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Intelligent Digital Redesign for Nonlinear Systems using a Guaranteed Cost Control Method

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Abstract: In this paper, a novel intelligent digital redesign (IDR) technique using the guaranteed cost control method is proposed for nonlinear systems which can be represented by a Takagi–Sugeno (T–S) fuzzy model. The IDR technique, which is one of the sampled-data fuzzy controller design methods, guarantees not only the stability condition of the sampled-data closed-loop system with the sampled-data fuzzy controller and the state-matching error is presented. By using the concept of the guaranteed cost control method, sufficient conditions are obtained for both minimization of the state-matching error and stabilization of the sampled-data closed-loop system and derived in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to verify the effectiveness of the proposed technique.

Keywords: Intelligent digital redesign (IDR), linear matrix inequality (LMI), sampled-data fuzzy controller, state-matching error, Takagi–Sugeno (T–S) fuzzy model.

1. INTRODUCTION

In recent years, as the nonlinearity problem has increased prominently in many engineering applications, the nonlinear control, which conquers the limitation of the linear control, has attracted many researchers' attention [1,2]. Among the many nonlinear control techniques, the Takagi–Sugeno (T–S) fuzzy control is regarded as a predominant method and has been adopted in many literatures, because it can effectively bridge the gap between nonlinear systems and various linear control theories [2]. However, in spite of many studies of the T–S fuzzy control [3-8], there still remain many issues, especially digital control issues, to be solved.

Apart from the nonlinearity issue, the digital redesign (DR) method has gathered much attention [9-14] as one of the most efficient sampled-data control techniques, because DR method not only guarantees the stability condition of the closed-loop system with a sampled-data controller but also maintains the performance of the well-designed analog controller in the sampled-data controller design. However, the DR method is limited in that it can only be applied in linear time-invariant systems. To overcome this limitation, Joo [15] developed an intelligent digital redesign (IDR) for the nonlinear systems by merging the fuzzy control technique and the DR method. After Joo's study [15], it has proposed many IDR techniques [16-19], such as a global state-matching approach [17], an observer-based output feedback approach [18], a robust stabilization approach [19], and others. In these papers, the state-matching conditions are guaranteed by using the minimization of the norm distance between the analog system matrix and the sampled-data system one. However, because this method cannot directly minimize the error between the analog state and the sampled-data state, it does not have the optimal performance for the state-matching condition. In [20], the study of the norm minimization of the state-matching error was proposed, but it still remains a challenging issue.

In this paper, we propose a novel IDR method using the guaranteed cost control method for nonlinear systems which can be modelled by T–S fuzzy systems. Discretized models of the analog closed-loop fuzzy system and the sampled-data closed-loop fuzzy one are presented, respectively, and the state-matching error is defined in order to achieve the state-matching condition. Based on the guaranteed cost control method, sufficient conditions are obtained for both the stability of the sampled-data closed-loop system with the state-matching error and the minimization of the state-matching error, and its constructive conditions are presented in terms of linear matrix inequalities (LMIs). Finally, through a simple example, it shows the validity of the proposed ideas, techniques and procedures.

This paper is organized as follows: Section 2 describes the discretized models of the T–S fuzzy systems for the

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sampled-data control system and the state-matching error. The stability and state-matching conditions are proposed with the LMI form in Section 3. In Section 4, an example is provided to demonstrate the design procedures. Finally, the conclusions are given in Section 5.

Notation: The subscripts i and j denote the fuzzy rule indices. $(\cdot)^T$ denotes the transpose of the argument. The notation * and He$\{A\}$ are used for the transposed element in symmetric positions and $A + A^T$, respectively. For simplicity, we will use $\mu_i(t)$ in place of $\mu_i(z(t))$.

2. T–S FUZZY MODEL AND DISCRETIZATION FOR NONLINEAR SYSTE

Consider a T–S fuzzy system in which the ith IF-THEN rule of the nonlinear system is represented by the following form:

$$\begin{aligned} &R_i : \text{IF } z_1 \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p \text{ is } \Gamma_{ip}, \\ &\quad \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) \end{aligned} \tag{1}$$

where $z_h, h \in \mathscr{I}_p$ is the premise variable; $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and control input, respectively; $\Gamma_{ih}, (i,h) \in \mathscr{I}_r \times \mathscr{I}_p$, is a fuzzy set for z_h; and A_i and B_i denote nominal system matrices with appropriate dimensions for the ith rule.

Using center-average defuzzification, product inference, and the singleton fuzzifier, the global dynamics is inferred as

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(t)\big(A_i x(t) + B_i u(t)\big), \tag{2}$$

where

$$\mu_i(t) = \omega_i(z(t)) \Big/ \sum_{i=1}^{r} \omega_i(z(t)),$$

$$\omega_i(z(t)) = \prod_{h=1}^{p} \Gamma_{ih}(z_h(t)),$$

in which $\Gamma_{ih}(z_h(t))$ is the fuzzy membership grade of z_h in Γ_{ih}.

