International Journal of Control, Automation, and Systems (2013) 11(4):683-691
DOI 10.1007/s12555-012-9114-4

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Stabilization of Stochastically Singular Nonlinear Jump Systems with Unknown Parameters and Continuously Distributed Delays

Quanxin Zhu

Abstract: In this paper, the problem of robustly asymptotic stabilization for a class of stochastically nonlinear singular jump systems is investigated. The jumping parameters are modeled as a continuous-time, finite-state Markov chain. Based on the Lyapunov-Krasovskii functional and stochastic analysis theory as well as a state feedback control technique, some new sufficient conditions are derived to ensure the asymptotic stability of the trivial solution in the mean square. A key feature of this paper is that singular, nonlinear, noise perturbations, unknown parameters and continuously distributed delays are all considered. In particular, the obtained stabilization criteria in this paper are expressed in terms of LMIs, which can be solved easily by recently developed algorithms. Finally, two numerical examples are presented to illustrate the effectiveness of the theoretical results. Moreover, the second example shows that delay-dependent stabilization criteria are less conservative than delay-independent criteria.

Keywords: Continuously distributed delay, Lyapunov functional, nonlinear jump system, robustly asymptotic stabilization, singular system, stochastic system.

1. INTRODUCTION

During the past decades, Markovian jump systems (MJSs) have received a great deal of research attention because they can be employed to model some plants whose structure is subject to random abrupt changes such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections, and changes of the operating point of a linearized model of a nonlinear system, etc. For instance, the exponential stability problem has been studied in [1-5], the robust stability problem has been discussed in [6-13], the stability and stabilization problem has been investigated in [8,10,14-16], and the feedback control problem has been considered in [7,9,11,15,17-20] for such a class of MJSs. It should be noted that almost all of the mentioned works are concentrated on the case of linear MJSs. For stochastically nonlinear jump systems, however, there are very few works to discuss the stability and stabilization of the equilibrium point [2,5,14].

On the other hand, singular systems have been recognized to be better for describing physical systems than regular ones. In fact, singular systems are referred to as implicit systems, descriptor systems, generalized state-space systems, differential-algebraic systems, and semi state systems [21-28]. Thus, many control and filter problems based on singular systems have been widely studied in the literature. For example, Xu and his coauthors studied the $\mathscr{H}_\infty$ filtering for singular systems in [29], and the reduced-order $\mathscr{H}_\infty$ filtering for singular systems in [30]; Xia and Jia [31] considered the $\mathscr{H}_\infty$ output feedback control of singular systems with time delays; In [32], the authors investigated the robust stability problem for a class of uncertain discrete-time singular fuzzy systems; Lu and Ho [33] discussed the generalized quadratic stability for continuous-time singular systems with nonlinear perturbation; Fang [34] considered the delay-dependent robust $\mathscr{H}_\infty$ control for uncertain singular systems with state delays; Wang *et al.* [35] studied the absolute stability criteria for a class of nonlinear singular systems with time delays; In [36], the authors discussed the improved results on delay-dependent $\mathscr{H}_\infty$ control for singular time-delay systems. However, all of the above mentioned papers do not consider MJSs. To the best of our knowledge, there are only a few works to study singular MJSs. Therefore, it is important to discuss the stability and stabilization problem of singular MJSs.

Recently, Boukas [14] discussed the stabilization problem for a class of stochastically singular MJSs, and he provided sufficient conditions to ensure the stochastic stability and robust stochastic stability for a class of continuous-time singular linear MJSs in [37]. More recently, Xia and his coauthors studied the problems of stability, state feedback control and static output feedback control for a class of discrete-time singular hybrid systems in [38], and they investigated the stability

---

Manuscript received March 16, 2012; revised January 3, 2013; accepted March 30, 2013. Recommended by Editorial Board member Duk-Sun Shim under the direction of Editor Yoshito Ohta.

This work was jointly supported by the National Natural Science Foundation of China, the National Natural Science Foundation of Zhejiang Province (LY12F03010), the Natural Science Foundation of Ningbo (2012A610032).

Quanxin Zhu is with the School of Mathematical Sciences and Institute of Finance and Statistics, Nanjing Normal University, Nanjing 210023, Jiangsu, China (e-mail: zqx22@126.com).

and state feedback stabilization for a class of continuous-time singular MJSs in [39]. But the authors in [14,37-39] did not consider time delays, whereas unknown parameters were removed in [14,38,39] and noise perturbations were not considered in [38,39]. As we know, time delays frequently occur in practical systems and are often the source of instability. Also, unknown parameters are the source of instability and poor performances in singular MJSs. In fact, the singular MJSs' parameters can not be known in prior in practice. In addition, noise perturbations are the source of instability and poor performances, too. It is shown that the singular MJSs can be stabilized or destabilized by certain stochastic inputs. Therefore, time delays, unknown parameters and noise perturbations should be taken into account when designing and investigating the stability and stabilization problem for a class of singular MJSs.

Motivated by the above discussion, in this paper we study the problem of robustly asymptotic stabilization for a class of stochastically nonlinear singular MJS with norm-bounded uncertainties and continuously distributed delays. Based on the Lyapunov-Krasovskii functional and stochastic analysis theory as well as a state feedback control technique, some new sufficient conditions are derived to ensure the asymptotic stability of the trivial solution in the mean square. A key feature of this paper is that singular, nonlinear, unknown parameters, noise perturbations and continuously distributed delays are all considered. Moreover, two numerical examples are given to illustrate the effectiveness of the theoretical results. In particular, the second example shows that delay-dependent stabilization criteria are less conservative than delay-independent criteria.

The rest of this paper is organized as follows. In Section 2, the model of a class of stochastically nonlinear singular MJSs with norm-bounded uncertainties and continuously distributed delays is introduced, and some necessary assumptions are given. By designing a linear feedback controller, both delay-independent and delay-dependent robustly asymptotic stabilization conditions are obtained in Section 3. In Section 4, two numerical examples are given to show the effectiveness of the obtained results. Finally, the paper is concluded with some general remarks in Section 5.

