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Delay-Dependent Robust Observer-based Control for Discrete-Time Uncertain Singular Systems with Interval Time-Varying State Delay

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Abstract: The problem of observer-based robust control design is studied for discrete-time singular systems with norm-bounded uncertainties and a time-varying delay. More precisely, a delay-dependent criterion is established that guarantees the admissibility of the considered systems, without resorting to its decomposition. Based on the proposed criterion and without the assumption that the considered systems are admissible, robust observer-based controllers are designed for discrete-time singular time-delay systems such that the closed-loop systems have the characteristics of regularity, causality and asymptotic stability. Seeking computational convenience, all the developed results are cast in the format of strict linear matrix inequalities (LMIs). Finally, some numerical examples are presented to show the feasibility of the proposed approach.

Keywords: Admissibility, discrete-time singular systems, interval time-varying delay, robust control, state observer.

1. INTRODUCTION

Time delays constitute an inherent feature of several dynamic systems; they are regarded as an important source of instability and performance degradation in a great number of important engineering problems involving material, information or energy transportation [1,2]. During the past three decades, considerable attention has been devoted to the analysis and synthesis of these time delay systems, and many research results have been reported in the literature (See, for example, [3,4] and references therein). When dealing with time-varying delays, a fundamental problem arises when estimating the upper bound of cross product terms, which tends to introduce a source of conservatism [5,6].

On the other hand, the descriptor formalism is very attractive for system modelling, since it can characterize a wide class of systems, including physical models with non-dynamic constraints (e.g., algebraic relations induced in interconnected systems such as power transfer networks or water distribution networks), or with jump behavior.

In recent years, the problems of stability analysis and controller design for descriptor systems have been extensively studied. This can be understood through the fact that the singular model preserves the structure of practical systems and describes a larger class of physical systems than the state-space ones. Compared with state-space systems, it is well known that the descriptor systems problems are more complicated to solve due to the regularity and absence of impulse (in continuous-time) or causality (in discrete-time) must be considered simultaneously [7-10].

Thus, as a special class of time delay systems, singular time delay systems have attracted attention from the mathematics and control community [11-13]. Due to its general description, the class of discrete-time singular systems with state-delay has been examined in [14-18] for stability and stabilization. From the literature, it seems that the stabilization problem for discrete-time singular and state-delay is often based on state feedback with the assumption that the state of the system is available for measurement. However, in practice this assumption is not realistic for many reasons, such as the non-existence of appropriate sensors to measure some of the states, or the limitation in the control strategies. In this regard, the observer-based output feedback control is probably well suited for feedback control, while the problem of designing observers for descriptor systems has also been investigated by a number of scholars: see, e.g., [19-22]. To the best of our knowledge, the observer design for uncertain discrete singular time-varying delay systems has received little attention [23,24].

Then, we focus in this paper on the observer-based control design problem for discrete-time singular systems with time-varying delays in the presence of model uncertainties. First, in the LMI framework, a delay-dependent admissibility criterion is established for the considered systems. Next, based on this criterion, the robust output feedback control problem is also solved

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and an explicit expression of the desired observer-based control law is given, which can be obtained by solving the feasibility problem of a strict LMI. Finally, the effectiveness and the reduced conservatism of the derived results are shown by several examples.

Notation: Throughout this paper, $X\in\mathbb{R}^n$ denotes the n–dimensional Euclidean space, while $X\in\mathbb{R}^{n\times m}$ refers to the set of all $n\times m$ real matrices. The notation $X>0$ (respectively, $X\geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol (*) stands for matrix block induced by symmetry and $sym(X)$ stands for $X+X^T$.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the class of singular discrete-time systems with state delay described by

$$\begin{aligned} Ex(k+1) &= (A+\Delta A)x(k)+(A_d+\Delta A_d)x(k-d(k)) \\ &\quad +(B+\Delta B)u(k), \\ x(k) &= \phi(k), \quad k\in[-d_M,0], \end{aligned} \tag{1}$$

where $xk\in\mathbb{R}^n$ is the state vector, $u(k)\in\mathbb{R}^m$ is the control input vector, $d(k)$ is a positive integer representing the time-varying delay that satisfies $d_m\leq d(k)\leq d_M$, where the bounds $d_m\geq 0$ and $d_M>0$ are known to be positive finite integers. $\phi(k)$ is a compatible initial condition. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular, and we shall assume that $rank(E)=r\leq n$. A, A_d and B are known real constant matrices with appropriate dimensions. ΔA, ΔA_d and ΔB are unknown matrices representing the parametric uncertainties, assumed to be of the form

$$[\Delta A \quad \Delta A_d \quad \Delta B] = MF(k)[N \quad N_d \quad N_u], \tag{2}$$

where M, N, N_d and N_u are known real constant matrices with appropriate dimensions, and $F(k)$ is an unknown matrix function satisfying

$$F^T(k)F(k)\leq I. \tag{3}$$

The nominal unforced discrete singular time-delay system of (1) is as follows:

$$\begin{aligned} Ex(k+1) &= Ax(k)+A_d x(k-d(k)), \\ x(k) &= \phi(k), \quad k\in[-d_M,0], \end{aligned} \tag{4}$$

Definition 1 [7,9,25]**:**

1) The pair (E, A) is said to be regular if $\det(zE-A)\neq 0$.
2) The pair (E, A) is said to be causal, if it is regular and $deg(det(zE-A))=rank(E)$.
3) For given positive scalars d_m and d_M, the discrete singular time-delay system (4) is said to be regular and causal for any time delay $d(k)$ satisfying $d_m\leq d(k)\leq d_M$, if the pair (E, A) is regular and causal.
4) The time-varying delay discrete singular system (4) is said to be admissible if it is regular, causal and stable.
5) The discrete singular time delay system (4) is said to be stable if, for any scalar $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$ such that, for any compatible initial condition $\phi(k)$ satisfying $\sup_{-d_M\leq k\leq 0}\|\phi(k)\|\leq\delta(\varepsilon)$, the solution $x(k)$ to system (4) des satisfies $\|x(k)\|\leq\varepsilon$ for any $k\geq 0$; moreover $\lim_{k\to\infty}x(k)=0$.

Without loss of generality, we introduce the following assumption for technical convenience.

Assumption 1: For a given $C_2\in\mathbb{R}^{q\times n}$ with rank(C_2) $=q$, there always exist two orthogonal matrices $U\in\mathbb{R}^{q\times q}$ and $V\in\mathbb{R}^{n\times n}$, such that

$$U^T C_2 V = [S \quad 0], \tag{5}$$

$S=diag\{s_1,s_2,\cdots,s_q\}$, where $s_i\ (i=1,\cdots,q)$ are nonzero singular values of C_2.

We end this section by recalling the following lemmas:

Lemma 1 [26]**:** Given matrices M, N and P of appropriate dimensions, with P symmetrical, then

$$P+MF(k)N+N^TF^T(k)M^T<0$$

for any $F(k)$ satisfying $F^T(k)F(k)\leq I$, if and only if there exists a scalar $\varepsilon>0$ such that

$$P+\varepsilon MM^T+\varepsilon^{-1}N^TN<0. \tag{6}$$

Lemma 2 [27]**:** For any matrix $M>0$, integers p and q satisfying $q>p$, and vector function $x:\mathbb{N}[p,q]\to\mathbb{R}^n$ such that the sums concerned are well defined, then:

$$\begin{aligned} &-(q-p+1)\sum_{s=p}^{q}x^T(s)Mx(s) \\ &\leq -\left(\sum_{s=p}^{q}x(s)\right)^T M\left(\sum_{s=p}^{q}x(s)\right). \end{aligned}$$

3. STABILITY ANALYSIS

In this section we provide a sufficient condition, written as LMIs in terms of a free-weighting-matrix, under which the nominal system (4) is regular, causal and stable. This condition will play a key role in solving the problems mentioned below.

