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On the State Observer Based Stabilization of Takagi-Sugeno Systems with Immeasurable Premise Variables

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Abstract: This paper presents two approaches of observer based stabilization for Takagi-Sugeno (T-S) systems with immeasurable premise variables in continuous time case. These approaches are based on the description of the state estimation error by a T-S model. To design the observer based stabilization law, the concept of PDC (Parallel Distributed Compensation) is employed, the sufficient stabilization conditions are proved and expressed in the form of Linear Matrix Inequalities (LMI). The performances of these approaches are tested by simulation for an illustrate example and a physical system representing a two-link robot.

Keywords: Immeasurable premise variables, LMI, PDC, T-S control, T-S observer.

1. NOMENCLATURE

In this paper, we denote the matrix identity by I, the positive definite symmetric matrix X by $X > 0$ (the positive semi definite symmetric matrix X by $X \geq 0$) and the transpose of X by X^T. We pose also:

$$\sum_{i,j}^{n} x_i x_j = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j, \quad \sum_{i,j,k}^{n} x_i x_j x_k = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} x_i x_j x_k.$$

2. INTRODUCTION

Most plants in the industries have severe nonlinearities, which make difficult to study such systems. In order to overcome this problem, various schemes have been developed in the last two decades [1-3], among them the Takagi-Sugeno (T-S) approach [4,5].

The T-S model approach consists to describe nonlinear or complex dynamic systems by means of interpolating the behaviour of several Linear Time Invariant (LTI) submodels. Each submodel contributes to the global model in particular subset of the operating space by a weighting or an activation function [5].

For few years, the trend of T-S control has been to develop some systematic design algorithms so as to guarantee the control performance and the system stability for the T-S model based controller [6-10].

Moreover, the knowledge of the state variables of such systems is necessary for the implementation of the industrial feedback control or the system supervision and diagnosis. However, all or some of the process state variables may be immeasurable. A solution to overcome this non-accessibility of the state variables consists in synthesizing a state observer [11-15].

Hence, the state observer based stabilization of non linear system becomes a focus of many researches in recent years [12-17]. The main key used for this goal is the direct Lyapunov approach based on different kinds of candidate functions is used as the quadratic function, the piecewise function or the polyquadratic function.

More particularly, a considerable interest has been paid for the stabilization by means of state observer and Lyapunov functions for nonlinear plants described by a T-S model.

However, the most published works about this subject consider that the weighting functions of T-S model depend on measurable premise variables. While, in most applications, like diagnosis and stabilization design, the weighting functions depend on the input, the immeasurable state variables or/and the output variables of the system. Therefore, it seems interesting to consider the case of weighting functions depending on unknown premise variables. This idea has been considered in few works [18,19], to synthesize a state observer. But it stills less developed for the stabilization problem [17].

The aim of our contribution is then to design an observer based stabilization law of T-S systems with immeasurable premise variables using a quadratic Lyapunov function. For this goal, we aim in this work to extend the results developed in [18] and [19] for the state observer to design a state observer based stabilization law. Thus, we consider the nonlinear systems represented by T-S models and we make use of the concept of parallel distributed compensation (PDC) [12,15,20] for the development of the proposed approaches.

This paper is organized as follows: Section 3 presents the structures of T-S model and T-S state observer with

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immeasurable premise variables. In section 4, we prove new conditions for T-S observer based stabilization with immeasurable premise variables. A numerical example and a two-link robot are considered in section 5 to check the stabilization performances of the developed approaches.

3. T-S SYSTEM AND OBSERVER DESCRIPTIONS

3.1. T-S System description

A continuous T-S model is based on the interpolation of several LTI local models as follows [5,12]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i\left(x(t)\right)\left(A_i x(t) + B_i u(t)\right), \\ y(t) = \sum_{i=1}^{n} \mu_i\left(x(t)\right) C_i x(t), \end{cases} \tag{1}$$

where n is the number of submodels, $x(t) \in \mathbb{R}^{\mathrm{p}}$ is the state vector, $y(t) \in \mathrm{R}^{\mathrm{l}}$ is the output vector, $u(t) \in \mathrm{R}^{\mathrm{m}}$ is the input vector and $\mu_i(x(t))$ is the activation function. $A_i \in \mathbb{R}^{\mathrm{p}\times\mathrm{p}}$, $B_i \in \mathbb{R}^{\mathrm{p}\times\mathrm{m}}$, $\mathrm{C}_i \in \mathbb{R}^{\mathrm{l}\times\mathrm{p}}$ are respectively the state matrix , the input matrix and the output matrix.

Different classes of models can be considered with respect to the choice of the premise variables and the type of the activation function.

Each linear consequent equation represented by $(A_i x(t) + B_i u(t))$ is called "subsystem" or "submodel".

The normalized activation function $\mu_i(x(t))$ corresponding to the i$^{\text{th}}$ submodel is such that

$$\begin{cases} \sum_{i=1}^{n} \mu_i(x(t)) = 1, \\ \mu_i(x(t)) \geq 0 \quad \forall\ i \in \{1,\ldots,n\}. \end{cases} \tag{2}$$

3.2. T-S observer description

For the state variables estimation of the T-S system (1), a Luenberger like observer is generally adopted. Such observer is described by the following system [12,19]

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{n} \mu_i\left(\hat{x}(t)\right)\left[A_i \hat{x}(t) + B_i u(t) + L_i\left(y(t) - \hat{y}(t)\right)\right], \\ \hat{y}(t) = \sum_{i=1}^{n} \mu_i\left(\hat{x}(t)\right) C_i \hat{x}(t), \end{cases} \tag{3}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ are respectively the estimated state vector and the corresponding output vector.

The normalized activation function $\mu_i(\hat{x}(t)),\ i = 1\ldots n$ corresponding to the ith observer of the ith submodel depending on the immeasurable premise variables considered here as the whole state vector $\hat{x}(t)$ such that

$$\begin{cases} \sum_{i=1}^{n} \mu_i(\hat{x}(t)) = 1, \\ \mu_i(\hat{x}(t)) \geq 0 \quad \forall\ i \in \{1,2,\ldots,n\}. \end{cases} \tag{4}$$

Let us definite the estimation error by

$$e(t) = x(t) - \hat{x}(t). \tag{5}$$

The T-S observer requires that the estimation error $e(t)$ converges to zero rapidly i.e., $e(t) \xrightarrow{\text{rapidly}} 0$ when t increases.

This T-S observer structure with immeasurable premise variables has been considered in few previous works. The complexity of the expression of the estimation error makes difficult to study the convergence. Hence, many works propose different approaches to study the convergence by considering the structure of the estimation error.

