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White Noise Estimators for Networked Systems with Packet Dropouts

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Abstract: This paper studies the optimal and suboptimal deconvolution problems over a network subject to random packet losses, which are modeled by an independent identically distributed Bernoulli process. By the projection formula, an optimal input white noise estimator is first presented with a stochastic Kalman filter. We show that this obtained deconvolution estimator is time-varying, stochastic, and it does not converge to a steady value. Then an alternative suboptimal input white-noise estimator with deterministic gains is developed under a new criterion. The estimator gain and its respective error covariance-matrix information are derived based on a new suboptimal state estimator. It can be shown that the suboptimal input white-noise estimator converges to a steady-state one under appropriate assumptions.

Keywords: Convergence analysis, networked system, packet dropout, white noise estimation.

1. INTRODUCTION

The problem of deconvolution has attracted considerable attention in recent years due to its wide applications ranging from seismology and channel equalization to speech processing. The purpose of the deconvolution problem is to estimate the unknown input signal of a system using the noise corrupted measurements, where the measurement is detected or transmitted via wireless sensor networks. A motivating example is given in seismic exploration, where a short duration seismic pulse is transmitted from the surface, reflected from boundaries between underground earth layers, and received by an array of sensors on the surface [1]. The received signals, called seismic traces, are analyzed to extract information about the underground structure of the layers in the explored area. As is well known that the phenomenon of packet losses is unavoidable in the signal communication over wireless channels. This happens in resource limited wireless sensor networks where communication between devices are power constrained and therefore limited in range and reliability. The main effect of the packet loss is that it will degrade the tracking performance and convergence of the white noise estimator. So in this paper, we will study the design method of the deconvolution estimation for networked systems with random packet losses, and analyze its convergence performance.

There exist several approaches to the deconvolution problem in the literature. The pioneer work of the deconvolution problems can be traced back to the study of white noise estimation with application to oil exploration based on the Kalman filtering method [2]. Latter, this method was successfully applied to the study of multi-sensor information fusion white noise filters [3,4] and self-tuning weighted measurement fusion deconvolution estimation [5]. A unified white noise estimation theory based on the modern time series analysis method was presented in [6], which included both the input white noise estimators and measurement white noise estimators. Alternatively, the polynomial systems approach is also an efficient method used to the optimal deconvolution estimator design in the frequency domain, in which the solutions are given in terms of spectral factorization and polynomial equations [7,8]. Concerning with the H_∞ deconvolution problems, the optimal estimators can also be derived via a polynomial system approach [9,10] or an innovation analysis approach [11]. In [12], a fixed-order H_∞ optimal deconvolution filter was designed by using the genetic algorithm. It can be seen that many research works have been obtained for the systems without packet losses.

The phenomenon of packet losses occurs in a number of engineering applications [13,14]. There have been a vast number of solutions for the state estimation problems of system with random packet losses. Recently, the Kalman filtering for systems with intermittent observations was studied in [15] and [16]. In [15] and [16], a stochastic Kalman filter was designed based on a set of intermittent observations, where the stability

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analysis of the filter with relation to the data arrival rate was given. In [17], the stability of Kalman filtering with Markovian packet losses was studied. The stability criteria were expressed by simple inequalities in terms of the largest eigenvalue of the open loop matrix and transition probabilities of the Markov process. In [18], an optimal H_2 filtering in networked control systems with multiple packet dropout was considered, where the random dropout was represented by two Bernoulli distributed white sequences which taking the values of 0 or 1, and the filter was derived by a convex optimization problem through a set of linear matrix inequalities (LMIs). In [19], Sun and Xie have presented a multiple packet dropout modeling method, and an optimal linear estimator was computed recursively in terms of the solution of a Riccati difference equation. In [20] and [21], the robust filtering and nonlinear H_∞ filtering were designed for the multiple missing measurement systems via the LMI techniques, respectively. Up to now, many research works on state estimation for the systems with packet dropouts have been obtained. However, there exist few results concerned with the white noise estimation for systems subject to random packet losses.