First, we suppose a pre-designed analog fuzzy controller for the fuzzy system (2)

$$u_c(t) = \sum_{i=1}^{r} \mu_i(t) K_{c_i} x_c(t), \tag{3}$$

where K_{c_i} denotes the analog control gain and the subscript 'c' indicates the analog control. Substituting (3) into (2), the closed-loop system is written as

$$\dot{x}_c(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(t)\mu_j(t)\Big(A_i + B_i K_{c_j}\Big) x_c(t). \tag{4}$$

Next, consider a sampled-data fuzzy controller for (2)

$$u_d(t) = u_d(kT) = \sum_{i=1}^{r} \mu_i(kT) K_{d_i} x_d(kT), \tag{5}$$

where $u_d(t)$ is the sampled-data control input to be determined in the time interval $t \in [kT, kT+T)$, $k \in \mathbb{Z}_{\geq 0}$, $T \in \mathbb{R}_{>0}$ is a sampling period, and the subscript 'd' denotes the sampled-data control. Substituting (5) into (2), the closed-loop system is obtained by

$$\dot{x}_d(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(t)\mu_j(kT)\Big(A_i x_d(t) + B_i K_{d_j} x_d(kT)\Big). \tag{6}$$

Supposing a sampling period T, the closed-loop fuzzy systems (4) and (6) are approximately discretized in the following forms, respectively:

$$x_c(kT+T) \approx \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(kT)\mu_j(kT)\Phi_{ij} x_c(kT), \tag{7}$$

$$x_d(kT+T) \approx \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(kT)\mu_j(kT)\Big(G_i + H_i K_{d_j}\Big) x_d(kT), \tag{8}$$

where

$$\Phi_{ij} = \exp\Big((A_i + B_i K_{c_j})T\Big),$$

$$G_i = \exp(A_i T),$$

$$H_i = (G_i - I)(A_i)^{-1} B_i.$$

Remark 1: The IDR, which is one of the sampled-data fuzzy control techniques, guarantees not only the stability condition of the sampled-data fuzzy control system, but also the state-matching condition. The guarantee of the state-matching condition is to minimize the trajectory error between the well-designed analog control system and the sampled-data fuzzy control one. Thus, from the state-matching condition, it is possible to convert the analog controller into the sampled-data one with maintaining the performance. That means, in cases such as H_∞ control, the performance of the well-designed analog H_∞ controller is possible to be maintained in the sampled-data fuzzy controller design.

Remark 2: In previous IDR studies [16-19], the state-matching condition was guaranteed by the minimization of the norm distance between the analog system matrix and the sampled-data one. However, this method cannot directly minimize the state-matching error, and it cannot know the performance of the state-matching error by theory, but only know by the simulation result. Thus, to conquer the problems of the previous technique, we propose a new IDR technique. To guarantee the state-matching condition, the state-matching error is defined and the dynamic model of the state-matching error is obtained. By the stability condition for the dynamic model of the state-matching error, it shows that the state-matching error is converged into zero. Also, based on the concept of the guaranteed cost control method, the state-matching error is minimized.

To guarantee the state-matching condition, we consider the state-matching error as follows:

$$e(t) = x_c(t) - x_d(t). \tag{9}$$

By substituting the closed-loop systems (7) and (8) into the state-matching error equation (9), we can obtain the following closed-loop system:

$$\chi(kT+T)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(kT)\mu_j(kT)\Psi_{ij}\chi(kT), \tag{10}$$

where

$$\chi(kT)=\begin{bmatrix}x_d(kT)\\ e(kT)\end{bmatrix},$$

$$\Psi_{ij}=\begin{bmatrix}G_i+H_iK_{d_j} & 0\\ \Phi_{ij}-G_i-H_iK_{d_j} & \Phi_{ij}\end{bmatrix}.$$

The main objective of this paper is to design a sampled-data fuzzy controller (5) to stabilize the closed-loop system (10) and to minimize the state-matching error trajectory.

3. NEW IDR BASED ON THE DISCRETIZED T–S FUZZY SYSTEM

In this paper, the main problem is addressed as follows:

Problem 1: Suppose that the equilibrium point $x_c(t) = 0$ of (4) is asymptotically stable by a well-constructed analog fuzzy controller (3). Then, the sampled-data fuzzy controller (5) has to satisfy the following conditions:

- The state-matching error $e(kT)$ is minimized for any $k\in\mathbb{Z}_{\geq 0}$.
- The sampled-data closed-loop system (6) is globally asymptotically stable.