The studies of these approaches have been led to the observer design without considering its application in the stabilization problem.

Since the availability of the separation principle between observation and control is not verified in the case of the immeasurable premise variables; it will be interesting to study the stabilization problem of T-S system provided with the observer (3).

Many works have formulated sufficient conditions of stabilization based on the observer state with measurable premise variables; one can cite [14], only few approaches have been developed in the case of the immeasurable premise variables to establish stabilization conditions in terms of linear matrix inequalities (LMI) [17].

4. T-S OBSERVER BASED STABILIZATION DESIGN

To simplify the notation, we denote $\mu_i(\hat{x}(t)) = \hat{\mu}_i$, $\mu_i(x(t)) = \mu_i$ and the time symbol t is omitted.

When the estimated state $\hat{x}(t)$ is available, the control law with the PDC technique can be written as follows:

$$u(t) = -\sum_{i=1}^{n} \hat{\mu}_i K_i \hat{x}(t). \tag{6}$$

So, the system (1) becomes

$$\begin{cases} \dot{x}(t) = \sum_{i,j}^{n} \mu_i \hat{\mu}_j \left[\left(A_i - B_i K_j\right) x(t) + B_i K_j e(t)\right], \\ y(t) = \sum_{i=1}^{n} \mu_i C_i x(t). \end{cases} \tag{7}$$

Considering the system (7) and the estimation error (5), an augmented system is obtained as follows [17]:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{e}(t) \end{pmatrix} = \sum_{i,j,k}^{n} \mu_i \hat{\mu}_j \hat{\mu}_k \begin{bmatrix} A_i - B_i K_j & B_i K_j \\ A_{ij} - L_k C_{ij} - B_{ij} K_k & A_j - L_j C_k + B_{ij} K_k \end{bmatrix} \begin{pmatrix} x(t) \\ e(t) \end{pmatrix}, \tag{8}$$

where $A_{ij} = A_i - A_j$, $B_{ij} = B_i - B_j$ and $C_{ij} = C_i - C_j$ for $i, j = 1,2,\ldots,n$.

In [17], nonlinear stabilization conditions of the system (8) have been derived. To overcome the non linearity problem, we have to choose several scalars, which lead to a linear problem but with a great number of Linear Matrix Inequalities (LMI).

In the next section, we present improved observer-based stabilization conditions for the studied system (1) provided with the control law (6), using the Lyapunov direct method that leads to the resolution of a minimum number of LMI compared with the results in [17].

4.1. T-S observer based stabilization conditions using the estimation error with unstructured perturbation

4.1.1 Augmented system description

This development is based on the result proved in [19], where the estimation error (5) is considered with a bounded uncertainty and $C_i = C$. Developing the time derivative of the estimation error, one obtains

$$\dot{e} = \sum_{i=1}^{n} \hat{\mu}_i \left(A_i - L_i C \right) e + \Delta\left(x, \hat{x}, u\right), \tag{9}$$

with

$$\Delta\left(x, \hat{x}, u\right) = \sum_{i=1}^{n} \left(\mu_i - \hat{\mu}_i \right)\left(A_i x + B_i u \right). \tag{10}$$

Note that if $e(t) \to 0$ then $\Delta(x, \hat{x}, u) \to 0$. So, the term $\Delta(x, \hat{x}, u)$ acts like an unstructured perturbation that is assumed to be bounded as follows [19,21]:

$$\left\| \Delta\left(x, \hat{x}, u\right) \right\| \le \beta \| e \|, \;\; \beta > 0. \tag{11}$$

Considering the system (7) and the estimation error (9), the augmented system can be written as follows:

$$\dot{x}_a = \sum_{i,j}^{n} \mu_i \hat{\mu}_j \left(\begin{bmatrix} G_{ij} & B_i K_j \\ 0 & R_j \end{bmatrix} x_a \right) + \begin{bmatrix} 0 \\ \Delta \end{bmatrix}, \tag{12}$$

where

$$x_a = \begin{pmatrix} x \\ e \end{pmatrix}, \;\; R_j = A_j - L_j C, \;\; G_{ij} = A_i - B_i K_j. \tag{13}$$

4.1.2 Improved stabilization conditions

The system (12) can be also written as follows:

$$\dot{x}_a = \sum_{i,j}^{n} \mu_i \hat{\mu}_j H_{ij} x_a + \begin{bmatrix} 0 \\ \Delta \end{bmatrix}, \tag{14}$$

where

$$H_{ij} = \begin{bmatrix} G_{ij} & B_i K_j \\ 0 & R_j \end{bmatrix} \;\; \text{for} \;\; i, j = 1, 2, ..., n. \tag{15}$$

The stabilization conditions for the system (14) are given in Theorem 1.

Theorem 1: The system (14) is globally asymptotically stable, if there exist two positive definite symmetric matrices X and P_2, scalars μ, θ, α and τ, matrices F_i and W_i for $i = 1, 2, ..., n$ such that:

$$\begin{cases} P_2 > 0, X > 0, \theta > 0, \alpha > 0, \mu > 0, \tau > 0, \\ \begin{bmatrix} X A_i^T - F_j^T B_i^T + A_i X - B_i F_j & B_i F_j \\ F_j^T B_i^T & -2\mu X + \mu^2 \theta^{-1} I \end{bmatrix} < 0, \\ \begin{bmatrix} A_j^T P_2 + P_2 A_j - C^T W_j^T - W_j C + (\tau + \theta) I & P_2 \\ P_2 & -\alpha I \end{bmatrix} < 0, \\ i, j = 1, 2, ..., n. \end{cases} \tag{16}$$

Proof: To prove the theorem 1, we make use of the following lemma:

Lemma 1 [1]**:** For any matrices A and B with appropriated dimensions, the following inequality holds for any positive scalar ε

$$AB^T + BA^T \le \varepsilon AA^T + \varepsilon^{-1} BB^T. \tag{17}$$

Considering the following Lyapunov quadratic function

$$V = x_a{}^T P x_a, \tag{18}$$

where P is a positive symmetric matrix and has the following form

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \;\; P_1 > 0, \;\; P_2 > 0, \tag{19}$$

and x_a is defined in (13).

