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Active Fault Detection for Open Loop Stable LTI SISO Systems

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Abstract: Active fault detection for a stable open-loop LTI SISO system is considered. The optimal active fault detection setup is developed around an estimator based architecture. The auxiliary signal and estimator are then designed in order to maximize detection performance. Equations are derived which relate the estimator design to the nominal residual signal covariance. The relationship between the auxiliary input and the system performance degradation constraint is considered. The effect of estimator gain and excitation signal frequency on the dual Youla-Jabr-Bongiorno-Kucera parameter is investigated. Finally, the effect of the excitation signal frequency on detector performance is investigated, and a minimum targeted detection time parameter is introduced. This set of equations are then used to minimise the fault detection time for fixed performance constraints and minimum targeted detection time.

Keywords: Active fault detection, fault tolerant systems, optimisation, state estimation.

1. INTRODUCTION

One way to increase the reliability of a control system is by employing Active Fault Tolerant Control (AFTC). AFTC involves the control system responding to faults by actively altering the control law. To be able to do this efficiently, faults must be detected in a reliable and timely manner. Fault detection schemes can be classified as either passive or active fault detection. For a brief discussion of many AFTC related topics please refer to [1] and [2]. AFTC has been applied to a number of practical problems, for example, a detailed study on applying AFTC to small unmanned aerial vehicles was conducted in [3].

Active fault detection (AFD) is the technique of detecting anomalous changes in a system by means of injecting an external excitation signal into the system and then monitoring the system's response to this excitation signal. With passive fault detection the system response is simply monitored, and the excitation is provided by control inputs or unknown external disturbances. For more details on passive fault detection please refer to [4-6].

Active fault detection has many advantages over a traditional passive approach. These include shorter average detection time, guaranteed detection time, and that the detection of a fault is not subject to unknown external excitation. Active fault detection does unfortunately have one major drawback. By injecting an auxiliary signal into the system, some nominal system performance may need to be sacrificed.

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A number of AFD techniques have been developed over the last few years, these include methods based on the linear matrix inequality (LMI) framework [7-9], as well as, methods based on the dual Youla parametrisation. The research presented in this article is to a large extent a continuation of the work presented in [10-12].

In [10] the dual Youla parametrisation is introduced. The link between the dual Youla parametrisation and system changes are considered and derived in explicit form. In addition to this the author considered the link between the dual Youla parametrisation and a number of standard uncertain models.

The theory developed in [10] is used in [11,12] to develop active fault detection and diagnosis (FDD) systems. In [12] the setup used for active fault detection is introduced, and active fault detection and diagnosis is briefly discussed for open-loop, closed-loop and reconfigured closed-loop systems. In [11] active FDD for a closed-loop system is considered in more detail. Furthermore, the optimal design of the external excitation signal was also considered for a fixed observer based feedback control system.

The subject of designing the auxiliary signal to optimize AFD performance has been discussed in a number of papers. The focus has however primarily been directed towards multi-model approaches. Examples of such research include [13,14].

In order to simplify the AFD system the focus is initially shifted to the open-loop stable SISO case, which leads to significant simplifications of the optimal AFD solution. This might seem restrictive at first but the resulting theory can easily be expanded to the MIMO case, or applied to unstable systems by first stabilising the system using feedback control.

For this paper it is considered that an estimator can be designed for the sole purpose of fault-detection. This allows the estimator to be optimised for AFD instead of being fixed due to control system requirements as is the case in [11,12].

With the extra degree of freedom this provides, the optimising equation of [11] yields a trivial solution. Therefore it is augmented with a final constraint that captures the detector dynamics. The theory is therefore extended beyond that of [8,9] in order to take the dynamics of the detector into consideration. A parameter called the minimum targeted detection time is introduced. This parameter is related to the detector dynamics, and places a lower limit on the excitation signal frequency, thereby allowing a non-trivial optimal AFD solution to be obtained.

Finally, the theory developed in this article is demonstrated by means of an illustrative example.

2. PRELIMINARIES

In this section the system setup used for AFD is derived, and a number of key concepts are introduced during this process. Amongst these concepts are: the introduction of an auxiliary signal; residual generation; and the Dual Youla parameter which describes the parametric faults in the system.