In [22], the optimal white noise filters were designed for the discrete time systems with packet dropouts. The solutions to the white noise filters were given in terms of a generalized Riccati equation plus a Lyapunov equation. No convergence analysis was given in [22]. In [23], an optimal deconvolution smoother was derived for systems with random parametric uncertainties, where the system uncertainties were time invariant and temporally correlated. The solution to the deconvolution smoother was given in terms of the state filter and the estimation of an auxiliary variable which concerned with the white noise input and random parametric uncertainties simultaneously. Convergence analysis for the derived smoother was provided. Note that the deconvolution filter designed in [22] requires solving an additional recursive Lyapunov equation related to the original state of the system, and the optimal deconvolution smoother designed in [23] requires solving an additional recursive equation concerning with the covariance of the introduced auxiliary variables.

In this paper, we will investigate the optimal and suboptimal input white noise estimation for the discrete-time systems with packet dropouts, which is described by an independent identically distributed (i.i.d) Bernoulli process. An optimal input white noise estimator is first presented based on the innovation analysis method, while its estimation gain and the respective error covariance-matrix information are obtained with a stochastic Kalman filter. Note that the estimator gain is time varying, stochastic, and it does not converge to a steady state. Then as a low-complexity solution, an alternative suboptimal input white noise estimator is developed under a new performance index, in which the estimator gain and its corresponding error covariance matrix are deterministic and can be derived from a new suboptimal state estimator. It can be shown that the suboptimal input white noise estimator converges to a steady-state white noise estimator under natural assumptions. Compared with the existing results in [22] and [23], the solutions to the deconvolution smoothers (both optimal and suboptimal) developed in this paper just require solving a stochastic Riccati equation or a generalized Riccati Equation. Especially for the suboptimal deconvolution smoother, all smother gains can be designed off-line, which reduces the online computation burden efficiently. In the case of no packet dropouts, the optimal and suboptimal white noise smoothers are reduced to those under complete measurement data in [2].

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the norm bounded linear space of all $m\times n$ matrices. For $L\in\mathbb{R}^{n\times n}$, L' stands for the transpose of L. As usual, $L\geq 0 (L>0)$ will mean that the symmetric matrix $L\in\mathbb{R}^{n\times n}$ is positive semi-definite (positive definite), respectively. Moreover, tr(.) indicates the trace operator, E(.) denotes the mathematical expectation operator, and Pr ob(.) means the occurrence probability of an event.

2. PROBLEM FORMULATIONS

Consider the following discrete-time system

$$x(t+1)=\Phi x(t)+Gw(t), x(0)=x_0, \tag{1}$$

$$y(t)=\gamma_t Hx(t)+\gamma_t v(t), \tag{2}$$

where $x(t)\in\mathbb{R}^n$ is the state, $w(t)\in\mathbb{R}^p$ is the input noise, $y(t)\in\mathbb{R}^m$ is the measurement and $v(t)\in\mathbb{R}^m$ is the measurement noise. γ_t is the packet arrival indicator. The following assumptions are made to the systems (1) and (2).

Assumption 1: The initial state x_0, $w(t)$, and $v(t)$ are null mean white noises with covariance matrices

$$\mathrm{E}[x_0x_0']=P_0,\quad \mathrm{E}[w(t)w'(s)]=Q\delta_{ts},$$
$$\mathrm{E}[v(t)v'(s)]=R\delta_{ts},$$

respectively. x_0, $w(t)$, and $v(t)$ are mutually independent.

Assumption 2: Measurements in (2) are time-stamped, and transmitted through a digital communication network. γ_t is a scalar quantity taking on values of 0 and 1 with $\Pr\mathrm{ob}(\gamma_t=1)=\rho$, $\Pr\mathrm{ob}(\gamma_t=0)=1-\rho$, and the random processes $w(t)$, $v(t)$, γ_t for all t and the initial state x_0 are mutually independent. γ_t can be observed at the present time t by employing the time-stamped technique. That is γ_t together with the observation $y(t)$ are available in the estimator design.