The time derivative developing of the function V applied to the system (14) gives:

$$\begin{aligned} \dot{V} &= \dot{x}_a{}^T P x_a + x_a{}^T P \dot{x}_a \\ &= \sum_{i,j}^{n} \mu_i \hat{\mu}_j x_a{}^T \left(H_{ij}^T P + P H_{ij} \right) x_a \\ &\quad + \begin{pmatrix} 0 \\ \Delta \end{pmatrix}^T P x_a + x_a{}^T P \begin{pmatrix} 0 \\ \Delta \end{pmatrix}. \end{aligned} \tag{20}$$

Considering (15) and (19) and developing (20), one obtains:

$$\begin{aligned} \dot{V} = \sum_{i,j}^{n} \mu_i \hat{\mu}_j \Big[& x^T \left(G_{ij}^T P_1 + P_1 G_{ij} \right) x + e^T \left(B_i K_j \right)^T P_1 x \\ & + x^T P_1 B_i K_j e + e^T \left(R_j^T P_2 + P_2 R_j \right) e \Big] \\ & + \Delta^T P_2 e + e^T P_2 \Delta. \end{aligned} \tag{21}$$

Using Lemma 1 for the scalars $\alpha > 0$ and $\theta > 0$ and the condition (11), one can write:

$$\begin{cases} \Delta^T P_2 e + e^T P_2 \Delta \le \alpha \Delta^T \Delta + \alpha^{-1} e^T P_2^2 e \\ \quad \le \alpha \beta^2 e^T e + \alpha^{-1} e^T P_2^2 e, \\ e^T (B_i K_j)^T P_1 x + x^T P_1 B_i K_j e \\ \quad \le \theta e^T e + \theta^{-1} x^T P_1 B_i K_j (B_i K_j)^T P_1 x. \end{cases} \tag{22}$$

From (21) and the inequalities (22), one can deduce:

$$\dot{V} \le \sum_{i,j}^{n} \mu_i \hat{\mu}_j \Big[x^T (G_{ij}^T P_1 + P_1 G_{ij} + \theta^{-1} P_1 B_i K_j (B_i K_j)^T P_1) x$$
$$+ e^T (R_j^T P_2 + P_2 R_j + (\alpha\beta^2 + \theta) I + \alpha^{-1} P_2^2) e \Big]. \quad (23)$$

To insure $\dot{V} < 0$, it is sufficient to have the following inequalities satisfied

$$G_{ij}^T P_1 + P_1 G_{ij} + \theta^{-1} P_1 B_i K_j (B_i K_j)^T P_1 < 0, \quad (24a)$$

$$R_j^T P_2 + P_2 R_j + (\alpha\beta^2 + \theta) I + \alpha^{-1} P_2^2 < 0. \quad (24b)$$

Using the Schur complement [22] for the inequalities (24) and the congruence with the full rank diagonal block matrix $diag[P_1^{-1} \quad P_1^{-1}]$ applied to (24a) leads to

$$\begin{bmatrix} P_1^{-1} A_i^T - P_1^{-1} K_j^T B_i^T + A_i P_1^{-1} - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -\theta (P_1^{-1})^2 \end{bmatrix} < 0. \quad (25)$$

Applying (17) with $A = \mu P^{-1}$, $B = I$ and $\varepsilon = \mu^{-2}\theta$ for a scalar $\mu > 0$ we get

$$P_1^{-1}(-\theta) P_1^{-1} \le -2\mu P_1^{-1} + \mu^2 \theta^{-1} I. \quad (26)$$

Using the property (26), the conditions (25) are verified if the following inequalities are satisfied

$$\begin{bmatrix} P_1^{-1} A_i^T + A_i P_1^{-1} - P_1^{-1} K_j^T B_i^T - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -2\mu P_1^{-1} + \mu^2 \theta^{-1} I \end{bmatrix} < 0. \quad (27)$$

Developing (24b) and considering (27), the stabilization conditions (24) of the system (14) can be written as follows:

$$\begin{cases} P_1 > 0, P_2 > 0, \alpha > 0, \beta > 0, \theta > 0, \mu > 0, \\ \begin{bmatrix} P_1^{-1} A_i^T + A_i P_1^{-1} - P_1^{-1} K_j^T B_i^T - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -2\mu P_1^{-1} + \mu^2 \theta^{-1} I \end{bmatrix} < 0, \\ \begin{bmatrix} A_j^T P_2 + P_2 A_j - C^T L_j^T P_2 - P_2 L_j C + (\alpha\beta^2 + \theta) I & P_2 \\ P_2 & -\alpha I \end{bmatrix} < 0, \\ i, j = 1, 2, \ldots, n. \end{cases} \quad (28)$$

Considering the following variables changes:

$$X = P_1^{-1}, \quad F_i = K_i P_1^{-1}, \quad W_i = P_2 L_i, \quad \tau = \alpha\beta^2, \quad (29)$$

and using the changes (29) in (28) , one obtains (16).

For fixed scalars μ and θ, the conditions (16) are solved in X, P_2, F_i, W_i ($i = 1,2,\ldots,n$) and the scalars $\alpha > 0$ and $\tau > 0$. So, the feedback gains, the observer gains and the bound of the observation error perturbation β are given by:

$$K_i = F_i X^{-1}, \quad L_i = P_2^{-1} W_i \quad \text{and} \quad \beta = \sqrt{\frac{\tau}{\alpha}}. \quad (30)$$

It is clear that the considered perturbation term on the estimation error depends on the input $u(t)$ and the state $x(t)$, so a large value of the input bound leads to a large value of the perturbation bound β. Then, in this case the LMI may be infeasible. To overcome these difficulties, another form for the state estimation error is proposed in the following section.

4.2. T-S observer based stabilization conditions using the bounded input

4.2.1 Augmented system description

We consider the system (1) with the control law (6) and the observer (3), the time derivative of the estimation error given by (5) where $C_i = C$ for $i = 1, 2, \ldots, n$ can be written as follows:

$$\dot{e} = \sum_{i=1}^{n} \big[A_i (\mu_i x - \hat{\mu}_i \hat{x}) + B_i u (\mu_i - \hat{\mu}_i) - \hat{\mu}_i L_i C e \big]. \quad (31)$$

To avoid the appearance of the sate bound in the expression of the estimation error, one can consider the following notation

$$A_0 = \frac{1}{n} \sum_{i=1}^{n} A_i \quad \text{and} \quad \overline{A_i} = A_i - A_0. \quad (32)$$

Then, equation (31) becomes

$$\dot{e} = \sum_{i=1}^{n} \Big[\big(A_0 + \overline{A_i} \big) (\mu_i x - \hat{\mu}_i \hat{x}) - \hat{\mu}_i L_i C e + B_i u (\mu_i - \hat{\mu}_i) \Big], \quad (33)$$

and the estimation error can be written as follows [18]:

$$\dot{e} = \sum_{i=1}^{n} \hat{\mu}_i \Psi_i e + \sum_{i=1}^{n} \big(B_i \Delta_i + \overline{A_i} \delta_i \big), \quad (34)$$

where

$$\Delta_i = (\mu_i - \hat{\mu}_i) u, \quad \delta_i = (\mu_i x - \hat{\mu}_i \hat{x}), \quad \Psi_i = (A_0 - L_i C). \quad (35)$$

Δ_i and δ_i verify the following hypothesis:

Hypothesis 1:

$$\begin{cases} \|u\| \le \eta, \\ \|\mu_i(x) x - \mu_i(\hat{x}) \hat{x}\| \le M_i \|x - \hat{x}\|, \\ \|\mu_i(x) - \mu_i(\hat{x})\| \le N_i \|x - \hat{x}\|, \end{cases} \quad (36)$$

M_i and N_i are the Lipchitz constants .