2. MODEL DESCRIPTION AND PROBLEM FORMULATION

Notations: Throughout this paper, the following notations will be used. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The superscript "T" denotes the transpose of a matrix or vector. Trace $(\cdot)$ denotes the trace of the corresponding matrix and I denotes the identity matrix with compatible dimensions. For any matrix A, $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) denotes the largest (respectively, smallest) eigenvalue of A. For square matrices M_1 and M_2, the notation $M_1 > (\geq, <, \leq) M_2$ denotes $M_1 - M_2$ is positive-definite (positive-semi-definite, negative, negative-semi-definite) matrix. Let $w(t) = (w_1(t), \cdots, w_m(t))^T$ be an m-dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$. Also, let $C((-\infty,0];\mathbb{R}^n)$ denote the family of continuous functions ϕ from $(-\infty,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\| = \sup_{\theta\leq 0}|\phi(\theta)|$. Denote by $L^2_{\mathscr{F}_0}((-\infty,0];\mathbb{R}^n)$ the family of all $\mathscr{F}_0$ measurable, $C((-\infty,0];\mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(\theta): -\infty < \theta \leq 0\}$ such that $\int_{-\infty}^0 \mathbb{E}|\xi(s)|^2 ds < \infty$, where $\mathbb{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure P.

Let $\{r(t), t\geq 0\}$ be a right-continuous Markov chain on a complete probability space $(\Omega, \mathscr{F}, P)$ taking values in a finite state space $S = \{1, 2, \cdots, N\}$ with generator $Q = (q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} q_{ij}\Delta t + o(\Delta t) & \text{if } i \neq j \\ 1 + q_{ii}\Delta t + o(\Delta t) & \text{if } i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t} = 0$. Here, $q_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $q_{ii} = -\sum_{j\neq i} q_{ij}$. As usual, we suppose that the Markov chain $\{r(t), t\geq 0\}$ is independent from the Brownian motion $\{w(t), t\geq 0\}$.

In this paper, we consider the following singular nonlinear system with Markovian switching and continuously distributed delays.

$$\begin{aligned} E(r(t))dx(t) = [&A(r(t))x(t) + B(r(t))x(t-\tau) \\ &+ C(r(t))\int_{-\infty}^t R(t-s)h(x(s))ds \\ &+ U(t, r(t)) + f(t, x(t), x(t-\tau)), r(t))]dt \\ &+ g(t, x(t), x(t-\tau), r(t))dw(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $U(t, r(t)) \in \mathbb{R}^{n\times n}$ is the control input, the matrices $A(r(t)) = (a_{ij}(r(t)))_{n\times n}$, $B(r(t)) = (b_{ij}(r(t)))_{n\times n}$, $C(r(t)) = (c_{ij}(r(t)))_{n\times n}$ and $E(r(t)) = (e_{ij}(r(t)))_{n\times n}$ are known matrix functions of the Markov jump process $\{r(t), t\geq 0\}$. In particular, the matrix $E(r(t))$ denotes a singular matrix such that $rank(E(r(t))) = n_{E(r(t))} < n$. The constant $\tau > 0$ denotes the time delay, and $R = diag(R_1, R_2, ..., R_n)$ denotes the delay kernel vector, where R_i is a real value non-negative continuous function defined on $[0, \infty)$ and such that $\int_0^\infty R_i(s)ds = 1$ for $i = 1, 2, \cdots, n$. $h(x(t)) = [h_1(x_1(t)), h_2(x_2(t)), \cdots, h_n(x_n(t))]^T$ denotes the activation function. $f: \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times S \to \mathbb{R}^n$ and $g: \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times S \to \mathbb{R}^{n\times m}$ denote the nonlinear uncertainties.

Throughout this paper, we make the following assumptions.

Assumption 1: Assume that the function h satisfies $h(0) \equiv 0$ and

$$u_i^- \leq \frac{h_i(\alpha)-h_i(\beta)}{\alpha-\beta} \leq u_i^+ \; \forall \alpha,\ \beta \in \mathbb{R},\ \alpha \neq \beta,\ i=1,2,\cdots,n,$$

where $U^- = diag(u_1^-, u_2^-, \cdots, u_n^-)$ and $U^+ = diag(u_1^+, u_2^+, \cdots, u_n^+)$ are two known diagonal matrices.

Assumption 2: There exist constant matrices R_{1i} and R_{2i} such that

$$| f(t,x(t),x(t-\tau),r(t)) | \leq | R_{1i}x(t) | + | R_{2i}x(t-\tau) |.$$

Assumption 3: There exist positive definite matrices Q_{1i} and $Q_{2i}\ (i \in S)$ such that

$$trace[g^T(t,x(t),x(t-\tau),i)g(t,x(t),x(t-\tau),i)] \leq x^T(t)Q_{1i}x(t) + x^T(t-\tau))Q_{2i}x(t-\tau).$$

Assumption 4: $f(t,0,0,r(t)) \equiv 0,\ g(t,0,0,r(t)) \equiv 0.$

In this paper, we will design a delay-independent memory less state feedback controller of the form $U(t,r(t)) = H(r(t))x(t)$ (the feedback gain $H(r(t))$ is a constant matrix for each fixed mode), which depends on the state $x(t)$ and the system mode $r(t)$. As usual, for any initial data $x(\theta) = \xi(\theta) \in L^2_{\mathscr{F}_0}((-\infty,0];\mathbb{R}^n)$ and $r(\theta) \equiv r(0)$ on $-\infty < \theta \leq 0$, we always suppose that the functions f, g, h satisfy enough conditions so that system (1) has a unique solution, which is denoted by $x(t;\xi)$. It is obvious that under Assumptions 1-4, system (1) admits a trivial solution $x(t;0) \equiv 0$ corresponding to the initial data $\xi = 0$. For simplicity, we write $x(t;\xi) = x(t)$.

Now we give the concept of asymptotic stability for system (1).

