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Subspace Identification for Fractional Order Hammerstein Systems Based on Instrumental Variables

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Abstract: This paper focuses on time-domain identification issues of multi-input multi-output (MIMO) fractional order Hammerstein systems which are the extension of traditional Hammerstein type models by allowing linear part to be fractional order systems. The principal component analysis (PCA) method in subspace family is extended to identify coefficient matrixes of fractional order systems. Singular value decomposition (SVD) is utilized to estimate the unknown parameters of nonlinear part of system directly. A proper instrumental variable is chosen to eliminate the bias of identification results. Numerical simulation validates the proposed method.

Keywords: Fractional order Hammerstein systems, instrumental variables, subspace identification.

1. INTRODUCTION

Fractional order calculus appears in three hundred years ago. However, due to the lack of physical and mechanical background, fractional order calculus is just considered as a pure mathematical theory and studied by mathematicians in the past years. With the development of science and technology, it has been shown that many real systems can be described or modeled more accurately by using fractional order calculus than traditional integer order calculus in recent years [1-4]. Therefore, the researches of fractional order calculus attract lots of attention. In the system control field, the identification issue of fractional order systems has become a hot spot.

Identification of fractional order systems was initiated in the late nineties. In [5], a method based on fractional order model discretization and least square (LS) was proposed. In [6,7], modal fractional models based on the diagonal representation form were skillfully used and the parameters to-be-identified were transformed into modal parameters. In [8], an instrumental variable was designed and used in identification process. In [9,10], a subspace identification method called multi-input multi-output output-error state space (MOESP) was extended to identifying fractional order systems. In [11], an overview of the identification issues of fractional order systems was presented. There have been several achievements for identifying common fractional order systems already, but less for systems with nonlinear part, such as fractional order Hammerstein systems which are the extension of traditional Hammerstein type models with linear part being fractional order systems. The identification issue of fractional order Hammerstein systems was firstly discussed in [12]. However, the proposed method is only suitable for single input single output (SISO) fractional order Hammerstein systems.

In this paper, we firstly discussed the identification issue of multi-input multi-output (MIMO) fractional order Hammerstein systems. A subspace method based on principal component analysis (PCA) is extended to identify coefficient matrixes of fractional order systems. Singular value decomposition (SVD) is used to estimate the unknown parameters of nonlinear part of system directly. A proper instrumental variable is chosen to eliminate the bias of identification results. Numerical simulation validates the proposed methods.

The whole paper is organized as follows: In Section 2, the mathematical background of fractional order calculus and fractional order Hammerstein systems are introduced. In Section 3, the identification method based on subspace orthogonal projection identification method via PCA (SOPIM+PCA) is proposed. In Section 4, a numerical simulation example is given to validate the proposed method. In Section 5, the conclusion is drawn finally.

2. MATHEMATICAL BACKGROUND

2.1. Fractional order calculus

Several definitions of fractional order differentiation or integration which are appropriate for system modeling and control will be introduced in this section.

The Caputo's definition is widely used in theoretical analysis of control systems, because the Laplace transform of this definition allows utilization of initial values of classical integer order derivatives with clear physical interpretations [1]. The Caputo's definition can be ex-

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pressed as follows [2]

$$ {}_a\mathcal{D}_t^{\gamma} f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\gamma)} \displaystyle\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\gamma+1-n}} d\tau, & (n-1 < \gamma < n) \\ \dfrac{1}{\Gamma(-\gamma)} \displaystyle\int_a^t \frac{f(\tau)}{(t-\tau)^{\gamma+1}} d\tau, & (\gamma < 0), \end{cases} \tag{1} $$

where $n \in \mathbb{N}$. γ is the fractional differential order or fractional integral order. ${}_a\mathcal{D}_t^{\gamma}$ denotes fractional order differential operator if $\gamma > 0$ or fractional order integral operator if $\gamma < 0$. a is the initial time. In general, the initial time is zeros and the operator can be written as $\mathcal{D}^{\gamma}$.

The Caputo's definition can not be easily calculated because it involves the calculation of high order derivative and integration. The Grünwald-Letnikov's definition is derived from traditional integer order differentiation and widely used in numerical calculation. The Grünwald-Letnikov's definition can be expressed as follows [1]:

$$ {}_a\mathcal{D}_t^{\gamma} f(t) = \lim_{h \to 0} \frac{1}{h^{\gamma}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} \omega_j^{(\gamma)} f(t - jh), \tag{2} $$

where $\omega_j^{(\gamma)} = \dfrac{(-1)^j \Gamma(\gamma+1)}{\Gamma(j+1)\Gamma(\gamma-j+1)}$, $[x]$ denotes the biggest integer which is no more than x. $\Gamma(\bullet)$ is the Gamma function defined by

$$ \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt. \tag{3} $$

A proper sample time T_s is usually chosen as the computational step. Define $t_k \triangleq kT_s$, $f(k) \triangleq f(kT_s)$, and the numerical calculation formula of fractional order differentiation or integration can be expressed as follows:

$$ \mathcal{D}^{\gamma} f(k) = \frac{1}{T_s^{\gamma}} \sum_{j=0}^{k} \omega_j^{(\gamma)} f(k-j). \tag{4} $$

From the definitions mentioned above, several properties of fractional order calculus can be easily deduced.

1) The fractional order operator is linear, that is

$$ \mathcal{D}^{\gamma} \left(\lambda f(t) + \mu g(t) \right) = \lambda \mathcal{D}^{\gamma} f(t) + \mu \mathcal{D}^{\gamma} g(t), \ \forall \lambda, \ \mu. \tag{5} $$

2) For fractional order integral, we have

$$ \mathcal{D}^{-\gamma_1} \mathcal{D}^{-\gamma_2} f(t) = \mathcal{D}^{-\gamma_2} \mathcal{D}^{-\gamma_1} f(t) = \mathcal{D}^{-(\gamma_1+\gamma_2)} f(t), \tag{6} $$

where $\gamma_1 > 0, \ \gamma_2 > 0$.