The calculus of the Lipchitz constant N_i can be led using the following elementary Taylor series development of $\mu_i(x)$:

$$\mu_i(x) = \mu_i(\hat{x}) + \int_{\hat{x}}^{x} \dot{\mu}_i(\alpha) d\alpha,$$
$$|\mu_i(x) - \mu_i(\hat{x})| \le \left| \int_{\hat{x}}^{x} \dot{\mu}_i(\alpha) d\alpha \right| \quad (37)$$
$$\le \int_{\hat{x}}^{x} |\dot{\mu}_i(\alpha)| d\alpha \le N_i \|x - \hat{x}\|,$$

where $N_i = \max_x \left|\dot{\mu}_i(x)\right|$.

In the same way, we determine the Lipchitz constant M_i using the function $f(x) = \mu_i(x)x$ where $M_i = \max_x \left|\dot{f}(x)\right|$.

Considering the systems (7) and (34), an augmented system can be written as the following form

$$\dot{x}_a = \sum_{i,j}^{n} \mu_i \hat{\mu}_j M_{ij} x_a + \sum_{i=1}^{n} \begin{bmatrix} 0 \\ B_i \Delta_i + \overline{A}_i \delta_i \end{bmatrix}, \tag{38}$$

where

$$M_{ij} = \begin{bmatrix} A_i - B_i K_j & B_i K_j \\ 0 & \Psi_j \end{bmatrix} \quad \text{for} \quad i, j = 1, 2, \ldots, n, \tag{39}$$

and x_a is defined in (13).

4.2.2 Improved stabilization conditions

The stabilization conditions for the system (38) are enounced in Theorem 2.

Theorem 2: The system (38) is globally asymptotically stable, if there exist two positive definite symmetric matrices X and P_2, scalars μ, λ, α, γ and τ, matrices F_i and W_i for $i = 1, 2, \ldots, n$ such that:

$$\begin{cases} X > 0, P_2 > 0, Q > 0, \alpha > 0, \lambda > 0, \mu > 0, \gamma > 0, \tau > 0, \\ A_0^T P_2 - C^T W_j^T + P_2 A_0 - W_j C < -Q, \\ \begin{bmatrix} X A_i^T - F_j^T B_i^T + A_i X - B_i F_j & B_i F_j \\ F_j^T B_i^T & -2\mu X + \mu^2 \lambda^{-1} I \end{bmatrix} < 0, \\ \begin{bmatrix} -\dfrac{Q}{n} + \left(\tau N_i^2 + \gamma M_i^2 + \dfrac{\lambda}{n} \right) I & P_2 \overline{A_i} & P_2 B_i \\ \overline{A_i}^T P_2 & -\gamma I & 0 \\ B_i^T P_2 & 0 & -\alpha \end{bmatrix} < 0, \\ i, j = 1, 2, \ldots, n. \end{cases} \tag{40}$$

Proof: Considering the Lyapunov quadratic function (18) with (19), then the development of the time derivative of the function V applied to the system (38) yields

$$\begin{aligned} \dot{V} = &\sum_{i,j}^{n} \mu_i \hat{\mu}_j \Big[x^T \left(\left(A_i - B_i K_j \right)^T P_1 + P_1 \left(A_i - B_i K_j \right) \right) x \\ &+ e^T \left(B_i K_j \right)^T P_1 x + x^T P_1 B_i K_j e + e^T \left(\Psi_j^T P_2 + P_2 \Psi_j \right) e \Big] \\ &+ \sum_{i=1}^{n} \left(B_i \Delta_i + \overline{A}_i \delta_i \right)^T P_2 e + e^T P_2 \left(B_i \Delta_i + \overline{A}_i \delta_i \right). \end{aligned} \tag{41}$$

Applying Lemma 1 (17) for scalars $\alpha > 0$, $\gamma > 0$ and $\lambda > 0$ and using the hypothesis 1 (36), one has

$$\begin{aligned} &\Delta_i^T B_i^T P_2 e + e^T P_2 B_i \Delta_i \le \alpha \Delta_i^T \Delta_i + \alpha^{-1} e^T P_2 B_i B_i^T P_2 e \\ &\quad \le \alpha \eta^2 N_i^2 e^T e + \alpha^{-1} e^T P_2 B_i B_i^T P_2 e, \\ &\delta_i^T \overline{A_i}^T P_2 e + e^T P_2 \overline{A_i} \delta_i \le \gamma \delta_i^T \delta_i + \gamma^{-1} e^T P_2 \overline{A_i A_i}^T P_2 e \\ &\quad \le \gamma M_i^2 e^T e + \gamma^{-1} e^T P_2 \overline{A_i A_i}^T P_2 e, \\ &e^T \left(B_i K_j \right)^T P_1 x + x^T P_1 B_i K_j e \\ &\quad \le \lambda e^T e + \lambda^{-1} x^T P_1 B_i K_j \left(B_i K_j \right)^T P_1 x. \end{aligned} \tag{42}$$

The inequalities (41) and (42) give:

$$\begin{aligned} \dot{V} \le\, & x^T \sum_{i,j}^{n} \mu_i \hat{\mu}_j \Big((A_i - B_i K_j)^T P_1 + P_1 (A_i - B_i K_j) \\ & + \lambda^{-1} P_1 B_i K_j (B_i K_j)^T P_1 \Big) x \\ & + e^T \sum_{i=1}^{n} \Big(\alpha \eta^2 N_i^2 I + \gamma M_i^2 I + \gamma^{-1} P_2 \overline{A_i A_i}^T P_2 \\ & + \alpha^{-1} P_2 B_i B_i^T P_2 + \frac{\lambda}{n} I \Big) e \\ & + e^T \sum_{j=1}^{n} \hat{\mu}_j \left(\Psi_j^T P_2 + P_2 \Psi_j \right) e. \end{aligned} \tag{43}$$

