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Sliding Mode Control for Uncertain Switched Systems Subject to Actuator Nonlinearity

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Abstract: This paper considers the problem of sliding mode control for a class of uncertain switched systems subject to sector nonlinearities and dead-zone. In the control systems, each subsystem is not required to share the same input channel, which is usually assumed in the previous works. By employing a weighted sum of the input matrices, a common sliding surface is designed and the corresponding sliding mode dynamics is obtained. A switching signal based on the average dwell time strategy is further proposed to ensure the exponential stability of the sliding mode dynamics. Moreover, it is shown that the reachability of the specified sliding surface can be ensured despite the presence of actuator nonlinearity, parameter uncertainties and external disturbances. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Keywords: Actuator nonlinearity, average dwell time, sliding mode control, switched systems.

1. INTRODUCTION

The past decades have witnessed considerable researches in the field of switched systems, since this kind of systems has numerous applications in the control of engineering systems, e.g., biomimetic robotic systems, chemical process systems, communication systems, and computer control systems [1]. It is noted that there are some special characteristics on switched systems. For example, the whole switched systems may be unstable, even if each subsystem is stable. These make some existing methods on non-switched systems cannot be directly extended to switched systems. Therefore, many efforts have been made on stability and stabilization of switched systems, and some effective methods, e.g., average dwell time technology, convex combined strategy, etc. have been proposed, see [2-4] and the reference therein. More recently, the sliding mode control (SMC) strategy has been extended to switched systems [5-7].

As is well known, SMC has many attractive properties, such as fast response, strong robustness against parameter uncertainties and external disturbances. In [5], Wu and Lam considered SMC for a class of uncertain switched systems with state delay, whose results were further extended to stochastic switched systems in [6]. Lian et al. [7] discussed the robust H_∞ SMC for a class of uncertain switched systems. More recently, Liu *et al.* [8] further considered the switched systems with different input matrices for each subsystem and constructed a common sliding surface via a weighted sum approach.

On the other hand, due to the physical limitations, actuators inevitably contain a source of nonlinearities, such as sectors, dead-zone, saturation, backlash, and so on. These kinds of nonlinear inputs may deteriorate the performance of the control systems, and even lead to instability. Hence, the problem of switched systems with actuator nonlinearities have been receiving more attention [9,10]. Moreover, some results based on SMC techniques have been also presented in [11-13] to deal with nonlinear inputs in the control systems. More recently, the results were further extended to Markovian jumping systems [14], in which a mode dependent sliding surface and a SMC law depending on the transition rates were designed. However, to the best of the authors' knowledge, there are few works involving in switched systems subject to actuator nonlinearities. Moreover, both the characteristics of switched systems and the structure of SMC make this work not be trivial and cannot be simply obtained from the existing results.

Motivated by the discussion above, we will consider the problem of SMC for a class of uncertain switched systems subject to sector nonlinearities and dead-zone. Moreover, each subsystem in the control systems is not required to share the same input channel, which is different from the existing works. A weighted sum of the input matrices is proposed to construct a common sliding surface. In addition, by designing a switching signal depending on the average dwell time, the exponential stability of the corresponding switched sliding mode dynamics can be guaranteed. Furthermore, the reachability of the sliding surface can be achieved by the designed sliding mode controller.

Notations: $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}^{m\times n}$ denotes the real $m\times n$ matrix space; $\|\cdot\|$ denotes

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the Euclidean norm of a vector or its induced matrix norm. For any vector x, x^T is its transpose. For a real symmetric matrix M, $M>0(<0)$ means positive (negative) definite, and I is used to represent an identity matrix of appropriate dimensions. The vector $\mathbf{1_n}\in\mathbb{R}^{\mathbf{n}}$ is consisted of ones, and $e_i\in\mathbb{R}^n$ is the i-th standard base vector. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximum and minimum eigenvalue of a real symmetric matrix, respectively, and $\otimes$ represents the Kronecker product. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

