)]^{\mathrm{T}} = [0\ -1\ 0]^{\mathrm{T}}$ (3000 iterations have been plotted).

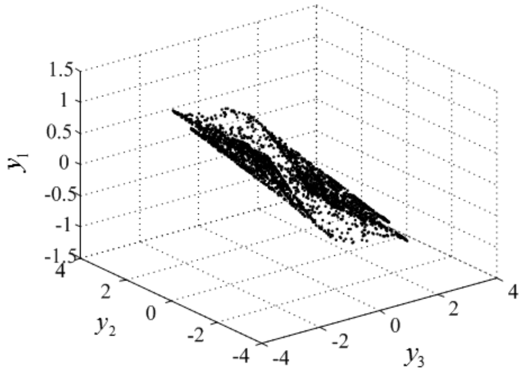

Fig. 6. Chaotic trajectory of the response RMLP (32) with the initial condition $[x_1(k)\ x_2(k)\ x_3(k)]^{\mathrm{T}} = [0\ -1\ 0]^{\mathrm{T}}$ for $-2 \le k \le 0$ (3000 iterations have been plotted).

$$\begin{cases} y(k+1) = W\tanh(V_1 y(k) + V_2 y(k-2) + u_2(k)) \\ \qquad\qquad + u_1(k) + Dw(k), \\ z_y(k) = Cy(k), \end{cases} \tag{32}$$

where $D = [1\ \ 1\ \ 1]^{\mathrm{T}}$, $V_1 = 0.9V$, $V_2 = 0.1V$, $w(k) = r\sin(2k)\exp(-0.03k)$. In the absence of disturbance $w(k)$ and controller inputs $u_1(k)$ and $u_2(k)$, the behavior of the delayed RMLPs described by (32) is shown in Fig. 6. We transform the chaotic RMLPs (32) into the system (3), where $A = 0_{3\times3}$, $A_d = 0_{3\times3}$, $B_p = W$, $C_q = V_1$, $C_{qd} = V_2$, $D_p = 0_{3\times3}$, $H = I_{3\times3}$, $[\phi_1(\zeta_1(k))\ \phi_2(\zeta_2(k))\ \phi_3(\zeta_3(k))]^{\mathrm{T}} = \tanh(V_1 y(k) + V_2 y(k-2))$. The controller (6) is employed to synchronize the RMLPs (31) with delayed RMLPs (32). According to Theorem 2, by solving the EVP (19)-(20), we obtain the gains of desired H_∞ synchronizer as follows:

$$K_1 = 10^{-9} \times \begin{bmatrix} 0.1222 & -0.0911 & 0.0296 \\ -0.2483 & 0.1866 & -0.0597 \\ -0.0731 & 0.0555 & -0.0174 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -1.8788 & -0.0156 & -1.0420 \\ -1.3722 & -0.2217 & -0.1457 \\ 0.1758 & 0.7691 & -1.4014 \end{bmatrix}.$$

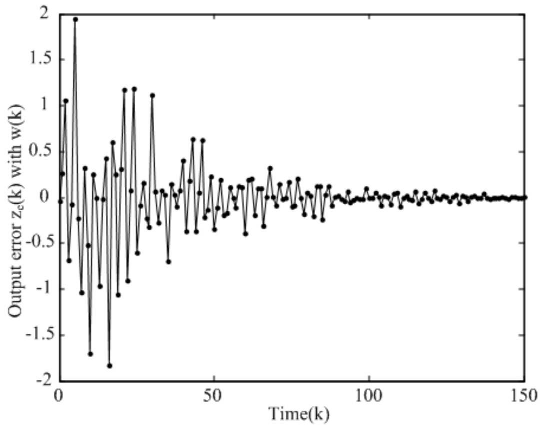

Fig. 7. The controlled output error $z_e(k)$ of discrete-time RMLPs with disturbance signal $w(k) = r\sin(2k) \times\exp(-0.03k)$.