Then the estimation problems considered in this paper can be stated as:

Problem 1 (Optimal white noise estimator)**:** Given the observation sequences $\{\{y(s)\}_{s=0}^t\}$ and $\{\{\gamma_s\}_{s=0}^t\}$, find a linear minimum mean square error (LMMSE) white noise estimation $\hat{w}(t\mid t+N)$ of $w(t)$, such that

$$\mathrm{E}_{w,v}\{[w(t)-\hat{w}(t\mid t+N)]\ [w(t)-\hat{w}(t\mid t+N)]'\} \tag{3}$$

is minimum, while the estimation gain is stochastic. Note that $N = 0$ is the filter, $N > 0$ is the smoother, and $N < 0$ is the predictor.

Problem 2 (Suboptimal white noise estimator)**:** Given the observation sequences $\{\{y(s)\}_{s=0}^{t}\}$, find a minimum mean square error white-noise estimation $\hat{w}_e(t \mid t+N)$, such that

$$\mathrm{E}_{w,v,\gamma}\{[w(t) - \hat{w}_e(t \mid t+N)]\,[w(t) - \hat{w}_e(t \mid t+N)]'\} \quad (4)$$

is minimum, while the estimation gain is deterministic. Note that $N = 0$ is the filter, $N > 0$ is the smoother, and $N < 0$ is the predictor.

Remark 1: As for the two problems, the expectation in (3) is only taken over on the addition noise w and v, and γ is assumed to be known, while the expectation in (4) is taken over on w, v, and γ, simultaneously. Therefore the estimator developed in Problem 1 has smaller estimation error. However, the estimation gain subject to problem 1 is stochastic, and the performance analysis of the estimator is difficult. The estimator developed in Problem 2 is with deterministic gains, and thus the convergence of the white-noise estimator will be guaranteed under appropriate assumptions. The precise definitions to the above problems will be given below.

Remark 2: In the optimal estimator design, the observation arrival process $\{\gamma_\tau\}_{\tau=0}^{t}$ is exactly known to the destination, and the estimator is designed online. The scheme is a TCP-like type of networking protocol, see Fig. 1. However, in the suboptimal estimator design, only the present arrival information γ_t (time-stamp) is known to the destination. And the filter gain is independent on the arrival process $\{\gamma_\tau\}_{\tau=0}^{t}$, so it can be designed offline. This scheme is similar to a UDP type of networking protocol, see Fig. 2.

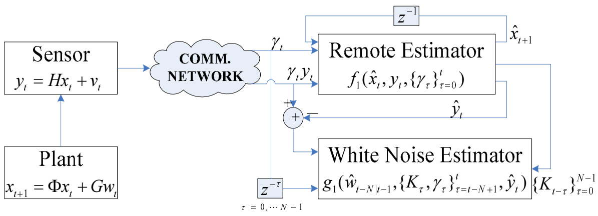

Fig. 1. Overview of Problem 1. We design an optimal state and white noise estimator online, where the observation travelling over an unreliable network and the arrival process $\{\gamma_\tau\}_{\tau=0}^{t}$ are exactly known to the destination.

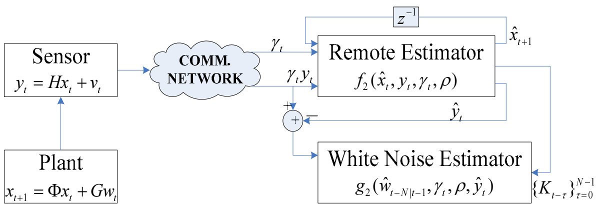

Fig. 2. Overview of Problem 2. We design an suboptimal state and white noise estimator offline, where only present time-stamp γ_t and arrival rate ρ are known to the destination.

3. OPTIMAL WHITE-NOISE ESTIMATOR

In this section, an analytical solution to the LMMSE white-noise estimation will be presented by applying the projection formula.

3.1. Design of the optimal state estimator

In the next, we first design an optimal state estimator, which will be used in the latter derivation of the optimal white-noise estimation. As in the Kalman filter design, an innovation sequence need to be defined, which is associated with the observation (2)

$$\eta(t) \triangleq y(t) - \hat{y}(t \mid t-1), \quad (5)$$

where $\hat{y}(t \mid t-1)$ is the LMMSE estimation of $y(t)$ given the observations $\{y(0), \cdots, y(t-1)\}$ and the information $\{\gamma_0, \cdots, \gamma_{t-1}\}$.