2. SYSTEM DESCRIPTION

Consider the following uncertain switched systems:

$$\dot{x}(t)=(A_\sigma+\tilde{A}_\sigma)x(t)+B_\sigma(\Phi_\sigma(u(t))+w_\sigma(x(t))), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the control input, $\Phi_\sigma(u(t)):\mathbb{R}^m\to\mathbb{R}^m$ is a nonlinear function of $u(t)$, A_σ and B_σ are known matrices, $\tilde{A}_\sigma$ is the parameter uncertainty, $w_\sigma(x(t))$ is the external disturbance, and $\sigma(t):\mathbb{R}\to\Lambda=\{1,2,\ldots,s\}$ is a piecewise constant function of time t called as switching signal.

The switching sequence $\{(i_0,t_0),(i_1,t_1),\ldots,(i_N,t_N)\mid i_k\in\Lambda\}$, corresponding to the switching signal $\sigma(t)=i_k$, means that the i_k-th subsystem is activated when $t\in[t_k, t_{k+1})$.

For simplicity, the parameters associated with the i-th subsystem are denoted as

$$A_\sigma\triangleq A_i,\quad \tilde{A}_\sigma\triangleq\tilde{A}_i,\quad B_\sigma\triangleq B_i,$$
$$w_\sigma(x(t))\triangleq w_i(x(t)),\quad \Phi_\sigma(u(t))\triangleq\Phi_i(u(t)).$$

In this work, it is assumed that the admissible uncertainty $\tilde{A}_i$ satisfies $\tilde{A}_i=E_iF_i(t)H_i$, where E_i and H_i are constant matrices, and $F_i(t)$ is an unknown matrix function satisfying $F_i^T(t)F_i(t)\le I$. The disturbance $w_i(x(t))$ is norm bounded and satisfies $\|w_i(x(t))\|<d_i$, with $d_i>0$ a known scalar. In addition, $\Phi_i(u(t))$ is a nonlinear function with $\Phi_i(u(t))=[\Phi_{i1}(u_{i1}(t)),\Phi_{i2}(u_{i2}(t)),\ldots,\Phi_{im}(u_{im}(t))]^T$ and satisfies

$$(u_{ij}(t)-u_{j^+})\Phi_{ij}(u_{ij}(t))\ge\alpha_j(u_{ij}(t)-u_{j^+})^2,\quad \text{as}\quad u_{ij}(t)>u_{j^+}, \tag{2}$$

$$(u_{ij}(t)+u_{j^-})\Phi_{ij}(u_{ij}(t))\ge\alpha_j(u_{ij}(t)+u_{j^-})^2,\quad \text{as}\quad u_{ij}(t)<-u_{j^-}, \tag{3}$$

$$\Phi_{ij}(u_{ij}(t))=0,\qquad \text{as}\quad -u_{j^-}\le u_{ij}(t)\le u_{j^+}. \tag{4}$$

Remark 1: It is worth noting from (2)-(4) that, for the case $u_{j^+}=u_{j^-}=0$, the dead-zone will disappear and the nonlinear inputs $\Phi_i(u(t))$ will be reduced to a special sector nonlinear function. Hence, system (1) with (2)-(4) is a more general form, which contains both sector nonlinearities and dead-zone. Additionally, by denoting $\alpha\le\min\{\alpha_j\}$, it can be shown that the nonlinear inputs outside the dead-zone have a lower bound α.

It is noted that the input matrix B_i for each subsystem is not necessarily the same, which brings challenge in designing a common sliding surface for system (1). To deal with the difficulty, a weighted sum of the input matrices is proposed as follows:

$$B\triangleq\sum_{i=1}^{s}\alpha_iB_i, \tag{5}$$

where α_i is a scalar satisfying $\underline{\alpha}\le\alpha_i\le\bar{\alpha}$, $i=1,2,\ldots,s$, with $\underline{\alpha}$ and $\bar{\alpha}$ being known scalars.

