International Journal of Control, Automation, and Systems (2012) 10(6):1119-1128
DOI 10.1007/s12555-012-0606-z
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Observer-Based Adaptive Fuzzy Fault-Tolerant Output Feedback Control of Uncertain Nonlinear Systems with Actuator Faults

Baoyu Huo, Shaocheng Tong*, and Yongming Li

Abstract: This paper develops an adaptive fuzzy control method for accommodating actuator faults in a class of unknown nonlinear systems with unmeasured states. The considered faults are modeled as lock-in-place (stuck at unknown place). With the help of fuzzy logic systems to approximate the unknown nonlinear functions, and K-filters are designed to estimate the unmeasured states. Combining the backstepping technique with the nonlinear fault-tolerant control theory, a novel adaptive fuzzy faults-tolerant control (FTC) approach is constructed. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded and the tracking error between the system output and the reference signal converges to a small neighborhood of zero by appropriate choice of the design parameters. Simulation results are provided to show the effectiveness of the control approach.

Keywords: Actuator faults, backstepping, fuzzy tolerant-control, K-filters, uncertain nonlinear systems.

1. INTRODUCTION

In the past decades, many approximation-based adaptive backstepping control approaches have been developed to deal with uncertain nonlinear strict-feedback systems via fuzzy logic systems, see for example [1-12] and references therein. Adaptive fuzzy backstepping control approaches in [1-5] are for single-input and single-output (SISO) nonlinear systems, and in [6-8] are for multiple-input and multiple-output (MIMO) nonlinear systems, while those in [9-12] are for SISO/MIMO nonlinear systems with immeasurable states. The main features of the above adaptive fuzzy control approaches are (i) they can be used to deal with those nonlinear systems without satisfying the so called the matching condition, and (ii) they do not require the unknown nonlinear functions are linearly parameterized. Therefore, the approximator-based adaptive fuzzy backstepping control has become one of the most popular design approaches in intelligent control field.

Although a great development has been achieved for the adaptive backstepping control, the aforementioned control approaches assume that all the components of the considered nonlinear systems are in good operating conditions. As we know, some faults, such as actuators and sensors usually occur in the real processes, which can degrade the control performances and even result in the instability of the control system or even catastrophic accidents [13,14]. It is thus important to develop a fault-tolerant control (FTC) scheme against actuator or sensor failures.

To handle the problem of nonlinear system with actuator or sensor faults, in recent years, many FTC approaches have been developed, see for example [15-24] and references therein. [15,16] presented adaptive fault–tolerant control for linear systems with both loss of effectiveness and lock-in-place actuator faults. [17,18] and [19] developed adaptive fault-tolerant controllers for a class of SISO nonlinear systems and MIMO nonlinear systems with the same actuator faults as in [15,16], while [20] and [21] developed observer-based adaptive backstepping fault-tolerant control approaches for some nonlinear systems with additive profile faults. However, the above mentioned fault-tolerant control scheme require that the considered nonlinear systems with the matching conditions or the nonlinear functions are known. To remove these limitations, authors in [22,23] investigated a class of unknown SISO nonlinear strict–feedback systems with both loss of effectiveness and lock-in-place actuator faults, in which fuzzy logic systems are employed to approximate the unknown functions, and based on the backstepping technique, two adaptive fuzzy backstepping FTC schemes were developed . The proposed control schemes guarantee not only the stability of the closed-loop system, but also keep the robust performance of the failed system. On the basis of the results of [22] and [23], authors in [24] proposed an adaptive fuzzy backstepping FTC scheme for unknown MIMO strict-feedback nonlinear systems, and the stability of the control system was given. However, the above fuzzy FTC methods [22-24] are all based on the assumption that the states of the nonlinear systems are directly measured. As what authors stated in [9-13],

Manuscript received April 19, 2012; revised July 8, 2012; accepted August 1, 2012. Recommended by Editorial Board member Bin Jiang under the direction of Editor Poogyeon Park.

This journal was supported by the National Natural Science Foundation of China (Nos. 61074014, 61203008), and Program for Liaoning Innovative Research Team in University (No. LT 2012013).

Baoyu Huo, Shaocheng Tong, and Yongming Li are with the Department of Basic Mathematics, Liaoning University of Technology, Jinzhou, Liaoning 121000, China (e-mails: huobaoyu2006 @126.com, jztsc@sohu.com, l_y_m_2004@ 163.com).

* Corresponding author.

in practice, state variables are often unmeasured for many practical nonlinear systems. Therefore, the existing approaches can not be implemented for the strict-feedback nonlinear systems with actuator faults and immeasurable states, which has motivated us for this study. It should be mentioned that in recent years, several fuzzy FTC approaches have been developed for fuzzy systems with actuator faults [25-29]. These fuzzy FTC scheme designs are mainly based on T-S fuzzy model, considered the actuators faults existing in nonlinear systems or fuzzy systems are not kinds of loss of effectiveness and lock-in-place like in [15,16,22-24]; therefore, they cannot be applied to deal with those nonlinear systems with actuator faults, immeasurable states and without satisfying the matching condition.

In this paper, an adaptive fuzzy fault-tolerant control method is developed for a class of unknown nonlinear systems with the actuator faults of the lock-in-place, and without assuming that the states are available for measurements. With the help of fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy filter is developed to estimate the unmeasured states. Using the designed fuzzy filter, and combining the backstepping technique with the fault-tolerant control methods [19-21], a novel adaptive fuzzy fault-tolerant scheme is constructed. It is shown that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded, and the tracking error converges to a small neighborhood of zero by appropriate choice of the design parameters.

2. PROBLEM FORMULATIONS AND FUZZY LOGIC SYSTEMS

2.1. Nonlinear system descriptions

Consider the following nonlinear system:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + f_1(x_1) + d_1(t)\\
\dot{x}_2 &= x_3 + f_2(\underline{x}_2) + d_2(t)\\
&\vdots\\
\dot{x}_{n-1} &= x_n + f_{n-1}(\underline{x}_{n-1}) + d_{n-1}(t)\\
\dot{x}_n &= f_n(\underline{x}_n) + \sum_{j=1}^{m} b_j\beta_j(y)u_j + d_n(t)\\
y &= x_1,
\end{aligned}
\tag{1}
$$

where u_j, $j = 1,2,\ldots,m$, are the control inputs whose actuators may fail during system operation, $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T$, $i = 1,2,\ldots,n$, are the vector of unmeasured states, y is the measured output, $b_j \neq 0$, $j = 1,2,\ldots,m$, are unknown constants, $\beta_j(y)$, $j = 1,2,\ldots,m$, are known nonlinear functions, and $\beta_j(y) \neq 0$ for $\forall y \in R$. $f_i(\underline{x}_i)$, $i = 1,2,\ldots,n$, are unknown smooth continuous nonlinear functions, $d_i(t)$, $i = 1,2,\ldots,n$ are external disturbances satisfying $|d_i(t)| \le d_i^*$, with d_i^* being unknown constants.

As in [6], the actuator failures are modeled as

$$u_j(t) = \bar{u}_j,\ t \ge t_j \quad j \in \{1,2,\ldots,m\}, \tag{2}$$

where the failure value $\bar{u}_j$ and failure time instant t_j are unknown, so is the failure index j.

The basic assumption for the actuator failure compensation problem is as follows.

Assumption 1: The system (1) is such that for any up to $m-1$ actuator failures, the remaining actuators can still achieve a desired control objective, when implemented with the knowledge of the plant and failure parameters.

Suppose p_k actuators fail at a time instant t_k, $k = 1,2, \ldots, q$, and $t_0 < t_1 < t_2 < \ldots < t_q < \infty$. Obviously, it follows from Assumption 1 that $\sum_{k=1}^{q} p_k \le m-1$. In other words, at time $t \in (t_k, t_{k+1})$, $k = 0,1,\ldots,q$, with $t_{q+1} = \infty$, there are $p = \sum_{i=1}^{k} p_i$ failed actuators, that is, $u_j(t) = \bar{u}_j$, $j = j_1, j_2, \ldots, j_p$, $0 \le p \le m-1$, and $u_j(t) = v_j(t)$, $j \ne j_1, j_2, \ldots, j_p$, where $v_j(t)$, $j = 1,2,\ldots,m$, are applied control inputs from some feedback control design.

Given a reference signal $y_r(t)$, and assume that $y_r(t)$ has up to nth order bounded derivatives. The control objective is to design an output feedback control scheme for the plant (1) with actuators failing at time instants t_k, $k = 1,2,\ldots,q$, such that the plant output $y(t)$ tracks the reference signal $y_r(t)$ as close as possible, and that all closed-loop signals are all bounded, despite the presence of unknown actuator failures and unknown functions.