3) The Laplace transform is as follows:

$$ \mathcal{L}\{\mathcal{D}^{-\gamma} f(t)\} = s^{-\gamma} F(s), \tag{7} $$

where $\gamma > 0, \ \mathcal{L}\{f(t)\} = F(s)$.

2.2. Fractional order Hammerstein systems

The equation of linear time-invariant fractional order system can be written as follows [2]:

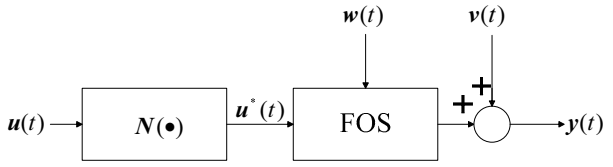

Fig. 1. Fractional order Hammerstein system.

$$ \sum_{k=1}^{n} a_k \mathcal{D}^{\gamma_k} y(t) = \sum_{k=1}^{m} b_k \mathcal{D}^{\bar{\gamma}_k} u(t). \tag{8} $$

If $\gamma_k, \ \bar{\gamma}_k = k\gamma, \ \gamma \in \mathbb{R}^+$, equation (8) can be simplified to the particular case referred to as commensurate fractional order system. γ is the commensurate fractional differential order. Consider a commensurate MIMO fractional order system. The state space equation can be described as follows [13]:

$$ \begin{cases} \mathcal{D}^{\gamma} \boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}^*(t) + \boldsymbol{w}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}^*(t) + \boldsymbol{v}(t), \end{cases} \tag{9} $$

where $0 < \gamma < 2$, $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, $\boldsymbol{B} \in \mathbb{R}^{n \times m}$, $\boldsymbol{C} \in \mathbb{R}^{l \times n}$, $\boldsymbol{D} \in \mathbb{R}^{l \times m}$. $\boldsymbol{w}(t)$ and $\boldsymbol{v}(t)$ are zero-mean Gaussian random noises.

The structure of fractional order Hammerstein system is shown in Fig. 1. It consists of a static nonlinear block $\boldsymbol{N}$ connected to a commensurate fractional order system. $\boldsymbol{N}(\bullet)$ is a nonlinear function which maps $\mathbb{R}^m \to \mathbb{R}^m$ and can be described by a linear combination of basis functions in the form [14]

$$ \boldsymbol{u}^*(t) = \boldsymbol{N}\left(\boldsymbol{u}(t)\right) = \sum_{k=1}^{r} \boldsymbol{\alpha}_k \boldsymbol{\varphi}_k\left(\boldsymbol{u}(t)\right), \tag{10} $$

where $r \in \mathbb{N}$, $\boldsymbol{\alpha}_k \in \mathbb{R}^{m \times m}$, $\boldsymbol{\varphi}_k(\bullet) : \mathbb{R}^m \to \mathbb{R}^m (k = 1, 2, ..., r)$.

For system (9) and (10), we make several assumptions as follows

A1) The input signal $\boldsymbol{u}(t)$ is persistently exciting.

A2) The fractional order system (9) is observable and controllable.

A3) The dimension of coefficient matrix $\boldsymbol{A}$ is known, that is rank($\boldsymbol{A}$) = n.

A4) The random noises $\boldsymbol{w}(t)$, $\boldsymbol{v}(t)$ are irrelevant to the input signals $\boldsymbol{u}(t)$ and $\boldsymbol{\varphi}_k(\boldsymbol{u}(t))$.

A5) The basis functions $\boldsymbol{\varphi}_k$ are known and the coefficient matrix $\boldsymbol{\alpha}_1$ is assumed to be identity matrix.

The problem studied in this paper is that determining coefficient matrixes $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$, fractional differential order γ and coefficient parameters $\boldsymbol{\alpha}_k$ from a group of input and output data $\{\boldsymbol{u}(k), \boldsymbol{y}(k)\}$, $k = 1, 2, ..., N$ in the presence of random noises disturbed.

Several notations are given to make subsequent analysis convenient. $\| \bullet \|_F$ denotes the Frobenius norm. Adopting the MATAB notation, $\boldsymbol{X}(r,:)$ and $\boldsymbol{X}(:,k)$ denote respectively the rth row and the kth column of $\boldsymbol{X}$. Similarly, $\boldsymbol{X}(i:j,:)$ and $\boldsymbol{X}(:,i:j)$ denote respectively the rows of $\boldsymbol{X}$ from the ith row to jth row and the columns

from the *i*th column to *j*th column. The orthogonal projection of the row space of $\boldsymbol{X}$ onto the row space of $\boldsymbol{Y}$ is defined as follows:

$$\boldsymbol{X}/\boldsymbol{Y} = \boldsymbol{X}\Pi_{\boldsymbol{Y}} = \boldsymbol{X}\boldsymbol{Y}^T(\boldsymbol{Y}\boldsymbol{Y}^T)^{-1}\boldsymbol{Y}, \tag{11}$$

where $\boldsymbol{\Pi}_{\boldsymbol{Y}}$ is the orthogonal projection operator.

The orthogonal projection of the row space of $\boldsymbol{X}$ onto the orthogonal complement of row space of $\boldsymbol{Y}$ is defined as follows:

$$\boldsymbol{X}/\boldsymbol{Y}^{\perp} = \boldsymbol{X}\Pi_{\boldsymbol{Y}}^{\perp}, \tag{12}$$

where $\boldsymbol{Y}^{\perp}$ denotes the orthogonal complement of row space of $\boldsymbol{Y}$ and $\Pi_{\boldsymbol{Y}}^{\perp} = \boldsymbol{I} - \Pi_{\boldsymbol{Y}}$.