Now, define

$$L(i)\triangleq(I_s-2e_ie_i^T)\otimes I_m,\quad F\triangleq 1_s\otimes I_m,$$
$$M\triangleq[(B-s\alpha_1B_1)(B-s\alpha_2B_2)\cdots(B-s\alpha_sB_s)],$$
$$\hat{M}\triangleq[M\quad \beta I_m],\quad \hat{L}(i)\triangleq\begin{bmatrix}L(i) & 0\\ 0 & \frac{1}{\beta}(1-s\alpha_i)B_i\end{bmatrix},$$
$$\hat{F}\triangleq\begin{bmatrix}F\\ I_m\end{bmatrix},\quad \beta\triangleq\max\{|s\underline{\alpha}-1|,|s\bar{\alpha}-1|\}\cdot\max_{1\le i\le s}\{\|B_i\|\}.$$

It can be shown that $B_i=B+\hat{M}\hat{L}(i)\hat{F}$, and $\|\hat{L}(i)\|\le 1$.

Remark 2: In the following sections, it will be seen that the above matrix transformation plays an important role in designing a common sliding surface. Besides, since the input matrix B_i for each subsystem is not required to be the same, this work has a wider application than some existing ones as in [5-7].

Remark 3: It should be pointed out that, when $\alpha_i=\frac{1}{s}$, $i=1,\ldots,s$, it is easily shown that $B_i=B+ML(i)F$ with $\|L(i)\|\le 1$.

Remark 4: It is worth noting that if B_i is full column rank, B in (5) is also full column rank generally. That is, for general choice of scalars $\alpha_i, i=1,\ldots,s$, it can be ensured that B is full column rank.

Taking the above transformation into account, system (1) can be rewritten as

$$\dot{x}(t)=(A_i+\tilde{A}_i)x(t)+(B+\hat{M}\hat{L}(i)\hat{F})(\Phi_i(u(t))+w_i(x(t))). \tag{6}$$

The objective of this work is to design a sliding mode controller such that system (1) is exponentially stable. To this end, the following preliminaries are first given.

Assumption 1: The matrix B_i is full column rank, i.e., $\mathrm{rank}(B_i)=m$.

Definition 1 [1]: For any $T_2>T_1>0$, let N_σ denote the number of switchings of $\sigma(t)$ over (T_1,T_2). If

$$N_\sigma\le N_0+\frac{(T_2-T_1)}{T_\sigma}$$

holds for $T_\sigma>0$ and $N_0\ge 0$, then T_σ is called the average dwell time.

In this work, let $N_0 = 0$, as is usually used in the existing literature.

Definition 2: The equilibrium $x^* = 0$ of system (1) is said to be exponentially stable if the solution $x(t)$ satisfies

$$\| x(t) \| \le \rho \| x(t_0) \| e^{-\lambda(t-t_0)}, \quad \forall\, t \ge t_0,$$

for scalars $\rho \ge 1$ and $\lambda > 0$.

3. SLIDING SURFACE DESIGN

In this section, a common sliding surface will be designed. It is worthy of noting that if a mode-dependent sliding surface is designed, it will be difficult to analyze the convergence of the state trajectories as they jump from a sliding surface to another one. Therefore, a common sliding function is desirable in application.

In view of the discussion above, the following integral sliding function is designed:

$$S(t) = Dx(t) + K\int_0^t x(s)ds, \tag{7}$$

where $D = (B^T B)^{-1} B^T$ and K will be designed later.
It can be derived from (6) that

$$\begin{aligned}\dot{S}(t) &= (DA_i + K + D\tilde{A}_i)x(t) + (I + D\hat{M}\hat{L}(i)\hat{F}) \\ &\quad \times \Phi_i(u(t)) + DB_i w_i(x(t)).\end{aligned} \tag{8}$$