Theorem 1: Given integers $d_m>0$ and $d_M>0$, for any delay $d(k)$ satisfying $d_m\leq d(k)\leq d_M$, system (4) is admissible if there exist matrices $P>0$, $Q>0$, $Q_1>0$, $Q_2>0$, $Z_1>0$, $Z_2>0$, X, Y, S, G_i, $i=1,2,3$, such that

$$\begin{bmatrix} \Phi+sym(\Phi_1) & \sqrt{\tau}X \\ * & -Z_2 \end{bmatrix}<0, \tag{7}$$

$$\begin{bmatrix} \Phi+sym(\Phi_1) & \sqrt{\tau}Y \\ * & -Z_2 \end{bmatrix}<0, \tag{8}$$

where $R \in \mathbb{R}^{n\times n-r}$ is any matrix with full column rank satisfying $E^T R = 0$ and

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & \Phi_{13} & \frac{1}{d_M}E^T Z_1 E & \Phi_{15} \\ * & -Q_1 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & \Phi_{35} \\ * & * & * & -Q_2 - \frac{1}{d_M}E^T Z_1 E & 0 \\ * & * & * & * & \Phi_{55} \end{bmatrix},$$

$$\Phi_{11} = Q_1 + Q_2 + (\tau+1)Q + sym(G_1^T(A-E)) - \frac{1}{d_M}E^T Z_1 E, \quad (9)$$

$$\Phi_{13} = G_1^T A_d + (A-E)^T G_2,$$
$$\Phi_{33} = -Q + sym(G_2^T A_d),$$
$$\Phi_{15} = E^T P + SR^T - G_1^T + (A-E)^T G_3,$$
$$\Phi_{35} = -G_2^T + A_d^T G_3,$$
$$\Phi_{55} = P + d_M Z_1 + \tau Z_2 - sym(G_3),$$
$$\Phi_1 = [0 \quad YE \quad XE - YE \quad -XE \quad 0],$$
$$\tau = d_M - d_m.$$

Proof: The proof of this theorem is divided into two parts. The first part is concerned with regularity and causality, while the second part treats the stability of system (4): Since rank $(E) = r \le n$, there always exist two nonsingular matrices $\bar{M}$ and $\bar{N} \in \mathbb{R}^{n\times n}$ such that

$$\bar{E} = \bar{M}E\bar{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \quad (10)$$

Then, R can be characterized as $R = \bar{M}^T \begin{bmatrix} 0 \\ \Phi \end{bmatrix}$, where $\Phi \in \mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.
We also define

$$\bar{A} = \bar{M}A\bar{N} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \quad \bar{S} = \bar{N}^T S = \begin{bmatrix} \bar{S}_{11} \\ \bar{S}_{21} \end{bmatrix},$$
$$\bar{A}_d = \bar{M}A_d\bar{N} = \begin{bmatrix} \bar{A}_{d11} & \bar{A}_{d12} \\ \bar{A}_{d21} & \bar{A}_{d22} \end{bmatrix}. \quad (11)$$

It follows from (31) and (32) that

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ * & \Psi_{22} \end{bmatrix} < 0, \quad (12)$$

where

$$\Psi_{11} = sym(G_1^T(A-E)) - \frac{1}{d_M}E^T Z_1 E,$$
$$\Psi_{12} = E^T P + SR^T - G_1^T + (A-E)^T G_3,$$
$$\Psi_{22} = -sym(G_3).$$

Pre- and post-multiplying (12) by $[I, A^T]$ and its transpose, respectively, we obtain

$$sym\left(E^T(P - G_3 - G_1)A - G_1 E - \frac{1}{d_M}E^T Z_1 E + SR^T A \right) < 0. \quad (13)$$

Pre- and post-multiplying (13) by $\bar{N}^T$ and $\bar{N}$, respectively, and then using the expression (10) and (11), we have that

$$sym(\bar{S}_{21}\Phi^T \bar{A}_{22}) < 0 \quad (14)$$

and thus $\bar{A}_{22}$ is nonsingular. Otherwise, suppose that the matrix $\bar{A}_{22}$ is singular, then there must exist a non-zero vector $\psi \in \mathbb{R}^{n-r}$ which ensures $\bar{A}_{22}\psi = 0.$ As a consequence, we conclude that $\psi^T sym(\bar{S}_{21}\Phi^T \bar{A}_{22})\psi = 0$ which contradicts (14), so $\bar{A}_{22}$ is nonsingular. Then, the pair (E, A) is regular and causal.

Next, under the conditions of the theorem, we will show that system (4) is stable. To this end, we select the Lyapunov-Krasovskii functional candidate

$$V = V_1(k) + V_3(k) + V_3(k) + V_4(k),$$
$$V_1(k) = x^T(k)E^T PEx(k),$$
$$V_2(k) = \sum_{s=k-d_m}^{k-1} x^T(s)Q_1 x(s) + \sum_{s=k-d_M}^{k-1} x^T(s)Q_2 x(s),$$
$$V_3(k) = \sum_{\theta=-d_M}^{-d_m}\sum_{s=k+\theta}^{k-1} x^T(s)Qx(s), \quad (15)$$
$$V_4(k) = \sum_{\theta=-d_M}^{-1}\sum_{s=k+\theta}^{k-1} \eta^T(s)E^T Z_1 E\eta(s) + \sum_{\theta=-d_M}^{-d_m-1}\sum_{s=k+\theta}^{k-1} \eta^T(s)E^T Z_2 E\eta(s),$$

where $\eta(k) = x(k+1) - x(k).$ In terms of the Lyapunov difference $\Delta V(k) = V(k+1) - V(k),$ one can obtain

$$\Delta V_1(k) = \eta^T(k)E^T PE\eta(k) + 2x^T(k)E^T PE\eta(k), \quad (16)$$

$$\Delta V_2(k) = x^T(k)(Q_1 + Q_2)x(k) - x^T(k-d_m)Q_1 x(k-d_m) - x^T(k-d_M)Q_2 x(k-d_M), \quad (17)$$

$$\Delta V_3(k) = (\tau+1)x^T(k)Qx(k) - \sum_{s=k-d_M}^{k-d_m} x^T(s)Qx(s) \le (\tau+1)x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k)), \quad (18)$$

$$\Delta V_4 k = \eta^T(k)E^T\left(d_M Z_1 + \tau Z_2\right)E\eta(k) - \sum_{s=k-d_M}^{k-1} \eta^T(s)E^T Z_1 E\eta(s) - \sum_{s=k-d_M}^{k-d_m-1} \eta^T(s)E^T Z_2 E\eta(s). \quad (19)$$

According to Lemma 2, we have that

$$\Delta V_4 k \le \eta^T(k)E^T\left(d_M Z_1 + \tau Z_2\right)E\eta(k) \quad (20)$$

$$-\frac{1}{d_M}\gamma^T(k)E^T Z_1 E\gamma(k)$$
$$-\sum_{s=k-d_M}^{k-d(k)-1}\eta^T(s)E^T Z_2 E\eta(s)$$
$$-\sum_{s=k-d(k)}^{k-d_m-1}\eta^T(s)E^T Z_2 E\eta(s),$$

where $\gamma(k)=x(k)-x(k-d_M)$.
Defining

$$\xi(k)=$$
$$[x^T(k)\, x^T(k-d_m)\, x^T(k-d(k))\, x(k-d_M)\, \eta^T(k)E^T]^T,$$

for any appropriately dimensioned matrix X, the following inequality holds

$$\sum_{s=k-d_M}^{k-d(k)-1}\begin{bmatrix}\xi(k)\\ E\eta(s)\end{bmatrix}^T\begin{bmatrix}XZ_2^{-1}X^T & X\\ X^T & Z_2\end{bmatrix}\begin{bmatrix}\xi(k)\\ E\eta(s)\end{bmatrix}\geq 0. \quad (21)$$