2.1. System setup

A generic two port model with uncertain parameters is given in transfer matrix form as

$$E(s) = G_{ed}(s,\Theta)D(s) + G_{eu}(s,\Theta)U(s), \quad (1)$$

$$Y(s) = G_{yd}(s,\Theta)D(s) + G_{yu}(s,\Theta)U(s). \quad (2)$$

By using a linear fractional transform [15], the uncertain model parameters are removed from the primary plant model and placed in a feedback path. This modified setup is shown in Fig. 1. A state-space realisation of this uncertain plant is now given by

$$\dot{x} = \mathbf{A}x + \mathbf{B}_w w + \mathbf{B}_u u + \mathbf{B}_d d, \quad (3)$$

$$z = \mathbf{C}_z x + \mathbf{D}_{zw} w + \mathbf{D}_{zu} u + \mathbf{D}_{zd} d, \quad (4)$$

$$y = \mathbf{C}_y x + \mathbf{D}_{yw} w + \mathbf{D}_{yu} u + \mathbf{D}_{yd} d, \quad (5)$$

$$e = \mathbf{C}_e x + \mathbf{D}_{ew} w + \mathbf{D}_{eu} u + \mathbf{D}_{ed} d, \quad (6)$$

where $d \in \mathbb{R}^r$ is the disturbance input, $e \in \mathbb{R}^q$ the error output, $u \in \mathbb{R}^m$ the actuator input, $y \in \mathbb{R}^p$ the sensor output, $w \in \mathbb{R}^k$ an external input and $z \in \mathbb{R}^k$ an external output. The loop from z to w is closed through Θ, where the diagonal elements of Θ describe the parametric faults of the system.

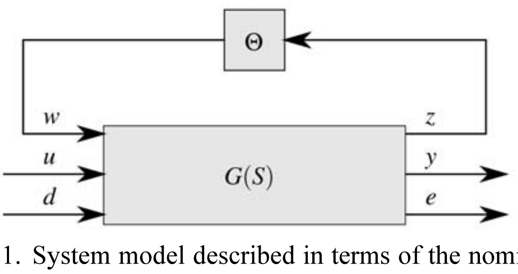

Fig. 1. System model described in terms of the nominal plant $G(s)$ and the deviations from the nominal plant given by Θ.

2.2. Coprime factorisation

A coprime factorisation of the system $G_{yu}(s)$ and $K(s)$ is given by [12]

$$G_{yu} = NM^{-1} = \tilde{M}^{-1}\tilde{N}N, M, \tilde{N}, \tilde{M} \in RH_\infty, \quad (7)$$

$$K = UV^{-1} = \tilde{V}^{-1}\tilde{U}U, V, \tilde{U}, \tilde{V} \in RH_\infty, \quad (8)$$

where the eight matrices must comply to the double Bezout equation, given by

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix}\begin{bmatrix} M & U \\ N & V \end{bmatrix} = \begin{bmatrix} M & U \\ N & V \end{bmatrix}\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \quad (9)$$

It is now assumed that G_{yu} is open-loop stable and therefore that AFD can be applied in the open-loop case. From this point on the value of F is assumed to be zero. Given these assumptions, the equations given in [16] can be simplified as follows:

$$\begin{bmatrix} M & U \\ N & V \end{bmatrix} = \left[\begin{array}{c|cc} A & B_u & -L \\ \hline 0 & I & 0 \\ C_y & D_{yu} & I \end{array}\right], \quad (10)$$

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \left[\begin{array}{c|cc} A + LC_y & -(B_u + LD_{yu}) & L \\ \hline 0 & I & 0 \\ C_y & -D_{yu} & I \end{array}\right]. \quad (11)$$

From (10) and (11) it can be shown that,

$$M = \tilde{V} = IU = \tilde{U} = 0. \quad (12)$$

Furthermore, it is easy to show that (10) and (11) comply with the requirements stipulated by (9).

2.3. Introducing the auxiliary input and residual output

A residual vector can be generated by using the co-prime factors as follows [11]

$$r = \tilde{M}y - \tilde{N}u. \quad (13)$$

It can be shown that this is equivalent to the vector r shown in Fig. 2 [17].

With reference to Fig. 2 an auxiliary signal η is

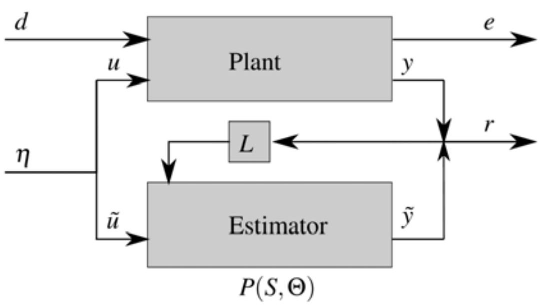

Fig. 2. System setup used for AFD in state space form. The plant is defined as in figure 1. The auxiliary input (η) as well as the residual signal (r) are also shown.

introduced which excites both the plant and estimator. This signal has no effect on r in the nominal case, but affects the residual signal if a fault has occurred. In this paper only single frequency periodic signals of the form

$$\eta = a\sin(\omega t) \tag{14}$$

will be considered.

According to [11] the four transfer functions from the two inputs to both outputs are given by

$$P_{ed} = G_{ed}(\Theta), \tag{15}$$

$$P_{e\eta} = G_{eu}(\Theta), \tag{16}$$

$$P_{rd} = G_{yd}(\Theta), \tag{17}$$

$$P_{r\eta} = G_{yu}(\Theta) - N, \tag{18}$$

where the simplifications obtained in (12) have been applied to the equations given in [11].