In view of the sliding mode theory, we have the following equivalent controller:

$$\begin{aligned}\Phi_{ieq}(u(t)) &= -(DB_i)^{-1}(DA_i + K + D\tilde{A}_i)x(t) \\ &\quad - w_i(x(t)).\end{aligned} \tag{9}$$

Substituting (9) into (6), we have the sliding mode dynamics as follows:

$$\begin{aligned}\dot{x}(t) &= (A_i - B_i(DB_i)^{-1}(DA_i + K))x(t) \\ &\quad + (I - B_i(DB_i)^{-1}D)\tilde{A}_i x(t).\end{aligned} \tag{10}$$

Remark 5: It is seen that the matrix DB_i is required to be non-singular. Hence, the parameters α_i, $i = 1,2,\dots,s$, should be selected such that the non-singularity condition can be satisfied.

4. MAIN RESULTS

In the sequel, the average dwell time approach is utilized to analyze the stability of the sliding mode dynamics (10).

Theorem 1: Consider the system (1) satisfying Assumptions 1. Given scalar $\beta > 0$ and matrix K such that $A_i - B_i(DB_i)^{-1}(DA_i + K)$ is stable, if there exist matrices $P_i > 0$ and scalars $\varepsilon_i > 0$, $i \in \Lambda$, satisfying the following linear matrix inequalities (LMIs):

$$\begin{bmatrix} \Theta_i & P_i(I - B_i(DB_i)^{-1}D)E_i \\ E_i^T(I - B_i(DB_i)^{-1}D)^T P_i & -\varepsilon_i I \end{bmatrix} < 0, \tag{11}$$

with

$$\begin{aligned}\Theta_i &= P_i(A_i - B_i(DB_i)^{-1}(DA_i + K)) \\ &\quad + (A_i - B_i(DB_i)^{-1}(DA_i + K))^T P_i + \beta P_i + \varepsilon_i H_i^T H_i,\end{aligned}$$

then with the parameter

$$\mu = \max_{i,j\in\Gamma, i\ne j} \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}, \tag{12}$$

and the average dwell time T_σ satisfying

$$T_\sigma > T_\sigma^* \ge \frac{\ln \mu}{\beta}, \tag{13}$$

the sliding mode dynamics (10) is exponentially stable for arbitrary switching signal $\sigma(t)$. Furthermore, the norm of the state obeys

$$\| x(t) \|^2 \le \gamma e^{-\kappa t} \| x(t_0) \|^2,$$

where

$$\begin{aligned}&\kappa = \frac{1}{2}\left(\beta - \frac{\ln \mu}{T_\sigma}\right), \qquad \gamma = \sqrt{\frac{b}{a}} \ge 1, \\ &a = \min_{i\in\Gamma}\{\lambda_{\min}(P_i)\}, \quad b = \max_{i\in\Gamma}\{\lambda_{\max}(P_i)\}.\end{aligned} \tag{14}$$

Proof: For the i-th subsystem, consider the multiple Lyapunov function of (10) as

$$V(i,t) = x^T(t)P_i x(t).$$

Thus, we have

$$\begin{aligned}&\dot{V}(i,t) + \beta V(i,t) \\ &\le x^T(t)[P_i(A_i - B_i(DB_i)^{-1}(DA_i + K)) \\ &\quad + (A_i - B_i(DB_i)^{-1}(DA_i + K))^T P_i]x(t) \\ &\quad + x^T(t)[\varepsilon_i^{-1}P_i(I - B_i(DB_i)^{-1}D)E_i \\ &\quad \times (P_i(I - B_i(DB_i)^{-1}D)E_i)^T + \varepsilon_i H_i^T H_i + \beta P_i]x(t) \\ &= x^T(t)\Pi_i x(t).\end{aligned} \tag{15}$$

By Shur's complement, it can be shown that $\Pi_i < 0$ is implied by (11), which together with (15) yields $\dot{V}(i,t) < -\beta V(i,t)$. This implies that

$$V(i,t) < \beta e^{-\beta(t-t_0)} V(i,t_0). \tag{16}$$

Therefore, each subsystem of the switched systems (10) is exponentially stable, which completes the proof.