Definition 1: The trivial solution of (1) is said to be asymptotically stable in the mean square if for every $\xi \in L^2_{\mathscr{F}_0}((-\infty,0];\mathbb{R}^n)$, the following equality holds:

$$\lim_{t\to\infty} \mathbb{E} | x(t;\xi) |^2 = 0.$$

Owing to the fact that it is common that some systems' parameters cannot be exactly known in prior in many applications, in this paper we consider the following stochastically singular nonlinear jump systems with unknown parameters:

$$\begin{aligned} E(r(t))dx(t) = \{&[A(r(t)) + \Delta A(r(t))]x(t) + [B(r(t)) \\ &+ \Delta B(r(t))]x(t-\tau) + [C(r(t)) + \Delta C(r(t))] \\ &\times \int_{-\infty}^{t} R(t-s)h(x(s))ds + H(r(t))x(t) \\ &+ f(t,x(t),x(t-\tau)),r(t))\}dt \\ &+ g(t,x(t),x(t-\tau),r(t))dw(t), \end{aligned} \tag{2}$$

where $\Delta A(r(t))$, $\Delta B(r(t))$ and $\Delta C(r(t))$ are unknown matrices denoting time-varying parameter uncertainties and such that the following condition:

$$\begin{aligned} &[\Delta A(r(t))\ \Delta B(r(t))\ \Delta C(r(t))] \\ &= M(r(t))F(t,r(t))[N_1(r(t)), N_2(r(t)), N_3(r(t))], \end{aligned} \tag{3}$$

where $M(r(t))$ and $N_k(r(t))(k=1,2,3)$ are known real constant matrices and $F(t,r(t))$ is the unknown time-varying matrix-valued function satisfying

$$F^T(t,r(t))F(t,r(t)) \leq I, \quad \forall t \geq 0. \tag{4}$$

Definition 2: The trivial solution of (2) is said to be robustly asymptotically stable in the mean square if the trivial solution of (2) is asymptotically stable in the mean square for all admissible unknown parameters.

The following lemmas are needed to prove our main results.

Lemma 1 [40]**:** For any real matrices X, Y, the following matrix inequality holds:

$$X^TY + Y^TX \leq X^TX + Y^TY.$$

In the sequel, for simplicity, when $r(t) = i$, the matrices $A(r(t))$, $B(r(t))$, $C(r(t))$, $M(r(t))$, $H(r(t))$, $U(t,r(t))$, $F(t,r(t))$ and $N_k(r(t))(k=1,2,3)$ will be written as A_i, B_i, C_i, M_i, H_i, $U(t,i)$, $F(t,i)$ and $N_{ki}\ (k=1,2,3)$, respectively.

3. MAIN RESULTS

In this section, the linear feedback controller $U(t,i) = H_ix(t)$ is designed to realize the robustly asymptotic stability in the mean square of the trivial solution for system (2).

Theorem 1: Under Assumptions 1-4, if there exist positive scalars $\lambda_i\ (i \in S)$, a positive diagonal matrix $L = diag\{l_1, l_2, ..., l_n\}$, and positive definite matrices $G, D_i, P_i\ (i \in S)$ such that the following conditions hold for all $i \in S$,

$$0 \leq E_i^T P_i = P_i^T E_i \leq \lambda_i I, \tag{5}$$

$$\begin{bmatrix} \Gamma_{11} & P_iB_i & P_iC_i & P_i & P_iM_i \\ \star & \Gamma_{22} & 0 & 0 & 0 \\ \star & \star & -L + N_{3i}^TN_{3i} & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -\frac{1}{3}I \end{bmatrix} < 0, \tag{6}$$

where

$$\begin{aligned} \Gamma_{11} &= P_iA_i + A_i^TP_i + N_{1i}^TN_{1i} + \lambda_iQ_{1i} + 2D_i \\ &\quad + G + ULU + \sum_{j=1}^{N} q_{ij}E_j^TP_j, \\ \Gamma_{22} &= -G + 2R_{2i}^TR_{2i} + N_{2i}^TN_{2i} + \lambda_iQ_{2i}, \end{aligned}$$

then the trivial solution of (2) is robustly asymptotically stable with the feedback controller gain $H_i = P_i^{-1}D_i$.

Proof: Let us consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t,x(t),i) &= x^T(t)E_i^TP_ix(t) + \int_{t-\tau}^{t} x^T(s)Gx(s)ds \\ &\quad + \sum_{j=1}^{n} l_j \int_0^{\infty} R_j(\theta) \int_{t-\theta}^{t} h_j^2(x_j(s))dsd\theta. \end{aligned}$$

Then, by the Itô's formula we have

$$dV(t,x(t),i)=\mathscr{L}V(t,x(t),i)dt+2x^T(t)E_i^TP_i \times g(t,x(t),x(t-\tau),r(t))dw(t),$$

where

$$\begin{aligned}\mathscr{L}V(t,x(t),i) &= 2x^T(t)P_i[(A_i+M_iF(t,i)N_{1i})x(t)\\&\quad+(B_i+M_iF(t,i)N_{2i})x(t-\tau)\\&\quad+(C_i+M_iF(t,i)N_{3i})\int_{-\infty}^{t}R(t-s)h(x(s))ds\\&\quad+H_ix(t)+f(t,i)]+\sum_{j=1}^{N}q_{ij}x^T(t)E_j^TP_jx(t)\\&\quad+trace[g^T(t,i)E_i^TP_ig(t,i)]\\&\quad+x^T(t)Gx(t)-x^T(t-\tau)Gx(t-\tau)\\&\quad+\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(\theta)h_j^2(x_j(t))d\theta\\&\quad-\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(\theta)h_j^2(x_j(t-\theta))d\theta\\&=2x^T(t)P_iA_ix(t)+2x^T(t)P_iM_iF(t,i)N_{1i}x(t)\\&\quad+2x^T(t)P_iB_ix(t-\tau)+2x^T(t)P_iM_iF(t,i)N_{2i}x(t-\tau)\\&\quad+2x^T(t)P_iC_i\int_{-\infty}^{t}R(t-s)h(x(s))ds+2x^T(t)D_ix(t)\\&\quad+2x^T(t)P_iM_iF(t,i)N_{3i}\int_{-\infty}^{t}R(t-s)h(x(s))ds\\&\quad+2x^T(t)P_if(t,i)+\sum_{j=1}^{N}q_{ij}x^T(t)E_j^TP_jx(t)\\&\quad+trace[g^T(t,i)E_i^TP_ig(t,i)]+x^T(t)Gx(t)\\&\quad-x^T(t-\tau)Gx(t-\tau)+h^T(x(t))Lh(x(t))\\&\quad-\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(\theta)h_j^2(x_j(t-\theta))d\theta.\end{aligned} \tag{7}$$