In [11] it is noted that $P_{r\eta}$ is equal to the dual Youla parameter S. According to [10,11] it is possible to rewrite S or $P_{r\eta}$ as

$$P_{r\eta}(\Theta) = \tilde{M}G_{yw}\Theta\left(I - G_{zw}\Theta\right)^{-1} G_{zu}, \tag{19}$$

where the simplifications obtained in (12) have been applied to the equations given in [11]. For further background on the Youla parameterisation please refer to [18,19].

3 OPTIMAL OPEN-LOOP AFD FOR SISO SYSTEMS

In this section optimal open-loop AFD is investigated by making use of the theory presented in the previous section. In the open-loop case accurate state estimation is not of primary importance, since a separate state estimator for the purpose of applying feedback control can be employed. Therefore, the estimator gains can be designed to optimise for AFD performance.

With AFD the injection of the auxiliary signal results in a reduction of the nominal system performance. The following design goals are considered important during the design of the optimal estimator and auxiliary signal pair:

1) Design the estimator gain so that $A + LC_2$ is stable.
2) Design the auxiliary signal, η, in such a way as to limit nominal performance degradation.
3) Design the estimator gain, L, and the auxiliary signal, η, in order to minimise the fault detection time.

3.1. A setup for active fault detection

Design goal 2 requires that the impact of the axillary signal on the system error output be known and kept within the stipulated design constraint. Now, consider design goal 3. This design goal requires that the impact of the axillary signal on the triggering output be known and maximised. Furthermore, this design goal requires that the impact of the estimator gain on the nominal detector noise level be known and minimised. A complying setup is shown in Fig. 3.

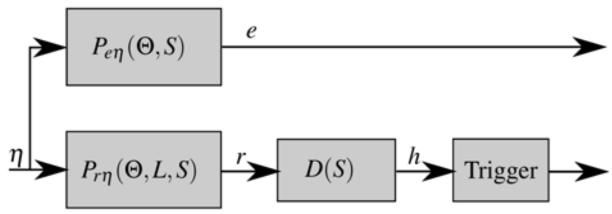

Fig. 3. Setup used for active fault detection. From left to right the following is shown: plant excitation dynamics; linearised detector dynamics; and fault trigger.

With reference to this figure a number of definitions are now made:

Define Λ_0 as the nominal system AFD disturbance constraint. Therefore $\sigma_{\max}(P_{e\eta}(0,S)\eta) \leq \Lambda_0$ must hold for all $t > 0$ and the postulated fault condition Θ_1.

- The signal η is the excitation signal used for the purpose of AFD, and is given by $\eta = a\sin(\omega t)$ where a is a simple gain such that the performance degradation constraint Λ_0 is adhered to.
- h is the signal on which thresholding is performed, and is given by, $h = D(s)r$. Where, $D(s)$ is a linear approximation of the detector dynamics.

From these definitions and the informal discussion provided earlier, the following optimisation criteria is now formulated.

Proposition 1: Find the estimator gain L, excitation frequency ω_η, and the admissible gain a which maximises the average fault signal to nominal noise ratio on $h(t)$ over a fixed time period t_d.

In the remainder of this section the subsystems making up the optimal AFD setup are considered and combined to arrive at the optimised AFD solution. With reference to Fig. 3 these subsystems are: the auxiliary signal and how it relates to the disturbance constraint; an approximated description of the detector dynamics and how it relates to the excitation frequency; a characterisation of the effect of the auxiliary signal on the residual output; and a description of the nominal detection noise.

3.2. Disturbance constraint and the auxiliary signal amplitude

Implementing AFD results in a reduction in nominal system performance. It is therefore of primary importance to limit this performance degradation. Therefore,

$$\Lambda_0 = c_0, \tag{20}$$

where c_0 is a constant.

The auxiliary signal must be designed to satisfy the single constraint Λ_0. Assuming maximum additional system disturbance, the maximum auxiliary signal amplitude is given by

$$a = c_0 \left| \frac{1}{P_{e\eta}(0, j\omega)} \right| = c_0 \left| \frac{1}{G_{eu}(0, j\omega)} \right|. \tag{21}$$

3.3. Approximated detector dynamics

The aim of this section is to approximately characterise the detector's response in terms of excitation frequency as a linear transfer function. The detector is responsible for detecting the small change in the residual signal due to a failure. Although not the only possible approach, in this paper the focus is on using an integrative change in mean detector.

As previously stated the auxiliary signal is a single frequency periodic signal of the following form:

$$\eta = a\sin(\omega_\eta t). \tag{22}$$

Given enough time for transient behaviour to have ceased, the residual output is of the form,

$$r(t) = a\sin(\omega_\eta t + \phi) + \upsilon, \tag{23}$$

where υ is a zero mean noise component.