Then, it is easy to verify that

$$\begin{aligned}&-\sum_{s=k-d_M}^{k-d(k)-1}\eta^T(s)E^T Z_2 E\eta(s)\\ &\leq (d_M-d(k))\xi^T(k)XZ_2^{-1}X^T\xi(k) \quad (22)\\ &\quad +2\xi^T(k)XE\big(x(k-d(k))-x(k-d_M)\big).\end{aligned}$$

Similarly, for any matrix Y we get

$$\begin{aligned}&-\sum_{s=k-d(k)}^{k-d_m-1}\eta^T(s)E^T Z_2 E\eta(s)\\ &\leq (d(k)-d_m)\xi^T(k)YZ_2^{-1}Y^T\xi(k) \quad (23)\\ &\quad +2\xi^T(k)YE\big(x(k-d_m)-x(k-d(k))\big).\end{aligned}$$

Setting $\rho(k)=\frac{d_M-d(k)}{\tau}$. From (22) and (23), it can be seen that

$$\begin{aligned}&-\sum_{s=k-d_M}^{k-d_m-1}\eta^T(s)E^T Z_2 E\eta(s)\\ &\leq \xi^T(k)\big\{\tau\rho(k)XZ_2^{-1}X^T+\tau(1-\rho(k))YZ_2^{-1}Y^T \quad (24)\\ &\quad +2[0 \quad YE \quad XE-YE \quad -XE \quad 0]\big\}\xi(k).\end{aligned}$$

From (4), the following equation holds for any matrix G with appropriate dimensions

$$\begin{aligned}&2[x^T(k)G_1^T+x^T(k-d(k))G_2^T+\eta^T(k)E^T G_3^T]\\ &\times[(A-E)x(k)+A_d x(k-d(k))-E\eta(k)]=0.\end{aligned} \quad (25)$$

On the other hand, it is clear that

$$2x^T(k)SR^T E\eta(k)=0. \quad (26)$$

From (16)-(26), we have

$$\Delta V(k)\leq \xi^T(k)\rho\big((k)\bar{\Phi}_1+(1-\rho(k))\bar{\Phi}_2\big)\xi(k), \quad (27)$$

where

$$\bar{\Phi}_1=\Phi+sym(\Phi_1)+\tau XZ_2^{-1}X^T,$$
$$\bar{\Phi}_2=\Phi+sym(\Phi_1)+\tau YZ_2^{-1}Y^T,$$

since $0\leq\rho(k)\leq 1$, $\rho(k)\bar{\Phi}_1+(1-\rho(k))\bar{\Phi}_2$ is a convex combination of $\bar{\Phi}_1$ and $\bar{\Phi}_2$. If (31)-(32) are satisfied, then by applying the Schur complement, it is possible to obtain that $\rho(k)\bar{\Phi}_1+(1-\rho(k))\bar{\Phi}_2<0$ and thus $\Delta V(k)<0$. According to Lyapunov stability theory, then there exists a scalar $\alpha>0$ such that

$$\Delta V(k)\leq -\alpha\,\|x(k)\|^2. \quad (28)$$

Therefore, we have

$$\sum_{i=0}^{k}\|x(i)\|^2\leq\frac{1}{\alpha}V(0)<\infty, \quad (29)$$

that is, the series $\sum_{i=0}^{k}\|x(i)\|^2$ converges, which implies that $\lim_{k\to\infty}x(k)=0$. Thus, according to Definition 2 system Q_2 des is stable.

Remark 1: A key feature of the proposed approach is that neither model transformation nor the bounding techniques are used, when estimating the upper bound of the cross product terms. In particular, none of the useful items are ignored when deriving our stability criterion. In fact, in some literature results, such as [28], the time delay term $d(k)$ is usually assumed to be d_M when estimating the upper bound of some cross terms and some useful terms such as $-\sum_{s=k-d_M}^{k-d(k)-1}\eta^T(s)E^T Z_2 E\eta(s)$ were ignored. This inevitably leads to increasing conservatism. Therefore, the results derived in this paper should be less conservative than some existing results.

Remark 2: Theorem 1 applied for $d_m=0$ may give a conservative result, this is due to the two redundant terms that appear in $V_4(k)$. Considering one only term in $V_4(k)$ with $Q_1=0$ in (15), the result can be improved. (See the Corollary that follows).

We now provide a Corollary that presents a delay-dependent admissibility criterion for the system (4) when $d_m=0$. This criterion can be established by the same procedure used for the proof of Theorem 1, with the following candidate Lyapunov functional

$$\begin{aligned}V(k)&=x^T(k)E^T\bar{P}Ex(k)+\sum_{s=k-d_M}^{k-1}x^T(s)\bar{Q}_1x(s)\\ &+\sum_{\theta=-d_M}^{0}\sum_{s=k+\theta}^{k-1}x^T(s)\bar{Q}x(s) \quad (30)\\ &+\sum_{\theta=-d_M}^{-1}\sum_{s=k+\theta}^{k-1}\eta^T(s)E^T\bar{Z}_1E\eta(s).\end{aligned}$$

Corollary 1: Given a integer $d_M>0$, for any delay $d(k)$ satisfying $0\leq d(k)\leq d_M$, system (4) is admissible if there exist matrices $\bar{P}>0$, $\bar{Q}>0$, $\bar{Q}_1>0$, $\bar{Z}_1>0$, $\bar{X}$,

$\bar{Y}$, $\bar{S}$, $\bar{G}_i$, $i = 1,2,3,$ such that

$$\begin{bmatrix} \bar{\Phi} + sym(\bar{\Phi}_1) & \sqrt{d_M}\bar{X} \\ * & -\bar{Z}_1 \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} \bar{\Phi} + sym(\bar{\Phi}_1) & \sqrt{d_M}\bar{Y} \\ * & -\bar{Z}_1 \end{bmatrix} < 0, \tag{32}$$

where $R \in \mathbb{R}^{n \times n-r}$ is any matrix with full column rank satisfying $E^T R = 0,$ and

$$\bar{\Phi} = \begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & 0 & \bar{\Phi}_{14} \\ & \bar{\Phi}_{22} & 0 & \bar{\Phi}_{24} \\ & * & -\bar{Q}_1 & 0 \\ & * & * & \bar{\Phi}_{44} \end{bmatrix},$$

$$\begin{aligned} &\bar{\Phi}_{11} = \bar{Q}_1 + (d_M + 1)\bar{Q} + sym(\bar{G}_1^T (A - E)), \\ &\bar{\Phi}_{12} = \bar{G}_1^T A_d + (A - E)^T \bar{G}_2, \\ &\bar{\Phi}_{22} = -\bar{Q} + sym(\bar{G}_2^T A_d), \\ &\bar{\Phi}_{14} = E^T \bar{P} + \bar{S}R^T - \bar{G}_1^T + (A - E)^T \bar{G}_3, \\ &\bar{\Phi}_{24} = -\bar{G}_2^T + A_d^T \bar{G}_3, \\ &\bar{\Phi}_{44} = \bar{P} + d_M \bar{Z}_1 - sym(\bar{G}_3), \\ &\bar{\Phi}_1 = [\bar{Y}E \quad \bar{X}E - \bar{Y}E \quad -\bar{X}E \quad 0]. \end{aligned} \tag{33}$$

4. OBSERVER-BASED CONTROL DESIGN

A state observer is usually used to reconstruct the states of a dynamic system and has very important applications in many aspects such as the realization of feedback control, system supervision and fault diagnosis. In many practical systems, the states of a system are not always measurable or have practical sense. Hence, observer-based control is well suited for feedback control. In this section, we aim to develop results to solve the output feedback control problem for the singular system (1).