2.2. Fuzzy logic systems

In the proposed design procedures and stability analysis, fuzzy logic systems will be used to approximate the unknown functions and construct a robust controller. Therefore, some useful Lemmas are first introduced as follows.

Lemma 1 [30]**:** Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a fuzzy logic system $\hat{f}(x|\theta) = \theta^T\phi(x)$ such as

$$\sup_{x\in\Omega}\left|f(x) - \theta^{\mathrm{T}}\phi(x)\right| \le \varepsilon, \tag{3}$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_N]^T$ is the ideal constant optimal vector and $\phi(x) = [\phi_1(x), \ldots, \phi_N(x)]^T$ is the basis function vector with $N > 1$ being the number of the fuzzy rules and ϕ_l are the basis functions defined as

$$\phi_l = \prod_{i=1}^{n}\mu_{F_i^l}(x_i) \Big/ \sum_{l=1}^{N}\Big(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\Big). \tag{4}$$

3. STATE OBSERVATION SCHEME

Since the nonlinear functions $f_i(\underline{x}_i)$ in (1) are unknown, by Lemma 1, we can assume that $f_i(\underline{x}_i)$ can be approximated by the following fuzzy logic systems

$$\hat{f}_i(\hat{\underline{x}}_i|\theta_i) = \theta_i^T\phi_i(\hat{\underline{x}}_i), \tag{5}$$

where $\hat{\underline{x}}_i = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i)^T$, $i = 1,2,\ldots,n-1$, and $\hat{x} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n)^T$ are the estimates of state vectors $\underline{x}_i$ and $\underline{x}_n$, which will be defined later.

Define the fuzzy approximation errors δ_i

$$f_i(\underline{x}_i) = \hat{f}_i(\hat{\underline{x}}_i \left| \theta_i^* \right.) + \delta_i, \tag{6}$$

where θ_i^* are optimal parameter vectors defined by [11-13].

$$\theta_i^* = \arg\min_{\theta_i \in \Omega_i}[\sup_{(\underline{x}_i, \hat{\underline{x}}_i) \in U_{i,1} \times U_{i,2}} \left| \hat{f}_i(\hat{\underline{x}}_i \mid \theta_i) - f_i(\underline{x}_i) \right|] \tag{7}$$

In (7), Ω_i, $U_{i,1}$ and $U_{i,2}$ are bounded compact sets for θ_i, $\underline{x}_i$ and $\hat{\underline{x}}_i$, respectively.

Denote $\delta = [\delta_1, ..., \delta_n]^T$, according to [11-13], we have the following assumption as follows.

Assumption 2: There exist an unknown constant δ^*, such that $\|\delta\| \le \delta^*$, $\|\cdot\|$ denotes the two-norm of a vector.

To develop a solution to the control problem, we use the proportional-actuation scheme [18,22,23]

$$v_j = sign[b_j]\frac{1}{\beta_j(y)}u_0, \quad j = 1, 2, ..., m, \tag{8}$$

where u_0 is the designed adaptive fuzzy controller to be designed by the backstepping technique.

To express the system (1) with actuator failures (2) under the actuation scheme (8), we define

$$k_1 = \sum_{j \ne j_1, ..., j_p} sign[b_j]b_j, \tag{9}$$

$$k_{2,j} = \begin{cases} b_j \bar{u}_j, & j = j_1, \cdots, j_p \\ 0, & j \ne j_1, \cdots, j_p. \end{cases} \tag{10}$$

Rewrite (1) in the state-space form as:

$$\begin{aligned} \dot{x} &= Ax + \phi^T(\hat{x})\Theta^* + B\sum_{j=1}^m k_{2,j}\beta_j(y) + Bk_1u_0 + \delta + d, \\ y &= C^T x, \end{aligned} \tag{11}$$

where

$$A = \begin{bmatrix} 0 & & \\ \vdots & I & \\ 0 & 0 \ \dots & 0 \end{bmatrix}, \quad x = [x_1, ..., x_n]^T,$$

$$B = [0, ..., 0, 1]^T, \quad \Theta^* = [\theta_1^*, ..., \theta_n^*]^T,$$

$$C = [1, 0, ..., 0]^T \in R^{n \times 1}, \quad \phi(\hat{x}) = diag[\phi_1(\hat{x}_1), ..., \phi_n(\hat{x})].$$

The system (11) is further rewritten as:

$$\begin{cases} \dot{x} = Ax + G^T\vartheta + \delta + B\sum_{j=1}^m k_{2,j}\beta_j(y) + d \\ y = C^T x, \end{cases} \tag{12}$$

where

$$\vartheta = \begin{bmatrix} k_1 \\ \Theta^* \end{bmatrix}_{(1+l) \times 1}, \quad G = \left[\begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix} u_0, \phi^T(\hat{x}) \right]^T_{(l+1)\times n}.$$

Choosing a vector $L = [L_1, L_2, ..., L_n]^T$ so that the matrix $A_0 = A - LC^T$ is a strict Hurwitz matrix, i.e., given a positive definite matrix $Q = Q^T > 0$ there exists a positive definite matrix $P = P^T > 0$ satisfying

$$PA_0 + A_0^T P = -Q. \tag{13}$$

Note that the states $x_2, \ldots, x_n$ in the system (1) are not measured directly, thus we need to design filters to estimate $x_2, \ldots, x_n$ and generate some signals for controller design.

With knowledge of $k_{2,j}$, $j = 1, 2, ..., m$, we have a virtual state observer as

$$z = \xi_0 + \xi^T\vartheta + \sum_{j=1}^m k_{2,j}\zeta_j \tag{14}$$

with the filters defined as

$$\dot{\xi}_0 = A_0\xi_0 + Ly, \tag{15}$$

$$\dot{\xi}^T = A_0\xi^T + G^T, \tag{16}$$

$$\dot{\zeta}_j = A_0\zeta_j + B\beta_j(y). \tag{17}$$

Since the signal ϑ in (14) is not available as Θ^* is not, and the actual state estimate should be

$$\hat{x} = \xi_0 + \xi^T\hat{\vartheta} + \sum_{j=1}^m \hat{k}_{2,j}\zeta_j, \tag{18}$$

where $\hat{\vartheta}$ and $\hat{k}_{2,j}$ are the estimate of ϑ and $k_{2,j}$, respectively. Denote μ is the first column of ξ^T. The vector μ is governed by

$$\dot{\mu} = A_0\mu + Bu_0. \tag{19}$$

In view of (16) and (19), ξ is expressed as

$$\xi = [\mu, \Xi]^T. \tag{20}$$

From (16), one obtains

$$\dot{\Xi} = A_0\Xi + \phi^T(\hat{x}). \tag{21}$$

Define the observation error as $e = x - z$, then from (12), (14), (15), (16) and (17), we have

$$\dot{e} = A_0 e + \delta + d. \tag{22}$$

Consider the following Lyapunov function $V_0 = e^TPe$, the time derivative of V_0 along with (22) is

$$\dot{V}_0 = e^T(A_0^T P + PA_0)e + 2e^TP(\delta + d). \tag{23}$$

By Young's inequality and Assumption 2, we have

$$2e^TP\delta \le 2\|e\|\|P\|\|\delta\| \le \|e\|^2 + \|P\|^2\delta^{*2}, \tag{24}$$

$$2e^TPd \le 2\|e\|\|P\|\|d\| \le \|e\|^2 + \|P\|^2\left(\sum_{i=1}^n d_i^{*2}\right). \tag{25}$$

Substituting (24) and (25) into (23), we obtain

$$\dot{V}_0 \le -(\lambda_{\min}(Q) - 2)\|e\|^2 + \|P\|^2\left(\delta^{*2} + \sum_{i=1}^n d_i^{*2}\right). \tag{26}$$

4. FAULT-TOLERANT CONTROL DESIGN AND STABILITY ANALYSIS

In this section, an adaptive fault-tolerant control scheme will be developed by the backstepping technique and the filters (15), (16), (17) and the stability of the closed-loop system will be given.

Define

$$k_2=[k_{2,1},k_{2,2},\ldots,k_{2,m}]^T \text{ and } \omega_i=[\zeta_{1,i},\zeta_{2,i},\ldots,\zeta_{m,i}]^T,$$

where $\zeta_{j,i}$, $j=1,2,\ldots,m$, $i=1,2,\ldots,n$ are the ith variable of ζ_j, $j=1,2,\ldots,m$.

From the first equation in (11), one obtains

$$\dot{x}_1=x_2+\theta_1^{*T}\phi_1(\hat{x}_1)+\delta_1+d_1. \tag{27}$$

Since x_2 is unavailable, it is replaced by available filter signals. From (14), x_2 is expressed as

$$\begin{aligned} x_2&=\xi_{0,2}+\xi_{(2)}^T\vartheta+\omega_2^Tk_2+e_2\\ &=k_1\mu_2+\xi_{0,2}+[0,\Xi_{(2)}]\vartheta+\omega_2^Tk_2+e_2, \end{aligned} \tag{28}$$

where $\Xi_{(2)}$ are the second rows of Ξ, respectively.