To minimize the state-matching error $e(kT)$, the guaranteed cost control method is used and the upper bound of the error cost function is minimized. For the guaranteed cost control, we consider the following error cost function:

$$J=\sum_{k=0}^{\infty}e(kT)^T Re(kT), \tag{11}$$

where R is the given positive-definite symmetric matrix for weighting the state-matching error. If there exist a sampled-data control law $u_d(t)$ and a scalar J_0 such that the closed-loop system is asymptotically stable and the value of the error cost function (11) satisfies $J\leq J_0$, then $u_d(t)$ is an optimal sampled-data control law for the IDR of the fuzzy system (2).

Before proceeding to our main results, the following lemmas and proposition will be needed throughout the proof:

Lemma 1 [21]**:** For any real matrices X_{ij}, Y_{ij} for $1\leq i\leq r$, and $S>0$ with appropriate dimensions, we have

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}h_ih_jh_mh_nX_{ij}^TSY_{mn} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\left(X_{ij}^TSX_{ij}+Y_{ij}^TSY_{ij}\right),$$

where $h_i\geq 0$ and $\sum_{i=1}^{r}h_i=1$.

Lemma 2 [22]**:** Given any matrices F and $P=P^T\succ 0$, we have

$$-F^TP^{-1}F\leq P-\mathrm{He}\{F\}.$$

Proposition 1: In the sampled-data closed-loop system (6), there exists some constant $\eta>0$ such that

$$\|\chi(t)\|\leq\eta\|\chi(kT)\|, \tag{12}$$

where

$$\eta=(1+T\beta)e^{\alpha T},$$

$$\alpha=\sup_{(i,j)\in\mathscr{I}_r\times\mathscr{I}_r}\left\|\begin{bmatrix}A_i & 0\\ -B_iK_{c_j} & A_i+B_iK_{c_j}\end{bmatrix}\right\|,$$

$$\beta=\sup_{(i,j)\in\mathscr{I}_r\times\mathscr{I}_r}\left\|\begin{bmatrix}B_iK_{d_j} & 0\\ -B_iK_{d_j} & 0\end{bmatrix}\right\|$$

for $t\in[kT,kT+T)$.

Proof: By using the continuous-time systems (4) and (6), we can obtain a continuous-time closed-loop system as follows:

$$\dot\chi(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(t)\mu_j(kT)\left(\Omega_{ij}\chi(t)+\Lambda_{ij}\chi(kT)\right), \tag{13}$$

where

$$\Omega_{ij}=\begin{bmatrix}A_i & 0\\ -B_iK_{c_j} & A_i+B_iK_{c_j}\end{bmatrix},$$

$$\Lambda_{ij}=\begin{bmatrix}B_iK_{d_j} & 0\\ -B_iK_{d_j} & 0\end{bmatrix}.$$

Integrating (13) from kT to t, the solution is given by

$$\chi(t)=\chi(kT)+\int_{kT}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\tau)\mu_j(kT)\times\left(\Omega_{ij}\chi(\tau)+\Lambda_{ij}\chi(kT)\right)d\tau$$

for $t\in[kT,kT+T)$. Taking the norms on both sides yields

$$\begin{aligned}\|\chi(t)\|&\leq\|\chi(kT)\|+\left\|\int_{kT}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\tau)\mu_j(kT)\times\left(\Omega_{ij}\chi(\tau)+\Lambda_{ij}\chi(kT)\right)d\tau\right\|\\ &\leq\|\chi(kT)\|+\int_{kT}^{t}\alpha\|\chi(\tau)\|+\beta\|\chi(kT)\|d\tau.\end{aligned}$$

An application of the Gronwall-Bellman inequality to $\chi(t)$ results in

$$\|\chi(t)\|=\sup_{(i,j)\in\mathscr{I}_r\times\mathscr{I}_r}(1+T\beta)e^{\alpha T}\|\chi(kT)\|=\eta\|\chi(kT)\|.$$

The main results are summarized to solve Problem 1 in the following theorems:

Theorem 1: If there exist some symmetric matrices Q_1, Q_3 and some matrices Q_2, X_1, X_2, M_i such that the following LMIs are satisfied, then $x_d(kT)$ of (8) closely matches $x_c(kT)$ of (7) and the equilibrium point of (6) is globally asymptotically stable.