Indeed, the sequence $\{\eta(t)\}$ is a white noise sequence with zero mean and covariance $Q_\eta(t)$, and spans the same linear space as $\{y(0), \cdots, y(t-1)\}$. Then the optimal state estimation based on the innovation sequences can be defined as follows:

Definition 1: Consider the given time instant t, the LMMSE state estimation $\hat{x}(t+1 \mid t)$ is defined as

$$\hat{x}(t+1 \mid t) = \Phi\hat{x}(t \mid t-1) + \Phi K(t)[y(t) - \gamma_t H\hat{x}(t \mid t-1)], \quad (6)$$

where $K(t)$ is to be determined such that

$$\mathrm{E}_{w,v}\left\| x(t+1) - \hat{x}(t+1 \mid t) \right\|^2 \quad (7)$$

is minimized. Further, define

$$P(t \mid t-1) \triangleq \mathrm{E}_{w,v}[\tilde{x}(t \mid t-1)\tilde{x}'(t \mid t-1)], \quad (8)$$

where

$$\tilde{x}(t \mid t-1) \triangleq x(t) - \hat{x}(t \mid t-1). \quad (9)$$

Remark 3: From the above definition, it can be observed that the state estimation $\hat{x}(t \mid t-1)$ is the projection of $x(t)$ onto the linear space

$$\mathscr{L}\{y(0), \cdots, y(t-1)\} = \mathscr{L}\{\eta(0), \cdots, \eta(t-1)\}, \quad (10)$$

and the innovation sequence can be rewritten as

$$\eta(t) = y(t) - \gamma_t H\hat{x}(t \mid t-1). \quad (11)$$

Based on the projection formula, we will obtain the result on optimal state estimation [15].

Lemma 1: Consider the system (1) and (2), the optimal state estimation $\hat{x}(t \mid t)$ is given by

$$\hat{x}(t \mid t) = \hat{x}(t \mid t-1) + K(t)[y(t) - \gamma_t H\hat{x}(t \mid t-1)], \quad (12)$$

$$\hat{x}(t+1 \mid t) = \Phi\hat{x}(t \mid t), \hat{x}(0 \mid -1) = 0, \quad (13)$$

where K(t) is the solution to the following equation

$$K(t)Q_\eta(t) = \gamma_t P(t \mid t-1)H' \quad (14)$$

with

$$Q_\eta(t) = \gamma_t HP(t \mid t-1)H' + \gamma_t R, \quad (15)$$

$$P(t+1|t)=\Phi P(t|t-1)\Phi' -\gamma_t P(t|t-1)H'K'(t)\Phi'+GQG'. \tag{16}$$

3.2. Design of the optimal white-noise estimator

Based on the innovation sequences $\eta(0),\cdots,\eta(t+N)$, we will derive the optimal white-noise estimator $\hat{w}(t|t+N)$.

For $N\le 0$, it can be observed that $w(t)$ is independent of $\eta(0),\cdots,\eta(t+N)$. Then the estimation of $w(t)$ based on $\eta(0),\cdots,\eta(t+N)$ is 0, that is

$$\hat{w}(t|t+N)=\mathrm{E}_{w,v}\{w(t)|\eta(0),\cdots,\eta(t+N)\} =\mathrm{E}_{w,v}\{w(t)\}=0. \tag{17}$$

For $N>0$, the optimal input white-noise smoother $\hat{w}(t|t+N)$ is defined as follows.

Definition 2: Consider the given time instant t, then for $N>0$, the optimal input white-noise smoother $\hat{w}(t|t+N)$ is defined as

$$\hat{w}(t|t+N)=\hat{w}(t|t+N-1) +M_w(t|t+N)\eta(t+N), \tag{18}$$

where $M_w(t|t+N)$ is to be determined, such that

$$\mathrm{E}_{w,v}\|w(t)-\hat{w}(t|t+N)\|^2 \tag{19}$$

is minimized.