Substituting (28) into (27) yields

$$\dot{x}_1=k_1\mu_2+\xi_{0,2}+\bar{\mathrm{M}}^T\vartheta+\omega_2^Tk_2+e_2+\delta_1+d_1, \tag{29}$$

where

$$\mathrm{M}=[\mu_2,\phi_{(1)}^T(\hat{x})+\Xi_{(2)}]^T, \tag{30}$$

$$\bar{\mathrm{M}}=[0,\phi_{(1)}^T(\hat{x})+\Xi_{(2)}]^T. \tag{31}$$

From (19), we obtain

$$\dot{\mu}_i=\mu_{i+1}-L_i\mu_1,\quad i=2,\ldots,n-1, \tag{32}$$

$$\dot{\mu}_n=u_0-L_n\mu_1. \tag{33}$$

Let μ_1 is the first row of μ.

Define a change of coordinates as

$$\chi_1=y-y_r, \tag{34}$$

$$\chi_i=\mu_i-\hat{\kappa}y_r^{(i-1)}-\alpha_{i-1},\quad i=2,\ldots,n, \tag{35}$$

where $\hat{\kappa}$ is the estimate of unknown constant $\kappa=1/k_1$.

Based on the above change of coordinates, the detailed adaptive backstepping control design is given by the following n-Steps.

Step 1: The time derivative of χ_1 is

$$\dot{\chi}_1=k_1\mu_2+\xi_{0,2}+\bar{\mathrm{M}}^T\vartheta+\omega_2^Tk_2+e_2+\delta_1+d_1-\dot{y}_r. \tag{36}$$

Consider the following Lyapunov function:

$$\begin{aligned} V_1&=V_0+\frac{1}{2}\chi_1^2+\frac{k_1}{2\gamma_1}\tilde{\kappa}^2+\frac{1}{2\gamma_2}\tilde{\delta}^{*2}\\ &+\frac{1}{2}\tilde{\vartheta}^T\Gamma_1^{-1}\tilde{\vartheta}+\frac{1}{2}\tilde{k}_2^T\Gamma_2^{-1}\tilde{k}_2, \end{aligned} \tag{37}$$

where $\Gamma_1=\Gamma_1^T>0$, $\Gamma_2=\Gamma_2^T>0$, $\gamma_1>0$ and $\gamma_2>0$ are design constants. $\tilde{\vartheta}=\vartheta-\hat{\vartheta}$, $\tilde{\delta}^{*2}=\delta^*-\hat{\delta}^*$, $\tilde{k}_2=k_2-\hat{k}_2$ and; $\hat{\vartheta}$, $\hat{\delta}^*$, $\hat{k}_2$ and $\hat{\kappa}$ are the estimates of ϑ, δ^*, k_2 and κ, respectively.

The time derivative of V_1 along (36) is

$$\begin{aligned} \dot{V}_1&=\dot{V}_0+\chi_1(k_1\mu_2+\xi_{0,2}+\bar{\mathrm{M}}^T\vartheta+\omega_2^Tk_2+\delta_1\\ &+d_1+e_2-\dot{y}_r)-\tilde{\vartheta}^T\Gamma_1^{-1}\dot{\hat{\vartheta}}-\frac{1}{\gamma_2}\tilde{\delta}^*\dot{\hat{\delta}}^*\\ &-\frac{1}{\gamma_1}k_1\tilde{\kappa}\dot{\hat{\kappa}}-\tilde{k}_2^T\Gamma_2^{-1}\dot{\hat{k}}_2. \end{aligned} \tag{38}$$

Substituting (35) into (38) results in

$$\begin{aligned} \dot{V}_1&=\dot{V}_0+k_1\chi_1\chi_2+\chi_1k_1\hat{\kappa}\dot{y}_r-\chi_1k_1\tilde{\kappa}\bar{\alpha}_1+\chi_1(\bar{\alpha}_1\\ &+\xi_{0,2}+\bar{\mathrm{M}}^T\vartheta-\dot{y}_r+\omega_2^Tk_2)+\chi_1(e_2+\delta_1+d_1)\\ &-\tilde{\vartheta}^T\Gamma_1^{-1}\dot{\hat{\vartheta}}-\frac{1}{\gamma_2}\tilde{\delta}^*\dot{\hat{\delta}}^*-\frac{1}{\gamma_1}k_1\tilde{\kappa}\dot{\hat{\kappa}}-\tilde{k}_2^T\Gamma_2^{-1}\dot{\hat{k}}_2, \end{aligned} \tag{39}$$

where $\alpha_1=\hat{\kappa}\bar{\alpha}_1$, $\bar{\alpha}_1$ will be designed later. Using Young's inequality, we have

$$\chi_1e_2\le|e_2|^2+\frac{1}{4}\chi_1^2\le\|e\|^2+\frac{1}{4}\chi_1^2, \tag{40}$$

$$\chi_1d_1\le\frac{1}{2}|d_1|^2+\frac{1}{2}\chi_1^2\le\frac{1}{2}d_1^{*2}+\frac{1}{2}\chi_1^2. \tag{41}$$

Substituting (26), (40), (41) and $\kappa=1/k_1$ into (39) yields

$$\begin{aligned} \dot{V}_1&\le-(\lambda_{\min}(Q)-3)\|e\|^2+\|P\|^2\left(\delta^{*2}+\sum_{i=1}^nd_i^{*2}\right)\\ &+k_1\chi_1\chi_2+\chi_1\left(\bar{\alpha}_1+\frac{3}{4}\chi_1+\xi_{0,2}+\bar{\mathrm{M}}^T\hat{\vartheta}+\omega_2^T\hat{k}_2\right.\\ &\left.+\chi_1\hat{\delta}^*\tanh\left(\frac{\chi_1}{\varsigma}\right)\right)+|\chi_1|\delta^*-\chi_1\delta^*\tanh\left(\frac{\chi_1}{\varsigma}\right)\\ &+\tilde{\delta}^*\left(\chi_1\tanh\left(\frac{\chi_1}{\varsigma}\right)-\frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right)+\tilde{\vartheta}^T\left(\chi_1\bar{\mathrm{M}}-\Gamma_1^{-1}\dot{\hat{\vartheta}}\right)\\ &+\tilde{\kappa}\left(-\chi_1k_1\bar{\alpha}_1-\chi_1k_1\dot{y}_r-\frac{1}{\gamma_1}k_1\dot{\hat{\kappa}}\right)\\ &+\tilde{k}_2^T\left(\chi_1\omega_2-\Gamma_2^{-1}\dot{\hat{k}}_2\right)+\frac{1}{2}d_1^{*2}, \end{aligned} \tag{42}$$

where $\varsigma>0$ is a design parameter.

For the convenience of the later derivations, we cite the following Lemma 2.

Lemma 2 [1]**:** The hyperbolic tangent function fulfills that for any $\varsigma>0$ and any $x\in R$,

$$|x|-x\tanh(x/\varsigma)\le0.2758\varsigma.$$

From Lemma 2, it can be seen that

$$|\chi_1|\delta^*-\chi_1\delta^*\tanh(\chi_1/\varsigma)\le0.2758\varsigma\delta^*=\varsigma'. \tag{43}$$

Choose the stabilizing control function $\bar{\alpha}_1$, tuning functions and parameters adaptation laws as

$$\bar{\alpha}_1 = -c_1\chi_1 - \frac{3}{4}\chi_1 - \omega_2^T\hat{k}_2 - \bar{\mathrm{M}}^T\hat{\vartheta} - \xi_{0,2} - \hat{\delta}^* \tanh\left(\frac{\chi_1}{\varsigma}\right), \tag{44}$$

$$\tau_1 = \bar{\mathrm{M}}\chi_1, \tag{45}$$

$$\sigma_1 = \chi_1 \tanh(\chi_1/\varsigma), \tag{46}$$

$$\nu_1 = \chi_1\omega_2, \tag{47}$$

$$\dot{\hat{\kappa}} = -\gamma_1(\chi_1(\bar{\alpha}_1 + \dot{y}_r) + \bar{\sigma}_\kappa\hat{\kappa}), \tag{48}$$

where c_1 is a positive design constant, $\bar{\sigma}_\kappa > 0$ is a small constant. Substituting (43)-(48) into (42) yields

$$\begin{aligned}\dot{V}_1 \le& -(\lambda_{\min}(Q) - 3)\|e\|^2 + k_1\chi_1\chi_2 - c_1\chi_1^2 \\ &+ \tilde{\delta}^*\left(\sigma_1 - \frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right) + \tilde{\vartheta}^T\left(\tau_1 - \Gamma_1^{-1}\dot{\hat{\vartheta}}\right) \\ &+ k_1\bar{\sigma}_\kappa\tilde{\kappa}\hat{\kappa} + \tilde{k}_2^T\left(\nu_1 - \Gamma_2^{-1}\dot{\hat{k}}_2\right) + D_1,\end{aligned} \tag{49}$$

where $D_1 = \|P\|^2\left(\delta^{*2} + \sum_{i=1}^{n} d_i^{*2}\right) + \frac{1}{2}d_1^{*2} + \varsigma'$.