To have $\dot{V} < 0$, it suffices that the following conditions are verified for a positive symmetric matrix Q

$$\Psi_j^T P_2 + P_2 \Psi_j < -Q, \tag{44a}$$

$$\begin{aligned} &\left(A_i - B_i K_j \right)^T P_1 + P_1 \left(A_i - B_i K_j \right) \\ &\quad + \lambda^{-1} P_1 B_i K_j \left(B_i K_j \right)^T P_1 < 0, \end{aligned} \tag{44b}$$

$$\begin{aligned} &\frac{-Q}{n} + \alpha \eta^2 N_i^2 I + \gamma M_i^2 I + \gamma^{-1} P_2 \overline{A_i A_i}^T P_2 \\ &\quad + \alpha^{-1} P_2 B_i B_i^T P_2 + \frac{\lambda}{n} I < 0. \end{aligned} \tag{44c}$$

Using the Schur complement [22], then a congruence with the full rank diagonal block matrix $diag[P_1^{-1}\; P_1^{-1}]$ applied to (44b), for $i, j = 1, 2, \ldots, n$ leads to the following:

$$\begin{bmatrix} P_1^{-1} A_i^T + A_i P_1^{-1} - P_1^{-1} K_j^T B_i^T - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -\lambda \left(P_1^{-1} \right)^2 \end{bmatrix} < 0. \tag{45}$$

Using the property (26), the conditions (45) are verified if the following inequalities hold:

$$\begin{bmatrix} P_1^{-1} A_i^T + A_i P_1^{-1} - P_1^{-1} K_j^T B_i^T - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -2\mu P_1^{-1} + \lambda^{-1} \mu^2 I \end{bmatrix} < 0. \tag{46}$$

Considering (46), developing (44a) and using the Schur complement for (44c), the stabilization conditions (44) for the system (38) can be enounced as follows:

$$\begin{cases} P_1 > 0, P_2 > 0, Q > 0, \eta > 0,\ \alpha > 0,\ \lambda > 0,\ \gamma > 0, \mu > 0, \\ A_0^T P_2 - C^T L_j^T P_2 + P_2 A_0 - P_2 L_j C < -Q, \\ \begin{bmatrix} P_1^{-1} A_i^T + A_i P_1^{-1} - \\ P_1^{-1} K_j^T B_i^T - B_i K_j P_1^{-1} & B_i K_j P_1^{-1} \\ P_1^{-1} K_j^T B_i^T & -2\mu P_1^{-1} + \lambda^{-1} \mu^2 I \end{bmatrix} < 0, \\ \begin{bmatrix} -\dfrac{Q}{n} + \left(\gamma M_i^2 + \alpha \eta^2 N_i^2 + \dfrac{\lambda}{n} \right) I & P_2 \overline{A_i} & P_2 B_i \\ \overline{A_i}^T P_2 & -\gamma I & 0 \\ B_i^T P_2 & 0 & -\alpha \end{bmatrix} < 0, \\ i, j = 1, 2, \ldots, n. \end{cases} \tag{47}$$

We consider the following variable changes

$$X = P_1^{-1},\quad F_i = K_i P_1^{-1},\quad W_i = P_2 L_i \text{ and } \tau = \alpha \eta^2. \tag{48}$$

Replacing (48) in (47), one obtains (40).

For fixed scalars μ and λ, the conditions (40) are solved in X, P_2, F_i, W_i ($i = 1, 2, \ldots, n$) and the scalars $\alpha > 0$ and $\tau > 0$. Therefore, the feedback gains, the observer gains and the input bound η are given by

$$K_i = F_i X^{-1},\ L_i = P_2^{-1} W_i \text{ and } \eta = \sqrt{\frac{\tau}{\alpha}}. \tag{49}$$

5. NUMERICAL EXAMPLES

5.1. Example 1

To illustrate the proposed development, we consider the system presented in [17], where the premise variables are considered immeasurable. The system is represented by the following equations

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i\left(x(t)\right)\left(A_i x(t) + B_i u(t)\right), \\ y(t) = C x(t), \\ \mu_i\left(x(t)\right) \ge 0, \quad \sum_{i=1}^{2} \mu_i\left(x(t)\right) = 1, \end{cases} \tag{50}$$

where

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},\quad A_1 = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix},\quad A_2 = \begin{bmatrix} 2.5 & 0 \\ -2.3 & -1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\quad C_1 = C_2 = \begin{bmatrix} 10 & 2 \end{bmatrix}.$$

$$\begin{cases} \mu_1\left(x(t)\right) = 0.5 + \dfrac{\arctan\left(x_2(t)\right)}{\pi}, \\ \mu_2\left(x(t)\right) = 1 - \mu_1\left(x(t)\right). \end{cases}$$

Note that the system (50) is unstable in open loop.

The state observer of the system (50) is given by

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{2} \mu_i\left(\hat{x}(t)\right)\left[A_i \hat{x}(t) + B_i u(t) + L_i\left(y(t) - \hat{y}(t)\right)\right], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{51}$$

where L_1 and L_2 are the observer gains.

The applied control law (6) has the following equation

$$u(t) = -\left(\mu_1\left(\hat{x}(t)\right) K_1 + \mu_2\left(\hat{x}(t)\right) K_2\right) \hat{x}(t), \tag{52}$$

where K_1 and K_2 are the local control gains.

For the improved stabilization conditions of Theorem 1, for $\mu = 8$ and $\theta = 5$; the resolution of (16) yields:

$$P_1 = \begin{bmatrix} 0.1225 & 0.0072 \\ 0.0072 & 0.1197 \end{bmatrix},\quad P_2 = \begin{bmatrix} 30.2056 & 17.1516 \\ 17.1516 & 65.4920 \end{bmatrix}.$$

Control gains:

$$K_1 = [6.5172 \quad -1.7995],\quad K_2 = [6.5172 \quad -1.7995].$$

Observer gains:

$$L_1 = \begin{bmatrix} 0.3802 \\ -0.1906 \end{bmatrix},\quad L_2 = \begin{bmatrix} 0.5286 \\ -0.3203 \end{bmatrix}.$$

$$\alpha = 132.3745 \text{ and } \tau = 19.8297.$$

Perturbation Bound: $\beta = 0.387$

It can be determined that if $\beta > 0.72$ the inequalities (16) become infeasible.