When the state feedback laws (6) with the above K_1 and K_2 are applied to the response chaotic RMLPs (32) with external disturbance $w(k)$, the response of the controlled output error $z_e(k)$ is shown in Fig. 7. It can be seen that the effect of the disturbance input $w(k)$ on the controlled output error $z_e(k)$ can be restricted on the lowest level.

We have noticed that, although Su *et al*. [24] have provided a common chaotic neural network model to describe several well-known discrete-time chaotic neural networks (such as Hopfield neural network, cellular neural network, Chua's circuit, etc), this model could not include RMLPs, and their approaches could not be used to solve the synchronization problem of chaotic RMLPs.

5.3. Synchronization of two hyperchaotic Hénon maps

Consider the following discrete-time hyperchaotic Hénon map [26]:

$$\begin{cases} x_1(k+1) = -0.1x_3(k) + 1.76 - x_2^2(k), \\ x_2(k+1) = x_1(k), \\ x_3(k+1) = x_2(k). \\ z_x(k) = x_1(k) + x_2(k) + x_3(k). \end{cases} \tag{33}$$

Hénon map is a typical chaotic system, which exhibits very rich complex dynamical dynamics shown in Fig. 8, where $x_2(k) \in [-d_1, d_1]$, with d_1 known already. If Hénon map (33) can be synchronized by the response system (23), where $D = [1\ 1\ 1]^T$, $w(k) \in l_2[0, \infty)$ is defined in (30), the states of the response system are finite, i.e., $y(k) \in [-d_2, d_2]$, where d_2 is a known constant. We transform Hénon map (33) into system (22), where

$$A = \begin{bmatrix} 0 & d_1 + d_2 & -0.1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},$$

$$C_q = [0 \quad 1 \quad 0], \quad D_p = 0, \quad H = 2(d_1 + d_2),$$

$$\phi(\xi(k)) = x_2^2(k) + (d_1 + d_2)x_2(k) - 1.76,$$

$$d_1 = 5, \text{ and } d_2 = 5.$$

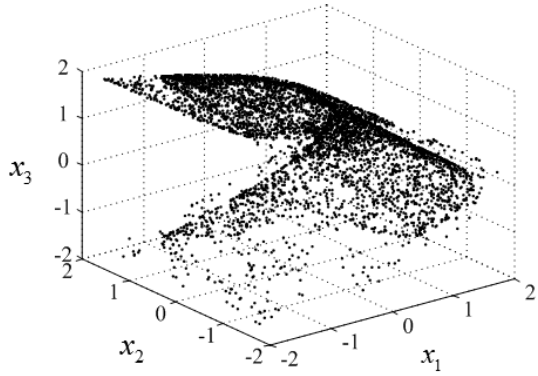

Fig. 8. Chaotic trajectory of Hénon map (33) with the initial condition $[x_1(0)\ x_2(0)\ x_3(0)]^T = [0.1\ \ 0.1\ \ 0.1]^T$ (5000 iterations have been plotted).

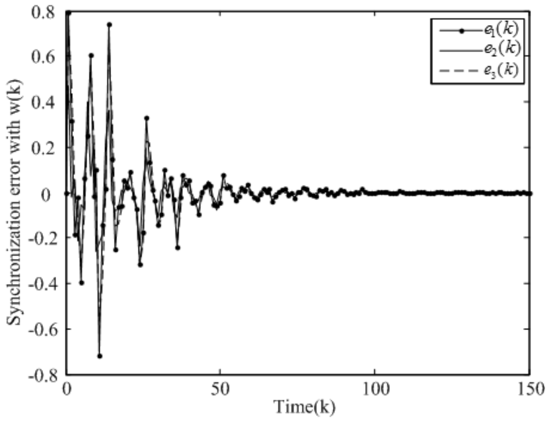

Fig. 9. The synchronization error of discrete-time Hénon maps with disturbance signal $w(k)$ defined as (30).

According to Corollary 1, the following gains of H_∞ synchronizer (24) are obtained:

$$K_1 = \begin{bmatrix} -0.4377 & -10.2365 & 1.5952 \\ -0.9324 & -0.2825 & -0.0720 \\ -0.0598 & 0.4777 & -0.5481 \end{bmatrix},$$

$$K_2 = [-0.0000 \quad -1.0000 \quad -0.0000].$$

Fig. 9 shows the synchronization error between drive and response systems with disturbance noises. It can be seen from Fig. 9 that the effect of disturbances on the synchronization error is reduced quickly.