Suppose that $t_k, k \in \{1,2,\dots,N_\sigma\}$, is the switching instant and the system (10) switches from the j-th subsystem to the i-th one. Hence, one has $\sigma(t_k^-) = j$ and $\sigma(t_k^+) = i$. In view of (16) and (12), it follows that

$$V(i,t) \le e^{-\beta(t-t_k)} V(i,t_k), \text{ and } V(i,t_k) \le \mu V(j,t_k^-). \tag{17}$$

Let $N_\sigma(t_0,t) \le N_0 + (t - t_0)/T_\sigma$. According to (17), we have

$$V(i,t) \le e^{-\beta(t-t_k)} \mu V(j,t_k^-) \tag{18}$$

$$\begin{aligned}&\vdots\\ &\le e^{-\beta(t-t_0)}\mu^{N_\sigma(t_0,t)}V(\sigma(t_0),t_0)\\ &\le e^{-(\beta-\ln\mu/T_\sigma)(t-t_0)}V(\sigma(t_0),t_0).\end{aligned}$$

In view of (14), one further has

$$a\|x(t)\|^2\le V(i,t),\ \text{and}\ V(\sigma(t_0),t_0)\le b\|x(t_0)\|^2. \quad (19)$$

Combining (18) and (19), yields

$$\|x(t)\|^2\le\frac{1}{a}V(i,t)\le\frac{b}{a}e^{-(\beta-\ln\mu/T_\sigma)(t-t_0)}\|x(t_0)\|^2. \quad (20)$$

In view of (13) and (20), it can be shown that the sliding mode dynamic system (10) is exponentially stable.

In the following theorem, a SMC law will be designed to guarantee the reachability of the specified sliding surface $S(t)=0$. Consider the system (1) satisfying Assumptions 1. A SMC law is designed as follows:

$$u_{ij}(t)=\begin{cases}-\dfrac{\bar{s}_{ij}(t)}{\|\bar{S}_i(t)\|}h_i(x(t))+u_{j^+}, & \text{if}\ \bar{s}_{ij}(t)<0,\\ 0, & \text{if}\ \bar{s}_{ij}(t)=0,\\ -\dfrac{\bar{s}_{ij}(t)}{\|\bar{S}_i(t)\|}h_i(x(t))-u_{j^-}, & \text{if}\ \bar{s}_{ij}(t)>0,\end{cases} \quad (21)$$
$$j\in\{1,2\ldots,m\},$$

where

$$\bar{S}_i(t)=(DB_i)^TS(t)=[\bar{s}_{i1}(t),\bar{s}_{i2}(t),\ldots,\bar{s}_{im}(t)], \quad (22)$$

and

$$\begin{aligned}h_i(x(t))=&\frac{1}{\alpha}(\|(DB_i)^{-1}(DA_i+K)\|\|x(t)\|\\ &+\|(DB_i)^{-1}DE_i\|\|H_ix(t)\|+d_i\|x(t)\|+\mu_i),\end{aligned}$$

with $\mu_i>0$ a designed parameter, then the sate trajectories can be driven onto the specified sliding surface $S(t)=0$ in finite time and remain there in the subsequent time.

Proof: Choose the Lyapunov function as

$$V(x,t)=\frac{1}{2}S^T(t)S(t). \quad (23)$$

Thus, it can be derived from (8) and (23) that

$$\begin{aligned}\dot{V}(x,t)=&\bar{S}_i^T(t)(DB_i)^{-1}(DA_i+K+\tilde{A}_i)x(t)\\ &+\bar{S}_i^T(t)\Phi_i(u(t))+\bar{S}_i^T(t)w_i(x(t)).\end{aligned} \quad (24)$$