3. IDENTIFICATION METHOD

3.1. Subspace method

Substituting (10) into (9), the state space equation of fractional order Hammerstein system can be written as follows:

$$\begin{cases} \mathcal{D}^{\gamma}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\sum_{k=1}^{r}\alpha_k\varphi_k\left(\boldsymbol{u}(t)\right) + \boldsymbol{w}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\sum_{k=1}^{r}\alpha_k\varphi_k\left(\boldsymbol{u}(t)\right) + \boldsymbol{v}(t). \end{cases} \tag{13}$$

Due to the existence of fractional differential order γ, the output of system is nonlinear with respect to parameters space. Therefore, several changes should be made to traditional subspace method in system identification. Consider the case fractional differential order γ known and make the following definitions:

$$\boldsymbol{\alpha} \triangleq \left[\alpha_1, \alpha_2, \cdots, \alpha_r\right],$$

$$\tilde{\boldsymbol{u}}(t) \triangleq \left[\varphi_1\left(\boldsymbol{u}(t)\right)^T, \varphi_2\left(\boldsymbol{u}(t)\right)^T, \cdots, \varphi_r\left(\boldsymbol{u}(t)\right)^T\right]^T.$$

Thus,

$$\begin{cases} \mathcal{D}^{\gamma}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{\alpha}\tilde{\boldsymbol{u}}(t) + \boldsymbol{w}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{\alpha}\tilde{\boldsymbol{u}}(t) + \boldsymbol{v}(t). \end{cases} \tag{14}$$

From the Caputo's definition, it is concluded that the term $\mathcal{D}^{-(j+1)\gamma}\boldsymbol{w}(t)$ exists. Thus, (14) can be rewritten as follows:

$$\begin{aligned} \boldsymbol{x}(t) = \boldsymbol{A}^k\mathcal{D}^{-k\gamma}\boldsymbol{x}(t) + \sum_{j=0}^{k-1}\boldsymbol{A}^j\boldsymbol{B}\boldsymbol{\alpha}\mathcal{D}^{-(j+1)\gamma}\tilde{\boldsymbol{u}}(t) \\ + \sum_{j=0}^{k-1}\boldsymbol{A}^j\mathcal{D}^{-(j+1)\gamma}\boldsymbol{w}(t) \quad (k \geq 1). \end{aligned} \tag{15}$$

Therefore, we have

$$\begin{aligned} \mathcal{D}^{-(i-1)\gamma}\boldsymbol{y}(t) = \boldsymbol{C}\mathcal{D}^{-(i-1)\gamma}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{\alpha}\mathcal{D}^{-(i-1)\gamma}\tilde{\boldsymbol{u}}(t) \\ + \mathcal{D}^{-(i-1)\gamma}\boldsymbol{v}(t) \\ \vdots \end{aligned}$$

$$\begin{aligned} \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{A}^{i-1}\mathcal{D}^{-(i-1)\gamma}\boldsymbol{x}(t) + \sum_{j=0}^{i-2}\boldsymbol{C}\boldsymbol{A}^j\boldsymbol{B}\boldsymbol{\alpha}\mathcal{D}^{-(j+1)\gamma}\tilde{\boldsymbol{u}}(t) \\ + \sum_{j=0}^{i-2}\boldsymbol{C}\boldsymbol{A}^j\mathcal{D}^{-(j+1)\gamma}\boldsymbol{w}(t) + \boldsymbol{D}\boldsymbol{\alpha}\tilde{\boldsymbol{u}}(t) + \boldsymbol{v}(t). \end{aligned}$$

The equations above can be written into matrix forms which is

$$\boldsymbol{Y}_i(t) = \Gamma_i\mathcal{D}^{-(i-1)\gamma}\boldsymbol{x}(t) + \boldsymbol{H}_i\tilde{\boldsymbol{U}}_i(t) + \boldsymbol{G}_i\boldsymbol{W}_i(t) + \boldsymbol{V}_i(t), \tag{16}$$

where Γ_i is the extended observable matrix and i is a fixed parameter with $i \geq n$,

$$\Gamma_i = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{i-1} \end{bmatrix}, \quad \boldsymbol{Y}_i(t) = \begin{bmatrix} \mathcal{D}^{-(i-1)\gamma}\boldsymbol{y}(t) \\ \mathcal{D}^{-(i-2)\gamma}\boldsymbol{y}(t) \\ \vdots \\ \boldsymbol{y}(t) \end{bmatrix},$$

$$\tilde{\boldsymbol{U}}_i(t) = \begin{bmatrix} \mathcal{D}^{-(i-1)\gamma}\tilde{\boldsymbol{u}}(t) \\ \mathcal{D}^{-(i-2)\gamma}\tilde{\boldsymbol{u}}(t) \\ \vdots \\ \tilde{\boldsymbol{u}}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{D}^{-(i-1)\gamma}\varphi_1\left(\mathrm{u}(t)\right) \\ \vdots \\ \mathcal{D}^{-(i-1)\gamma}\varphi_r\left(\boldsymbol{u}(t)\right) \\ \hdashline \mathcal{D}^{-(i-2)\gamma}\varphi_1\left(\boldsymbol{u}(t)\right) \\ \vdots \\ \mathcal{D}^{-(i-2)\gamma}\varphi_r\left(\boldsymbol{u}(t)\right) \\ \hdashline \vdots \\ \hdashline \varphi_1\left(\boldsymbol{u}(t)\right) \\ \vdots \\ \varphi_r\left(\boldsymbol{u}(t)\right) \end{bmatrix},$$

$$\boldsymbol{H}_i = \begin{bmatrix} \boldsymbol{D}\boldsymbol{\alpha} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{C}\boldsymbol{B}\boldsymbol{\alpha} & \boldsymbol{D}\boldsymbol{\alpha} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}\boldsymbol{A}^{i-2}\boldsymbol{B}\boldsymbol{\alpha} & \boldsymbol{C}\boldsymbol{A}^{i-3}\boldsymbol{B}\boldsymbol{\alpha} & \cdots & \boldsymbol{D}\boldsymbol{\alpha} \end{bmatrix},$$

$$\boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{C} & \boldsymbol{I} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}\boldsymbol{A}^{i-2} & \boldsymbol{C}\boldsymbol{A}^{i-3} & \cdots & \boldsymbol{I} \end{bmatrix}.$$

The matrix layout of $\boldsymbol{W}_i(t)$, $\boldsymbol{V}_i(t)$ are similar to $\boldsymbol{Y}_i(t)$.