Although [26,27] provided synchronization methods of discrete-time generalized Hénon maps, the influence of the noise or disturbance on synchronization controller hasn't been considered. Besides, the methods proposed in [26,27] cannot be applied to other chaotic systems. Therefore, our method provides an improvement and is more applicable.

6. CONCLUSION

In this paper, we have proposed H_∞ synchronization algorithms for a class of chaotic systems where the response systems have time delays which are either different from or the same as the drive systems. Central to our design is the introduction of the unified model, which interconnects a linear dynamic system with static nonlinear operators, and the transformation of the discrete-time chaotic system into this unified model. By employing the Lyapunov functional method combined with the H_∞ control concept, the novel state feedback controllers were designed to asymptotically synchronize two unified models with different or identical time delays and reduce the H_∞-norm from the disturbance input to the output error within the lowest level. Illustrative examples show that most discrete-time chaotic systems can be converted into this unified model (1) (or (22)), and optimal H_∞ synchronization controllers can be designed by Theorem 2 and Corollary 1 (or Corollary 2). Here it should be noted that systems such as recurrent neural networks [28] and cellular networks [29] are also accessible to our method if synchronization problems are required.

REFERENCES

[1] S. Wen, Z. Zeng, T. Huang, and Y. Chen, "Fuzzy modeling and synchronization of different memristor-based chaotic circuits," *Physics Letters A*, vol. 377, no. 34-36, pp. 2016-2021, November 2013.

[2] S. Wen, Z. Zeng, and T. Huang, "Adaptive synchronization of memristor-based Chua's circuits," *Physics Letters A*. vol. 376, no. 44, pp. 2775-2780, September 2012.

[3] S.-Y. Li and Z.-M. Ge, "Fuzzy modeling and synchronization of two totally different chaotic systems via novel fuzzy model," *IEEE Trans. on Systems, Man, and Cybernetics-Part B*, vol. 41, no. 4, pp. 1015-1026, August 2011.

[4] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Physical Review Letters*, vol. 64, no. 8, pp. 821-824, February 1990.

[5] J. Cao, G. Chen, and P. Li, "Global synchronization in an array of delayed neural networks with hybrid coupling ," *IEEE Trans. on Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 38, no. 2, pp. 488-498, April 2008.

[6] X. Yang, J. Cao, and J. Lu "Stochastic synchronization of complex networks with nonidentical nodes via hybridadaptive and impulsive control," *IEEE Trans. on Circuits and Systems I-Regular Papers*, vol. 59, no. 2, pp. 371-384, February 2012.

[7] W. Ding, M. Han, and M. Li, "Exponential lag synchronization of delayed fuzzy cellular neural networks with impulses," *Physics Letters A*, vol. 373, no. 8-9, pp. 832-837, February 2009.

[8] C.-C. Yang, "Synchronization of second-order chaotic systems via adaptive terminal sliding mode control with input nonlinearity," *Journal of the Franklin Institute*, vol. 349, no. 6, pp. 2019-2032, August 2012.

[9] X. Feng, F. Zhang, and W. Wang, "Global exponential synchronization of delayed fuzzy cellular neural networks with impulsive effects," *Chaos, Solitons & Fractals*, vol. 44, no. 1-3, pp. 9-16, January-March 2011.

[10] N. Mahdavi and M. B. Menhaj, "A new set of sufficient conditions based on coupling parameters for synchronization of Hopfield like chaotic neural networks," *International Journal of Control, Automation, and Systems*, vol. 9, no. 1, pp. 104-111, January 2011.

[11] J. A. K. Suykens, J. Vandewalle, and L. O. Chua, "Nonlinear H_∞ synchronization of chaotic Lur'e systems," *International Journal of Bifurcation and Chaos*, vol. 7, no. 6, pp. 1323-1335, June 1997.