Further, define the covariance matrix of the estimation error as

$$P_w(t|t+N)\triangleq \mathrm{E}_{w,v}\{[w(t)-\hat{w}(t|t+N)] [w(t)-\hat{w}(t|t+N)]'\}.$$

Remark 4: In fact, the smoother $\hat{w}(t|t+N)$ defined in Definition 2 is the projection of $w(t)$ based on the linear space of $\eta(0),\cdots,\eta(t+N)$. The expectation in (19) is just taken over on the white noise $w(t)$ and $v(t)$, so there exists the packet arrival indicator γ_t in the smoother gains.

In light of the projection formula, we can obtain the optimal recursive input white-noise smoother $\hat{w}(t|t+N)$ as follows.

Theorem 1: Consider the system (1) and (2), the optimal recursive input white-noise smoother is given by

$$\hat{w}(t|t+N)=\hat{w}(t|t+N-1)+M_w(t|t+N)\eta(t+N), \tag{20}$$

where the initial value $\hat{w}(t|t)=0$, $N=1,2,\cdots$, and the smoother gain $M_w(t|t+N)$ satisfies the following equation

$$M_w(t|t+N)Q_\eta(t+N)=\gamma_{t+N}QG'\left\{\prod_{i=1}^{N-1}\Psi'_p(t+i)\right\}H' \tag{21}$$

with

$$\Psi_p(t+i)=\Phi[I_n-\gamma_{t+i}K(t+i)H]. \tag{22}$$

And the covariance matrix $P_w(t|t+N)$ can be calculated recursively as

$$P_w(t|t+N)=P_w(t|t+N-1)-M_w(t|t+N) \times Q_\eta(t+N)M'_w(t|t+N) \tag{23}$$

with the initial value $P_w(t|t)=Q$.

Proof: From the projection formula, we have

$$\hat{w}(t|t+N)=\hat{w}(t|t+N-1) +M_w(t|t+N)\eta(t+N), \tag{24}$$

where $M_w(t|t+N)$ to be determined. Note that

$$\eta(t+N)=\gamma_{t+N}H\tilde{x}(t+N|t+N-1)+\gamma_{t+N}v(t+N), \tag{25}$$

$$\tilde{x}(t+1|t)=\Psi_p(t)\tilde{x}(t|t-1)+Gw(t)-\gamma_t\Phi K(t)v(t), \tag{26}$$

where

$$\Psi_p(t)=\Phi[I_n-\gamma_t K(t)H]. \tag{27}$$

From (26), we have

$$\tilde{x}(t+N|t+N-1)=\Psi(t+N,t)\tilde{x}(t|t-1) +\sum_{i=t+1}^{t+N}\Psi(t+N,i)[Gw(i-1) -\gamma_{i-1}\Phi K(i-1)v(i-1)], \tag{28}$$

where

$$\Psi(t+N|t+N)=I_n,$$
$$\Psi(t+N|i)=\Psi_p(t+N-1)\cdots\Psi_p(i), i<t+N.$$

Substitute (28) into (25), yields

$$\eta(t+N)=\gamma_{t+N}H\{\Psi(t+N,t)\tilde{x}(t|t-1) +\sum_{i=t+1}^{t+N}\Psi(t+N,i)\times[Gw(i-1) -\gamma_{i-1}\Phi K(i-1)v(i-1)]\}+\gamma_{t+N}v(t+N). \tag{29}$$

Under Assumptions 1 and 2, and given the above equation, we have

$$\mathrm{E}[w(t)\eta'(t+N)]=\gamma_{t+N}QG'\Psi'(t+N,t+1)H' =\gamma_{t+N}QG'\left\{\prod_{i=1}^{N-1}\Psi'_p(t+i)\right\}H'. \tag{30}$$

Case 1: When the covariance matrix of the innovation $\eta(t+N)$, denoted by $Q_\eta(t+N)$, is invertible, we have

$$M_w(t|t+N) =\mathrm{E}_{w,v}[w(t)\eta'(t+N)]\mathrm{E}_{w,v}[\eta(t+N)\eta'(t+N)]^{-1} =\gamma_{t+N}QG'\left\{\prod_{i=1}^{N-1}\Psi'_p(t+i)\right\}H'Q_\eta^{-1}(t+N),$$

which satisfies (21).