Step i $(2 \le i \le n-1)$**:** The time derivative of χ_i is

$$\begin{aligned}\dot{\chi}_i =& \mu_{i+1} - L_i\mu_1 - \hat{\kappa}y_r^{(i)} - \frac{\partial\alpha_{i-1}}{\partial x_1}(k_1\mu_2 + \xi_{0,2} + \bar{\mathrm{M}}^T\vartheta \\ &+ \omega_2^T k_2 + e_2 + \delta_1 + d_1) + H_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\vartheta}}(\dot{\hat{\vartheta}} - \Gamma_1\tau_{i-1} \\ &+ \Gamma_1\bar{\sigma}_1\hat{\vartheta}) - \frac{\partial\alpha_{i-1}}{\partial\hat{\delta}^*}(\dot{\hat{\delta}}^* - \gamma_2\sigma_{i-1} + \gamma_2\bar{\sigma}_2\hat{\delta}^*) \\ &- \frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}(\dot{\hat{k}}_2 - \Gamma_2\nu_{i-1} + \Gamma_2\bar{\sigma}_3\hat{k}_2),\end{aligned} \tag{50}$$

where $\bar{\sigma}_1 > 0$, $\bar{\sigma}_2 > 0$ and $\bar{\sigma}_3 > 0$ are small constants.

$$\begin{aligned}H_i =& -\frac{\partial\alpha_{i-1}}{\partial\xi_0}(A_0\xi_0 + Ly) - \frac{\partial\alpha_{i-1}}{\partial\Xi}(A_0\Xi + \phi^T(\hat{x})) \\ &- \frac{\partial\alpha_{i-1}}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_{i-1}}{\partial\mu}\dot{\mu} - \left(y_r^{(i-1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\kappa}}\right)\dot{\hat{\kappa}} \\ &- \frac{\partial\alpha_{i-1}}{\partial\hat{\vartheta}}\Gamma_1(\tau_{i-1} - \bar{\sigma}_1\hat{\vartheta}) - \frac{\partial\alpha_{i-1}}{\partial\hat{\delta}^*}\gamma_2(\sigma_{i-1} - \bar{\sigma}_2\hat{\delta}^*) \\ &- \frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}\Gamma_2(\nu_{i-1} - \bar{\sigma}_3\hat{k}_2)\end{aligned}$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}\chi_i^2. \tag{51}$$

The time derivative of V_i along the solutions of (50) is

$$\begin{aligned}\dot{V}_i =& \dot{V}_{i-1} + \chi_i\Bigg[\mu_{i+1} - L_i\mu_1 - \hat{\kappa}y_r^{(i)} - \frac{\partial\alpha_{i-1}}{\partial x_1}(\xi_{0,2} \\ &+ \mathrm{M}^T\vartheta + \omega_2^T k_2 + e_2 + \delta_1 + d_1) + H_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\vartheta}}(\dot{\hat{\vartheta}} \\ &- \Gamma_1\tau_{i-1} + \Gamma_1\bar{\sigma}_1\hat{\vartheta}) - \frac{\partial\alpha_{i-1}}{\partial\hat{\delta}^*}(\dot{\hat{\delta}}^* - \gamma_2\sigma_{i-1} \\ &+ \gamma_2\bar{\sigma}_2\hat{\delta}^*) - \frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}(\dot{\hat{k}}_2 - \Gamma_2\nu_{i-1} + \Gamma_2\bar{\sigma}_3\hat{k}_2)\Bigg].\end{aligned} \tag{52}$$

Using Young's inequality, we have

$$-\chi_i\frac{\partial\alpha_{i-1}}{\partial x_1}e_2 \le \|e\|^2 + \frac{1}{4}\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2\chi_i^2, \tag{53}$$

$$-\chi_i\frac{\partial\alpha_{i-1}}{\partial x_1}d_1 \le \frac{1}{2}d_1^{*2} + \frac{1}{2}\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2\chi_i^2. \tag{54}$$

Substituting (53) and (54) into (52), and by Applying Lemma 2, the (52) can be rewritten as

$$\begin{aligned}\dot{V}_i \le& -\left(\lambda_{\min}(Q) - (i+2)\right)\|e\|^2 - \sum_{q=1}^{i-1}c_q\chi_q^2 + \tilde{\vartheta}^T\left(\tau_i - \Gamma_1^{-1}\dot{\hat{\vartheta}}\right) \\ &+ \tilde{\delta}^*\left[\sigma_i - \frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right] + \tilde{k}_2^T\left(\nu_i - \Gamma_2^{-1}\dot{\hat{k}}_2\right) \\ &+ D_i + k_1\bar{\sigma}_\kappa\tilde{\kappa}\hat{\kappa} + \sum_{j=i}^{n}\sum_{k=1}^{i-2}\Big(\Lambda_{k,j} + \mathrm{A}_{k,j} \\ &+ E_{k,j}\Big)\chi_{k+1}\chi_j + \chi_i\Bigg[\chi_{i+1} + \frac{3}{4}\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2\chi_i + \alpha_i \\ &- L_i\mu_1 - \frac{\partial\alpha_{i-1}}{\partial x_1}\left(\xi_{0,2} + \mathrm{M}^T\hat{\vartheta} + \omega_2^T\hat{k}_2\right) + H_i \\ &- \frac{\partial\alpha_{i-1}}{\partial\hat{\vartheta}}\left(\dot{\hat{\vartheta}} - \Gamma_1\tau_{i-1} + \Gamma_1\bar{\sigma}_1\hat{\vartheta}\right) - \frac{\partial\alpha_{i-1}}{\partial\hat{\delta}^*}\Big(\dot{\hat{\delta}}^* \\ &- \gamma_2\sigma_{i-1} + \gamma_2\bar{\sigma}_2\hat{\delta}^*\Big) - \frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}\Big(\dot{\hat{k}}_2 - \Gamma_2\nu_{i-1} \\ &+ \Gamma_2\bar{\sigma}_3\hat{k}_2\Big) + \eta_i\hat{\delta}^*\tanh\left(\frac{\chi_i\eta_i}{\varsigma}\right)\Bigg].\end{aligned} \tag{55}$$

When $i=2$,

$$\begin{aligned}\dot{V}_2 \le& -\left(\lambda_{\min}(Q) - 4\right)\|e\|^2 - c_1\chi_1^2 + \tilde{\delta}^*\left(\sigma_2 - \frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right) \\ &+ \tilde{\vartheta}^T\left(\tau_2 - \Gamma_1^{-1}\dot{\hat{\vartheta}}\right) + k_1\bar{\sigma}_\kappa\tilde{\kappa}\hat{\kappa} + \tilde{k}_2^T\left(\nu_2 - \Gamma_2^{-1}\dot{\hat{k}}_2\right) \\ &+ D_2 + \chi_2\Bigg[\chi_3 + \hat{k}_1\chi_1 + \alpha_2 + \frac{3}{4}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2\chi_2 - L_2\mu_1 \\ &- \frac{\partial\alpha_1}{\partial x_1}\left(\xi_{0,2} + \mathrm{M}^T\hat{\vartheta} + \omega_2^T\hat{k}_2\right) + H_2 - \frac{\partial\alpha_1}{\partial\hat{\vartheta}}\Big(\dot{\hat{\vartheta}} - \Gamma_1\tau_1 \\ &+ \Gamma_1\bar{\sigma}_1\hat{\vartheta}\Big) - \frac{\partial\alpha_1}{\partial\hat{\delta}^*}\left(\dot{\hat{\delta}}^* - \gamma_2\sigma_1 + \gamma_2\bar{\sigma}_2\hat{\delta}^*\right) - \frac{\partial\alpha_1}{\partial\hat{k}_2}\Big(\dot{\hat{k}}_2 \\ &- \Gamma_2\nu_1 + \Gamma_2\bar{\sigma}_3\hat{k}_2\Big) + \eta_2\hat{\delta}^*\tanh\left(\frac{\chi_2\eta_2}{\varsigma}\right)\Bigg],\end{aligned}$$

where

$$D_i = D_{i-1} + \frac{1}{2}d_1^{*2} + \varsigma', \quad \tau_i = \tau_{i-1} - \chi_i \frac{\partial \alpha_{i-1}}{\partial x_1}\mathrm{M}(i \ge 3),$$

$$\tau_2 = \tau_1 - \chi_2\left(\frac{\partial \alpha_1}{\partial x_1}\mathrm{M} - \lambda\right),\ \lambda = [\chi_1, 0, ..., 0]^T,\ \eta_i = \frac{\partial \alpha_{i-1}}{\partial x_1},$$

$$\nu_i = \nu_{i-1} - \chi_i \frac{\partial \alpha_{i-1}}{\partial x_1}\omega_i, \quad \sigma_i = \sigma_{i-1} + \chi_i \eta_i \tanh\left(\frac{\chi_i \eta_i}{\varsigma}\right).$$