Choose the proper sample time T_s and set $t_k = kT_s$, $k = 1, 2, ..., N$, $\boldsymbol{y}(k) \triangleq \boldsymbol{y}(t_k)$, $\boldsymbol{u}(k) \triangleq \boldsymbol{u}(t_k)$, we can obtain

$$\begin{aligned} \boldsymbol{Y}_{i,N-k+1} = \Gamma_i\boldsymbol{X}_{N-k+1} + \boldsymbol{H}_i\tilde{\boldsymbol{U}}_{i,N-k+1} \\ + \boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1} + \boldsymbol{V}_{i,N-k+1}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \boldsymbol{X}_{N-k+1} = [\mathcal{D}^{-(i-1)\alpha}\boldsymbol{x}(k) \;\; \mathcal{D}^{-(i-1)\alpha}\boldsymbol{x}(k+1) \;\; \cdots \\ \mathcal{D}^{-(i-1)\alpha}\boldsymbol{x}(N)], \end{aligned}$$

$$\boldsymbol{Y}_{i,N-k+1} = [\boldsymbol{Y}_i(k) \;\; \boldsymbol{Y}_i(k+1) \;\; \cdots \;\; \boldsymbol{Y}_i(N)] \in \mathbb{R}^{li\times(N-k+1)},$$

$$\tilde{\boldsymbol{U}}_{i,N-k+1} = [\tilde{\boldsymbol{U}}_i(k) \;\; \tilde{\boldsymbol{U}}_i(k+1) \;\; \cdots \;\; \tilde{\boldsymbol{U}}_i(N)] \in \mathbb{R}^{rmi\times(N-k+1)}.$$

The matrix layout of $\boldsymbol{W}_{i,N-k+1}$, $\boldsymbol{V}_{i,N-k+1}$ are similar to $\boldsymbol{Y}_{i,N-k+1}$.

Indeed, (17) can be rewritten as another form

$$\begin{aligned}&\left[\boldsymbol{I} \mid -\boldsymbol{H}_i\right]\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\\ &\quad=\boldsymbol{\Gamma}_i\boldsymbol{X}_{N-k+1}+\boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}+\boldsymbol{V}_{i,N-k+1}.\end{aligned}\tag{18}$$

Pre-multipying (18) with $(\Gamma_i^{\perp})^T$, the orthogonal complement of $\boldsymbol{\Gamma}_i$ with full column rank [15], and moving the input matrix term, (18) becomes

$$\begin{aligned}&\left[\left(\Gamma_i^{\perp}\right)^T \mid -\left(\Gamma_i^{\perp}\right)^T\boldsymbol{H}_i\right]\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\\ &\quad=\left(\Gamma_i^{\perp}\right)^T\boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}+\left(\Gamma_i^{\perp}\right)^T\boldsymbol{V}_{i,N-k+1}.\end{aligned}\tag{19}$$

The main information of fractional order Hammerstein system is contained in matrix $\left[(\Gamma_i^{\perp})^T \mid -(\Gamma_i^{\perp})^T\boldsymbol{H}_i\right]$ from which a group of basis of the extended observable matrix can be determined directly. Consider the case that there is no random noise disturbed in fractional order Hammerstein system, and (19) can be written as

$$\left[\left(\Gamma_i^{\perp}\right)^T \mid -\left(\Gamma_i^{\perp}\right)^T\boldsymbol{H}_i\right]\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]=0.\tag{20}$$

Therefore, $\left[(\Gamma_i^{\perp})^T \mid -(\Gamma_i^{\perp})^T\boldsymbol{H}_i\right]$ is in the left null space of $\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]$. Utilizing PCA method, we can obtain

$$\left[\begin{array}{c}\Gamma_i^{\perp}\\ \hline -\boldsymbol{H}_i^T\Gamma_i^{\perp}\end{array}\right]=\boldsymbol{K}\boldsymbol{Q}=\left[\begin{array}{c}\boldsymbol{K}_1\\ \hline \boldsymbol{K}_2\end{array}\right]\boldsymbol{Q},\tag{21}$$

where $\boldsymbol{K}$ is the loading matrix with submatrices $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$. $\boldsymbol{Q}$ is non-singular matrix which usually be chosen as identity matrix.

The loading matrix $\boldsymbol{K}$ can be estimated by using SVD, that is

$$\begin{bmatrix}\boldsymbol{Y}_{i,N-k+1}\\ \tilde{\boldsymbol{U}}_{i,N-k+1}\end{bmatrix}=\begin{bmatrix}\bar{\boldsymbol{U}}_1 & \bar{\boldsymbol{U}}_2\end{bmatrix}\begin{bmatrix}\Sigma_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\bar{\boldsymbol{V}}_1\\ \bar{\boldsymbol{V}}_2\end{bmatrix}.\tag{22}$$

The rank of $\boldsymbol{\Sigma}_1$ should be $li+n$ without random noises disturbed [16]. Therefore, we can obtain $\boldsymbol{K}=\bar{\boldsymbol{U}}_2$. Then, (21) can be written as

$$\hat{\boldsymbol{\Gamma}}_i=\boldsymbol{K}_1^{\perp},\tag{23}$$

$$\hat{\boldsymbol{H}}_i=\left(\boldsymbol{K}_2\boldsymbol{K}_1^{\perp}\right)^T.\tag{24}$$