$$\begin{bmatrix} Q_1-\mathrm{He}\{X_1\} & * & * & * & * \\ Q_2 & Q_3-\mathrm{He}\{X_2\} & * & * & * \\ \Xi_{ii} & 0 & -Q_1 & * & * \\ W_{ii} & \Phi_{ii}X_2 & -Q_2 & -Q_3 & * \\ 0 & X_2 & 0 & 0 & -R^{-1} \end{bmatrix} \prec 0, \quad (14)$$

$$\begin{bmatrix} Q_1-2\mathrm{He}\{X_1\} & * & * & * & * \\ Q_2 & Q_3-2\mathrm{He}\{X_2\} & * & * & * \\ \Xi_{ij}+\Xi_{ji} & 0 & -Q_1 & * & * \\ W_{ij}+W_{ji} & \left(\Phi_{ij}+\Phi_{ji}\right)X_2 & -Q_2 & -Q_3 & * \\ 0 & 2X_2 & 0 & 0 & -R^{-1} \end{bmatrix} \prec 0, \quad (15)$$

$$\begin{bmatrix} Q_1 & * \\ Q_2 & Q_3 \end{bmatrix} \succ 0, \quad (16)$$

where

$$\Xi_{ij} = G_iX_1 + H_iM_j,$$
$$W_{ij} = \Phi_{ij}X_1 - G_iX_1 - H_iM_j$$

for $\forall\{(i,j,k)\in \mathscr{I}_r\times\mathscr{I}_r\times\mathscr{I}_r \,|\, 1\le i\le j\le r\}$.

Proof: Suppose the Lyapunov function as follows:

$$V(kT)=\chi(kT)^T P\chi(kT). \quad (17)$$

Then, the rate of increase of (17) along (8) is computed by

$$\begin{aligned}\Delta V(kT) &= V(kT+T)-V(kT)\\ &= \chi(kT+T)^T P\chi(kT+T)-\chi(kT)^T P\chi(kT)\\ &= \sum_{i=1}^r\sum_{j=1}^r\sum_{m=1}^r\sum_{n=1}^r \mu_i(kT)\mu_j(kT)\mu_m(kT)\mu_n(kT)\\ &\quad\times\chi(kT)^T\left(\Psi_{ij}^T P\Psi_{mn}-P\right)\chi(kT). \end{aligned} \quad (18)$$

By using Lemma 1, the equality (18) becomes

$$\begin{aligned}&\Delta V(kT)\\ &\le \sum_{i=1}^r\sum_{j=1}^r \mu_i(kT)\mu_j(kT)\chi(kT)^T\left(\Psi_{ij}^T P\Psi_{ij}-P\right)\chi(kT)\\ &= \sum_{i=1}^r \mu_i(kT)^2\chi(kT)^T\left(\Psi_{ii}^T P\Psi_{ii}-P\right)\chi(kT)\\ &\quad+\frac{1}{2}\sum_{i<j}^r \mu_i(kT)\mu_j(kT)\\ &\quad\times\chi^T(kT)\left(\left(\Psi_{ij}+\Psi_{ji}\right)^T P\left(\Psi_{ij}+\Psi_{ji}\right)-4P\right)\chi(kT). \end{aligned} \quad (19)$$

By decomposing the positive matrix P into sub-matrices, the inequality (19) can be rewritten by

$$\begin{aligned}&\Delta V(kT)\\ &\le \sum_{i=1}^r \mu_i(kT)\chi(kT)^T\left(\begin{bmatrix}\Upsilon_{ii} & 0\\ \Phi_{ii}-\Upsilon_{ii} & \Phi_{ii}\end{bmatrix}^T\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\right.\\ &\quad\left.\times\begin{bmatrix}\Upsilon_{ii} & 0\\ \Phi_{ii}-\Upsilon_{ii} & \Phi_{ii}\end{bmatrix}-\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\right)\chi(kT)\\ &\quad+\frac{1}{2}\sum_{i<j}^r \mu_i(kT)\mu_j(kT)\chi(kT)^T\\ &\quad\times\left(\begin{bmatrix}\Upsilon_{ij}+\Upsilon_{ji} & 0\\ \Phi_{ij}+\Phi_{ji}-\Upsilon_{ij}-\Upsilon_{ji} & \Phi_{ij}+\Phi_{ji}\end{bmatrix}^T\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\right.\\ &\quad\left.\times\begin{bmatrix}\Upsilon_{ij}+\Upsilon_{ji} & 0\\ \Phi_{ij}+\Phi_{ji}-\Upsilon_{ij}-\Upsilon_{ji} & \Phi_{ij}+\Phi_{ji}\end{bmatrix}-4\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\right)\\ &\quad\times\chi(kT),\end{aligned}$$

where

$$\Upsilon_{ij}=G_i+H_iK_{d_j}.$$

If the following inequalities are satisfied

$$\begin{aligned}&\begin{bmatrix}\Upsilon_{ii} & 0\\ \Phi_{ii}-\Upsilon_{ii} & \Phi_{ii}\end{bmatrix}^T\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\begin{bmatrix}\Upsilon_{ii} & 0\\ \Phi_{ii}-\Upsilon_{ii} & \Phi_{ii}\end{bmatrix}\\ &\quad-\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}+\begin{bmatrix}0 & *\\ 0 & R\end{bmatrix}\prec 0\end{aligned} \quad (20)$$