Define

$$-\frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}(\dot{\hat{\vartheta}} - \Gamma_1 \tau_{i-1} + \Gamma_1 \bar{\sigma}_1 \hat{\vartheta}) = \sum_{j=i}^{n} \Lambda_{i-1,j}\chi_j, \tag{56}$$

$$-\frac{\partial \alpha_{i-1}}{\partial \hat{\delta}^*}(\dot{\hat{\delta}}^* - \gamma_2 \sigma_{i-1} + \gamma_2 \bar{\sigma}_2 \hat{\delta}^*) = \sum_{j=i}^{n} \mathrm{A}_{i-1,j}\chi_j, \tag{57}$$

$$-\frac{\partial \alpha_{i-1}}{\partial \hat{k}_2}(\dot{\hat{k}}_2 - \Gamma_2 \nu_{i-1} + \Gamma_2 \bar{\sigma}_3 \hat{k}_2) = \sum_{j=i}^{n} \mathrm{E}_{i-1,j}\chi_j, \tag{58}$$

where

$$\Lambda_{i,j} = \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}\Gamma_1 \frac{\partial \alpha_{j-1}}{\partial x_1}\mathrm{M},$$

$$\mathrm{A}_{i,j} = \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}^*}\gamma_2 \eta_j \tanh\left(\frac{\eta_j \chi_j}{\varsigma}\right),$$

$$\mathrm{E}_{i,j} = \frac{\partial \alpha_{i-1}}{\partial \hat{k}_2}\Gamma_2 \frac{\partial \alpha_{j-1}}{\partial x_1}\omega_2.$$

Choose the stabilizing control function α_i as

$$\begin{aligned} \alpha_i = &-c_i \chi_i - \frac{3}{4}\left(\frac{\partial \alpha_{i-1}}{\partial x_1}\right)^2 \chi_i \\ &+ \frac{\partial \alpha_{i-1}}{\partial x_1}(\xi_{0,2} + \omega_2^T \hat{k}_2 + \mathrm{M}^T \hat{\vartheta}) \\ &+ L_i \mu_1 - \sum_{k=1}^{i-1}(\mathrm{A}_{k,i} + \Lambda_{k,i} + \mathrm{E}_{k,i})\chi_{k+1} \\ &- H_i - \eta_i \hat{\delta}^* \tanh\left(\frac{\chi_i \eta_i}{\varsigma}\right). \end{aligned} \tag{59}$$

When $i = 2$,

$$\begin{aligned} \alpha_2 = &-\hat{k}_1 \chi_1 - c_2 \chi_2 - \frac{3}{4}\left(\frac{\partial \alpha_1}{\partial x_1}\right)^2 \chi_2 \\ &+ \frac{\partial \alpha_1}{\partial x_1}(\xi_{0,2} + \omega_2^T \hat{k}_2 + \mathrm{M}^T \hat{\vartheta}) \\ &+ L_2 \mu_1 - (\mathrm{A}_{1,2} + \Lambda_{1,2} + \mathrm{E}_{1,2})\chi_2 \\ &- H_2 - \eta_2 \hat{\delta}^* \tanh\left(\frac{\chi_2 \eta_2}{\varsigma}\right), \end{aligned}$$

where $c_i > 0$ are design constants. Substituting (56)-(59) into (55) results in

$$\begin{aligned} \dot{V}_i \le &-(\lambda_{\min}(Q) - (i+2))\|e\|^2 - \sum_{q=1}^{i} c_q \chi_q^2 \\ &+ \tilde{\vartheta}^T(\tau_i - \Gamma_1^{-1}\dot{\hat{\vartheta}}) + \tilde{\delta}^*\left(\sigma_i - \frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right) \\ &+ \tilde{k}_2^T(\nu_i - \Gamma_2^{-1}\dot{\hat{k}}_2) + k_1 \bar{\sigma}_\kappa \tilde{\kappa}\hat{\kappa} + \chi_i \chi_{i+1} \\ &+ \sum_{j=i+1}^{n}\sum_{k=1}^{i-1}(\Lambda_{k,j} + \mathrm{A}_{k,j} + E_{k,j})\chi_{k+1}\chi_j + D_i. \end{aligned} \tag{60}$$

Step n: The time derivative of χ_n along (35) is

$$\begin{aligned} \dot{\chi}_n = &u_0 - L_n \mu_1 - \hat{\kappa} y_r^{(n)} - \dot{\hat{\kappa}} y_r^{(n-1)} \\ &- \frac{\partial \alpha_{n-1}}{\partial x_1}(\xi_{0,2} + M^T \vartheta + \omega_2^T k_2 + e_2 + \delta_1 + d_1) \\ &+ H_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}}(\dot{\hat{\vartheta}} - \Gamma_1 \tau_{n-1} + \Gamma_1 \bar{\sigma}_1 \hat{\vartheta}) \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}^*}(\dot{\hat{\delta}}^* - \gamma_2 \sigma_{n-1} + \gamma_2 \bar{\sigma}_2 \hat{\delta}^*) \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{k}_2}(\dot{\hat{k}}_2 - \Gamma_2 \nu_{n-1} + \Gamma_2 \bar{\sigma}_3 \hat{k}_2), \end{aligned} \tag{61}$$

where H_n is given in H_i with $i = n$.
Consider the Lyapunov function as

$$V = V_n = V_{n-1} + \frac{1}{2}\chi_n^2. \tag{62}$$

Using (62), the time derivative of V_n is

$$\begin{aligned} \dot{V}_n = \dot{V}_{n-1} + \chi_n \Big[&u_0 - L_n \mu_1 - \hat{\kappa} y_r^{(n)} - \dot{\hat{\kappa}} y_r^{(n-1)} \\ &- \frac{\partial \alpha_{n-1}}{\partial x_1}(\xi_{0,2} + M^T \vartheta + \omega_2^T k_2 + e_2 + \delta_1 + d_1) + H_n \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}}(\dot{\hat{\vartheta}} - \Gamma_1 \tau_{n-1} + \Gamma_1 \bar{\sigma}_1 \hat{\vartheta}) \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}^*}(\dot{\hat{\delta}}^* - \gamma_2 \sigma_{n-1} + \gamma_2 \bar{\sigma}_2 \hat{\delta}^*) \\ &- \frac{\partial \alpha_{n-1}}{\partial \hat{k}_2}(\dot{\hat{k}}_2 - \Gamma_2 \nu_{n-1} + \Gamma_2 \bar{\sigma}_3 \hat{k}_2) \Big]. \end{aligned} \tag{63}$$

Using Young's inequality, we have

$$-\chi_n \frac{\partial \alpha_{n-1}}{\partial x_1} e_2 \le \|e\|^2 + \frac{1}{4}\left(\frac{\partial \alpha_{n-1}}{\partial x_1}\right)^2 \chi_n^2, \tag{64}$$

$$-\chi_n \frac{\partial \alpha_{n-1}}{\partial x_1} d_1 \le \frac{1}{2}d_1^{*2} + \frac{1}{2}\left(\frac{\partial \alpha_{n-1}}{\partial x_1}\right)^2 \chi_n^2. \tag{65}$$