For the improved stabilization conditions of Theorem 2, for $\mu = 5$, $\lambda = 5$, $M_1 = M_2 = 1$, $N_1 = N_2 = 1$ and for an input bound $\eta = 5$; the resolution of (40) yields:

$$P_1 = \begin{bmatrix} 0.0021 & -0.0002 \\ -0.0002 & 0.0014 \end{bmatrix},\quad P_2 = \begin{bmatrix} 38.609 & 9.7204 \\ 9.7204 & 5518.4 \end{bmatrix},$$

$$Q = \begin{bmatrix} 7522.8 & 820.71 \\ 820.71 & 7394.4 \end{bmatrix}.$$

Control gains:

$$K_1 = [5.7568 \quad -1.4431],\quad K_2 = [5.7568 \quad -1.4431].$$

Observer gains:

$$L_1 = \begin{bmatrix} 15.451 \\ -0.1834 \end{bmatrix},\quad L_2 = \begin{bmatrix} 15.451 \\ -0.1834 \end{bmatrix}.$$

$$\gamma = 3526.1 \text{ and } \alpha = 0.9515.$$

Fig. 1 illustrates the state variables evolutions of the system (50) for both approaches (Theorems 1 and 2), with the initial conditions $x_o = [100 \quad 100]$ and $\hat{x}_0 = [0 \quad 0]$.

It appears on these curves that the state variables of the system (50) with control law (52) (unstable in open loop) converge rapidly which proves the availability of the developed stabilization conditions with estimated state and immeasurable premise variables. Moreover, the convergence of the sate variables in the proposed approaches is more rapid than that in [17].

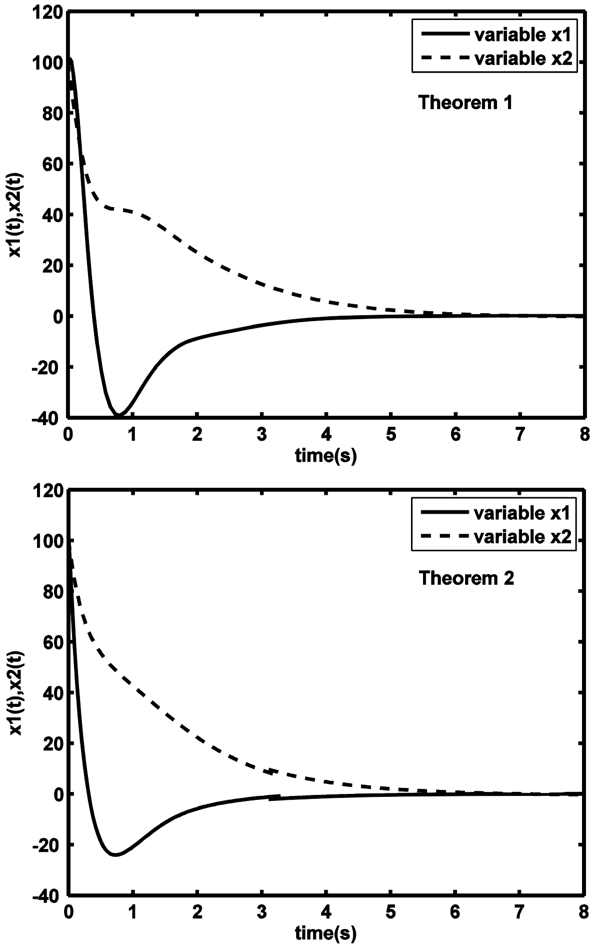

Fig. 1. State variables evolutions of the system (50) with control law (52) designed by the two studied approaches: the approach given by Theorem 1 and that given by Theorem 2.

5.2. Example 2

We consider two-link robot system, as shown in Fig. 2, characterized by the following dynamic equation [23]

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=r, \tag{53}$$

where

$$M(q)=\begin{bmatrix}(m_1+m_2)l_1^2 & m_2l_1l_2(s_1s_2+c_1c_2)\\ m_2l_1l_2(s_1s_2+c_1c_2) & m_2l_2^2\end{bmatrix},$$

$$C(q,\dot{q})=m_2l_1l_2(c_1s_2-s_1c_2)\begin{bmatrix}0 & -\dot{q}_2\\ -\dot{q}_1 & 0\end{bmatrix},$$

$$G(q)=\begin{bmatrix}-(m_1+m_2)l_1gs_1\\ -m_2l_2gs_2\end{bmatrix}$$

$G(g)$ is the gravitational force, $q=[q_1\ q_2]^T$ are the vector of generalized coordinates, $M(q)$ is the moment of inertia and $C(q,\dot{q})$ includes Coriolis and centripetal forces.

The parameters of the two-link robot are the link mass m_1 and $m_2(Kg)$ and the link length l_1 and $l_2(m)$. The notations are also used $s_1=\sin(q_1)$, $s_2=\sin(q_2)$, $c_1=\cos(q_1)$ and $c_2=\cos(q_2)$.

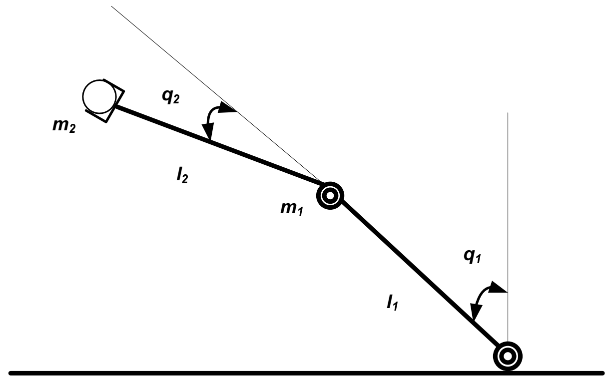

Fig. 2. Structure of two-link robot systems.

Table 1. Parameters values of the robot.

Parameter	Value
m_1	1 Kg
m_2	1 Kg
l_1	1 m
l_2	1 m

The angular positions are q_1 and q_2 (rad), the applied torques is $r=[r_1\ r_2]^T$ (N-m) and the acceleration due to gravity is g (m/s^2).