Considering (21), we have

$$u_{ij}(t)>u_{j^+},\qquad \text{if}\ \bar{s}_{ij}(t)<0, \quad (25)$$

$$u_{ij}(t)<-u_{j^-},\qquad \text{if}\ \bar{s}_{ij}(t)>0. \quad (26)$$

In view of (2)-(4), one further has

$$\Phi_{ij}(u_{ij}(t))>\alpha(u_{ij}(t)-u_{j^+}),\ \text{if}\ u_{ij}(t)>u_{j^+}, \quad (27)$$

$$\Phi_{ij}(u_{ij}(t))<\alpha(u_{ij}(t)+u_{j^-}),\ \text{if}\ u_{ij}(t)<-u_{j^-}. \quad (28)$$

According to (25)-(28), there holds

$$\begin{aligned}\bar{s}_{ij}(t)\Phi_{ij}(u_{ij}(t))&<\alpha\bar{s}_{ij}(t)(u_{ij}(t)-u_{j^+})\\ &=-\frac{\alpha\bar{s}_{ij}^2(t)}{\|\bar{S}_i(t)\|}h_i(x(t)),\ \text{if}\ u_{ij}(t)>u_{j^+},\\ \bar{s}_{ij}(t)\Phi_{ij}(u_{ij}(t))&<\alpha\bar{s}_{ij}(t)(u_{ij}(t)+u_{j^-})\\ &=-\frac{\alpha\bar{s}_{ij}^2(t)}{\|\bar{S}_i(t)\|}h_i(x(t)),\ \text{if}\ u_{ij}(t)<-u_{j^-}.\end{aligned}$$

Therefore, it can be derived that

$$\begin{aligned}\bar{S}_i^T(t)\Phi_i(u(t))&=\sum\nolimits_{j=1}^{m}\bar{s}_{ij}(t)\Phi_{ij}(u_{ij}(t))\\ &\le-\alpha\|\bar{S}_i^T(t)\|h_i(x(t)).\end{aligned} \quad (29)$$

Substituting (29) into (24) yields

$$\begin{aligned}\dot{V}(x,t)\le&\bar{S}_i^T(t)(DB_i)^{-1}(DA_i+K+\tilde{A}_i)x(t)\\ &-\alpha\|\bar{S}_i^T(t)\|h_i(x(t))+\bar{S}_i^T(t)w_i(x(t))\\ \le&-\mu_i\|\bar{S}_i(t)\|.\end{aligned}$$

This means that the state trajectories of system (1) will be globally driven onto the sliding surface $S(t)=0$ in finite time and remain there in the subsequent time.

5. SIMULATION

Consider the switched systems (1) with two modes and parameters as follows:
Subsystem 1:

$$A_1=\begin{bmatrix}-3.5 & 2.5 & 2.5\\ -2.5 & 3.5 & 3.5\\ 3.5 & -4.5 & -5.5\end{bmatrix},\quad B_1=\begin{bmatrix}1 & 1\\ 2 & 1\\ 2 & 4\end{bmatrix},$$

$$E_1=[-0.5\ \ 0.5\ \ -0.5]^T,\quad F_1(t)=\sin(t),$$

$$H_1=[-1\ \ 1\ \ -1],\quad w_1=\begin{bmatrix}\sin(t)\\ 0\end{bmatrix},$$

Subsystem 2:

$$A_2=\begin{bmatrix}2.5 & -7.5 & -4.5\\ 2.5 & -3.5 & -5.5\\ 2.5 & 10.5 & -3.5\end{bmatrix},\quad B_2=\begin{bmatrix}2 & -1\\ -1 & 1\\ -1 & -1\end{bmatrix},$$

$$E_2=[-0.5\ \ 0.5\ \ -0.5]^T,\quad F_2(t)=\cos(t),$$

$$H_2=[-1\ \ 1\ \ 1],\quad w_2=\begin{bmatrix}\cos(t)\\ 0\end{bmatrix}.$$

It is assumed that the nonlinear actuators are as follows:

$$\Phi_{11}(u_{11}(t))=\begin{cases}(u_{11}(t)-1)(2+2e^{|\sin(t)|}+2\sin(t)),\\ \text{if}\quad u_{11}(t)>1,\\ (u_{11}(t)+2)(2+2e^{|\sin(t)|}+2\sin(t)),\\ \text{if}\quad u_{11}(t)<-2,\\ 0,\quad \text{if}\ \ -2\le u_{11}(t)\le 1,\end{cases}$$

$$\Phi_{12}(u_{12}(t)) = \begin{cases} (u_{12}(t)-2)(2+2e^{|\cos(t)|}+2\cos(t)), \\ \text{if} \quad u_{12}(t)>2, \\ (u_{12}(t)+3)(2+2e^{|\cos(t)|}+2\cos(t)), \\ \text{if} \quad u_{12}(t)<-3, \\ 0, \quad \text{if} \quad -3 \le u_{12}(t) \le 2, \end{cases}$$

$$\Phi_{21}(u_{21}(t)) = \begin{cases} (u_{21}(t)-1)(2+2e^{|\sin(t)|}+2\sin(t)), \\ \text{if} \quad u_{21}(t)>1, \\ (u_{21}(t)+2)(2+2e^{|\sin(t)|}+2\sin(t)), \\ \text{if} \quad u_{21}(t)<-2, \\ 0, \quad \text{if} \quad -2 \le u_{21}(t) \le 1, \end{cases}$$

$$\Phi_{22}(u_{22}(t)) = \begin{cases} (u_{22}(t)-2)(2+2e^{|\cos(t)|}+2\cos(t)), \\ \text{if} \quad u_{22}(t)>2, \\ (u_{22}(t)+3)(2+2e^{|\cos(t)|}+2\cos(t)) \\ \text{if} \quad u_{22}(t)<-3 \\ 0, \quad \text{if} \quad -3 \le u_{22}(t) \le 2. \end{cases}$$

By choosing $\alpha_1 = \alpha_2 = \frac{1}{2}$, it can be verified that the non-singularity of DB_i can be guaranteed. Furthermore, for $\beta = 1.5$ and $K = \begin{bmatrix} -1 & 1 & 3 \\ -1 & -2 & 2 \end{bmatrix}$, solving LMIs (11) yields

$$\varepsilon_1 = 0.3245, \quad \varepsilon_2 = 1.2049,$$

$$P_1 = \begin{bmatrix} 1.0870 & -0.3440 & -0.5315 \\ -0.3440 & 0.3879 & 0.3420 \\ -0.5315 & 0.3420 & 0.6991 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.9448 & -0.6235 & -0.5100 \\ -0.6235 & 0.7393 & 0.4009 \\ -0.5100 & 0.4009 & 0.4573 \end{bmatrix}.$$

Choose the parameters $\mu_1 = \mu_2 = 0.05$ and $d_1 = d_2 = 1$.

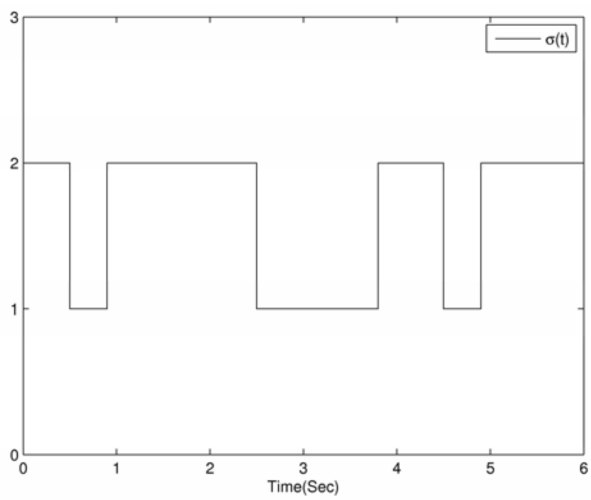

Fig. 1. Switching signal $\sigma(t)$.