From (23), the coefficient matrixes $\boldsymbol{A}$ and $\boldsymbol{C}$ can be easily determined.

$$\hat{\boldsymbol{C}}=\hat{\boldsymbol{\Gamma}}_i(1:l,:),\tag{25}$$

$$\hat{\boldsymbol{A}}=\hat{\boldsymbol{\Gamma}}_i^{+}\left(1:end-l,:\right)\hat{\boldsymbol{\Gamma}}_i\left(l+1:end,:\right).\tag{26}$$

From (24), we can obtain $\widehat{\boldsymbol{B}\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{D}\boldsymbol{\alpha}}$ directly, that is

$$\begin{bmatrix}\boldsymbol{l}_1 & \boldsymbol{l}_2 & \cdots & \boldsymbol{l}_i\\ \boldsymbol{l}_2 & \boldsymbol{l}_3 & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{l}_i & \boldsymbol{0} & \cdots & \boldsymbol{0}\end{bmatrix}\left[\begin{array}{c|c}\boldsymbol{I} & \boldsymbol{0}\\ \hline \boldsymbol{0} & \hat{\boldsymbol{\Gamma}}_i\left(1:l(i-1)\right)\end{array}\right]\begin{bmatrix}\widehat{\boldsymbol{D}\boldsymbol{\alpha}}\\ \widehat{\boldsymbol{B}\boldsymbol{\alpha}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{h}_1\\ \boldsymbol{h}_2\\ \vdots\\ \boldsymbol{h}_i\end{bmatrix},\tag{27}$$

where

$$-\boldsymbol{K}_1^T=[\boldsymbol{l}_1 \quad \boldsymbol{l}_2 \quad \cdots \quad \boldsymbol{l}_i],\ \boldsymbol{l}_k\in\mathbb{R}^{(li-m)\times l},$$

$$\boldsymbol{K}_2^T=[\boldsymbol{h}_1 \quad \boldsymbol{h}_2 \quad \cdots \quad \boldsymbol{h}_i],\ \boldsymbol{h}_k\in\mathbb{R}^{(li-m)\times m},\ k=1,2,...,i.$$

Therefore, the least square solution of $\boldsymbol{B}\boldsymbol{\alpha}$, $\boldsymbol{D}\boldsymbol{\alpha}$ can be calculated respectively.

In order to determine coefficient matrixes $\boldsymbol{B}$, $\boldsymbol{D}$, $\boldsymbol{\alpha}$ accurately, we consider this identification problem as an optimization problem of the form [17]

$$\left(\hat{\boldsymbol{B}},\hat{\boldsymbol{D}},\hat{\boldsymbol{\alpha}}\right)=\arg\min_{\boldsymbol{B},\boldsymbol{D},\boldsymbol{\alpha}}\left\{\left\|\begin{bmatrix}\widehat{\boldsymbol{B}\boldsymbol{\alpha}}\\ \widehat{\boldsymbol{D}\boldsymbol{\alpha}}\end{bmatrix}-\begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{D}\end{bmatrix}\boldsymbol{\alpha}\right\|_2^2\right\}.\tag{28}$$

The problem can be solved by using the SVD method, that is

$$\begin{bmatrix}\widehat{\boldsymbol{B}\boldsymbol{\alpha}}\\ \widehat{\boldsymbol{D}\boldsymbol{\alpha}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{U}_{s_1} & \boldsymbol{U}_{s_2}\end{bmatrix}\begin{bmatrix}\Sigma_{s_1} & \boldsymbol{0}\\ \boldsymbol{0} & \Sigma_{s_2}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_{s_1}^T\\ \boldsymbol{V}_{s_2}^T\end{bmatrix}.\tag{29}$$

The identified results of $\boldsymbol{\alpha}$ should be unique under assumption A5. Therefore, we have

$$\hat{\boldsymbol{\alpha}}=\left(\boldsymbol{V}_{s_1}^T(1:m,1:m)\right)^{-1}\boldsymbol{V}_{s_1}^T,\tag{30}$$

$$\begin{bmatrix}\hat{\boldsymbol{B}}\\ \hat{\boldsymbol{D}}\end{bmatrix}=\boldsymbol{U}_{s_1}\Sigma_{s_1}\left(\boldsymbol{V}_{s_1}^T(1:m,1:m)\right)^{-1}.\tag{31}$$

Consider the case which fractional differential order γ is unknown. Nonlinear optimization techniques can be used to estimate the fractional differential order. We can choose the quadratic criterion as follows:

$$\hat{\gamma}=\arg\min_{0<\gamma<2}\boldsymbol{J}(\gamma)=\arg\min_{0<\gamma<2}\frac{\left\|\hat{\boldsymbol{y}}(\gamma)-\boldsymbol{y}\right\|_F^2}{\left\|\boldsymbol{y}\right\|_F^2},\tag{32}$$

where $\hat{\boldsymbol{y}}(\alpha)$ is the output of identified model and $\boldsymbol{y}$ is the output of original model. From (32), the estimation of fractional differential order is transformed into nonlinear optimization problem. As a matter of convenience, we choose traversal method to estimate the fractional differential order.

3.2. Instrumental variables

Actually, (20) can not be established because of the existence of random noises disturbed in fractional order Hammerstein system. The estimation results are usually biased without considering the influence of random noises. Instrumental variables method is often used to elimi-

nate this bias. An instrumental variable $\boldsymbol{Z}$ based on past inputs is chosen in this paper, where $p < k$, $p \in \mathbb{N}$ and it can be chosen as $p = k - 1$.