In the similar derivation procedures as Step i, the (63) can be rewritten as

$$\begin{aligned} \dot{V}_n \le &-(\lambda_{\min}(Q) - (n+2))\|e\|^2 - \sum_{q=1}^{n-1} c_q \chi_q^2 + \tilde{\vartheta}^T(\tau_n \\ &- \Gamma_1^{-1}\dot{\hat{\vartheta}}) + \tilde{\delta}^*\left[\sigma_n - \frac{1}{\gamma_2}\dot{\hat{\delta}}^*\right] + \tilde{k}_2^T(\nu_n - \Gamma_2^{-1}\dot{\hat{k}}_2) \\ &+ \sum_{j=n-1}^{n}\sum_{k=1}^{n-3}(\Lambda_{k,j} + \mathrm{A}_{k,j} + E_{k,j})\chi_{k+1}\chi_j \end{aligned} \tag{66}$$

$$+k_1\bar{\sigma}_\kappa\tilde{\kappa}\hat{\kappa}+D_n+\chi_n\left[u_0+\frac{3}{4}\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2\chi_n\right.$$
$$-L_n\mu_1-\frac{\partial\alpha_{n-1}}{\partial x_1}(\xi_{0,2}+M^T\hat{\vartheta}+\omega_2^T\hat{k}_2)+H_n$$
$$-\frac{\partial\alpha_{n-1}}{\partial\hat{\vartheta}}(\dot{\hat{\vartheta}}-\Gamma_1\tau_{n-1}+\Gamma_1\bar{\sigma}_1\hat{\vartheta})-\frac{\partial\alpha_{n-1}}{\partial\hat{\delta}^*}(\dot{\hat{\delta}}^*$$
$$-\gamma_2\sigma_{n-1}+\gamma_2\bar{\sigma}_2\hat{\delta}^*)-\frac{\partial\alpha_{n-1}}{\partial\hat{k}_2}(\dot{\hat{k}}_2-\Gamma_2\nu_{n-1}$$
$$\left.+\Gamma_2\bar{\sigma}_3\hat{k}_2)+\eta_n\hat{\delta}^*\tanh\left(\frac{\chi_n\eta_n}{\varsigma}\right)\right],$$

where

$$D_n=D_{n-1}+\frac{1}{2}d_1^{*2}+\varsigma',\quad \tau_n=\tau_{n-1}-\chi_n\frac{\partial\alpha_{n-1}}{\partial x_1}M,$$
$$\nu_n=\nu_{n-1}-\chi_n\frac{\partial\alpha_{n-1}}{\partial x_1}\omega_n,\quad \eta_n=\frac{\partial\alpha_{n-1}}{\partial x_1},$$
$$\sigma_n=\sigma_{n-1}-\chi_n\eta_n\tanh\left(\frac{\chi_n\eta_n}{\varsigma}\right).$$

Choose the actual control u_0 and parameters adaptive laws as

$$u_0=-c_n\chi_n-\frac{3}{4}\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2\chi_n+\frac{\partial\alpha_{n-1}}{\partial x_1}(\xi_{0,2}+\omega_2^T\hat{k}_2$$
$$+M^T\hat{\vartheta})+L_n\mu_1-\sum_{k=1}^{n-1}(\mathrm{A}_{k,n}+\Lambda_{k,n}+\mathrm{E}_{k,n})\chi_{k+1}\quad(67)$$
$$-H_n-\eta_n\hat{\delta}^*\tanh\left(\frac{\chi_n\eta_n}{\varsigma}\right),$$

$$\dot{\hat{\vartheta}}=\Gamma_1(\tau_n-\bar{\sigma}_2\hat{\vartheta}),\quad(68)$$

$$\dot{\hat{\delta}}^*=\gamma_2(\sigma_n-\bar{\sigma}_3\hat{\delta}^*),\quad(69)$$

$$\dot{\hat{k}}_2=\Gamma_2(\nu_n-\bar{\sigma}_4\hat{k}_2),\quad(70)$$

where $c_n>0$ is a design constant. Substituting (67)-(70) into (66) yields

$$\dot{V}_n\le-(\lambda_{\min}(Q)-(n+2))\|e\|^2-\sum_{q=1}^{n}c_q\chi_q^2+\bar{\sigma}_2\tilde{\vartheta}^T\hat{\vartheta}$$
$$+\bar{\sigma}_3\tilde{\delta}^*\hat{\delta}^*+\bar{\sigma}_4\tilde{k}_2{}^T\hat{k}_2+k_1\bar{\sigma}_\kappa\tilde{\kappa}\hat{\kappa}+D_n.\quad(71)$$

Using Young's inequality, we have

$$\dot{V}_n\le-(\lambda_{\min}(Q)-(n+2))\|e\|^2-\sum_{q=1}^{n}c_q\chi_q^2+D$$
$$-\frac{1}{2}(\bar{\sigma}_2\|\tilde{\vartheta}\|^2+\bar{\sigma}_3|\tilde{\delta}^*|^2+\bar{\sigma}_4\|\tilde{k}_2\|^2+k_1\bar{\sigma}_\kappa|\tilde{\kappa}|^2),\quad(72)$$

where

$$D=D_n+\frac{1}{2}(\bar{\sigma}_2\|\vartheta\|^2+\bar{\sigma}_3|\delta^*|^2+\bar{\sigma}_4\|k_2\|^2+k_1\bar{\sigma}_\kappa|\kappa|^2).$$

Let $\lambda_{\min}(Q)-(n+2)>0$, and define

$$C=\min\left\{\frac{\lambda_{\min}(Q)-(n+2)}{\lambda_{\max}(P)},2c_q,\frac{\bar{\sigma}_2}{\lambda_{\max}(\Gamma_1^{-1})},\frac{\bar{\sigma}_4}{\lambda_{\max}(\Gamma_2^{-1})},\gamma_1\bar{\sigma}_\kappa,\gamma_2\bar{\sigma}_3,q=1,\ldots,n\right\}.$$

Then (72) can be written as

$$\dot{V}_n\le-CV_n+D,\quad(73)$$

which implies that

$$V_n\le V_n(0)e^{-Ct}+D/C.\quad(74)$$

From (74), it can be concluded that for each $i=1,\ldots,n,$ the signals $\underline{x}_i$, $\underline{\hat{x}}_i$, χ_i, $\hat{\vartheta}$, $\hat{\delta}^*$, $\hat{\kappa}$, $\hat{k}_2$ and $u_0(u)$ are bounded, and that $|y(t)-y_r(t)|\le\sqrt{2V(0)}e^{-(C/2)t}+\sqrt{2D/C}$. As $t\to\infty$, $e^{-(C/2)t}\to0$, it follows that $|y(t)-y_r(t)|\le\sqrt{2D/C}$. Moreover, as stated in [12,17,19], the constant $\sqrt{2D/C}$ can be made as small as possible by choosing the positive definite matrix Q, and the design parameters c_q, γ_1, γ_2, $\bar{\sigma}_\kappa$, $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $\bar{\sigma}_3$, $q=1,\ldots,n.$

From the above design procedures and analysis, the following theorem is summarized as.

Theorem 1: For nonlinear system (1) with actuator fault (2), if Assumptions 1-2 are satisfied, the controller (67) with the filters (15), (16), (17) and parameter adaptive laws (48), (68), (69), and (70) can guarantee that all the signals in the closed-loop system remain bounded and the output error converges to a small neighborhood of the origin by choosing the design parameters appropriately.

5. SIMULATION STUDY

In this section, two examples are given to show the effectiveness of the proposed adaptive fuzzy fault-tolerant method.

Example 1 (Numerical example)**:** Consider the following uncertain nonlinear system:

$$\dot{x}_1=x_2+f_1(x_1)+d_1(t),$$
$$\dot{x}_2=f_2(x)+b_1\beta_1(y)u_1+b_2\beta_2(y)u_2+d_2(t),\quad(75)$$
$$y=x_1,$$

where $f_1(x_1)=\cos x_1^2$, $f_2(x)=e^{x_1}+x_2$, $b_1=3$, $\beta_1(y)=\beta_2(y)=1$, $d_1(t)=d_2(t)=\sin(t)$ and $b_2=5$.

Define fuzzy membership as follows:

$$\mu_{F_1^l}(\hat{x}_1)=\exp[-(\hat{x}_1-3+l)^2/4],\quad l=1,\cdots,5,$$
$$\mu_{F_2^l}(\hat{x}_1,\hat{x}_2)=\exp[-(\hat{x}_1-3+l)^2/1]\times\exp[-(\hat{x}_2-3+l)^2/4].$$

Specify the observer gain vector $L=[L_1,L_2]^T=[7,7]^T$, such that A_0 is a strict Hurwitz matrix.

Construct filters (15), (16), (17) and choose the control u_0 (67), the stabilizing control $\bar{\alpha}_1$ (44) and the adaptive laws $\hat{\kappa}$ (48), $\hat{\vartheta}$ (68), $\hat{\delta}^*$ (69) and $\hat{k}_2$ (70).

The design parameters in the controller and adaptation laws are chosen as $\bar{\sigma}_1 = \bar{\sigma}_2 = \bar{\sigma}_3 = \bar{\sigma}_\kappa = 0.9$, $c_1 = 4$, $c_2 = 4$, $\gamma_1 = 0.03$, $\gamma_2 = 0.4$, $\varsigma = 1$, $\Gamma_1 = 11I$, $\Gamma_2 = 2I$.

In this simulation, the actuator faults introduced for simulation are $u_1 = 0.2$ when $t \geq 15$. The tracking reference signal is chosen as $y_r(t) = \sin(t)$.