To generate a detection signal with a zero mean in the nominal case and a non-zero mean in the faulty case the residual is multiplied by a single frequency periodic signal of frequency ω_η. The detection signal is therefore given by

$$\nu_0 = \upsilon\sin(\omega_\eta t + \phi_0) \tag{24}$$

in the nominal case, and

$$\nu_1 = \upsilon\sin(\omega_\eta t + \phi_1) + a_1\sin^2(\omega_\eta t + \phi_1) \tag{25}$$

in the faulty case.

Now, consider the second term of (25). This term has a non-zero mean value, and is used for fault detection. Since it is not known at what time a fault will occur it is further assumed that $\phi_1 = 0$. Of primary interest is the effect of the excitation frequency on detector performance. This parameter can be designed for, while the phase at which a failure occurs is random.

The detector integrates ν_1, and after a time t_d seconds the output is given by

$$\begin{aligned} h &= \int_0^{t_d} a\sin^2(\omega t)dt \\ &= \frac{t_d}{2} - \frac{\sin(2\omega t_d)}{4\omega}. \end{aligned} \tag{26}$$

It can be seen that as ω becomes large, h approaches $\frac{t_d}{2}$. Furthermore, when ω becomes small h approaches zero.

It is now shown that the detector's integration action can be closely approximated by a second order transfer function of the form

$$D(S) \approx \frac{t_d S^2}{2\left(S^2 + 2\left(\frac{a}{t_d}\right)S + \left(\frac{a}{t_d}\right)^2\right)}, \tag{27}$$

where a is a constant approximately equal to one.

The value of a for small values of ω is calculated by setting (26) equal to the magnitude of (27)

$$\frac{t_d}{2} - \frac{2\omega t_d - \frac{(2\omega t_d)^3}{3!}}{4\omega} = \left| \frac{t_d (j\omega)^2}{2\left((j\omega)^2 + 2\left(\frac{a}{t_d}\right)(j\omega) + \left(\frac{a}{t_d}\right)^2\right)} \right|, \tag{28}$$

where $S = j\omega$ and $\sin(2\omega t_d)$ is approximated by a second order Taylor series expansion.

Now, assuming that a, t_d and ω are positive, real numbers, (28) is simplified as

$$2\left(a^2 + t_d^2\omega^2\right) = 3. \tag{29}$$

Therefore, as ω approaches zero

$$a = \sqrt{\frac{3}{2}}. \tag{30}$$

From Fig. 4, equations (30) and (27) it can be seen that auxiliary signal frequencies lower than $1/t_d$ are severely penalised by the detector dynamics. It is therefore up to the control engineer to select a reasonable minimum targeted detection time. Please note it is not suggested that it is impossible to detect a fault in less time than t_d, merely that doing so may require a much larger change in ν. Furthermore, the optimal auxiliary signal frequency is not simply a function of the detector dynamics, but is also influenced by other factors, such as $P_{r\eta}(\Theta)$.

It is rarely possible to know the system model with perfect accuracy, therefore it is often necessary to employ a leaky detector. From Fig. 4 it can be seen that employing a leaky detector causes a severe performance drop in the low cut-off region of the detector dynamics. Since this area is typically excluded as an optimal solution due to the detector dynamics the extra dynamics can usually be ignored, as is the case in the examples shown later.

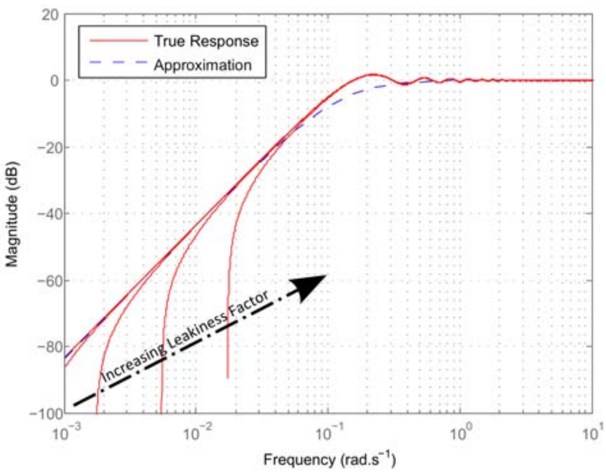

Fig. 4. True frequency response of detector versus second order approximated response. For this plot t =1 and the magnitude is normalised so that $h(\infty)$ =1. Furthermore, the effect of using a leaky detector is shown. As the leakiness increases the severe performance drop moves closer to $1/t_d$.