Utilizing orthogonal projection, (19) can be written as

$$\begin{aligned}&\left[\left(\Gamma_i^{\perp}\right)^T \;\middle|\; -\left(\Gamma_i^{\perp}\right)^T \boldsymbol{H}_i\right]\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hdashline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\boldsymbol{\Pi}_{\boldsymbol{Z}}\\ &\quad=\left(\Gamma_i^{\perp}\right)^T \boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}\boldsymbol{\Pi}_{\boldsymbol{Z}}+\left(\Gamma_i^{\perp}\right)^T \boldsymbol{V}_{i,N-k+1}\boldsymbol{\Pi}_{\boldsymbol{Z}}.\end{aligned} \tag{33}$$

Calculate the first term at the right side of (33), we can obtain

$$\begin{aligned}&\left(\Gamma_i^{\perp}\right)^T \boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}\boldsymbol{\Pi}_{\boldsymbol{Z}}\\ &\quad=\left(\Gamma_i^{\perp}\right)^T \boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}\boldsymbol{Z}^T\left(\boldsymbol{Z}\boldsymbol{Z}^T\right)^{-1}\boldsymbol{Z},\end{aligned}$$

$$\boldsymbol{Z}=\begin{bmatrix}\tilde{\boldsymbol{u}}(k-p) & \tilde{\boldsymbol{u}}(k+1-p) & \cdots & \tilde{\boldsymbol{u}}(N-p)\\ \tilde{\boldsymbol{u}}(k+1-p) & \tilde{\boldsymbol{u}}(k+2-p) & \cdots & \tilde{\boldsymbol{u}}(N+1-p)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\boldsymbol{u}}(k-1) & \tilde{\boldsymbol{u}}(k) & \cdots & \tilde{\boldsymbol{u}}(N-1)\end{bmatrix}$$

$$=\left[\begin{array}{c|c|c|c}\varphi_1(\boldsymbol{u}(k-p)) & \varphi_1(\boldsymbol{u}(k+1-p) & & \varphi_1(\boldsymbol{u}(N-p))\\ \vdots & \vdots & \cdots & \vdots\\ \varphi_r(\boldsymbol{u}(k-p)) & \varphi_r(\boldsymbol{u}(k+1-p)) & & \varphi_r(\boldsymbol{u}(N-p))\\ \hline \varphi_1(\boldsymbol{u}(k+1-p)) & \varphi_1(\boldsymbol{u}(k+2-p)) & & \varphi_1(\boldsymbol{u}(N+1-p))\\ \vdots & \vdots & \cdots & \vdots\\ \varphi_r(\boldsymbol{u}(k+1-p)) & \varphi_r(\boldsymbol{u}(k+2-p)) & & \varphi_r(\boldsymbol{u}(N+1-p))\\ \hline \vdots & \vdots & \ddots & \vdots\\ \hline \varphi_1(\boldsymbol{u}(k-1)) & \varphi_1(\boldsymbol{u}(k)) & & \varphi_1(\boldsymbol{u}(N-1))\\ \vdots & \vdots & \cdots & \vdots\\ \varphi_r(\boldsymbol{u}(k-1)) & \varphi_r(\boldsymbol{u}(k)) & & \varphi_r(\boldsymbol{u}(N-1))\end{array}\right].$$

Consider the r-th row and h-th column block matrix of $\boldsymbol{W}_{i,N-k+1}\boldsymbol{Z}^T$. It is obvious that random noises $\boldsymbol{w}(t)$ and $\tilde{\boldsymbol{u}}(t)$ are irrelevant from assumptions, therefore we have

$$\begin{aligned}&\frac{1}{N}\sum_{j=k}^{N}\left[\mathcal{D}^{-(i-r)\gamma}\boldsymbol{w}(k)\right]\tilde{\boldsymbol{u}}^T(j-p+h-1)\\ &\quad=\frac{1}{N}\left(\frac{1}{T_s}\right)^{-(i-r)\gamma}\sum_{j=k}^{N}\sum_{i_1=0}^{k}\omega_{i_1}^{(-i\gamma+r\gamma)}\boldsymbol{w}(k-i_1)\\ &\qquad\cdot\tilde{\boldsymbol{u}}^T(j-p+h-1)\to 0\left(N\to\infty\right).\end{aligned}$$

Thus,

$$\lim_{N\to\infty}\frac{1}{N}\left(\Gamma_i^{\perp}\right)^T \boldsymbol{G}_i\boldsymbol{W}_{i,N-k+1}\boldsymbol{\Pi}_Z=0. \tag{34}$$

Similarly, we have

$$\lim_{N\to\infty}\frac{1}{N}\left(\Gamma_i^{\perp}\right)^T \boldsymbol{V}_{i,N-k+1}\boldsymbol{\Pi}_Z=0. \tag{35}$$

Therefore, (33) can be written as

$$\left[\left(\Gamma_i^{\perp}\right)^T \;\middle|\; -\left(\Gamma_i^{\perp}\right)^T \boldsymbol{H}_i\right]\lim_{N\to\infty}\frac{1}{N}\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hdashline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\boldsymbol{\Pi}_Z=0. \tag{36}$$

The orthogonal column space of $\lim_{N\to\infty}\frac{1}{N}\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hdashline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\boldsymbol{\Pi}_Z$ is equal to the column space of $\left[(\Gamma_i^{\perp})^T \;\middle|\; -(\Gamma_i^{\perp})^T \boldsymbol{H}_i\right]$ and the rank of $\lim_{N\to\infty}\frac{1}{N}\left[\begin{array}{c}\boldsymbol{Y}_{i,N-k+1}\\ \hdashline \tilde{\boldsymbol{U}}_{i,N-k+1}\end{array}\right]\boldsymbol{\Pi}_Z$ is equal to $li + n$.

Utilizing instrumental variable, the influence of random noises will be eliminated only if the length of identification data is enough. Also, the results of identification are unbiased. The proposed method is referred to as subspace orthogonal projection identification method via PCA, abbreviated SOPIM+PCA.