The initial conditions are chosen as $x_1(0) = 0.1$, $x_2(0) = 0.1$, $\xi_{0,1}(0) = 0.1$, $\xi_{0,2}(0) = 0.1$, $\hat{\vartheta}(0) = [0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$, and the other initial values are chosen as zeros. The simulation results are shown by Figs. 1 and 2, respectively.

From Figs. 1 and 2, it can be seen that the proposed fault-tolerant control method can guarantee that all the variables are bounded and the output $y(t)$ can track the given reference signal y_r.

Remark 1: It should be pointed out that adaptive fuzzy backstepping faults-tolerant control approaches have been recently developed by [22-24] for a class of uncertain nonlinear systems in strict-feedback form. However, these adaptive fuzzy faults-tolerant control methods are all based on the assumption that the states of the nonlinear systems are directly measured. Therefore,

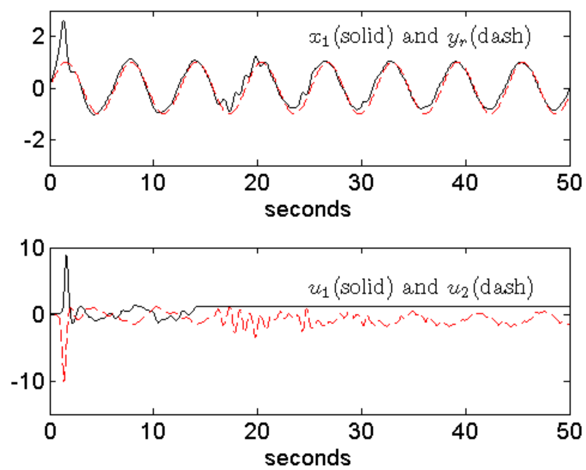

Fig. 1. The curves of x_1 (solid), y_r (dash) and curves of u_1 (solid), u_2 (dash).

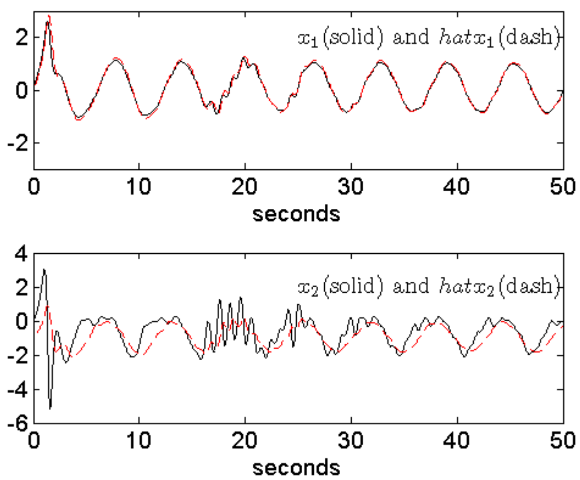

Fig. 2. The curves of x_i (solid), $\hat{x}_i$ (dash), $i = 1,2$.

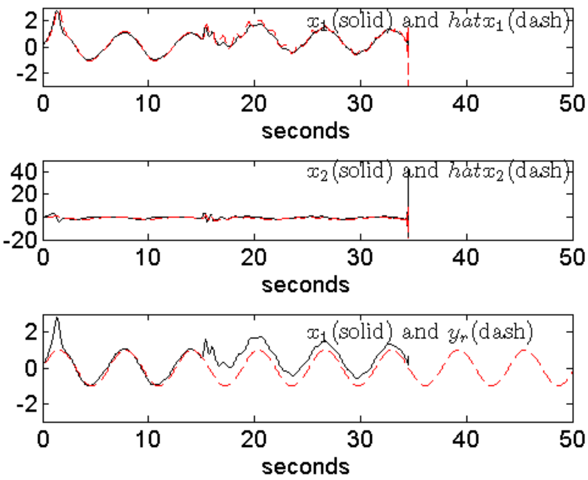

Fig. 3. The curves of x_i (solid), $\hat{x}_i$ (dash), $i = 1,2$; curves of x_1 (solid) and y_r (dash).

these approaches in [22-24] can not be implemented for the systems with actuator faults and immeasurable states.

In order to demonstrate the effectiveness of our fault-tolerant control scheme, the tracking, observer errors and control input curves of the proposed adaptive fuzzy control method are plotted in Fig. 3 without fault-tolerant technique. It can be seen that the closed-loop system becomes unstable.

Example 2 (Applied example) [32]**:** To further illustrate the effectiveness of the proposed adaptive fuzzy control approach. Let us apply the proposed adaptive control scheme to a pendulum system with disturbances. The equation of motion of the pendulum is given by:

$$ml\ddot{q} = -mg\sin q - kl\dot{q} = \frac{1}{l}u, \tag{76}$$

where $u \in IR$ is the torque applied to the pendulum, $q \in IR$ is the anticlockwise angle between the vertical axis through the pivot point and the rod, g is the gravity acceleration, and the constants k, l and m denote a coefficient of friction, the length of the rod, and the mass of the bob, respectively. It is assumed that the constants k, l and m are unknown. Let $x_1 = ml^2(q - \pi)$, $x_2 = ml^2\left(\dot{q} + \frac{k}{m}(q - \pi)\right)$. By choosing k, l and m so that $ml^2 = 1$, that is, $m = g^{-2}$, $k = g^{-2}$, and $l = g$.

The nonlinear system can be expressed as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1, \\ \dot{x}_2 &= b_1 u_1 + b_2 u_2 + \sin x_1, \\ y &= x_1. \end{aligned} \tag{77}$$

Define fuzzy membership as follows:

$$\mu_{F_1^l}(\hat{x}_1) = \exp[-(\hat{x}_1 - 2 + l)^2 / 6], \quad l = 1, \cdots, 5.$$

$$\mu_{F_2^l}(\hat{x}_1, \hat{x}_2) = \exp[-(\hat{x}_1 - 2 + l)^2 / 2] \times \exp[-(\hat{x}_2 - 2 + l)^2 / 6]$$

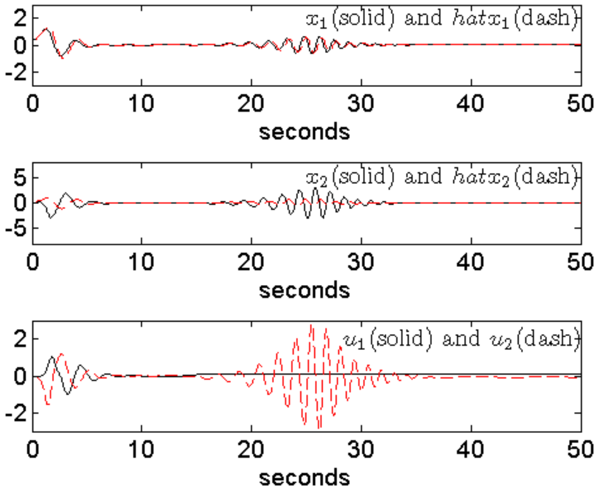

Fig. 4. The curves of x_i (solid), $\hat{x}_i$ (dash), $i=1,2$; curves of u_1 (solid) and u_2 (dash).

Given the reference signal $y_r(t)=0$, the actuator faults introduced for simulation are $u_1=0.2$ when $t\geq 15$. Construct filters (15), (16), (17) and choose the control u_0(67), the stabilizing control $\bar{\alpha}_1$(44) and the adaptive laws $\hat{\kappa}$(48), $\hat{\vartheta}$(68), $\hat{\delta}^*$(69) and $\hat{k}_2$(70).

The design parameters are chosen as $L_1=5$, $L_2=5$, $c_1=1$, $c_2=1$, $\gamma_1=0.04$, $\gamma_2=0.5$, $\varsigma=0.8$, $\Gamma_1=11I$, $\Gamma_2=2I$, $\bar{\sigma}_1=\bar{\sigma}_2=\bar{\sigma}_3=\bar{\sigma}_\kappa=0.7$.

In the simulation, the initial conditions are chosen as, $x_1(0)=0.314$, $x_2(0)=0$, $\hat{\vartheta}(0)=[0.5,0,0,0,0,0,0,0,0,0,0]^T$, and the other initial values are chosen as zeros. The simulation results are shown by Fig. 4, respectively.

From the above simulation studies, it is clearly know that the proposed adaptive fuzzy fault-tolerant control method can guarantee the all the variables are bounded and can achieve the good control performances even in the presence of the actuator faults, unknown functions and immeasurable states.

6. CONCLUSION

This paper has developed an adaptive fuzzy faults-tolerant control method for accommodating actuator faults in a class of unknown nonlinear systems with unmeasured states. The considered faults are modeled as the lock-in-place. With the help of fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy filter has been first established for estimating the unmeasured states. Based on the backstepping technique and the nonlinear tolerant-fault control theory, a novel adaptive fuzzy faults-tolerant control approach has been constructed. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded and the tracking error between the system output and the reference signal converges to a small neighborhood of zero by appropriate choice of the design parameters. The detailed simulation studies have been provided to show the effectiveness of the control approach.