$$\begin{aligned}&\begin{bmatrix}\Upsilon_{ij}+\Upsilon_{ji} & 0\\ \Phi_{ij}+\Phi_{ji}-\Upsilon_{ij}-\Upsilon_{ji} & \Phi_{ij}+\Phi_{ji}\end{bmatrix}^T\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\\ &\quad\times\begin{bmatrix}\Upsilon_{ij}+\Upsilon_{ji} & 0\\ \Phi_{ij}+\Phi_{ji}-\Upsilon_{ij}-\Upsilon_{ji} & \Phi_{ij}+\Phi_{ji}\end{bmatrix}-4\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}\\ &\quad+4\begin{bmatrix}0 & *\\ 0 & R\end{bmatrix}\prec 0.\end{aligned} \quad (21)$$

Then $\Delta V(kT)$ is majorized by

$$\Delta V(kT)\le -e(kT)^T Re(kT)<0. \quad (22)$$

Applying the Schur complement to (20) and the congruence transformation with $\mathrm{diag}\{X_1, X_2, I, I\}$, we obtain

$$\begin{bmatrix} Y & * \\ Z_{ii} & \begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}^{-1}\end{bmatrix}\prec 0,$$

where

$$Y=\begin{bmatrix}X_1 & 0\\ 0 & X_2\end{bmatrix}^T\left(\begin{bmatrix}P_1 & *\\ P_2 & P_3\end{bmatrix}+\begin{bmatrix}0 & *\\ 0 & R\end{bmatrix}\right)\begin{bmatrix}X_1 & 0\\ 0 & X_2\end{bmatrix},$$

$$Z_{ij} = \begin{bmatrix} \Upsilon_{ij} X_1 & 0 \\ \Phi_{ij} X_1 - \Upsilon_{ij} X_1 & \Phi_{ij} X_2 \end{bmatrix}.$$

By using the Schur complement and Lemma 1, and denoting

$$K_{d_i} X_1 = M_i,$$

$$\begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix}^{-1} = \begin{bmatrix} Q_1 & * \\ Q_2 & Q_3 \end{bmatrix},$$

we obtain (15).

We can again establish a similar argument to (21) in order to obtain (15), as follows:

$$(21) \Leftrightarrow \begin{bmatrix} 4Y & * \\ Z_{ij} + Z_{ji} & \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix}^{-1} \end{bmatrix} \prec 0$$
$$\Leftrightarrow (15).$$

To find the error cost value, inequality (22) is summed from 0 to ∞:

$$\sum_{k=0}^{\infty} e(kT)^T Re(kT) \le V(\chi(0)) - V(\chi(\infty)).$$

Because of $V(\chi(\infty)) = 0$, we obtain the following inequality:

$$\sum_{k=0}^{\infty} e(kT)^T Re(kT) \le \chi(0)^T P \chi(0) = J^*. \tag{23}$$

By Proposition 1, if LMIs (14), (15), and (16) are satisfied, the closed-loop system (6) is asymptotically stable and the state-matching error is guaranteed to be less than J^*. The proof is complete.

Remark 3: As the initial conditions of $x_c(t)$ and $x_d(t)$ are same, the initial condition of $e(t)$ is 0. Thus, the inequality (23) can be rewritten as follows:

$$\sum_{k=0}^{\infty} e(kT)^T Re(kT) \le x_d(0)^T P_1 x_d(0) = J^*.$$

By Theorem 1, we can know the upper bound of the error cost function (11) of the closed-loop system (10), but it does not provide the optimal value of the upper bound. Thus, we desire to find the sampled-data control law minimizing the upper bound and to get the minimized upper bound. This optimization problem is solved by the following theorem:

Theorem 2: Consider the system (10) associated with the cost function (11). If the following optimization problem

$$\min_{\lambda} \psi, \quad \lambda \in (\psi, Q_1, Q_2, Q_3, X_1, X_2, M_i) \tag{24}$$

subject to

1) LMIs (14), (15) and (16) of Theorem 1, (25)

$$2) \begin{bmatrix} -\psi & * & * \\ x(0) & -Q_1 & * \\ 0 & -Q_2 & -Q_3 \end{bmatrix} \prec 0 \tag{26}$$

has a solution, then the sampled-data fuzzy controller is an optimal controller for the IDR of the fuzzy system (2), and $J^* = \psi$ is the minimized upper bound of the error cost function (11).

Proof: By Theorem 1, the first condition of Theorem 2 is clear. In addition, by the Schur complement, the second condition is equivalent to $\chi(0)^T P \chi(0) < \psi$. Thus, the minimization of ψ implies the minimization of the state-matching error for the system (2).