To achieve this objective, we use the following observer-based controller:

$$\begin{cases} E\hat{x}(k+1) = A\hat{x}(k) + A_d \hat{x}(k - d(k)) + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C_2 \hat{x}(k) \\ u(k) = K\hat{x}(k) \\ \hat{x}(k) = \psi(k), \quad \forall k \in [-d_M, 0], \end{cases} \tag{34}$$

where $\hat{x}(k)$ is the state estimation of $x(k)$, $\hat{y}(k)$ is the observer output, and $L \in \mathbb{R}^{n \times q}$ and $K \in \mathbb{R}^{p \times n}$ are, respectively, the observer and the controller constant gain matrices, to be determined.

Let us denote the estimation error as $e(k) = x(k) - \hat{x}(k)$ and $\tilde{x}^T(k) = [\hat{x}^T(k) \quad e^T(k)]$. Combining (1) with (34), the augmented closed-loop system is written as

$$\begin{cases} \tilde{E}\tilde{x}(k+1) = \tilde{\mathbb{A}}\tilde{x}(k) + \tilde{\mathbb{A}}_d \tilde{x}(k - d(k)) \\ \tilde{x}(k) = [\psi^T(k), (\phi(k) - \psi(k))^T]^T, \ \forall k \in [-d_M, 0] \end{cases} \tag{35}$$

with $\tilde{\mathbb{A}} = \tilde{A} + \tilde{M}\tilde{F}(k)\tilde{N}$, $\tilde{\mathbb{A}}_d = \tilde{A}_d + \tilde{M}_d \bar{F}(k)\tilde{N}_d$, $\tilde{F}(k) = diag(F(k), F(k), F(k))$, $\bar{F}(k) = diag(F(k), F(k))$,

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A + B_2 K & LC_2 \\ 0 & A - LC_2 \end{bmatrix},$$

$$\tilde{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \quad \tilde{M} = \begin{bmatrix} M & 0 & M \\ 0 & M & 0 \end{bmatrix},$$

$$\tilde{N} = \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_u K & 0 \end{bmatrix}, \ \tilde{M}_d = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}, \tilde{N}_d = \begin{bmatrix} N_d & 0 \\ 0 & N_d \end{bmatrix}.$$

Theorem 2: For given integers d_m, d_M with $d_M \geq d_m > 0$, system (35) is admissible if there exist matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Z}_1 > 0$, $\tilde{Z}_2 > 0$, $\tilde{X}$, $\tilde{Y}$, $\tilde{S}$, $\tilde{G}_i$, $i = 1,2,3,$ such that

$$\begin{bmatrix} \tilde{\Phi} + sym(\tilde{\Phi}_1) & \sqrt{\tau}\tilde{X} & \tilde{\Upsilon}_1 \\ * & -\tilde{Z}_2 & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{36}$$

$$\begin{bmatrix} \tilde{\Phi} + sym(\tilde{\Phi}_1) & \sqrt{\tau}\tilde{Y} & \tilde{\Upsilon}_1 \\ * & -\tilde{Z}_2 & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{37}$$

where $\tilde{R} \in \mathbb{R}^{n \times n-r}$ is any matrix with full column rank satisfying $\tilde{E}^T \tilde{R} = 0$ and

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & 0 & \tilde{\Phi}_{13} & \frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & \tilde{\Phi}_{15} \\ * & -\tilde{Q}_1 & 0 & 0 & 0 \\ * & * & \tilde{\Phi}_{33} & 0 & \tilde{\Phi}_{35} \\ * & * & * & -\tilde{Q}_2 - \frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & 0 \\ * & * & * & * & \tilde{\Phi}_{55} \end{bmatrix},$$

$$\tilde{\Upsilon}_1 = \begin{bmatrix} -\tilde{G}^T \tilde{N}^T & -\tilde{G}^T \tilde{N}_d^T \\ 0 & 0 \\ -\mu_1 \tilde{G}^T \tilde{N}^T & -\mu_1 \tilde{G}^T \tilde{N}_d^T \\ 0 & 0 \\ -\mu_2 \tilde{G}^T \tilde{N}^T & -\mu_2 \tilde{G}^T \tilde{N}_d^T \end{bmatrix},$$

$$\begin{aligned} &\tilde{\Phi}_{11} = \tilde{Q}_1 + \tilde{Q}_2 + (\tau + 1)Q + sym((\tilde{A} - \tilde{E}^T)\tilde{G}) \\ &\qquad - \frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T + \tilde{H}\tilde{H}^T, \\ &\tilde{\Phi}_{13} = \tilde{G}^T \tilde{A}_d^T + \mu_2(\tilde{A} - \tilde{E}^T)\tilde{G}, \\ &\tilde{\Phi}_{33} = -\tilde{Q} + \mu_1 sym(\tilde{A}_d \tilde{G}) + \varepsilon \tilde{H}_d \tilde{H}_d^T, \\ &\tilde{\Phi}_{15} = \tilde{E}P + \tilde{S}R^T - \tilde{G}^T + \mu_2(\tilde{A} - \tilde{E}^T)\tilde{G}, \\ &\tilde{\Phi}_{35} = -\mu_1 \tilde{G}^T + \mu_2 \tilde{A}_d \tilde{G}, \\ &\tilde{\Phi}_{55} = \tilde{P} + d_M \tilde{Z}_1 + \tau \tilde{Z}_2 - \mu_2 sym(\tilde{G}), \\ &\tilde{\Phi}_1 = \begin{bmatrix} 0 & \tilde{Y}\tilde{E}^T & \tilde{X}\tilde{E}^T - \tilde{Y}\tilde{E}^T & -\tilde{X}\tilde{E}^T & 0 \end{bmatrix}. \end{aligned} \tag{38}$$

Proof: Now consider the following singular delay system

$$\tilde{E}^T\zeta(k+1)=\left(\tilde{\mathbb{A}}^T\zeta(k)+\tilde{\mathbb{A}}_d^T\zeta(k-d(k))\right). \quad (39)$$

Note that $\det(z\tilde{E}-\tilde{\mathbb{A}})=\det(z\tilde{E}^T-\tilde{\mathbb{A}}^T)$, then the pair $(\tilde{E},\tilde{\mathbb{A}})$ is regular, impulse-free and stable if and only if the pair $\tilde{E}^T$, $\tilde{\mathbb{A}}^T$ is regular, impulse-free and stable. Moreover, $\det z(z\tilde{E}-\tilde{\mathbb{A}}-z^{-d_M}\tilde{\mathbb{A}}_d)=0$ and $\det(z\tilde{E}^T-\tilde{\mathbb{A}}^T-z^{-d_M}\tilde{\mathbb{A}}_d^T)=0$ have the same solution.