3.4. Dual Youla parameter

As previously stated the transfer function from auxiliary input to residual output is given by the dual Youla parameter. With reference to (19), this parameter is given by

$$P_{r\eta}(\Theta) = \tilde{M}G_{yw}\Theta(I - G_{zw}\Theta)^{-1}G_{zu}. \quad (31)$$

As is evident from (11) the transfer function $\tilde{M}$ is dependant on L, therefore

$$P_{r\eta}(\Theta, L) = \tilde{M}(L)G_{yw}\Theta(I - G_{zw}\Theta)^{-1}G_{zu}. \quad (32)$$

$P_{r\eta}$ is an indication of the effectiveness of the auxiliary signal in altering the residual, therefore it is advantageous to maximise $P_{r\eta}$.

$$\max_{L}(P_{r\eta}(\Theta, L)) \quad (33)$$

3.5. Noise covariance on nominal residual signal

From the proposition it would seem that the noise covariance on $h(t)$ is required. However if it is assumed that the system has been on for a long time the dynamic effect of the detector is of minimal importance, therefore it is acceptable to consider the noise covariance on the nominal residual instead.

The nominal residual noise power determines the maximum attainable detector sensitivity for a given false detection rate (NOTE: Calculating statistical metrics, such as the false detection rate, is beyond the scope this paper. The interested reader should refer to literature such as [20]. It is assumed that a simple change detector is used. If this is not the case refer to literature on detector theory and the universally most powerful test, such as [21].). It is therefore desirable to minimise the residual covariance. It is not suggested that the optimal AFD system simply minimizes output noise covariance, but that in the final optimisation a lower output noise covariance is advantageous as it leads to a higher signal to noise ratio.

The covariance on the residual prior to a fault occurring can be calculated using the H_2 system norm. The H_2 norm for a stable proper continuous system is given by

$$\|H\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{Trace}\left[\mathrm{H}(\mathrm{j}\omega)^{\mathrm{H}}\mathrm{H}(\mathrm{j}\omega)\right]\mathrm{d}\omega}. \quad (34)$$

From (17) (with $\Theta = 0$) and (34) the output noise power is dependant on the estimator gain, and is given by

$$\| P_{rd_0}(L) \|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{Trace}\left[\mathrm{P}_{\mathrm{rd}_0}(\mathrm{j}\omega,\mathrm{L})^{\mathrm{P}_{\mathrm{rd}_0}(\mathrm{L})}\mathrm{P}_{\mathrm{rd}_0}(\mathrm{j}\omega,\mathrm{L})\right]\mathrm{d}\omega}. \quad (35)$$

The estimator gain which minimises the nominal residual covariances satisfies

$$\max_{L}\left(\frac{1}{\| P_{rd_0}(L) \|_2}\right). \quad (36)$$

3.6. Combining results

In this section the components of the optimal open-loop AFD setup have been investigated. Firstly the input shaping filter was introduced and discussed for a number of different scenarios. Next approximated detector dynamics were derived. Then the noise power on the residual of nominal system was minimised. This allows the sensitivity of the detector to be maximised. Lastly equations were derived which can be used to maximise the effect of η on r. This allows the detector threshold to be reached as fast as possible.

From the equations given in Sections 3.2, 3.3 and 3.4, the transfer function from input η_h to the thresholding signal h is given by

$$P_{h\eta_h}(L, S) = aD(S)P_{r\eta}(L, S). \quad (37)$$

The peak gain in the frequency response of a linear system is given by the H_∞ norm

$$\| H(s) \|_\infty = \max_{\omega}|H(j\omega)|. \quad (38)$$

Therefore,

$$\| P_{h\eta}(L) \|_\infty = \max_{\omega}\left|aD(j\omega)P_{r\eta}(L, j\omega)\right|. \quad (39)$$

Therefore the estimator gain which leads to the largest h is given by

$$\max_{L}\left(\| P_{h\eta_h}(L) \|_\infty\right). \quad (40)$$

As was stated in *proposition 1* the average fault signal to nominal noise ratio on $h(t)$ must be maximised, therefore from (36) and (40),

$$\max_{L}\left(\frac{\| P_{h\eta}(L) \|_\infty}{\| P_{rd_0}(L) \|_2}\right), \quad (41)$$

where the optimal estimator gain, L_{opt} satisfies (41), while the optimal auxiliary signal frequency satisfies $\| P_{h\eta}(L_{opt}) \|_\infty$.

3.7. Discussion

3.7.1 Solving the optimisation problem

For simple first and second order problems the optimisation problem can easily be solved by producing plots using a computer. However, for more complex problems drawing simple plots becomes impractical. In these cases the optimisation problem can be solved using a number of standard numerical techniques. For a detailed discussion of these and other techniques please refer to [22,23]. Ultimately even the most efficient algorithm will be undone by the exponential time complexity of the optimisation problem as the number of states increase.

If, however, a slightly suboptimal solution is acceptable the time complexity problem can be easily overcome by restricting the estimator error dynamics to that of a simple low-pass filter. This modified problem leads to the following proposition:

Proposition 2: Find the estimator bandwidth ω_L, excitation frequency ω_η, and admissible gain a which maximises the average fault signal to nominal noise ratio on $h(t)$ over a fixed time period t_d.