Taking $y(t)=|R_{1i}x(t)|+|R_{2i}x(t-\tau)|$, from Lemma 1 and Assumption 2 we have

$$\begin{aligned}2x^T(t)P_if(t,i)&\le x^T(t)P_i^2x(t)+f^T(t,i)f(t,i)\\&\le x^T(t)P_i^2x(t)+y^T(t)y(t)\\&\le x^T(t)P_i^2x(t)+2x^T(t)R_{1i}^TR_{1i}x(t)\\&\quad+2x^T(t-\tau)R_{2i}^TR_{2i}x(t-\tau),\end{aligned} \tag{8}$$

$$\begin{aligned}&2x^T(t)P_iM_iF(t,i)N_{1i}x(t)\\&\le x^T(t)P_iM_iF(t,i)F^T(t,i)M_i^TP_ix(t)\\&\quad+x^T(t)N_{1i}^TN_{1i}x(t)\\&\le x^T(t)P_iM_iM_i^TP_ix(t)+x^T(t)N_{1i}^TN_{1i}x(t),\end{aligned} \tag{9}$$

$$\begin{aligned}&2x^T(t)P_iM_iF(t,i)N_{2i}x(t-\tau)\\&\le x^T(t)P_iM_iF(t,i)F^T(t,i)M_i^TP_ix(t)\\&\quad+x^T(t-\tau)N_{2i}^TN_{2i}x(t-\tau)\\&\le x^T(t)P_iM_iM_i^TP_ix(t)+x^T(t-\tau)N_{2i}^TN_{2i}x(t-\tau),\end{aligned} \tag{10}$$

$$\begin{aligned}&2x^T(t)P_iM_iF(t,i)N_{3i}\int_{-\infty}^{t}R(t-s)h(x(s))ds\\&\le x^T(t)P_iM_iF(t,i)F^T(t,i)M_i^TP_ix(t)+(\int_{-\infty}^{t}R(t-s)\\&\quad\times h(x(s))ds)^TN_{3i}^TN_{3i}\int_{-\infty}^{t}R(t-s)h(x(s))ds\\&\le x^T(t)P_iM_iM_i^TP_ix(t)+(\int_{-\infty}^{t}R(t-s)h(x(s))ds)^T\\&\quad\times N_{3i}^TN_{3i}\int_{-\infty}^{t}R(t-s)h(x(s))ds.\end{aligned} \tag{11}$$

On the other hand, by Assumption 3 and the condition (5) we obtain

$$\begin{aligned}&trace[g^T(t,i)E_i^TP_ig(t,i)]\\&\le\lambda_{\max}(E_i^TP_i)trace[g^T(t,i)g(t,i)]\\&\le\lambda_i trace[g^T(t,i)g(t,i)]\\&\le\lambda_ix^T(t)Q_{1i}x(t)+\lambda_ix^T(t-\tau)Q_{2i}x(t-\tau).\end{aligned} \tag{12}$$

By the well-known Cauchy-Schwarz inequality, we get

$$\begin{aligned}&\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(\theta)h_j^2(x_j(t-\theta))d\theta\\&=\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(u)du\int_0^{\infty}R_j(\theta)h_j^2(x_j(t-\theta))d\theta\\&\ge\sum_{j=1}^{n}l_j[\int_0^{\infty}R_j(\theta)h_j(x_j(t-\theta))d\theta]^2\\&=(\int_0^{\infty}R(\theta)h(x(t-\theta))d\theta)^T\\&\quad\times L(\int_0^{\infty}R(\theta)h(x(t-\theta))d\theta)\\&=(\int_{-\infty}^{t}R(t-s)h(x(s))ds)^T\\&\quad\times L(\int_{-\infty}^{t}R(t-s)h(x(s))ds).\end{aligned} \tag{13}$$

It follows from Assumption 1 that

$$h^T(x(t))Lh(x(t))\le x^T(t)ULUx(t). \tag{14}$$

Submitting (8)-(14) into (7), it can be derived

$$\mathbb{E}[\mathscr{L}V(t,x(t),i)]\le\mathbb{E}[\zeta^T(t)\Pi_i\zeta(t)], \tag{15}$$

where

$$\zeta^T(t)=[\,x^T(t)\ \ x^T(t-\tau)\ \ (\int_{-\infty}^{t}R(t-s)h(x(s))ds)^T\,],$$

$$\Pi_i=\begin{bmatrix}\Gamma_{11} & P_iB_i & P_iC_i\\ \star & \Gamma_{22} & 0\\ \star & \star & -L+N_{3i}^TN_{3i}\end{bmatrix},$$

$$\Gamma_{11}=P_iA_i+A_i^TP_i+N_{1i}^TN_{1i}+\lambda_iQ_{1i}+2D_i+G+ULU+\sum_{j=1}^{N}q_{ij}E_j^TP_j+P_i^2+3P_iM_iM_i^TP_i,$$

$$\Gamma_{22}=-(1-\rho_1)F_1+E_{1i}^T\sum_{j=1}^{N}q_{ij}P_jE_{1i}+\lambda_iR_{2i}.$$

By the condition (5) and the Schur complement lemma, we have $\Pi_i < 0$. Therefore, there must exist a scalar $\alpha_i > 0 (i \in S)$ such that $\Pi_i + \alpha_i I \leq 0$. Setting $\alpha = \min_{i \in S} \alpha_i$, it is clear that $\alpha > 0$. This fact together with (15) yields

$$\begin{aligned} \mathbb{E}\mathscr{L}V(t,x(t),i) &\leq \mathbb{E}\zeta^T(t)\Pi_i\zeta(t) \\ &\leq -\alpha_i\mathbb{E}|x(t;\xi)|^2 \qquad (16) \\ &\leq -\alpha\mathbb{E}|x(t;\xi)|^2 . \end{aligned}$$

Applying the Dynkin formula and from (16), it follows that

$$\begin{aligned} &\mathbb{E}V(t,x(t),i) - \mathbb{E}V(0,x(0),r(0)) \\ &= \int_0^t \mathbb{E}\mathscr{L}V(s,x(s),r(s))ds \\ &\leq -\alpha \int_0^t \mathbb{E}|x(s)|^2\, ds, \end{aligned}$$

and so

$$\begin{aligned} \int_0^t \mathbb{E}|x(s)|^2\, ds &\leq -\frac{1}{\alpha}\mathbb{E}V(t,x(t),i) \\ &\quad + \frac{1}{\alpha}\mathbb{E}V(0,x(0),r(0)) \\ &\leq \frac{1}{\alpha}\mathbb{E}V(0,x(0),r(0)), \end{aligned}$$

which implies that the trivial solution of (2) is robustly asymptotic stability in the mean square. This completes the proof.