As long as the regularity, being impulse-free and stability are concerned, we can consider system (39) instead of (35). Then, applying Theorem 3 to system (39) and setting $G_1=\tilde{G}$, $G_2=\mu_1\tilde{G}$ and $G_3=\mu_2\tilde{G}$, the following inequalities hold for all $F(k)$ satisfying $F^T(k)F(k)\le I$.

$$\begin{bmatrix}\tilde{\Phi}(k)+sym(\tilde{\Phi}_1) & \sqrt{\tau}\tilde{X}\\ * & -\tilde{Z}_2\end{bmatrix}<0, \quad (40)$$

$$\begin{bmatrix}\tilde{\Phi}(k)+sym(\Phi_1) & \sqrt{\tau}\tilde{Y}\\ * & -\tilde{Z}_2\end{bmatrix}<0 \quad (41)$$

with

$$\tilde{\Phi}(k)=\begin{bmatrix}\tilde{\Phi}_{11}(k) & 0 & \tilde{\Phi}_{13}(k) & \frac{1}{d_M}\tilde{E}Z_1\tilde{E}^T & \tilde{\Phi}_{15}(k)\\ * & -\tilde{Q}_1 & 0 & 0 & 0\\ * & * & \tilde{\Phi}_{33}(k) & 0 & \tilde{\Phi}_{35}(k)\\ * & * & * & -\tilde{Q}_2-\frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & 0\\ * & * & * & * & \tilde{\Phi}_{55}\end{bmatrix},$$

$$\tilde{\Phi}_{11}(k)=\tilde{Q}_1+\tilde{Q}_2+(\tau+1)Q+sym((\tilde{\mathbb{A}}-\tilde{E})\tilde{G})-\frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T,$$

$$\tilde{\Phi}_{13}(k)=\tilde{G}^T\tilde{\mathbb{A}}_d^T+\mu_2(\tilde{\mathbb{A}}-\tilde{E})\tilde{G},$$

$$\tilde{\Phi}_{33}(k)=-\tilde{Q}+\mu_1 sym(\tilde{G}^T\tilde{\mathbb{A}}_d^T),$$

$$\tilde{\Phi}_{15}(k)=\tilde{E}P+SR^T-\tilde{G}^T+\mu_2(\tilde{\mathbb{A}}-\tilde{E})\tilde{G},$$

$$\tilde{\Phi}_{35}(k)=-\mu_1\tilde{G}^T+\mu_2\tilde{\mathbb{A}}_d\tilde{G},$$

which can be written as

$$\tilde{\Phi}+sym\left(\tilde{\Psi}_1\tilde{\mathscr{F}}(k)\tilde{\Upsilon}_1^T\right)<0, \quad (42)$$

where

$$\tilde{\Phi}=\begin{bmatrix}\tilde{\Phi}_{11} & 0 & \tilde{\Phi}_{13} & \frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & \tilde{\Phi}_{15}\\ * & -\tilde{Q}_1 & 0 & 0 & 0\\ * & * & \tilde{\Phi}_{33} & 0 & \tilde{\Phi}_{35}\\ * & * & * & -\tilde{Q}_2-\frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & 0\\ * & * & * & * & \tilde{\Phi}_{55}\end{bmatrix},$$

$$\tilde{\Psi}_1=\begin{bmatrix}\tilde{M} & 0\\ 0 & 0\\ 0 & \tilde{M}_d\\ 0 & 0\\ 0 & 0\end{bmatrix}.$$

By Lemma 1, and applying the Schur complement, the conditions (36)-(37) hold for $\varepsilon>0$.

Theorem 3: Consider the system (39) with the observer-based control (34). For given integers d_m, d_M with $d_M\ge d_m>0$, a positive scalar ε, and scalar tuning parameters $\mu_i\neq 0$, $i=1,2$, if there exist matrices $\tilde{P}>0$, $\tilde{Q}>0$, $\tilde{Q}_1>0$, $\tilde{Q}_2>0$, $\tilde{Z}_1>0$, $\tilde{Z}_2>0$, $\tilde{X}$, $\tilde{Y}$, $\tilde{S}$, $\hat{\boldsymbol{G}}_{11}\in\mathbb{R}^{q\times q}$, $\hat{\boldsymbol{G}}_{22}\in\mathbb{R}^{(n-q)\times(n-q)}$, $\hat{\boldsymbol{G}}_{21}\in\mathbb{R}^{(n-q)\times n}$, $\mathscr{G}\in\mathbb{R}^{n\times n}$, $\mathbf{K}\in\mathbb{R}^{m\times n}$, $\mathbf{L}\in\mathbb{R}^{n\times q}$, such that the following LMIs hold:

$$\begin{bmatrix}\tilde{\Psi}+sym(\tilde{\Psi}_1) & \sqrt{\tau}\tilde{X} & \tilde{\Gamma}_1\\ * & -\tilde{Z}_2 & 0\\ * & * & -\varepsilon I\end{bmatrix}<0, \quad (43)$$

$$\begin{bmatrix}\tilde{\Psi}+sym(\tilde{\Psi}_1) & \sqrt{\tau}\tilde{Y} & \tilde{\Gamma}_1\\ * & -\tilde{Z}_2 & 0\\ * & * & -\varepsilon I\end{bmatrix}<0, \quad (44)$$

where

$$\tilde{\Psi}=\begin{bmatrix}\tilde{\Psi}_{11} & 0 & \tilde{\Psi}_{13} & \frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & \tilde{\Psi}_{15}\\ * & -\tilde{Q}_1 & 0 & 0 & 0\\ * & * & \tilde{\Psi}_{33} & 0 & \tilde{\Psi}_{35}\\ * & * & * & -\tilde{Q}_2-\frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T & 0\\ * & * & * & * & \tilde{\Psi}_{55}\end{bmatrix},$$

$$\tilde{\Gamma}_1=\begin{bmatrix}-\mathbf{N}^T & -\mathbf{N}_d^T\\ 0 & 0\\ -\mu_1\mathbf{N}^T & -\mu_1\mathbf{N}_d^T\\ 0 & 0\\ -\mu_2\mathbf{N}^T & -\mu_2\mathbf{N}_d^T\end{bmatrix},$$

$$\tilde{\Psi}_{11}=\tilde{Q}_1+\tilde{Q}_2+(\tau+1)Q+sym(\mathbf{A}-\tilde{E}\tilde{G})-\frac{1}{d_M}\tilde{E}\tilde{Z}_1\tilde{E}^T+\varepsilon M,$$

$$\tilde{\Psi}_{13}=\mathbf{A}_d^T+\mu_2(\mathbf{A}-\tilde{E}\tilde{G}),$$

$$\tilde{\Psi}_{33}=-\tilde{Q}+\mu_1 sym(\mathbf{A}_d)+\varepsilon\mathbf{M}_d,$$

$$\tilde{\Psi}_{15}=\tilde{E}^TP+SR^T-\tilde{G}^T+\mu_2(\mathbf{A}-\tilde{E}\tilde{G}),$$

$$\tilde{\Psi}_{35}=-\mu_1\tilde{G}^T+\mu_2\mathbf{A}_d,$$

$$\tilde{\Psi}_{55}=\tilde{P}+d_M\tilde{Z}_1+\tau\tilde{Z}_2-\mu_2 sym(\tilde{G}),$$

$$\tilde{\Psi}_1=\begin{bmatrix}0 & \tilde{Y}\tilde{E}^T & \tilde{X}\tilde{E}^T-\tilde{Y}\tilde{E}^T & -\tilde{X}\tilde{E}^T & 0\end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} A\mathcal{G} + B_2 Y & FC_2 \\ 0 & A\mathbf{G} - FC_2 \end{bmatrix}, \quad \mathbf{A}_d = \begin{bmatrix} A_d\mathcal{G} & 0 \\ 0 & A_d\mathbf{G} \end{bmatrix},$$

$$\hat{\mathbf{G}} = \begin{bmatrix} \hat{\mathbf{G}}_{11} & 0 \\ \hat{\mathbf{G}}_{21} & \hat{\mathbf{G}}_{22} \end{bmatrix}, \quad \mathbf{G} = V\hat{\mathbf{G}}V^T, \quad \tilde{G} = \begin{bmatrix} \mathcal{G} & 0 \\ 0 & \mathbf{G} \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 2MM^T & 0 \\ 0 & MM^T \end{bmatrix}, \quad \mathbf{M}_d = \begin{bmatrix} MM^T & 0 \\ 0 & MM^T \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} N\mathcal{G} & 0 \\ 0 & N\mathbf{G} \\ N_u Y & 0 \end{bmatrix}, \quad \mathbf{N}_d = \begin{bmatrix} N_d\mathcal{G} & 0 \\ 0 & N_d\mathbf{G} \end{bmatrix}.$$

Then the closed-loop singular system (35) is regular, impulse-free and asymptotically stable; the gain matrices that provide these properties are $K = Y\mathcal{G}^{-1}$ and $L = FUS\hat{\mathbf{G}}_{11}^{-1}S^{-1}U^T$, where U, V and S come from (5).