4. SIMULATION EXAMPLE

Consider a two order commensurate fractional order Hammerstein system described by state space equation as follows:

$$\begin{cases}\mathcal{D}^{0.9}\boldsymbol{x}(t)=\begin{bmatrix}0 & -0.1\\ 1 & -0.2\end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\boldsymbol{u}(t)+\boldsymbol{w}(t)\\ \boldsymbol{y}(t)=\begin{bmatrix}0 & 0.1\\ 0.5 & -0.1\end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\boldsymbol{u}(t)+\boldsymbol{v}(t).\end{cases} \tag{37}$$

Preceded by a static nonlinearity described by a third order polynomial of the form

$$\boldsymbol{N}\left(\boldsymbol{u}(t)\right)=\boldsymbol{u}(t)+\begin{bmatrix}0.7 & -0.2\\ 0 & 0.5\end{bmatrix}\boldsymbol{u}(t)^2+\begin{bmatrix}-0.5 & 0\\ 0 & 0.2\end{bmatrix}\boldsymbol{u}(t)^3. \tag{38}$$

The sample time $T_s = 0.1$sec and the input signals are random sequences with length $N = 2047$. System (37) is disturbed by Gauss white noises with SNR=20dB. Choose parameter $i = 2$, $p = 50$. The traversal step is 0.0001. The identified results are as follows:

$$\hat{\gamma}=0.9022,$$

$$\hat{\boldsymbol{A}}=\begin{bmatrix}0.0212 & 0.4029\\ -0.2590 & -0.2260\end{bmatrix},\ \hat{\boldsymbol{B}}=\begin{bmatrix}0.3574 & -0.1510\\ -0.3792 & 0.0091\end{bmatrix},$$

$$\hat{\boldsymbol{C}}=\begin{bmatrix}-0.6554 & -0.6144\\ 0.7184 & -0.6459\end{bmatrix},\ \hat{\boldsymbol{D}}=\begin{bmatrix}-0.0006 & 0.0031\\ 0.0002 & -0.0003\end{bmatrix},$$

$$\hat{\boldsymbol{\alpha}}_2=\begin{bmatrix}0.7083 & -0.1817\\ 0.0008 & 0.5564\end{bmatrix},\ \hat{\boldsymbol{\alpha}}_3=\begin{bmatrix}-0.5074 & 0.0070\\ -0.0038 & 0.2124\end{bmatrix}.$$

Because the identified results of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ based on subspace method are not unique, we need transform state space models of fractional order system into transfer functions and compare the coefficients of transfer functions. The transfer functions of fractional order system are as follows:

$$G_{i,j}(s)=\frac{e_{i,j}s^{2\gamma}+d_{i,j}s^{\gamma}+c_{i,j}}{s^{2\gamma}+bs^{\gamma}+a},\quad i,j=1,2. \tag{39}$$

Table 1 shows the eigenvalues comparison of original model and identified model. Table 2 shows the theoretical values and identified results of coefficients of transfer functions. Fig. 2 shows the output curves of original model and identified model. Fig. 3 shows the output error curves of original model and identified model. In order to check the statistical property of the SOPIM+PCA, the Monte Carol experiments are carried out with 50 runs under different random noises. Table 3 shows the mean and variance of identified results. Compared with theoretical results, the identified results are unbiased. Therefore, the SOPIM+PCA can be used to identify fractional order Hammerstein systems precisely.

Table 1. The eigenvalues comparison of original model and identified model.

Original model	Identified model
-0.1000 + 0.3000i	-0.1024 + 0.2984i
-0.1000 - 0.3000i	-0.1024 - 0.2984i

Table 2. The theoretical values and identified results of coefficients of transfer functions (SNR=20dB).

	Theoretical value	Identified value
γ	0.9	0.9022
a	0.2	0.2048
b	0.1	0.0995
$c_{1,1}$	0.1	0.0991
$d_{1,1}$	0	-0.0014
$e_{1,1}$	0	-0.0006
$c_{1,2}$	0	-0.0036
$d_{1,2}$	0.1	0.0940
$e_{1,2}$	0	0.0031
$c_{2,1}$	0	0.0029
$d_{2,1}$	0.5	0.5017
$e_{2,1}$	0	0.0002
$c_{2,2}$	-0.05	-0.0471
$d_{2,2}$	-0.1	-0.1144
$e_{2,2}$	0	-0.0003
$\boldsymbol{\alpha}_2$	$\begin{bmatrix} 0.7 & -0.2 \\ 0 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.7083 & -0.1817 \\ 0.0008 & 0.5564 \end{bmatrix}$
$\boldsymbol{\alpha}_3$	$\begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} -0.5074 & 0.0070 \\ -0.0038 & 0.2124 \end{bmatrix}$

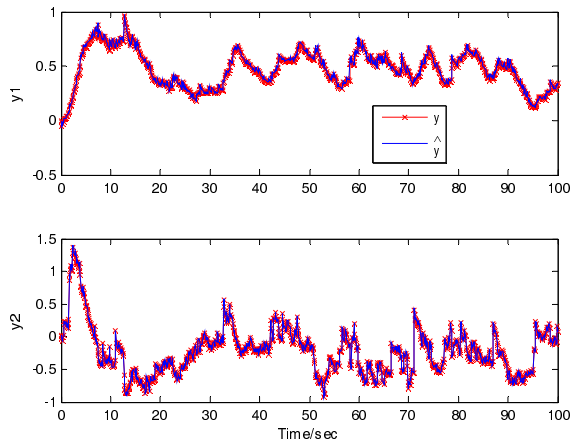

Fig. 2. The output curves of original model and identified model (SNR=20dB).