Remark 4: It is noticed that

- We propose a novel IDR technique, which can directly minimize the state-matching error and theoretically obtain the performance of the state-matching condition, by using the concept of the guaranteed cost control method.
- Through the methodologies in this paper, better performance for the state-matching condition can be expected in various control problems such as robust control, H_∞ control, and others.

4. NUMERICAL EXAMPLE

To verify the proposed technique, we consider the Duffing-like chaotic oscillator system, which can be represented by [17,23]:

$$\begin{aligned} &\ddot{y}_1(t) - ay_1(t) + by_1(t)|y_1(t)| \\ &\quad = \varepsilon(-\zeta \dot{y}_1(t) + cy_2(t)) + u_1(t), \\ &\dot{y}_2(t) = \omega y_3(t) + u_2(t), \\ &\dot{y}_3(t) = -\omega y_2(t), \end{aligned} \tag{18}$$

where $a = 1.1$, $b = 1$, $c = 21$, $\zeta = 3$, $\varepsilon = 0.1$ and $\omega = 1.8$.

Assuming $y_1(t) \in [-M, M]$ and choosing $M = 2.5$, which is a reasonable value based on Fig. 1, and $x = [y_1, \dot{y}_1, y_2, y_3]^T$, the T–S fuzzy system can be constructed as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(t)(A_i x(t) + B_i u(t)),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a & -\varepsilon\zeta & \varepsilon c & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a - bM & -\varepsilon\zeta & \varepsilon c & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix},$$

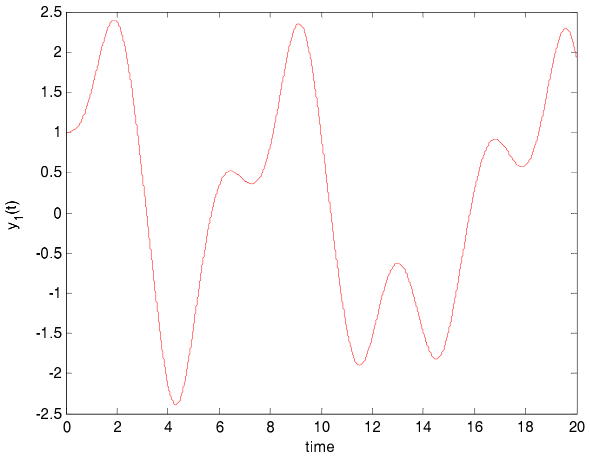

Fig. 1. The time response of $y_1(t)$ for the uncontrolled Duffing-like chaotic oscillator system.

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mu_1(t) = 1 - \frac{|x_1(t)|}{M}, \quad \mu_2(t) = \frac{|x_1(t)|}{M}.$$

We assume the initial state conditions $x(0) = [1,0,0,1]^T$ and the weighting matrix $R = I$. From Theorem 1 in [3], well-constructed analog fuzzy control gain matrices can be obtained as follows:

$$K_{c_1} = [-9.1263 \quad -2.6634 \quad -1.3199 \quad 0.0400],$$
$$K_{c_2} = [-8.3869 \quad -3.5096 \quad -0.7655 \quad 0.3501].$$

Based on the above analog control gains, we can obtain the sampled-data control gain matrices by using Theorem 2. First, if we assume a sampling period $T = 0.3$, then the sampled-data fuzzy control gains are

$$K_{d_1} = [-4.8138 \quad -2.0952 \quad -1.2528 \quad -0.3050],$$
$$K_{d_2} = [-4.3379 \quad -2.4752 \quad -1.0967 \quad -0.3921].$$

The time responses of the analog controlled system, the proposed controlled system and the sampled-data controlled system by Lee's IDR method [17] are shown in Figs. 2, 3, 4 and 5. As shown in these figures, we can know not only that all state variables converge to the origin, but also that the proposed IDR method has a better state-matching condition than Lee's IDR method. To demonstrate the superiority of the proposed method, it compares the proposed method with Lee's IDR method by using the following performance measure function:

$$\mathscr{P} = \sum_{i=1}^{4} \left(\int_0^6 \left| x_{c_i}(t) - x_{d_i}(t) \right| dt \right).$$

The results of the performance measure function are shown in Table 1.