Proof: Under the conditions of the theorem, it follows from $\tilde{\Psi}_{55} < 0$ that $\tilde{G}$ is nonsingular. Thus, $\mathbf{G}$ is also nonsingular. Setting $F = LUS\hat{\mathbf{G}}_{11}S^{-1}U^T = L\hat{\mathbf{G}}$ and $Y = K\mathcal{G}^{-1}$. Under the condition of Assumption 1, we have that

$$\begin{aligned} \hat{\mathbf{G}}C_2 &= US\hat{\mathbf{G}}_{11}S^{-1}U^T U[S \quad 0]V^T \\ &= U[S\hat{\mathbf{G}}_{11} \quad 0]V^T \\ &= U[S \quad 0]V^T V \begin{bmatrix} \hat{\mathbf{G}}_{11} & 0 \\ \hat{\mathbf{G}}_{21} & \hat{\mathbf{G}}_{22} \end{bmatrix} V^T \\ &= C_2\mathbf{G}. \end{aligned}$$

Then, the augmented matrices can be written as

$$\mathbf{A} = \begin{bmatrix} A\mathcal{G} + B_2 K\mathcal{G} & LC_2\mathbf{G} \\ 0 & A\mathbf{G} - LC_2\mathbf{G} \end{bmatrix} = \tilde{A}\tilde{G},$$

$$\mathbf{A}_d = \begin{bmatrix} A_d\mathcal{G} & 0 \\ 0 & A_d\mathbf{G} \end{bmatrix} = \tilde{A}_d\tilde{G},$$

$$\mathbf{N} = \begin{bmatrix} N\mathcal{G} & 0 \\ 0 & N\mathbf{G} \\ N_u K\mathcal{G} & 0 \end{bmatrix} = \tilde{N}\tilde{G},$$

$$\mathbf{N}_d = \begin{bmatrix} N_d\mathcal{G} & 0 \\ 0 & N_d\mathbf{G} \end{bmatrix} = \tilde{N}_d\tilde{G}.$$

Then, from Theorem 2, the closed-loop singular system (35) is regular, impulse-free and asymptotically stable.

5. NUMERICAL EXAMPLES

In this section we provide some examples to show the effectiveness of our proposed method.

Example 1: Consider an unforced singular time-delay system with parameters as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \; A_d = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}.$$

Table 1. Maximum allowed delays d_M for various d_m.

d_m	0	3	6	9	12	15
[25]	7	8	10	13	15	18
[29]	17	18	19	21	23	25
[30]	15	16	19	22	25	28
Theorem 1	18	18	21	24	27	30

Our purpose is to determine the allowable time delay upper bounds d_M for various d_m such that the system (4) des will be admissible. Table 1 gives a more detailed comparison of results on the maximum allowed bounds for d_M via the methods in [25,29,30] and Theorem 1 (or Corollary 1 for $d_m = 0$) in this paper.

In terms of conservatism, the results in Table 1 clearly show that the result in this paper outperforms those in [25,29,30].

Example 2: Consider the singular time-delay system in desc-sys with the following parameters

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, \; A_d = \begin{bmatrix} 0 & -0.02 \\ 0.1 & 0.15 \end{bmatrix},$$
$$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \; C_2 = [5 \quad 1], \; M = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$N = [-0.25 \quad 0], \; N_d = [0.2 \quad 0.2], \; N_u = 0.25.$$

In this example we choose $d_m = 1$, $d_M = 4$, $\mu_1 = 0.001$, $\mu_2 = 0.8$ and $F(k) = r_0$. r_0 is a random number taken from a uniform distribution over $[-1,1]$.

We record that the open-loop system is unstable, since its eigenvalues are outside the unit disc. Implementation of the LMIs (43), (44) yields the following feasible solution:

$$\begin{aligned} \mathcal{G} &= \begin{bmatrix} 37.5841 & -15.8172 \\ -12.4015 & 55.9098 \end{bmatrix}, \\ \hat{\mathbf{G}} &= \begin{bmatrix} 35.9478 & 0 \\ -41.2878 & 31.8568 \end{bmatrix}, \\ K &= \begin{bmatrix} -0.5831 & -0.8594 \end{bmatrix}, \; L = \begin{bmatrix} 0.0330 \\ 0.1768 \end{bmatrix}. \end{aligned} \tag{45}$$

Given the initial conditions

$$\phi(k) = \begin{bmatrix} -0.1\sin(k) & -0.75e^{k-d_M} \end{bmatrix}^T \text{ and}$$

$$\psi(k) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T,$$

the simulation results are presented in Fig. 1. From the plotted graphs, it is quite clear that the generated control law guarantees regulation to the zero level.

Example 3: Consider the system (1) with the following parameters

$$E = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} 0.1530 & 0.0450 & 0.0690 \\ 0.1560 & 0.2520 & 0.1560 \\ 0.1350 & -0.1710 & -0.6360 \end{bmatrix},$$

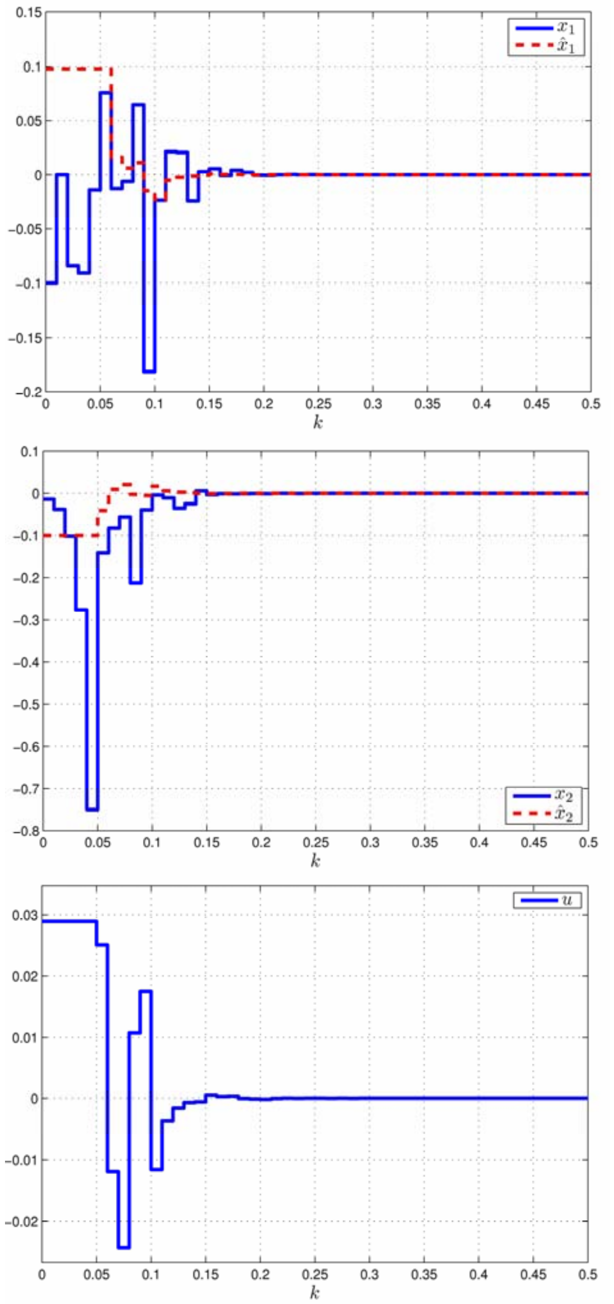

Fig. 1. State and control input trajectories for Example 2.