Then, for the initial state $x(0) = [-0.1 \ 0.1 \ -0.1]^T$, the simulation results with the proposed SMC law can be seen in Figs. 1-4. It can be seen from Fig. 2 that the system is stable despite the presence of actuator nonlinearity, parameter uncertainties and external disturbances.

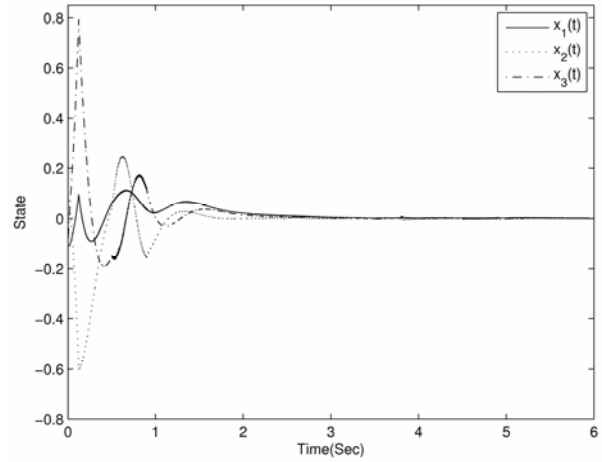

Fig. 2. State trajectories $x(t)$.

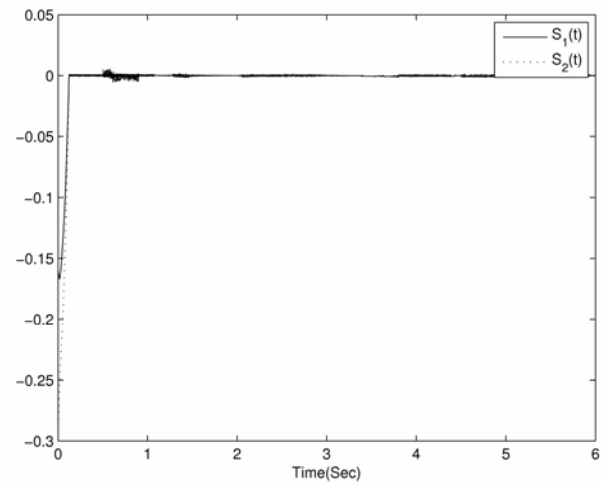

Fig. 3. Sliding variables $S(t)$.

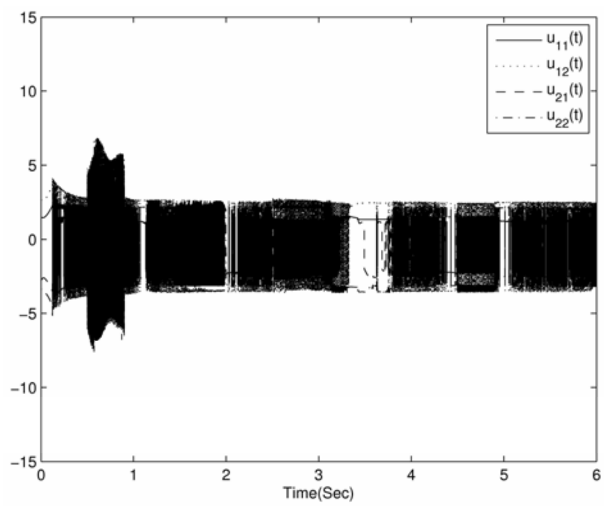

Fig. 4. Control signals $u(t)$.

6. CONCLUSION

This paper has been concerned with SMC for a class of uncertain switched systems with nonlinear inputs, in which the weighted sum approach has been utilized to construct a common sliding surface. It is noted that the parameters α_i should be chosen to ensure the non-singularity of the matrix DB_i. In practical applications, whether are there relations between the parameters α_i and the system performance? How to give an effective designing method for parameters α_i? All of these should be further considered in the future research.

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