When using this modified proposition the estimator gain in (41) is determined via the optimal bandwidth and the chosen filter topology, therefore the time complexity of the numerical solution is now independent of the number of states. Furthermore, using this problem setup allows the use of the same simple plots to solve higher order problems as was suggested previously for first order problems.

3.7.2 Choosing the targeted detection time

One of the parameters required by the optimisation problem presented is the targeted detection time. This parameter places a soft lower bound on the optimal excitation frequency. When choosing this parameter the following should be considered:

- Changing t_d provides a mechanism for exchanging small error detection performance (sensitivity) for detection speed and vice versa. A longer t_d provides more sensitivity but slower detection speed for larger errors while a shorter t_d provides less sensitivity but faster detection speed for larger errors.
- The shortest reasonable t_d is determined by the inverse of the system bandwidth. As the minimum excitation frequency is pushed beyond the system bandwidth the efficacy of the excitation signal starts to diminish.
- The longest reasonable t_d is determined by the fault detection specification.

Fig. 5 shows the effect of varying the targeted detection time. It can be seen from the figure that as t_d is shortened the detection gain is reduced and vice versa.

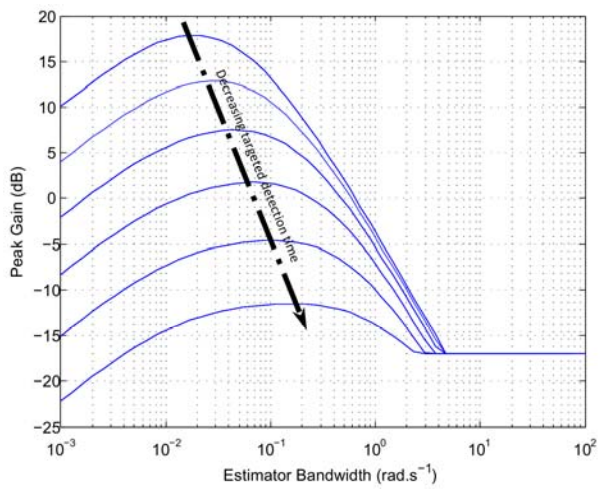

Fig. 5. Peak gain plot showing the effect of the targeted detection time. As t_d becomes shorter the optimal estimator bandwidth increases, while the peak gain is significantly reduced. This is indicated by the arrow in the figure.

3.7.3 Comparison to multi-model approaches

When comparing the method developed in this paper to the commonly used multiple-model fault detection method, a number of important advantages are identified. In the multiple-model method a bank of Kalman filters are usually employed. Each one of these Kalman filters are then designed to match a postulated failure case. If a large number of non-nominal models need to be detected, this method can quickly result in a large computational load. Additionally it is difficult to envisage all possible failure cases. The method shown in this paper only requires a single estimator for each fault class which must be optimised for. Non time critical faults are still detected even if they were not explicitly designed for. Detection in such cases merely occurs in a sub-optimal manner.

One major disadvantage to consider is that when a fault is detected by a multiple-model setup, the properties of the new model are immediately known. With the method presented in this paper system identification needs to be performed as a separate step. In a computationally constrained environment this is most likely a compromise worth making.

4. EXAMPLES

A few examples are used to illustrate the application of the theory developed in this paper.

4.1. Example 1

This simple example will demonstrate the problem with simply applying the existing theory to the open loop optimal AFD problem. It will be shown that using the existing theory in this manner leads to a trivial solution, because the existing theory was intended to be used with a pre-existing estimator, and not to be used in designing an estimator for optimal AFD.

In general a first order linear differential equation is given by the following equation:

$$\dot{X} = aX + bU. \tag{42}$$

This can be represented by the following state space representation.

$$\dot{X} = [a]X + [b]\delta_A, \tag{43}$$

$$y = [1]X. \tag{44}$$

Consider the following fault model:

$$a = a_0(1+\theta_a), \tag{45}$$

$$b = b_0(1+\theta_b), \tag{46}$$

where θ_a and θ_b are zero in the nominal case.

Using an upper linear fractional transform the system can be written as

$$\dot{X} = [a_0]X + [a_0 \quad b_0]w + [b]U, \tag{47}$$

$$z = \begin{bmatrix}1\\0\end{bmatrix}X + \begin{bmatrix}0\\1\end{bmatrix}U, \tag{48}$$

$$y = [1]X \tag{49}$$

with

$$w = \begin{bmatrix} \theta_a & 0 \\ 0 & \theta_b \end{bmatrix} z. \tag{50}$$

Next, the disturbance model is introduced. In this example the following properties will be assumed:

- Zero mean white process noise which enters the system in the same manner as the control input
- Bandwidth limited zero mean white measurement noise
- The error signal is equal to the plant output

Adding the disturbance model results in the following three port model:

$$\begin{bmatrix} \dot{X} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_0 & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} X \\ x_n \end{bmatrix} + \begin{bmatrix} a_0 & b_0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} U + \begin{bmatrix} k_P \\ -c \end{bmatrix} d, \tag{51}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U, \tag{52}$$

$$y = [1 \quad k_m] \begin{bmatrix} X \\ x_n \end{bmatrix}, \tag{53}$$

$$e = [1 \quad k_m] \begin{bmatrix} X \\ x_n \end{bmatrix}. \tag{54}$$

The values for a_0, b_0, c, k_m and k_p are now chosen as, –1, –5, –10000, 0.01, 0.1 respectively.