To give the state space representation, we use the state variables $x_1=q_1$, $x_2=\dot{q}_1$, $x_3=q_2$ and $x_4=\dot{q}_2$. The generalized coordinates q_1 and q_2 are measurable through the optical encoder attached on the robot. So, equation (53) can be written as follows:

$$\begin{cases}\dot{x}_1=x_2,\\ \dot{x}_2=f_1(x)+g_{11}(x)r_1+g_{12}r_2,\\ \dot{x}_3=x_4,\\ \dot{x}_4=f_2(x)+g_{21}(x)r_1+g_{22}r_2,\\ y_1=x_1,\\ y_2=x_3.\end{cases} \tag{54}$$

The expressions $f_1(x)$, $g_{11}(x)$, g_{12}, $f_2(x)$, $g_{21}(x)$ and g_{22} of (54) are given in [23].

The generalized coordinates q_1 and q_2 are constrained within $\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$. The parameters of the robot are given in Table 1.

The critical values of q_1 and q_2 are $-\frac{\pi}{2}$, 0 and $\frac{\pi}{2}$. So, for these extreme values, a T-S fuzzy model for the system (54) is given by the following nine-rule fuzzy model [20]:

Rule i: If x_1 is about a and x_3 is about b, then

$$\begin{cases}\dot{x}=A_ix+B_iu,\\ y=Cx,\end{cases}\quad i=1,2,\ldots,9,$$

$$a\in\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}\ \text{and}\ b\in\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}.$$

The global system is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{9} \mu_i\left(x(t)\right)\left(A_i x(t) + B_i u(t)\right), \\ y(t) = Cx(t), \\ \mu_i\left(x(t)\right) \geq 0, \quad \sum_{i=1}^{9} \mu_i\left(x(t)\right) = 1, \end{cases} \tag{55}$$

where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$, $u = [r_1 \quad r_2]^T$ and $y = [y_1 \quad y_2]^T$.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 5.927 & -0.001 & -0.315 & -8.4e-006 \\ 0 & 0 & 0 & 1 \\ -6.859 & 0.002 & 3.155 & 6.2e-006 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3.0428 & -0.0011 & 0.1791 & -0.0002 \\ 0 & 0 & 0 & 1 \\ 3.5436 & 0.0313 & 2.5611 & 1.14e-005 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.2728 & 0.003 & 0.4339 & -0.0001 \\ 0 & 0 & 0 & 1 \\ 9.1041 & 0.0158 & -1.0574 & -3.2e-005 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.4535 & 0.0017 & 1.2427 & 0.0002 \\ 0 & 0 & 0 & 1 \\ -3.1873 & -0.0306 & 5.1911 & -1.8e-005 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 11.1336 & 0 & -1.8145 & 0 \\ 0 & 0 & 0 & 1 \\ -9.0918 & 0 & 9.1638 & 0 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.1702 & -0.001 & 1.687 & -0.0002 \\ 0 & 0 & 0 & 1 \\ -2.3559 & 0.0314 & 4.5298 & 1.1e-005 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.1206 & -0.0041 & 0.6205 & 0.0001 \\ 0 & 0 & 0 & 1 \\ 8.8794 & -0.0193 & -1.0119 & 4.4e-005 \end{bmatrix},$$

$$A_8 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3.6421 & 0.0018 & 0.0721 & 0.0002 \\ 0 & 0 & 0 & 1 \\ 2.429 & -0.0305 & 2.9832 & -1.9e-005 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.2933 & -0.0009 & -0.2188 & -1.2e-005 \\ 0 & 0 & 0 & 1 \\ -7.4649 & 0.0024 & 3.2693 & 9.2e-006 \end{bmatrix}.$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ -1 & 2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}.$$

$$B_9 = B_5 = B_1, \quad B_4 = B_6 = B_8 = B_2, \quad B_7 = B_3 = B_1.$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The state observer of the system (55) is given by:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{9} \mu_i\left(\hat{x}(t)\right)\begin{bmatrix} A_i\hat{x}(t) + B_i u(t) + \\ L_i\left(y(t) - \hat{y}(t)\right) \end{bmatrix}, \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{56}$$

where L_i for $i = 1, 2, \ldots, 9$ are the observer gains.
The control law has the following form

$$u(t) = -\sum_{i=1}^{9} \mu_i\left(\hat{x}(t)\right) K_i \hat{x}(t). \tag{57}$$

For the design covariance, triangle type membership functions are adapted for Rule 1 to Rule 9 [23].

For the improved stabilization conditions of theorem 1, with the scalars μ =1 and θ =10 the resolution of (16) yields:

$\tau = 69.802$ and $\alpha = 2499.3$.

Perturbation Bound: $\beta = 0.1671$

Control gains:

$$K_1 = \begin{bmatrix} 18.562 & 10.957 & 1.9174 & 1.0311 \\ 1.0177 & 0.7053 & 10.739 & 8.2123 \end{bmatrix},$$

$$K_2 = K_3 = K_4 = K_5 = K_6 = K_7 = K_8 = K_9 = K_1.$$

$$P_1 = \begin{bmatrix} 0.0478 & 0.0131 & 0.0077 & 0.0009 \\ 0.0131 & 0.0147 & 0.0037232 & 0.0006 \\ 0.0077 & 0.0037 & 0.061576 & 0.0123 \\ 0.0009 & 0.0006 & 0.0123 & 0.0131 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 838.17 & -544.33 & 0.0832 & -0.0179 \\ -544.33 & 538.41 & -0.018 & -0.0693 \\ 0.0832 & -0.018 & 838.35 & -544.2 \\ -0.0179 & -0.0693 & -544.2 & 537.93 \end{bmatrix}.$$

Observer gains:

$$L_1 = \begin{bmatrix} 6.2032 & 6.1837 \\ 13.198 & 5.936 \\ -6.1863 & 6.2058 \\ -13.116 & 10.435 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 6.2041 & -3.2411 \\ 10.316 & -3.1281 \\ 3.1846 & 6.205 \\ 6.7668 & 9.839 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 6.1967 & -8.2216 \\ 13.536 & -7.8928 \\ 8.1954 & 6.2047 \\ 17.396 & 6.2194 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 6.1982 & 4.247 \\ 13.717 & 5.5686 \\ -4.1873 & 6.2057 \\ -7.4223 & 12.471 \end{bmatrix},$$

$$L_5 = \begin{bmatrix} 6.2012 & 6.8796 \\ 18.402 & 5.142 \\ -6.8784 & 6.2059 \\ -16.049 & 16.444 \end{bmatrix}, \quad L_6 = \begin{bmatrix} 6.2034 & 3.7632 \\ 13.442 & 5.4611 \\ -3.8211 & 6.2056 \\ -6.2201 & 11.809 \end{bmatrix},$$

$$L_7 = \begin{bmatrix} 6.2103 & -7.7701 \\ 13.404 & -7.2143 \\ 7.8105 & 6.2046 \\ 16.782 & 6.2648 \end{bmatrix}, \quad L_8 = \begin{bmatrix} 6.1985 & -2.1668 \\ 10.907 & -2.0864 \\ 2.2275 & 6.2051 \\ 4.6835 & 10.261 \end{bmatrix},$$

$$L_9 = \begin{bmatrix} 6.2029 & 6.846 \\ 13.564 & 6.7014 \\ -6.8494 & 6.2058 \\ -14.393 & 10.55 \end{bmatrix}.$$

It can be also checked that for $\beta > 0.99$, the inequalities (16) become infeasible.