[12] Y. Y. Hou, T.-L. Liao, and J.-J. Yan, "H_∞ synchronization of chaotic systems using output feedback control design," *Physica A: Statistical Mechanics and Its Applications*, vol. 379, no. 1, pp. 81-89, June 2007.

[13] S. M. Lee, D. H. Ji, J. H. Park, and S. C. Won, "H_∞ synchronization of chaotic systems via dynamic feedback approach," *Physics letters A*, vol. 372, no. 29, pp. 4905-4012, July 2008.

[14] M. Mackey and L. Glass, "Oscillation and chaos in physiological control systems," *Science*, vol. 197, no. 4300, pp. 287-289, July 1977.

[15] H. R. Karimi and H. Gao, "New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time delays," *IEEE Trans. on Systems, Man, and Cybernetics, Part B-Cybernetics*, vol. 40, no.1, pp.173-185, February 2010.

[16] H. R. Karimi and P. Maass, "Delay-range-dependent exponential H_∞ synchronization of a class of delayed neural networks," *Chaos, Solitons & Fractals*, vol. 41, no. 3, pp. 1125-1135, August 2009.

[17] J. H. Park, D. H. Ji, S. C. Won, and S. M. Lee, "H_∞ synchronization of time-delayed chaotic systems," *Applied Mathematics and Computation*, vol. 204, no. 1, pp. 170-177, October 2008.

[18] D. Qi, M. Liu, M. Qiu, and S. Zhang, "Exponential H_∞ synchronization of general discrete-time chaotic neural networks with or without time delays," *IEEE Trans. on Neural Networks*, vol. 21, no. 8, pp. 1358-1365, August 2010.

[19] M. Liu, S. Zhang, Z. Fan, and M. Qiu, "H_∞ state estimation for discrete-time chaotic systems based on a unified model," *IEEE Trans. on Systems, Man and Cybernetics, Part B: Cybernetics*, vol. 42, no. 4, pp. 1053-1063, August 2012.

[20] M. Liu, S. Zhang, Z. Fan, S. Zheng, and W. Sheng, "Exponential H_∞ synchronization and state estimation for chaotic systems via a unified model," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 24, no. 7, pp. 1114-1126, July 2013.

[21] S. Anton, *The H_∞ Control Problem*, Prentice-Hall, New York, 1992.

[22] S. P. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, PA, 1994.

[23] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chi-

lali, *LMI Control Toolbox- for use with Matlab*, MA: The MATH Works, Inc., Natick, 1995.

[24] H. Su and X. H. Ding, "Synchronization in time-discrete delayed chaotic systems," *Neurocomputing*, vol. 73, no. 1-3, pp. 478-483, December 2009.

[25] J. A. K. Suykens, J. P. L. Vandewalle, and B. L. R. De Moor, *Artificial Neural Networks for Modeling and Control of Non-linear Systems*, Boston, Kluwer Academic Publishers, Dordrecht, London, 1996.

[26] J. Lu, "Generalized (complete, lag, anticipated) synchronization of discrete-time chaotic systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 13, no. 9, pp. 1851-1859, November 2008.

[27] Y. Zheng, Y. Nian, and F. Sun, "Synchronization of discrete-time chaotic systems based on Takagi-Sugeno fuzzy model, " *Proc. of the International Conference on Computational Intelligence and Natural Computing*, Wuhan, China, no. 1, pp. 320-323, June 6-7, 2009.

[28] Z. Zeng, T. Huang, and W. Zheng, "Multistability of recurrent neural networks with time-varying delays and the piecewise linear activation function," *IEEE Trans. on Neural Networks*, vol. 21, no. 8, pp. 1371-1377, August 2010.

[29] Z. Zeng and J. Wang, "Associative memories based on continuous-time cellular neural networks designed using space-invariant cloning templates," *Neural Networks*, vol. 22, no. 5-6, pp. 651-657, July/August 2009.

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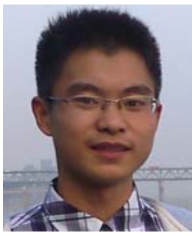

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