Suppose that the plant suffers damage which results in

$$\Theta = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.1 \end{bmatrix}. \tag{55}$$

Therefore, there is a 40% reduction in damping and a 10% reduction in control authority.

From Fig. 6 it can be seen that simply applying the theory from [11] leads to incomplete results. The results obtained suggests that the best estimator is the open-loop case, and that the optimal excitation frequency is 0 rad $\cdot$s^{-1}. This would lead to an infinite detection time.

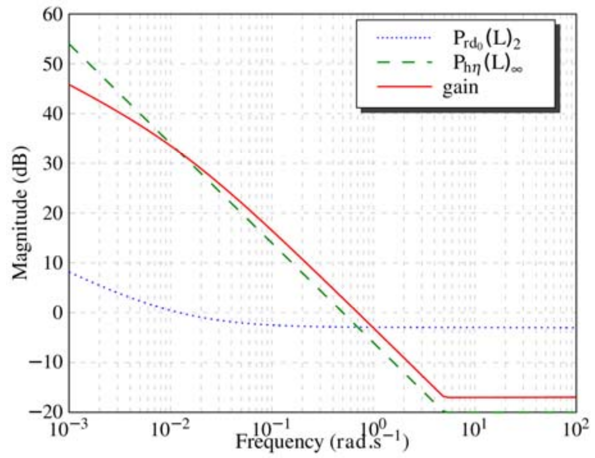

Fig. 6. AFD performance as a function of estimator bandwidth. $L_{P_{opt}}$ is given by peak of $\dfrac{\| P_{h\eta}(L) \|_\infty}{\| P_{rd_0}(L) \|_2}$.

4.2. Example 2

Example 1 is now repeated for the Optimal AFD problem, using the augmented theory described in this paper where detector dynamics are also taken into consideration.

This example uses the same model as in example 1, and the model is therefore not restated here.

Now, using the equations derived in this work, a frequency plot can be easily produced showing the AFD performance as a function of the estimator bandwidth. The results are shown in Fig. 7. From this figure it is easy to determine the optimal estimator bandwidth as

$$\omega_{L_{opt}} = -0.0455 \text{ rad} \cdot \text{s}^{-1}. \tag{56}$$

Finally, with $L_{\omega_{opt}}$ known, a plot of $P_{h\eta}(L_{opt})$ can be produced. The result is shown in Fig. 8. From the figure the optimal excitation frequency is

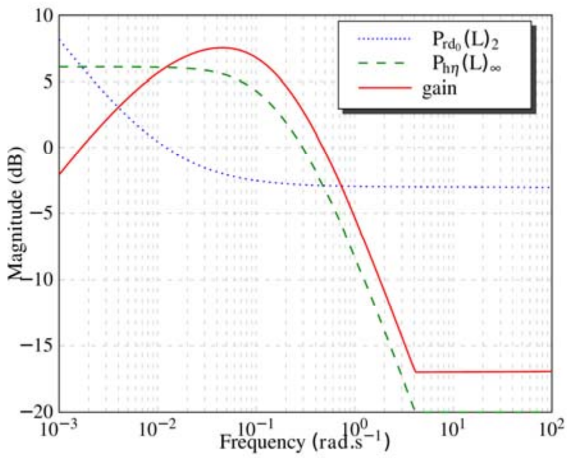

Fig. 7. AFD performance as a function of estimator bandwidth. $L_{P_{opt}}$ is given by peak of $\dfrac{\| P_{h\eta}(L) \|_\infty}{\| P_{rd_0}(L) \|_2}$.

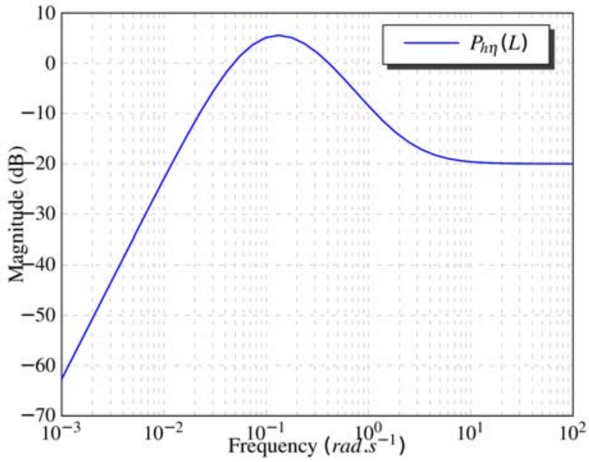

Fig. 8. Magnitude response of $P_{h\eta}(L_{opt})$. The optimal excitation frequency ($\omega_{\eta_{opt}}$) is given by the peak of the magnitude response.