$$A_d = \begin{bmatrix} 0.15 & 0 & 0 \\ 0.1 & -0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.15 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.1 & 0 & 0.1 \end{bmatrix}, \quad N_d = \begin{bmatrix} 0.2 & -0.15 & 0.1 \end{bmatrix},$$

$$N_u = 0.$$

Assume that $d_m = 2$, $d_M = 5$, $\mu_1 = 0.01$ and $\mu_2 = 1.3$. Theorem 3 gives a feasible solution to the corresponding LMIs with the following parameters:

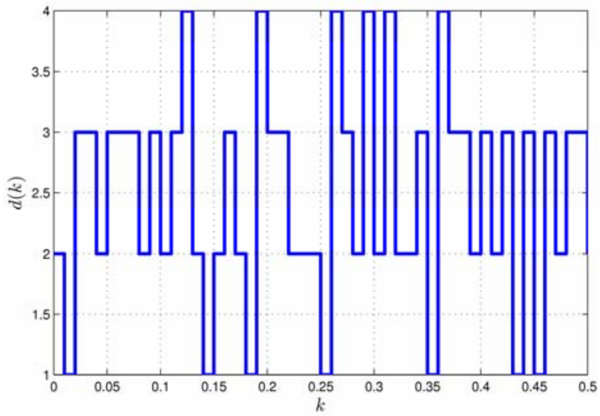

Fig. 2. Time-varying delay.

$$\mathcal{G} = \begin{bmatrix} 6.8127 & -1.5564 & 3.2015 \\ -4.6247 & 6.1177 & -0.4742 \\ 4.0353 & 7.7814 & 10.7793 \end{bmatrix},$$

$$\hat{\mathbf{G}} = \begin{bmatrix} 2.9653 & -2.7765 & 0 \\ -1.1280 & 4.0407 & 0 \\ -0.4658 & -1.6755 & 4.5012 \end{bmatrix}, \tag{46}$$

$$K = \begin{bmatrix} 18.6321 & 13.8631 & -6.1571 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.2636 & 0.4099 \\ -0.5833 & -0.1515 \\ -0.8431 & -0.4853 \end{bmatrix}. \tag{47}$$

For simulation we select $F(k) = 0.8 + 0.2sin(k\pi/2)$. The simulation results depicted in Fig. 3 show that the closed-loop behavior of the system with the above controller for the following initial conditions:

$\phi(k) = [0.35sin(k) \quad -0.15sin(k) \quad -0.2]^T$ and

$\psi(k) = [0.05 \quad 0 \quad 0]^T$,

tends to zero, which is in accordance with the analysis in this paper.

Example 4: Consider the linear uncertain discrete singular delay system in (1) with parameters as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 1.2 & 0 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0.4 \\ 0.1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix},$$

$$N = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad N_d = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \quad N_u = 0.$$

Because the (2,2) -th entry of A is 0, it follows that the matrix pair (E, A) must not be causal, and hence the unforced part of the considered system is not admissible for all the delay $d(k)$.

Assume that $d_m = 3$, $d_M = 5$ and $d(k)$ is a repeating of sequence [5,3,4,4]. Table 2 presents the allowable controller gains calculated by Theorem 3 for $\mu_1 = 0.002$, $\mu_2 = 0.9$ and different values of α.

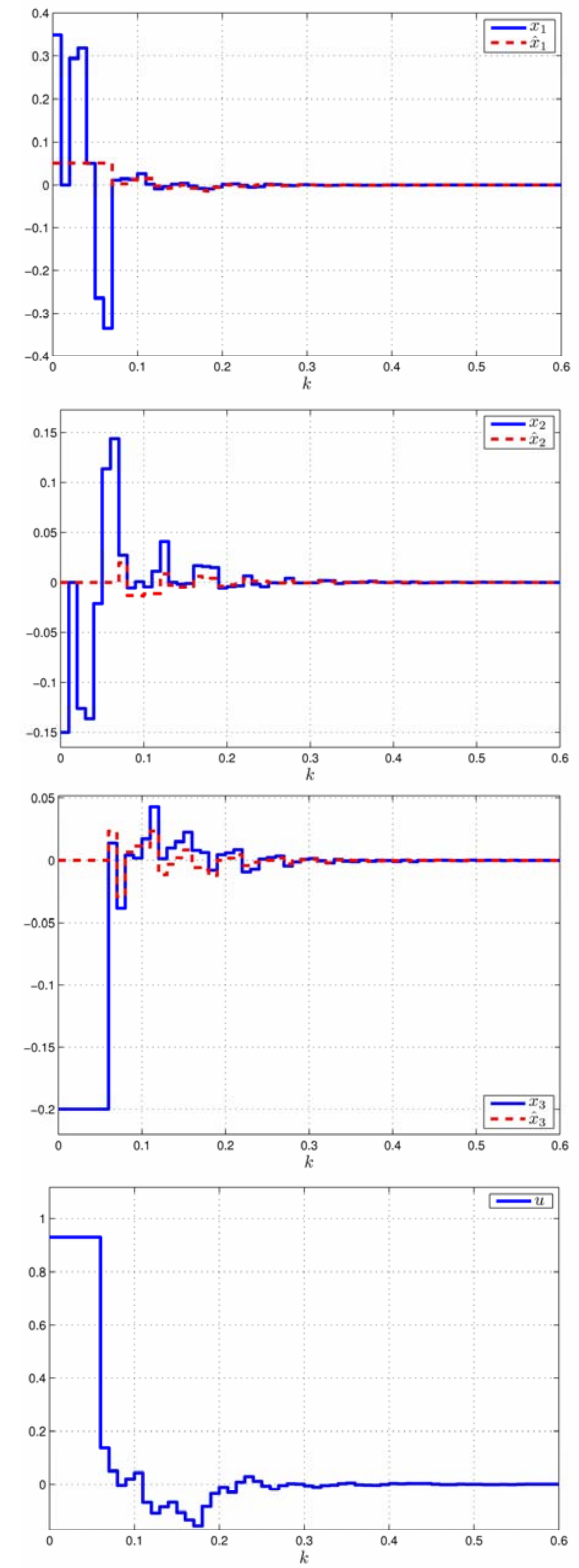

Fig. 3. State and control input trajectories for Example 3.

With the initial functions

$$\phi(k) = [-0.15sin(k) \quad 0.45e^{k/d_M}]^T \quad \text{and}$$
$$\psi(k) = [0.1 \quad -0.2]^T,$$

Table 2. Allowable controller gains for various α.