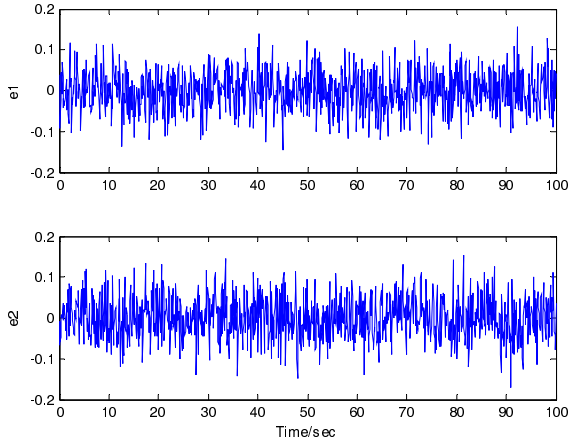

Fig. 3. The output error curves of original model and identified model (SNR=20dB).

Table 3. The mean and variance of identified results with 50 Monte Carlo runs.

	Theoretical value	Mean	Variance/10^{-3}
γ	0.9	0.8999	0.0142
a	0.2	0.2016	0.0350
b	0.1	0.1004	0.0034
$c_{1,1}$	0.1	0.1010	0.0036
$d_{1,1}$	0	0.0011	0.0661
$e_{1,1}$	0	0.0001	0.0111
$c_{1,2}$	0	0.0001	0.0087
$d_{1,2}$	0.1	0.0994	0.0335
$e_{1,2}$	0	-0.0003	0.0123
$c_{2,1}$	0	-0.0008	0.0328
$d_{2,1}$	0.5	0.5023	0.1038
$e_{2,1}$	0	-0.0006	0.0113
$c_{2,2}$	-0.05	-0.0499	0.0098
$d_{2,2}$	-0.1	-0.0994	0.2035
$e_{2,2}$	0	-0.0003	0.0117
$\boldsymbol{\alpha}_2$	$\begin{bmatrix} 0.7 & -0.2 \\ 0 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.6963 & -0.2000 \\ -0.0121 & 0.5080 \end{bmatrix}$	$\begin{bmatrix} 0.2568 & 0.2400 \\ 3.2954 & 1.3177 \end{bmatrix}$
$\boldsymbol{\alpha}_3$	$\begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} -0.4997 & -0.0001 \\ 0.0040 & 0.2030 \end{bmatrix}$	$\begin{bmatrix} 0.0659 & 0.1057 \\ 0.4615 & 0.5824 \end{bmatrix}$

5. CONCLUSION

The identification issues of MIMO fractional order Hammerstein systems are discussed in this paper. The SOPIM+PCA is proposed to identify coefficient matrixes of fractional order systems and unknown parameters of nonlinear part. The identification results are unbiased by using an instrumental variable. Simulation results show that the SOPIM+PCA can effectively identify fractional order Hammerstein systems.

REFERENCES

[1] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York, 1999.

[2] C. A. Monje, Y. Q. Chen, and B. M. Vinagre, *et al., Fractional-order Systems and Controls: Fundamentals and Applications*, Springer-Verlag, London, 2010.

[3] T. Pritz, "Five-parameter fractional derivative model for polymeric damping materials," *Journal of Sound and Vibration*, vol. 265, no. 5, pp. 935-952, March 2003.
[4] A. Carpinnteri and F. Mainardi, *Fractals and Fractinal Calculus in Continuum Mechanics*, Springer, Wien, 1997.
[5] A. Oustaloup, L. Le Lay, and B. Mathieu, "Identification of non integer order system in the time domain," *Proc. of SMC IMACS Multiconference, Computational Engineering in Systems Application, Symposium on Control, Optimisation and Supervision*, Lille, France, 1996.
[6] O. Cois, F. Levron, and A. Oustaloup, "Complex-fractional systems: modal decomposition and stability condition," *Proc. of the 6th Conf. European Control*, September 2001.
[7] O. Cois, A. Oustaloup, and E. Battaglia, "Non integer model from modal decomposition for time domain system identification," *Proc. of Symp. System Identification*, pp. 989-994, June 2000.
[8] R. Malti, S. Victor, A. Oustaloup, and H. Garnier, "An optimal instrumental variable method for continuous time fractional model identification," *Proc. of the 17th IFAC World Congress*, pp. 14379-14384, July 2008.
[9] M. Thomassin and R. Malti, "Subspace method for continuous-time fractional system identification," *Proc. of the 15th IFAC SYSID 2009*, May 2009.
[10] M. Thomassin and R. Malti, "Multivariable identification of continuous time fractional system," *Proc. of ASME 2009 IDETC/CIEC*, September 2009.
[11] A. Oustaloup, J. Sabatier, P. Lanusse, R. Malti, P. Melchior, X. Moreau, and M. Moze, "An overview of the CRONE approach in system analysis, modeling and identification, observation and control," *Proc. of the 17th IFAC World Congress*, pp. 14254-14265, July 2008.
[12] M. Aoun, R. Maltl, O. Cois, and A. Oustaloup, "System identification using fractional Hammerstein models," *Proc. of the 15th IFAC World Congress*, pp. T-We-M21, July 2002.
[13] L. Wang, P. Cheng, and Y. Wang, "Frequency domain subspace identification of commensurate fractional order input time delay systems," *International Journal of Control, Automation and Systems*, vol. 9, no. 2, pp. 310-316, February 2011.
[14] M. Verhaegen and D. Westwick, "Identifying MIMO Hammerstein systems in the context of subspace model identification methods," *International Journal of Control*, vol. 63, no. 2, pp. 331-349, February 1996.
[15] J. Wang and S. J. Qin, "A new subspace identification approach based on principal component analysis," *Journal of Process Control*, vol. 12, no. 8, pp. 841-855, December 2002.
[16] B. Huang, S. X. Ding, and S. J. Qin, "Closed-loop subspace identification: an orthogonal projection approach," *Journal of Process Control*, vol. 15, no. 1, pp. 53-66, February 2005.
[17] J. C. Gómez and E. Baeyens, "Subspace identification of mulitvariable Hammerstein and Wiener models," *Proc. of the 15th IFAC World Congress*, pp. T-Th-M01, July 2002.

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