Remark 1: Obviously, the criteria given in [1,3,7-11,15-20,37] fail in Theorem 1 since they ignored the nonlinear term and the singular term. Actually, if using the linear term to replace the nonlinear term and considering the singular term one can get a corollary, which generalizes and improves the corresponding results obtained in [1,3,7-11,15-20,37].

Remark 2: The criteria obtained in [2,5] do not hold in Theorem 1 since they did not consider the mixed time delays and the singular term, and so Theorem 1 extends and improves those given in [2,5].

Remark 3: Since the authors in [14,37-39] did not consider time delays, whereas unknown parameters were removed in [14,38,39] and noise perturbations were not considered in [38,39], the criteria obtained in [14,37-39] do not hold in Theorem 1.

Remark 4: The criteria obtained in [11,12,30,32-35,39] do not hold in Theorem 1 since they ignored noise perturbations and did not consider Markov jump parameters.

Setting $\Delta A(r(t)) = \Delta B(r(t)) = \Delta C(r(t)) \equiv 0$, the system (2) is reduced to the system (1). Thus, by Theorem 1 we obtain the following result.

Corollary 1: Suppose that Assumptions 1-4 hold. If there exist positive scalars $\lambda_i\,(i \in S)$, a positive diagonal matrix $L = diag\{l_1, l_2, ..., l_n\}$, and positive definite matrices $G, D_i, P_i\,(i \in S)$ such that the following conditions hold for all $i \in S$,

$$0 \leq E_i^T P_i = P_i^T E_i \leq \lambda_i I, \qquad (17)$$

$$\begin{bmatrix} \Gamma_{11} & P_iB_i & P_iC_i & P_i & P_iM_i \\ \star & \Gamma_{22} & 0 & 0 & 0 \\ \star & \star & -L & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -\frac{1}{3}I \end{bmatrix} < 0, \qquad (18)$$

where

$$\begin{aligned} \Gamma_{11} &= P_iA_i + A_i^TP_i + \lambda_iQ_{1i} + 2D_i \\ &\quad + G + ULU + \sum_{j=1}^N q_{ij}E_j^TP_j, \\ \Gamma_{22} &= -G + 2R_{2i}^TR_{2i} + \lambda_iQ_{2i}, \end{aligned}$$

then the trivial solution of (1) is robustly asymptotically stable with the feedback controller gain $H_i = P_i^{-1}D_i$.

Theorem 1 and Corollary 1 provide two delay-independent stabilization criteria. Generally speaking, delay-independent stabilization criteria are more conservative than delay-dependent stabilization criteria when the delay is small. Next, we try to obtain some new delay-dependent stabilization conditions for the system (2) based on the linear feedback controller $U(t,i) = H_ix(t)$.

Theorem 2: Suppose that Assumptions 1-4 hold. If there exist positive scalars $\lambda_i\,(i \in S)$, a positive diagonal matrix $L = diag\{l_1, l_2, ..., l_n\}$, and positive definite matrices $G, K, D_i, P_i\,(i \in S)$ such that the following conditions hold for all $i \in S$,

$$0 \leq E_i^T P_i = P_i^T E_i \leq \lambda_i I, \qquad (19)$$

$$\begin{bmatrix} \Gamma_{11} & P_iB_i & P_iC_i & P_i & P_iM_i \\ \star & \Gamma_{22} & 0 & 0 & 0 \\ \star & \star & -\tau L + N_{3i}^TN_{3i} & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -\frac{1}{3}I \end{bmatrix} < 0, \qquad (20)$$

where

$$\begin{aligned} \Gamma_{11} &= P_iA_i + A_i^TP_i + N_{1i}^TN_{1i} + \lambda_iQ_{1i} + 2D_i \\ &\quad + \tau G + \tau K + \tau ULU + \sum_{j=1}^N q_{ij}E_j^TP_j, \\ \Gamma_{22} &= -\tau G + 2R_{2i}^TR_{2i} + N_{2i}^TN_{2i} + \lambda_iQ_{2i}, \end{aligned}$$

then the trivial solution of (2) is robustly asymptotically stable with the feedback controller gain $H_i = P_i^{-1}D_i$.

Proof: Let us consider the following Lyapunov-Krasovskii functional:

$$V(t,x(t),i)=x^T(t)E_i^TP_ix(t)+\tau\int_{t-\tau}^{t}x^T(s)Gx(s)ds$$
$$+\int_{-\tau}^{0}d\theta\int_{t+\theta}^{t}x^T(s)Kx(s)ds$$
$$+\tau\sum_{j=1}^{n}l_j\int_0^{\infty}R_j(\theta)\int_{t-\theta}^{t}h_j^2(x_j(s))dsd\theta.$$

The rest is similar to the proof of Theorem 1, and so we omit it. This completes the proof.

Remark 5: As discussed in Remarks 1-4, the criteria obtained in [1-3,5,7-12,14-20,30,32-34,37,39] do not hold in Theorem 2, and Theorem 2 extends and improves those given in [1-3,5,7-12,14-20,30,32-34,37,39].

Letting $\Delta A(r(t))=\Delta B(r(t))=\Delta C(r(t))\equiv 0$ in Theorem 2, we can obtain the following results.