Next, we again simulate the oscillator with a sampling period $T = 0.6$ to emphasize the performance improvement of the proposed method. Based on Theorem 2, the sampled-data control gain matrices can be obtained as follows:

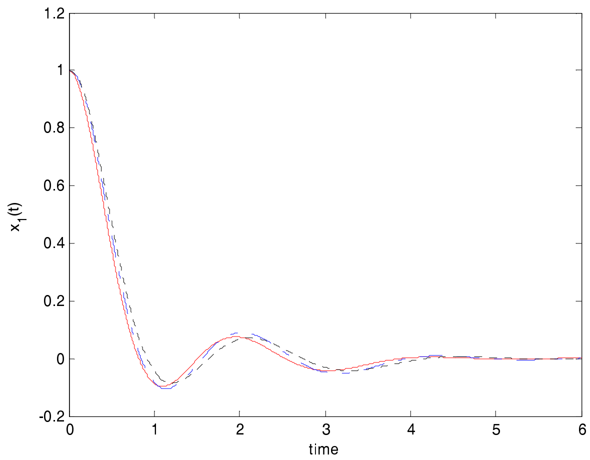

Fig. 2. The time responses of $x_1(t)$ for the Duffing-like chaotic oscillator system for $T = 0.3$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

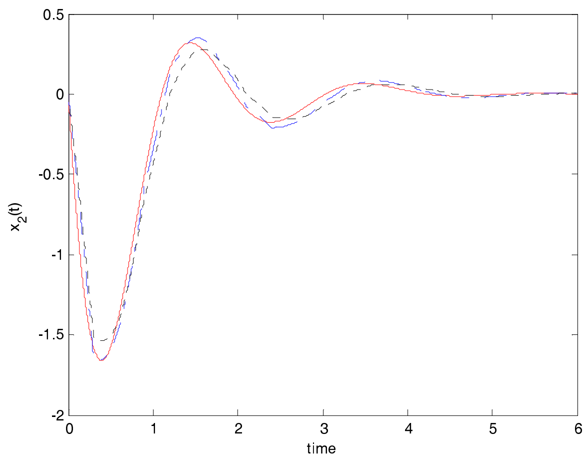

Fig. 3. The time responses of $x_2(t)$ for the Duffing-like chaotic oscillator system for $T = 0.3$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

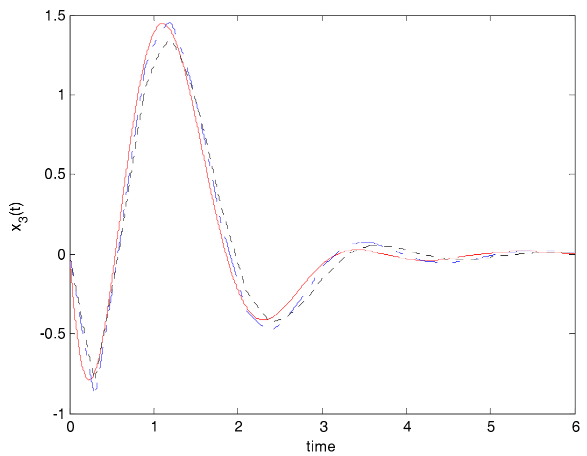

Fig. 4. The time responses of $x_3(t)$ for the Duffing-like chaotic oscillator system for $T = 0.3$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

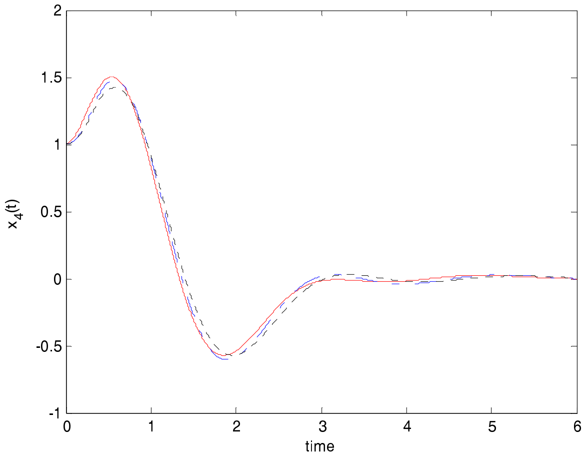

Fig. 5. The time responses of $x_4(t)$ for the Duffing-like chaotic oscillator system for $T = 0.3$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

Table 1. Comparison of the performance $\mathscr{P}$ of the proposed method with the previous IDR method.

	Lee *et. al*. [17]	Proposed
$T = 0.3$	1.2280	0.7925
$T = 0.6$	3.2602	3.0408

$$K_{d_1} = [-2.0057 \quad -1.4080 \quad -1.0768 \quad -0.5475],$$
$$K_{d_2} = [-1.8538 \quad -1.5009 \quad -1.1087 \quad -0.5397].$$

The simulation results are shown in Figs. 6, 7, 8 and 9. As shown in the figures, we can clearly see that the proposed IDR method is better than the previous IDR method. To summarize the simulation results, the proposed IDR method guarantees that the sampled-data fuzzy controller satisfies the asymptotic stability of the closed-loop system and provides better performance for the state-matching condition than the previous IDR method.