Case 2: When the covariance matrix $Q_\eta(t+N)$ is singular, the matrix $M_w(t|t+N)$ is chosen to minimize $\|\hat{w}(t|t+N)-w(t)\|^2$, such that

$$M_w(t\mid t+N)Q_\eta(t+N)=\gamma_{t+N}QG'\left\{\prod_{i=1}^{N-1}\Psi'_p(t+i)\right\}H',$$

which is (21).

Next, we start to derive the expression of $P_w(t\mid t+N)$. Recalling from (20), we have

$$\tilde{w}(t\mid t+N)=\tilde{w}(t\mid t+N-1)-M_w(t\mid t+N)\eta(t+N).$$

Note that $\tilde{w}(t\mid t+N)\perp\eta(t+N)$, yields (23) with the initial value $P_w(t\mid t)=Q$.

Remark 5: In practice, the smoother developed in Theorem 1 is a fixed-point smoother, which is optimal for any packet generating process. The solution to this smoother is based on a stochastic Kalman filter, which can be obtained from Lemma 1 directly.

Further, we can derive the optimal nonrecursive input white-noise smoother from Theorem 1 directly.

Corollary 1: Consider the system (1) and (2), the optimal nonrecursive input white-noise smoother is given by

$$\hat{w}(t\mid t+N)=\sum_{j=1}^{N}M_w(t\mid t+j)\eta(t+j), N>0,$$

and the covariance matrix of the estimation error is given as

$$P_w(t\mid t+N)=Q-\sum_{j=1}^{N}M_w(t\mid t+j)Q_\eta(t+j)M'_w(t\mid t+j).$$

Remark 6: The smoother developed in Corollary 1 is an optimal fixed-lag smoother. However, it is need to remark that different from the standard white-noise smoother for a time-invariant system, neither the optimal gain nor the error covariance converges to a steady state value, therefore a new suboptimal white-noise smoother will be derived in the next section. Under appropriate assumptions, the suboptimal smoother will converge to a stationary one.

4. SUBOPTIMAL WHITE-NOISE ESTIMATOR

In this section, we will propose a new suboptimal input white-noise estimator with deterministic gains by minimizing the mean square estimation error where the statistics of the observation packet arrival random variable is used.

4.1. Design of the suboptimal state estimator

Before proceeding along the development of the suboptimal white-noise estimation, we first define the following state estimation, which will be used for the latter design of the white-noise estimator.

Definition 3: Given the system (1) and (2), the linear suboptimal state estimation $\hat{x}_e(t\mid t-1)$ of $x(t)$ with deterministic gain is defined as

$$\begin{aligned}\hat{x}_e(t+1\mid t)&\triangleq\Phi\hat{x}_e(t\mid t-1)\\&\quad+\Phi K(t)[y(t)-\gamma_t H\hat{x}_e(t\mid t-1)],\end{aligned}\tag{31}$$

$$\hat{x}_e(0\mid -1)=0,$$

where $K(t)$ is to be determined such that

$$\mathrm{E}_{\gamma,w,v}\left\|x(t+1)-\hat{x}_e(t+1\mid t)\right\|^2\tag{32}$$

is minimized. Further, we define the covariance matrix of the estimation error

$$P(t+1\mid t)\triangleq\mathrm{E}_{\gamma,w,v}[\tilde{x}_e(t+1\mid t)\tilde{x}'_e(t+1\mid t)],\tag{33}$$

where

$$\tilde{x}_e(t+1\mid t)=x(t+1)-\hat{x}_e(t+1\mid t).\tag{34}$$

Remark 7: Note from the criteria index of (32) that, the estimation $\hat{x}_e(t\mid t-1)$ defined in Definition 3 is different from the stochastic Kalman filter developed in Definition 1. The expectation in (32) is taken over w, v, and γ simultaneously.