Corollary 2: Suppose that Assumptions 1-4 hold. If there exist positive scalars $\lambda_i\,(i\in S)$, a positive diagonal matrix. $L=diag\{l_1,l_2,...,l_n\}$, and positive definite matrices $G,D_i,P_i\,(i\in S)$ such that the following conditions hold for all $i\in S$,

$$0\le E_i^TP_i=P_i^TE_i\le\lambda_iI, \tag{21}$$

$$\begin{bmatrix}\Gamma_{11} & P_iB_i & P_iC_i & P_i & P_iM_i\\ \star & \Gamma_{22} & 0 & 0 & 0\\ \star & \star & -\tau L & 0 & 0\\ \star & \star & \star & -I & 0\\ \star & \star & \star & \star & -\frac{1}{3}I\end{bmatrix}<0, \tag{22}$$

where

$$\Gamma_{11}=P_iA_i+A_i^TP_i+\lambda_iQ_{1i}+2D_i+\tau G$$
$$+\tau H+\tau ULU+\sum_{j=1}^{N}q_{ij}E_j^TP_j,$$
$$\Gamma_{22}=-\tau G+2R_{2i}^TR_{2i}+\lambda_iQ_{2i},$$

then the trivial solution of (1) is robustly asymptotically stable with the feedback controller gain $H_i=P_i^{-1}D_i$.

4. ILLUSTRATIVE EXAMPLES

In this section, two numerical examples and their simulations are given to illustrate the effectiveness of the obtained results.

Example 1: Consider a two dimensional stochastically singular nonlinear jump systems with unknown parameters and continuously distributed delays:

$$E_idx(t)=\{[A_i+\Delta A(r(t))]x(t)+[B_i+\Delta B(r(t))]$$
$$\times x(t-0.2)+[C_i+\Delta C(r(t))]$$
$$\times\int_0^3\frac{e^{-s}}{1-e^{-3}}h(x(t-s))ds+H_ix(t)) \tag{23}$$
$$+f(t,x(t),x(t-0.2)),i)\}dt$$
$$+g(t,x(t),x(t-0.2),i)dw(t),$$

where $x(t)=(x_1(t),x_2(t))^T$, $w(t)$ is a two dimensional Brownian motion, and $r(t)$ is a right-continuous Markov chain taking values in $S=\{1,2\}$ with generator

$$Q=\begin{bmatrix}-11 & 11\\ 8 & -8\end{bmatrix}.$$

Take

$$h(x(t))=0.05(|x(t)+1|-|x(t)-1|),$$
$$f(x(t),x(t-0.2),t,i)$$
$$=0.1[\sin(x(t))+\sin(x(t-0.2))](i=1,2),$$
$$g(x(t),x(t-0.2),t,1)$$
$$=\begin{pmatrix}0.6x_1(t) & 0.6x_2(t-0.2)\\ 0.3(x_1(t)+x_1(t-0.2)) & 0.1(x_1(t)+x_2(t-0.2))\end{pmatrix},$$
$$g(x(t),x(t-0.2),t,2)$$
$$=\begin{pmatrix}0.8x_1(t-0.2) & 0.3x_2(t)\\ 0.6x_2(t) & 0.1(x_2(t)+x_1(t-0.2))\end{pmatrix}.$$

It is easy to check that the system (23) satisfies Assumptions 1-4. Other parameters of the system (23) are given as follows:

$$A_1=\begin{bmatrix}-2.2 & 0\\ 0 & -1.5\end{bmatrix},\ B_1=\begin{bmatrix}0.2 & -0.1\\ 0.1 & -0.2\end{bmatrix},$$
$$C_1=\begin{bmatrix}-0.5 & 0.2\\ -0.3 & 0.3\end{bmatrix},\ E_1=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix},\ A_2=\begin{bmatrix}-2.1 & 0\\ 0 & -2.2\end{bmatrix},$$
$$B_2=\begin{bmatrix}-0.3 & 0.2\\ -0.1 & 0.5\end{bmatrix},\ C_2=\begin{bmatrix}0.2 & 0.1\\ -0.1 & -0.2\end{bmatrix},\ E_2=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix},$$
$$M=\begin{bmatrix}-0.5 & 0.2\\ -0.1 & -0.4\end{bmatrix},\ N=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$
$$\Delta A(t)=\Delta B(t)=\Delta C(t)=M\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix}N.$$

Proposition 1: By using the Matlab LMI toolbox, we can get the following feasible solution for the LMIs (5)-(6):

$$L=10^4\times\begin{bmatrix}1.2047 & 0\\ 0 & 1.2047\end{bmatrix},$$
$$G=10^3\times\begin{bmatrix}3.6602 & 2.0426\\ 2.0426 & 8.9552\end{bmatrix},$$
$$P_1=10^3\times\begin{bmatrix}1.9540 & 1.9540\\ 1.9540 & 8.6427\end{bmatrix},$$
$$P_2=10^3\times\begin{bmatrix}1.3089 & 1.3089\\ 1.3089 & 7.0888\end{bmatrix},$$
$$D_1=10^3\times\begin{bmatrix}2.5753 & 2.2324\\ 2.2324 & 2.1979\end{bmatrix},$$
$$D_2=\begin{bmatrix}128.4549 & 38.2899\\ 38.2899 & 79.2501\end{bmatrix},$$
$$\lambda_1=439.7213,\quad \lambda_2=107.8501,$$

and the feedback gains are as follows

$$H_1 = \begin{bmatrix} 1.3693 & 1.1476 \\ -0.0513 & -0.0052 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0.1137 & 0.0222 \\ -0.0156 & 0.0071 \end{bmatrix}.$$

Therefore, by Theorem 1 we see that the system (23) is robustly asymptotically stable.

Proposition 2: By using the Matlab LMI toolbox, we can get the following feasible solution for the LMIs (19)-(20):

$$L = 10^3 \times \begin{bmatrix} 1.6953 & 0 \\ 0 & 1.6953 \end{bmatrix},$$

$$G = \begin{bmatrix} 305.0049 & 21.8400 \\ 21.8400 & 547.0824 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 136.5653 & 136.5653 \\ 136.5653 & 426.2974 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 46.1877 & 46.1877 \\ 46.1877 & 264.0816 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 398.3320 & 332.3297 \\ 332.3297 & 338.5776 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 14.8937 & 9.3731 \\ 9.3731 & 33.7326 \end{bmatrix},$$

$$K = \begin{bmatrix} 116.5477 & 66.5601 \\ 66.5601 & 196.1952 \end{bmatrix},$$

$$\lambda_1 = 38.3468, \quad \lambda_2 = 19.2070,$$

and the feedback gains are as follows

$$H_1 = \begin{bmatrix} 3.1446 & 2.4119 \\ -0.2278 & 0.0216 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0.3478 & 0.0911 \\ -0.0253 & 0.1118 \end{bmatrix}.$$

Therefore, by Theorem 2 we see that the system (23) is robustly asymptotically stable.