α	0.1	0.3	0.5
K	$[1.078 \quad -2.976]$	$[1.168 \quad -3.283]$	$[1.349 \quad -3.684]$
L	$\begin{bmatrix} -3.410 & 1.591 \\ -7.706 & 4.156 \end{bmatrix}$	$\begin{bmatrix} -3.902 & 1.857 \\ -8.798 & 4.762 \end{bmatrix}$	$\begin{bmatrix} -4.612 & 2.245 \\ -10.611 & 5.730 \end{bmatrix}$

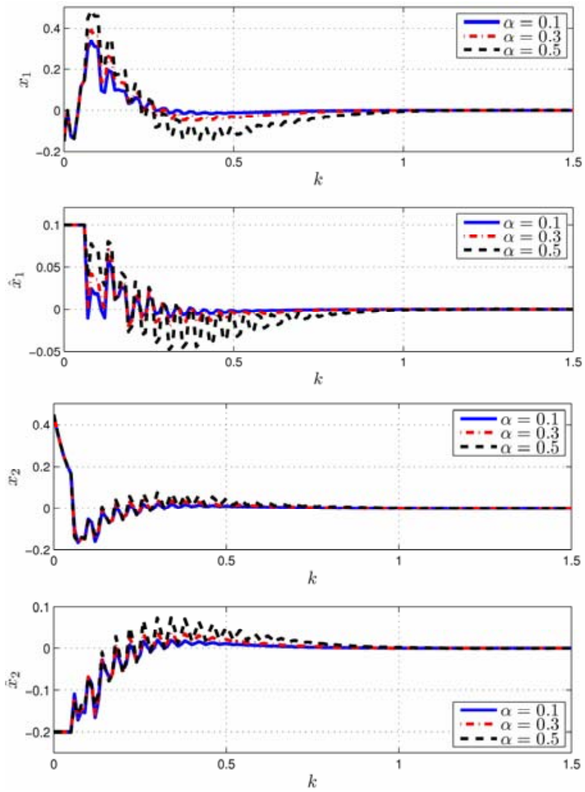

Fig. 4. Closed-loop responses for various α.

the control results are depicted in Fig. 4 for various values of α. It is clear that the observer-based controller (34) stabilizes the system which validates the theoretical finding, even some remarkable oscillation occurs when α becomes important.

6. CONCLUSION

The design of robust output feedback controllers has been studied for the class of discrete-time singular systems with time-varying delays and uncertainties. First, the problem of admissibility has been considered, and a delay-dependent criterion is derived ensuring the considered system to be regular, causal, and stable has been developed in terms of LMIs, without using decomposition or equivalent transformations. Using this condition, the problem of robust output feedback stabilization is then solved. The proposed results have been applied to three examples, showing the efficacy of the method.

It must be pointed out that in the present study the proposed control design is based on the assumption of system linearity. Further work is being pursued to solve the equivalent problem for nonlinear systems, also with time-varying delays.

REFERENCES

[1] Y. Xia, M. Fu, and P. Shi, *Analysis and Synthesis of Dynamical Systems with Time-Delays*, Springer-Verlag, Berlin, 2009.

[2] K. Gu, V. Kharitonov, and J. Chen, *Stability of Time Delay Systems*, Springer-Verlag, Berlin, 2003.

[3] J. Chen, C. Yang, C. Lien, and J. Horng, "New delay-dependent non-fragile H_∞ observer-based control for continuous time-delay systems," *Information Sciences*, vol. 178, pp. 4699-4706, 2008.

[4] M. Kchaou, A. Toumi, and M. Souissi, "Robust reliable guaranteed cost piecewise fuzzy control for discrete-time nonlinear systems with time-varying delay and actuator failures," *International Journal of General Systems*, vol. 40, no. 5, pp. 531-558, 2011.

[5] J. Sun, G. Liu, J. Chena, and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays," *Automatica*, vol. 46, pp. 466-470, 2010.

[6] H. Shao, "Improved delay-dependent stability criteria for systems with a delay varying in a range," *Automatica*, vol. 44, no. 12, pp. 3215-3218, 2008.

[7] L. Dai, *Singular Control Systems*, vol. 118 of Lecture Notes in Control and Information Sciences, Springer, New York, 1989.

[8] P. Muller, "Linear mechanical descriptor systems: identification, analysis and design," *Proc. of Conference on Control of Independent Systems*, pp. 501-506, IFAC, Belfort, France, 1997.

[9] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer-Verlag, 2006.

[10] G. Duan, *Analysis and Design of Descriptor Linear Systems*, Springer Verlag, New York, 2010.

[11] E. Boukas, "Singular linear systems with delay: H_∞ stabilization," *Optimal Control Applications and Methods*, vol. 28, pp. 259-274, 2007.

[12] R. Zhong and Z. Yang, "Delay-dependent robust control of descriptor systems with time delay," *Asian Journal of Control*, vol. 8, no. 1, pp. 36-44, 2008.

[13] M. Kchaou, M. Souissi, and A. Toumi, "Delay dependent stability and robust L_2-L_∞ control for a class of fuzzy descriptor systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 23, pp. 284-304, 2013.

[14] S. Ma, C. Zhang, and Z. Wu, "Delay-dependent stability and H_∞ control for uncertain discrete switched singular systems with time-delay," *Applied Mathematics and Computation*, vol. 206, pp. 413-424, 2008.

[15] S. Chen and J. Chou, "Robust D-stability analysis for linear uncertain discrete singular systems with state delay," *Applied Mathematics Letters*, vol. 19, pp. 197-205, 2006.

[16] C. Huang, "Stability analysis of discrete singular fuzzy systems," *Fuzzy Sets and Systems*, vol. 151, pp. 155-165, 2005.

[17] J. Kim, "New reduced-order H_∞ filter design method for discrete-time singular systems with lossy measurements by strict LMI approach," *International Journal of Control, Automation, and Systems*, vol. 9, pp. 1095-1102, 2011.

[18] J. Zhao, Z. Hu, and L. Zhang, "Stability and stabilization for discrete-time singular systems with infinite distributed delays and actuator failures," *International Journal of Control, Automation, and Systems*, vol. 10, pp. 721-726, 2012.

[19] J. Feng, P. Cui, and Z. Cheng, "H_∞ output feedback control for descriptor systems with delayed states," *Journal of Control Theory and Applications*, vol. 4, pp. 341-347, 2005.

[20] J. Feng, S. Zhu, and Z. Cheng, "Observer design for linear singular time-delay systems," *Proc. of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii USA, December 2003.

[21] Z. Wu, H. Su, and J. Chu, "Output feedback stabilization for discrete singular systems with random abrupt changes," *International Journal of Robust and Nonlinear Control*, vol. 20, pp. 1945-1957, 2010.

[22] Z. Wang and H. Zhang, "Design of a bilinear fault detection observer for singular bilinear systems," *Journal of Control Theory and Applications*, vol. 5, no. 1, pp. 28-36, 2007.

[23] D. Koenig, B. Marx, and O. Sename, "Unknown inputs proportional integral observers for descriptor systems with multiple delays and unknown inputs," *Proc. of the American Control Conference*, Boston. Massachusetts, June 2004.

[24] Z. Wang, Y. Shen, X. Zhang, and Q. Wang, "Observer design for discrete-time descriptor systems: an LMI approach," *Systems & Control Letters*, vol. 61, pp. 683-687, 2012.

[25] S. Ma, Z. Cheng, and C. Zhang, "Delay-dependent robust stability and stabilization for uncertain discrete singular systems with time-varying delay," *IET Control Theory & Applications*, vol. 1, no. 4, pp. 1086-1095, 2007.

[26] I. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Systems & Control Letters*, vol. 8, pp. 35-357, 1987.

[27] X. Zhu, Y. Wang, and G. Yang, "New delay dependent stability results for discrete-time recurrent neural networks with time-varying delay," *Neurocomputing*, vol. 72, pp. 3376-3383, 2009.

[28] S. Ma, E. Boukas, and Y. Chinniah, "Stability and stabilization of discrete-time singular Markov jump systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 20, pp. 531-543, 2010.

[29] H. Wang, X. Zhao, A. Xue, and R. Lu, "Delay dependent robust control for uncertain discrete singular systems with time-varying delay," *Journal of Zhejiang University Science A*, vol. 9, no. 8, pp. 1034-1042, 2008.

[30] M. Fang, "Delay-dependent stability analysis for discrete singular systems with time-varying delays," *Acta Automatica Sinica*, vol. 36, no. 5, pp. 751-755, 2010.

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