In what follows, we introduce a new sequence

$$e(t)=y(t)-\gamma_t H\hat{x}_e(t\mid t-1),\tag{35}$$

where $\hat{x}_e(t\mid t-1)$ is as in Definition 3. It can be shown that the sequence $\{\{e(s)\}_{s=0}^{t}\}$ is mutually uncorrelated, which is with zero mean and covariance matrix

$$Q_e(t)=\rho HP(t\mid t-1)H'+\rho R.\tag{36}$$

Then one has the following result on the suboptimal state estimation [24].

Lemma 2: The suboptimal state estimation $\hat{x}_e(t\mid t)$ with deterministic gains is given by

$$\hat{x}_e(t\mid t)=\hat{x}_e(t\mid t-1)+K(t)[y(t)-\gamma_t H\hat{x}_e(t\mid t-1)],\tag{37}$$

$$\hat{x}_e(t+1\mid t)=\Phi\hat{x}_e(t\mid t),\hat{x}_e(0\mid -1)=0,\tag{38}$$

where

$$K(t)=\rho P(t\mid t-1)H'[\rho HP(t\mid t-1)H'+\rho R]^{-1},\tag{39}$$

and $P(t\mid t-1)$ satisfies the following generalized Riccati equation

$$\begin{aligned}P(t+1\mid t)&=\Phi P(t\mid t-1)\Phi'-\rho\Phi P(t\mid t-1)H'\\&\quad\times(HP(t\mid t-1)H'+R)^{-1}HP(t\mid t-1)\Phi'\\&\quad+GQG'\end{aligned}\tag{40}$$

with $P(0\mid -1)=P_0$.

Remark 8: The advantage of the suboptimal filter is that it leads to a deterministic time-varying filter which is easy to be implemented and all of its calculations (the gain matrices) can be done off line. The deterministic gain allows us to analyze the convergence and mean square stability of the filter. If the suboptimal estimation method developed in this paper and the estimation method employed in [22] are both designed based on the same packet loss measurement model as (1), the suboptimal estimator developed in this paper has smaller MSEs since it explores additional information on the arrival sequence γ_t. The detailed proof can be seen from Lemma 2 in [24].

4.2. Design of the suboptimal white-noise estimator

Based on the innovation sequences $e(0),\cdots,e(t+N)$, we will introduce a new linear recursive suboptimal input white-noise estimator $\hat{w}_e(t\,|\,t+N)$.

For $N\le 0$, it can be observed that $w(t)$ is independent of $e(0),\cdots,e(t+N)$, since $e(0),\cdots,e(t+N)$ are the linear combinations of the observations $y(0),\cdots,y(t+N)$. Then the estimation of $w(t)$ based on $e(0),\cdots,e(t+N)$ is 0, that is

$$\begin{aligned}\hat{w}_e(t\,|\,t+N)&=\mathrm{E}_{\gamma,w,v}\{w(t)\,|\,e(0),\cdots,e(t+N)\}\\&=\mathrm{E}_{\gamma,w,v}\{w(t)\}=0.\end{aligned}\tag{41}$$

For $N>0$, the suboptimal white-noise smoother $\hat{w}_e(t\,|\,t+N)$ is defined as follows.

Definition 4: Consider the given time t and assume that $N>0$, a linear recursive suboptimal estimation $\hat{w}_e(t\,|\,t+N)$ is defined as

$$\begin{aligned}\hat{w}_e(t\,|\,t+N)&\triangleq\hat{w}_e(t\,|\,t+N-1)\\&\quad+\bar{M}_w(t\,|\,t+N)e(t+N),\end{aligned}\tag{42}$$

where $\bar{M}_w(t\,|\,t+N)$ is to be determined, such that

$$\mathrm{E}_{\gamma,w,v}\left\|w(t)-\hat{w}_e(t\,|\,t+N)\right\|^2$$

is minimized.

Further, define

$$\begin{aligned}P_w(t\,|\,t+N)&\triangleq\mathrm{E}_{\gamma,w,v}\{[w(t)-\hat{w}_e(t\,|\,t+N)]\\&\quad\times[w(t)-\hat{w}_e(t\,|\,t+N)]'\}.\end{aligned}\tag{43}$$

In the next, we will derive the suboptimal white-noise smoother $\hat{w}_e(t\,|\,t+N)(N>0)$ defined in Definition 4 by using the minimum mean squared error method.