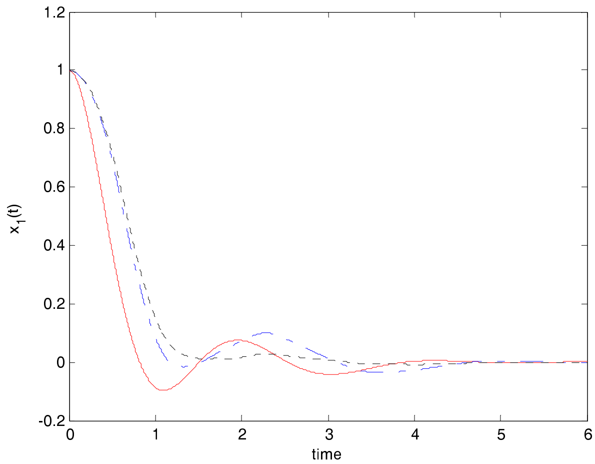

Fig. 6. The time responses of $x_1(t)$ for the Duffing-like chaotic oscillator system for $T = 0.6$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

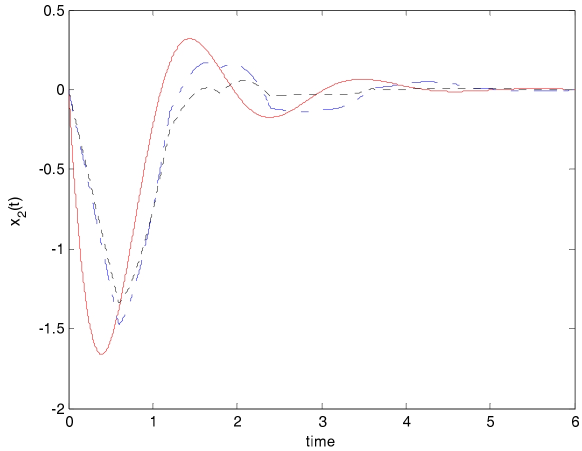

Fig. 7. The time responses of $x_2(t)$ for the Duffing-like chaotic oscillator system for $T = 0.6$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

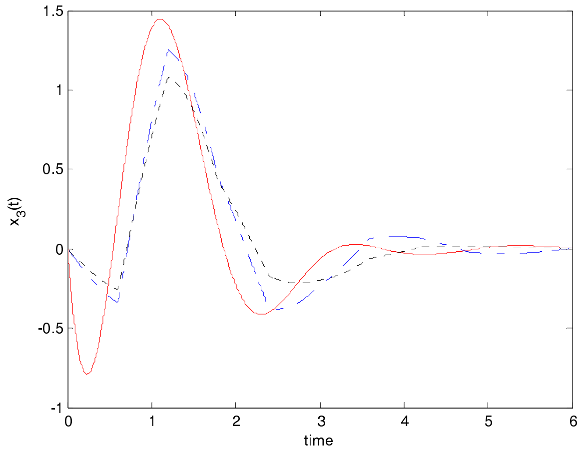

Fig. 8. The time responses of $x_3(t)$ for the Duffing-like chaotic oscillator system for $T = 0.6$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

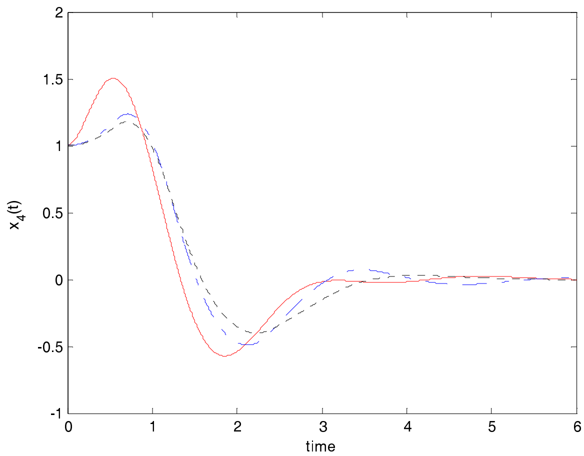

Fig. 9. The time responses of $x_4(t)$ for the Duffing-like chaotic oscillator system for $T = 0.6$s: analog (solid), proposed (dashed), Lee's method [17] (dash-dotted).

5. CONCLUSIONS

This paper has established an IDR technique using the guaranteed cost control method for nonlinear systems. Using the T--S fuzzy model, we presented the sampled-data closed-loop fuzzy system, the state-matching error system and their discretized models. Based on the discretized models, it was shown that the IDR technique can find the sampled-data fuzzy control gains to minimize the state-matching error and stabilize the sampled-data closed-loop system. In addition, sufficient design conditions were derived and formulated in the LMI format with the optimization problem. Finally, by a simulation example we demonstrated that the results of this paper are effective and valuable.

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