Example 2: In Example 1, if we take

$$A_1 = \begin{bmatrix} -2.2 & 0 \\ 0 & -1.2 \end{bmatrix}$$

and the other parameters do not change, then we can apply Theorem 2 to obtain the following result.

Proposition 3: By using the Matlab LMI toolbox, we can get the following feasible solution for the LMIs (19)-(20):

$$L = 10^3 \times \begin{bmatrix} 4.4297 & 0 \\ 0 & 4.4297 \end{bmatrix},$$

$$G = 10^3 \times \begin{bmatrix} 0.7691 & 0.1160 \\ 0.1160 & 1.6188 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 154.9369 & 154.9369 \\ 154.9369 & 898.6387 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 108.3664 & 108.3664 \\ 108.3664 & 625.5130 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 219.1967 & 173.7457 \\ 173.7457 & 229.9655 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 37.3560 & 21.4856 \\ 21.4856 & 32.2606 \end{bmatrix},$$

$$K = \begin{bmatrix} 266.3663 & 152.5461 \\ 152.5461 & 240.5075 \end{bmatrix},$$

$$\lambda_1 = 89.0625, \quad \lambda_2 = 25.8461,$$

and the feedback gains are as follows

$$H_1 = \begin{bmatrix} 1.4759 & 1.0458 \\ -0.0611 & 0.0756 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0.3754 & 0.1774 \\ -0.0307 & 0.0208 \end{bmatrix}.$$

Thus, by Theorem 2 we see that the system (23) is robustly asymptotically stable.

Remark 6: It is worth pointing out that the criterion obtained in Theorem 1 fails in Example 2. Therefore, Example 2 has shown that delay-dependent stabilization criteria are less conservative than delay-independent criteria especially when the delay is small.

Remark 7: As discussed in Remarks 1-5, the criteria obtained in [1-3,5,7-12,14-20,30,32-34,37,39] fail in Examples 1 and 2.

5. CONCLUDING REMARKS

In this paper, we have studied the problem of robustly asymptotic stabilization for a class of stochastically nonlinear singular jump systems with unknown parameters and continuously distributed delays. Based on the Lyapunov-Krasovskii functional and stochastic analysis theory as well as a state feedback control technique, some new delay-independent and delay-dependent conditions are derived to guarantee the robustly asymptotic stability of the trivial solution or zero solution in the mean square. A key feature of this paper is that singular, nonlinear, noise perturbations, unknown parameters and continuously distributed delays are all considered. It is worth pointing out that nonlinear and time delays have seldom been considered in singular MJSs, let alone be continuously distributed delays. Actually, continuously distributed delays have never been used to investigate the asymptotic stability in stochastic singular MJSs owing to their complexities. Therefore, the results obtained in this paper are less conservatism and generalize those given in the previous literature. Moreover, two numerical examples are provided to illustrate the effectiveness of the obtained results. In particular, the second example has shown that delay-dependent stabilization criteria are less conservative than delay-independent criteria.

REFERENCES

[1] E. K. Boukas and H. Yang, "Exponential stabilizability of stochastic systems with Markovian jumping parameters," *Automatica*, vol. 35, no. 8, pp. 1437-1441, 1999.

[2] Y. He, Y. Zhang, M. Wu, and J. She, "Improved exponential stability for stochastic Markovian jump systems with nonlinearity and time-varying delay," *Int. J. Robust Nonlinear Control*, vol. 20, no. 1, pp. 16-26, 2010.

[3] X. Liao and X. Mao, "Exponential stability and instability of stochastic neural networks," *Stoch. Anal. Appl.*, vol. 14, no. 2, pp. 165-185, 1996.

[4] G. Wei, Z. Wang, and H. Shu, "Nonlinear $\mathscr{H}_\infty$ control of stochastic time-delay systems with Markovian switching," *Chaos, Solitons Fractals*, vol. 35, no. 3, pp. 442-451, 2008.

[5] D. Yue and Q. Han, "Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity and Markovian switching," *IEEE Trans. Automat. Control*, vol. 50, no. 2, pp. 217-222, 2005.

[6] E. K. Boukas and Z. Liu, "Robust stability and stability of Markov jump linear uncertain systems with mode dependent time delays," *J. Optim. Theory Appl.*, vol. 109, no. 3, pp. 587-600, 2001.

[7] P. Shi and E. K. Boukas, "$\mathscr{H}_\infty$ control for Markovian jumping linear systems with parametric uncertainty," *J. Optim. Theory Appl.*, vol. 95, no. 1, pp. 75-99, 1997.

[8] C. E. de Souza, "Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems," *IEEE Trans. Automat. Control*, vol. 51, no. 5, pp. 836-841, 2006.

[9] Y. Wang and H. Zhang, "$\mathscr{H}_\infty$ control for uncertain Markovian jump systems with mode-dependent mixed delays," *Progress in Natural Science*, vol. 18, no. 3, pp. 309-314, 2008.

[10] Z. Wang, H. Qiao, and K. J. Burnham, "On stabilization of bilinear uncertain time-delay systems with Markovian jumping parameters," *IEEE Trans. Automat. Control*, vol. 47, no. 4, pp. 640-646, 2002.

[11] J. Wu, T. Chen, and L. Wang, "Delay-dependent robust stability and $\mathscr{H}_\infty$ control for jump linear systems with delays," *Syst. Control Lett.*, vol. 55, no. 11, pp. 939-948, 2006.

[12] S. Xu, T. Chen, and J. Lam, "Robust $\mathscr{H}_\infty$ filtering for uncertain Markovian jump systems with mode-dependent time delays," *IEEE Trans. Automat. Control*, vol. 48, no. 5, pp. 900-907, 2003.

[13] Q. Zhu, F. Xi, and X. Li, "Robust exponential stability of stochastically nonlinear jump systems with mixed time delays," *J. Optim. Theory Appl.*, vol. 154, no. 1, pp 154-174, 2012.