For the improved stabilization conditions of Theorem 2, for $\mu = 10$, $\lambda = 5$, $M_i = 0.5$, $N_i = 0.5$ ($i = 1, 2, ..., 9$) and an input bound $\eta = 5$; the resolution of (40) yields:

$$P_1 = \begin{bmatrix} 0.0527 & 0.0183 & 0.0051 & 0.0010 \\ 0.0183 & 0.0184 & 0.0047492 & 0.0013 \\ 0.0051 & 0.0047 & 0.0579 & 0.0165 \\ 0.0010 & 0.0013 & 0.0165 & 0.0159 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 3501.4 & -72.972 & -293.31 & -0.9220 \\ -72.972 & 2.9011 & -2.5762 & 0.1872 \\ -293.31 & -2.5762 & 5430.9 & -62.228 \\ -0.9220 & 0.1872 & -62.228 & 1.6245 \end{bmatrix},$$

$$Q = \begin{bmatrix} 4413.2 & -126.93 & -609.33 & 4.38 \\ -126.93 & 126.2 & 4.3167 & 1.951 \\ -609.33 & 4.3167 & 7267.8 & -156.69 \\ 4.38 & 1.951 & -156.69 & 114.88 \end{bmatrix}.$$

Control gains:

$$K_1 = \begin{bmatrix} 19.574 & 10.949 & 2.526 & 1.614 \\ 0.80589 & 0.83693 & 10.798 & 7.6609 \end{bmatrix},$$

$$K_2 = K_3 = K_4 = K_5 = K_6 = K_7 = K_8 = K_9 = K_1.$$

Observer gains:

$$L_1 = \begin{bmatrix} 53.347 & -3.3373 \\ 2542.4 & -504.25 \\ -1.8086 & 68.063 \\ -509.66 & 5913.4 \end{bmatrix},$$

$$L_2 = L_3 = L_4 = L_5 = L_6 = L_7 = L_8 = L_9 = L_1.$$
$$\gamma = 174.48 \text{ and } \alpha = 9.1596.$$

Fig. 3 illustrates the state variables evolutions of the system (55) for both approaches (Theorems 1 and 2), with the initial conditions $x_0 = [0.5 \;\; 0.5 \;\; -0.5 \;\; 0.5]^T$ and $\hat{x}_0 = [0 \;\; 0 \;\; 0 \;\; 0]^T$.

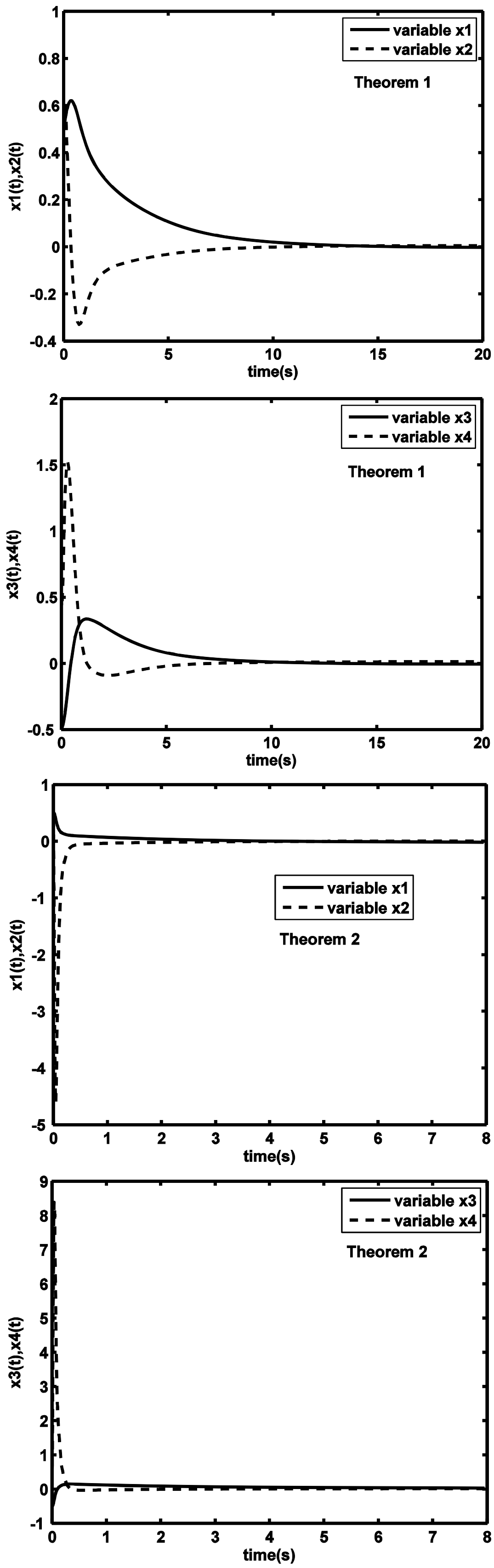

Fig. 3. State variables evolutions of the system (55) with control law (57) designed by the two studied approaches: the approach given by Theorem 1 and that given by Theorem 2.

It appears on these curves that the state variables of the system (55) with control law (57) converge also rapidly, which proves the availability of the developed stabilization conditions with estimated state and immeasurable premise variables.

6. CONCLUSION

In this paper, improved stabilization conditions for T-S observer based controlled systems, where the premise variables are immeasurable, have been presented in form of Linear Matrix Inequalities (LMI).

These conditions are based on two methods for state estimation error expressions. In the first one, we express the state estimation error with an unstructured uncertainty and in the second one; we use the input bound and the application of the Lipchitz constant to the activation function. The first approach leads to conservative conditions limiting the uncertainty bound for which the conditions are feasible. That difficulty is overcome by the second approach that the feasibility of the conditions obtained is not limited. These approaches are less conservative than in previous works by the expression of stabilization conditions with a minimum numbers of parameters and LMI.

The availability of the proposed design techniques has been checked on an illustrative numerical example that is unstable in open loop and on a physical system representing a two-link robot.

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