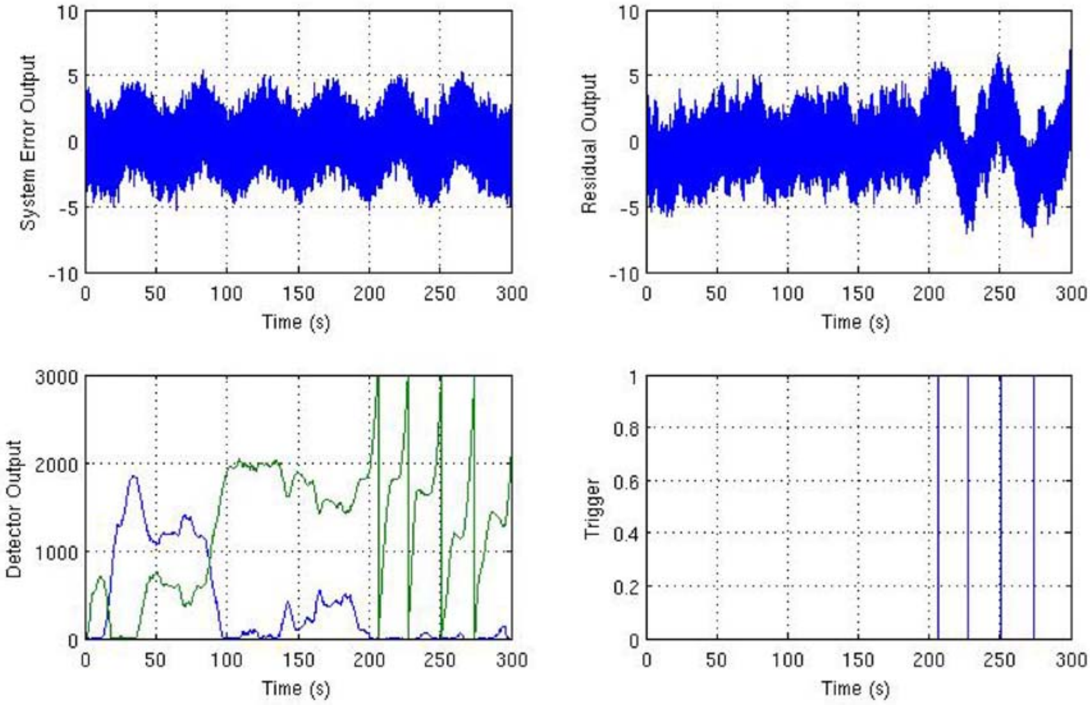

Fig. 9. Simulation results with a large amount of noise in the correct ratio. Failure occurs at 186 seconds and is detected 20 seconds later. Note that each time the threshold is reached the detector is reset, and the trigger value is set to one for a single sample period.

$$\omega_{\eta_{opt}} = 0.1365 \text{ rad} \cdot \text{s}^{-1}. \quad (57)$$

These results are used to set up a simulation of the optimal AFD system. The simulation results for a case with a large amount of noise is shown in Fig. 9. From the simulation results it can be seen that the AFD system is very robust against the effects of white noise.

It is important to note that model inaccuracies are a major hurdle to effective fault detection. Parametric uncertainties are mathematically identical to faulty parameters, and therefore much effort should be spent in determining accurate system models. Not withstanding this there are a number of steps that can be taken in order to deal with parameter uncertainties. These include employing the system as part of a intelligent active fault tolerant control system and/or using a leaky detector.

5. CONCLUSION

Optimal open-loop active fault detection was investigated for a stable SISO system. The design of the estimator was considered an integral part of the AFD optimisation process instead of being a fixed controller attribute. It is suggested that a separate estimator should be used for optimal state estimation, if desired. The research presented in [10,11,18] and [12] was simplified for the open-loop case considered. Equations were derived to minimise the noise covariance on the nominal residual output as well as to maximise the Dual Youla parameter. In order to realise a non-trivial solution, the theory was extended to include dynamical effects of the detector. It was found that this effect can be closely approximated by a second order transfer function, with a low-pass cut-off frequency determined by the minimum targeted detection time design parameter. The theory developed was applied to a few simple illustrative examples.

As was previously stated, a simplification was made by considering the system in the open-loop case. Closing a control loop around the system reduces AFD performance due to the controller's disturbance rejection. Future research will therefore deal with formalizing this effect as well as expanding the research to MIMO systems.

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