[14] E. K. Boukas, "Stabilization of stochastic singular nonlinear hybrid systems," *Nonlinear Anal.*, vol. 64, no. 2, pp. 217-228, 2006.

[15] Y. Cao, J. Lam, and L. Hu, "Delay-dependent stochastic stability and $\mathscr{H}_\infty$ analysis for time-delay systems with Markovian jumping parameters," *J. Franklin Inst.*, vol. 340, no. 6-7, pp. 423-434, 2003.

[16] Z. Fei, H. Gao, and P. Shi, "New results on stabilization of Markovian jump systems with time delay," *Automatica*, vol. 45, no. 10, pp. 2300-2306, 2009.

[17] D. P. de Farias, J. C. Geromel, J. B. R. do Val, and O. L. V. Costa, "Output feedback control of Markov jump linear systems in continuous-time," *IEEE Trans. Automat. Control*, vol. 45, no. 5, pp, 944-949, 2000.

[18] M. Mariton and P. Bertrand, "Output feedback for a class of linear systems with stochastic jump parameters," *IEEE Trans. Autom. Control*, vol. 30, no. 9, pp. 898-900, 1985.

[19] N. N. Krasovskii and E. A. Lidskii, "Analysis design of controller in systems with random attributes-Part 1," *Automat. Remote Control*, vol. 22, no. 9, pp. 1021-1025, 1961.

[20] N. N. Krasovskii, E. A. Lidskii, "Analysis design of controller in systems with random attributes-Part 2," *Automat. Remote Control*, vol. 22, no. 9, pp. 1141-1146, 1961.

[21] U. Ascher and L. Petzold, "The numerical solution of delay differential algebraic equations of retarded and neutral type," *SIAM J. Numer. Anal.*, vol. 32, no. 5, pp. 635-1657, 1995.

[22] L. Dai, *Singular Control Systems*, Springer-Verlag, Berlin, Germany, 1989.

[23] C. Fang and F. Chang, "Analysis of stability robustness for generalized state-space systems with structured perturbations," *Syst. Control Lett.*, vol. 21, no. 2, pp. 109-114, 1993.

[24] E. Fridman, "$\mathscr{H}_\infty$ control of linear state-delay descriptor systems: an LMI approach," *Linear Algebra Appl.*, vol. 351-352, pp. 271-302, 2002.

[25] F. L. Lewis, "A survey of linear singular systems," *Circuits Syst. Signal Processing*, vol. 5, no. 1, pp. 3-36, 1986.

[26] I. Masubuchi, Y. Kamitane, A. Ohara, and N. Suda, "$\mathscr{H}_\infty$ control for descriptor systems: a matrix inequalities approach," *Automatica*, vol. 33, no. 4, pp. 669-673, 1997.

[27] E. Uezato and M. Ikeda, "Strict LMI conditions for stability, robust stabilization, and $\mathscr{H}_\infty$ control of descriptor systems," *Proc. of the 38th IEEE Conf. Decision Control*, Phoenix, AZ, USA, pp. 4092-4097, 1999.

[28] W. Zhu and L. Petzold, "Asymptotic stability of linear delay-differential-algebraic equations and numerical methods," *Applied Numerical Mathematics*, vol. 24, no. 2-3, pp. 247-264, 1997.

[29] S. Xu, J. Lam, and Y. Zou, "$\mathscr{H}_\infty$ filtering for singular systems," *IEEE Trans. Automat. Control*, vol. 48, no. 12, pp. 2217-2222, 2003.

[30] S. Xu and J. Lam, "Reduced-order $\mathscr{H}_\infty$ filtering for singular systems," *Sys. Control Lett.*, vol. 56, no. 1, pp. 48-57, 2007.

[31] Y. Xia and Y. Jia, "$\mathscr{H}_\infty$ output feedback control of singular systems with time delay," *Control Theory and Applications*, vol. 20, pp. 323-328, 2003.

[32] S. Xu, B. Song, J. Lu, and J. Lam, "Robust stability of uncertain discrete-time singular fuzzy systems," *Fuzzy Sets and Systems*, vol. 158, no. 20, pp. 2306-

2316, 2007.
[33] G. Lu and D. W. C. Ho, "Generalized quadratic stability for continuous-time singular systems with nonlinear perturbation," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 818-823, 2006.
[34] M. Fang, "Delay-dependent robust $\mathscr{H}_\infty$ control for uncertain singular systems with state delay," *Acta Automatica Sinica*, vol. 35, no. 1, pp. 65-70, 2009.
[35] H. Wang, A. Xue, and R. Lu, "Absolute stability criteria for a class of nonlinear singular systems with time delay," *Nonlinear Anal.*, TMA, vol. 70, no. 2, pp. 621-630, 2009.
[36] Z. Wu, H. Su, and J. Chu, "Improved results on delay-dependent $\mathscr{H}_\infty$ control for singular time-delay systems," *Acta Automatica Sinica*, vol. 35, no. 8, pp. 1101-1106, 2009.
[37] E. K. Boukas, "On robust stability of singular systems with random abrupt changes," *Nonlinear Anal.*, vol. 63, no. 3, pp. 301-310, 2005.
[38] Y. Xia, J. Zhang, and E. K. Boukas, "Control for discrete singular hybrid systems," *Automatica*, vol. 44, no. 10, pp. 2635-2641, 2008.
[39] Y. Xia, E. K. Boukas, P. Shi, and J. Zhang, "Stability and stabilization of continuous-time singular hybrid systems," *Automatica*, vol. 45, no. 6, pp. 1504-1509, 2009.
[40] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, PA 1994.

Quanxin Zhu received his Ph.D. degree from Sun Yat-Sen University, Guangzhou, China, in 2005. He is currently a Professor with Nanjing Normal University. He is the author or co-author of more than 50 research papers, a member of IEEE and a reviewer of Mathematical Reviews, Zentralblatt Math. His current research interests include random processes, stochastic controls, stochastic differential equations, stochastic partial differential equations, Markovian jump systems, and stochastic neural networks. Prof. Zhu is an Associate Editor of Transnational Journal of Mathematical Analysis and Applications, and a reviewer of more than 30 other journals.