REFERENCES

[1] B. Chen, X. P Liu, and P. Shi, "Direct adaptive fuzzy control for nonlinear systems with time-varying delays," *Inf. Sci.*, vol. 180, no. 5, pp. 776-792, 2010.
[2] B. Chen, X. P. Liu, K. F. Liu, and C. Lin, "Direct adaptive fuzzy control of nonlinear strict-feedback systems," *Automatica*, vol. 45, no. 6, pp. 1530-1535, 2009.
[3] M. Wang, B. Chen, X. P. Liu, and P. Shi, "Adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear time-delay systems," *Fuzzy Sets Syst.*, vol. 159, no. 8, pp. 949-967, 2008.
[4] M. Wang, B. Chen, and P. Shi, "Adaptive neural control for a class of perturbed strict-feedback nonlinear time-delay systems," *IEEE Trans. Syst., Man, Cybern. B*, vol. 38, no. 3, pp. 721-730, 2008.
[5] W. S. Chen, L. C. Jiao, R. H. Li, and J. Li, "Adaptive backstepping fuzzy control for nonlinearly parameterized systems with periodic disturbances," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 4, pp. 674-685, 2010.
[6] B. Chen and X. P. Liu, "Adaptive fuzzy output tracking control of MIMO nonlinear uncertain systems," *IEEE Trans. on Fuzzy Systems*, vol. 15, no. 2, pp. 287-300, 2007.
[7] T. S. Li, S. C. Tong, and G. Feng, "A novel robust adaptive-fuzzy-tracking control for a class of nonlinear MIMO systems," *IEEE Trans. on Fuzzy Systems*, vol. 18, no. 1, pp. 150-160, 2010.
[8] H. Lee, "Robust adaptive fuzzy control by backstepping for a class of MIMO nonlinear systems," *IEEE Trans. on Fuzzy Systems*, vol. 19, no. 2, pp. 265-275, 2011.
[9] C. C. Hua, Q. G. Wang, and X. P. Guan, "Adaptive fuzzy output-feedback controller design for nonlinear time-delay systems with unknown control direction," *IEEE Trans. Syst., Man, Cybern. B*, vol. 39, no. 2, pp. 363-374, 2009.
[10] S. C. Tong and Y. M. Li, "Observer-based fuzzy adaptive control for strict-feedback nonlinear systems," *Fuzzy Sets Syst.*, vol. 160, no. 12, pp. 1749-1764, 2009.
[11] S. C. Tong, C. L. Liu, and Y. M. Li, "Robust adaptive fuzzy filters output feedback control of strict-feedback nonlinear systems," *Int. J. Appl. Math. Comput. Sci.*, vol. 20, no. 4, pp. 637-653, 2010.
[12] S. C. Tong, C. Y. Li, and Y. M. Li, "Fuzzy adaptive observer backstepping control for MIMO nonlinear systems," *Fuzzy Sets Syst.*, vol. 160, no. 19, pp. 2755-2775, 2009.
[13] S. C. Tong, G. J. Yang, and W. Zhang, "Observer-based fuzzy fault-tolerant control for fuzzy systems with time delays," *Int. J. Appl. Math. Comput. Sci.*, vol. 21, no. 4, pp. 617-627, 2011.
[14] D. Xu, B. Jiang, and P. Shi, "Nonlinear actuator fault estimation observer: An inverse system approach via a T-S fuzzy model," *Int. J. Appl. Math. Comput. Sci.*, vol. 22, no. 1, pp. 183-196, 2012.
[15] D. Ye and G. Yang, "Adaptive fault-tolerant track-

ing control against actuator faults with application to flight control," *IEEE Trans. on Control Systems Technology*, vol. 14, no. 6, pp. 1088-1096, 2006.
[16] G. Yang and D. Ye, "Adaptive fault-tolerant H_∞ control via state feedback for linear systems against actuator faults," *Proc. of the IEEE Conference on Decision and Control*, pp. 3530-3535, 2006.
[17] G. Tao, S. M. Joshi, and X. L. Ma, "Adaptive state feedback and tracking control of systems with actuator failures," *IEEE Trans. Autom. Contr.*, vol. 46, no. 1, pp. 78-95, Jan. 2001.
[18] X. D. Tang, G. Tao, and S. M. Joshi, "An adaptive control scheme for output feedback nonlinear systems with actuator failures," *Proc. of the 15th IFAC World Congress*, T-Tu-A03, July, 2002, Barcelona, Spain.
[19] X. Tang, G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application," *Automatica*, vol. 43, no. 11, pp. 1869-1883, 2007.
[20] M. Lingya and B. Jiang, "Backstepping-based active fault-tolerant control for a class of uncertain SISO nonlinear systems," *Journal of Systems Engineering and Electronic*, vol. 20, no. 6, pp. 1263-1270, 2009.
[21] B. Jiang, Z. F. Gao, and P. Shi, "Observer-based integrated robust fault estimation and accommodation design for discrete-time systems," *International Journal of Control*, vol. 83, no. 6, pp. 1167-1181, 2010.
[22] P. Li and G. Yang, "Backstepping adaptive fuzzy control of uncertain nonlinear systems against actuator faults," *Control Theory Appl.*, vol. 7, no. 3, pp. 248-256, 2009.
[23] P. Li and G. Yang, "Adaptive fuzzy control of unknown nonlinear systems with actuator failures for robust output tracking," *Proc. of the American Control Conference*, pp. 4898-4903, 2008.
[24] P. Li and G. Yang, "An adaptive fuzzy design for fault-tolerant control of MIMO nonlinear uncertain systems," *Control Theory Appl.*, vol. 9, no. 2, pp. 244-250, 2011.
[25] S. C. Tong, T. C. Wang, and W. Zhang, "Fault tolerant control for uncertain fuzzy systems with actuator failures," *International Journal of Innovative Computing, Information and Control*, vol. 4, no. 10, pp. 2461-2474, 2008.
[26] H. Wu and M. Z. Bai, "Stochastic stability analysis and synthesis for nonlinear fault tolerant control systems based on T-S fuzzy model," *International Journal of Innovative Computing, Information and Control*, vol. 6, no. 9, pp. 3989-4000, 2010.
[27] K. Zhang, B. Jiang, and P. Shi, "Fault estimation observer design for discrete-time Takagi-Sugeno fuzzy systems based on piecewise Lyapunov functions," *IEEE Trans. on Fuzzy Systems*, vol. 20, no. 1, pp. 192-200, 2012.
[28] K. Nguang, P. Shi, and S. Ding, "Fault detection for uncertain fuzzy systems: an LMI approach," *IEEE Trans on Fuzzy Systems*, vol. 15, no. 6, pp. 1251-1262, 2007.
[29] W. J. Chang, C. C. Ku, and P. H. Huang, "Robust fuzzy control via observer feedback for passive stochastic fuzzy systems with time-delay and multiplicative noise," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 1, pp. 345-364, 2011.
[30] L. X. Wang, *Adaptive Fuzzy Systems and Control*, Prentice Hall, Englewood Cliffs, NJ, 1994.
[31] H. G. Zhang and Y. B. Quan, "Modeling, identification, and control of a class of nonlinear systems," *IEEE Trans. on Fuzzy Systems*., vol. 9, no. 2, pp. 349-354, 2001.
[32] Z. P. Jiang and D. J. Dill, "A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics," *IEEE Trans. Autom. Contr.*, vol. 44, no. 9, pp. 1705-1711, 1999.

Baoyu Huo received her BS degree from Mathematics & Information Science College, Hebei Normal University, Shijiazhuang, China, in 2010. She is now working towards an ME degree in Applied Mathematics, from Liaoning University of Technology, Jinzhou, China. Her current research interests include nonlinear adaptive control, fuzzy control and FTC.

Shaocheng Tong received his BA degree in Mathematics from Jinzhou Normal College, Jinzhou, China, an MA degree in Fuzzy Mathematics from Dalian Marine University, PRC, and a Ph.D. degree in Fuzzy Control from Northeastern University, PRC, in 1982, 1988 and 1997, respectively. Currently, he is a Professor in the Department of Basic Mathematics, Liaoning University of Technology, Jinzhou, PRC. His research interests include fuzzy control theory, nonlinear adaptive control, and intelligent control.

Yongming Li received his BS and MS degrees in Applied Mathematics from Liaoning University of Technology, Jinzhou, China, in 2004 and 2007, respectively. He is currently a lecturer in the Department of Basic Mathematics, Liaoning University of Technology. His current research interests include fuzzy and neural networks control and nonlinear adaptive control.