Theorem 2: Consider the system (1) and (2), the suboptimal recursive input white-noise smoother is given by

$$\begin{aligned}\hat{w}_e(t\,|\,t+N)&=\hat{w}_e(t\,|\,t+N-1)\\&\quad+\bar{M}_w(t\,|\,t+N)e(t+N),\end{aligned}\tag{44}$$

where the initial value $\hat{w}_e(t\,|\,t)=0$, and the smoother gain $\bar{M}_w(t\,|\,t+N)$ is given by

$$\bar{M}_w(t\,|\,t+N)=\rho QG'\left\{\prod_{i=1}^{N-1}\bar{\Psi}'_p(t+i)\right\}H'Q_e^{-1}(t+N)\tag{45}$$

with

$$\bar{\Psi}_p(t+i)=\mathrm{E}_\gamma\{\Psi_p(t+i)\}=\Phi[I_n-\rho K(t+i)H].\tag{46}$$

Meanwhile, the covariance matrix $P_w(t\,|\,t+N)$ can be calculated recursively by the following equation

$$\begin{aligned}P_w(t\,|\,t+N)&=P_w(t\,|\,t+N-1)-\bar{M}_w(t\,|\,t+N)\\&\quad\times Q_e(t+N)\bar{M}'_w(t\,|\,t+N),\end{aligned}\tag{47}$$

where the initial value $P_w(t\,|\,t)=Q$.

Proof: From Definition 4, we have

$$\begin{aligned}\hat{w}_e(t\,|\,t+N)&=\hat{w}_e(t\,|\,t+N-1)\\&\quad+\bar{M}_w(t\,|\,t+N)e(t+N),\end{aligned}\tag{48}$$

where $\bar{M}_w(t\,|\,t+N)$ to be determined. Note that

$$e(t+N)=\gamma_{t+N}H\tilde{x}_e(t+N\,|\,t+N-1)+\gamma_{t+N}v(t+N),\tag{49}$$

while

$$\begin{aligned}&\tilde{x}_e(t+N\,|\,t+N-1)\\&=\Psi(t+N,t)\tilde{x}_e(t\,|\,t-1)+\sum_{i=t+1}^{t+N}\Psi(t+N,i)\\&\quad\times[Gw(i-1)-\gamma_{i-1}\Phi K(i-1)v(i-1)]\end{aligned}\tag{50}$$

with

$$\begin{aligned}&\Psi(t+N,t+N)=I_n,\\&\Psi(t+N,i)=\Psi_p(t+N-1)\cdots\Psi_p(i),\\&\Psi_p(t+i)=\Phi[I_n-\gamma_{t+i}K(t+i)H].\end{aligned}$$

Substitute (50) into (49), yields

$$\begin{aligned}e(t+N)&=\gamma_{t+N}H\{\Psi(t+N,t)\tilde{x}_e(t\,|\,t-1)\\&\quad+\sum_{i=t+1}^{t+N}\Psi(t+N,i)[Gw(i-1)-\gamma_{i-1}\Phi\\&\quad\times K(i-1)v(i-1)]\}+\gamma_{t+N}v(t+N).\end{aligned}\tag{51}$$

In view of (48) and (51), we have

$$\begin{aligned}&\mathrm{E}_{\gamma,w,v}\{[w(t)-\hat{w}_e(t\,|\,t+N)][w(t)-\hat{w}_e(t\,|\,t+N)]'\}\\&=[\bar{M}_w(t\,|\,t+N)Q_e(t+N)-\rho QG'\bar{\Psi}'(t+N,t+1)H']\\&\quad\times Q_e^{-1}(t+N)\\&\quad[\bar{M}_w(t\,|\,t+N)Q_e(t+N)-\rho QG'\bar{\Psi}'(t+N,t+1)H']'\\&\quad+P_w(t\,|\,t+N-1)-\rho^2QG'\bar{\Psi}'(t+N,t+1)H'\\&\quad\times Q_e^{-1}(t+N)H\bar{\Psi}(t+N,t+1)GQ,\end{aligned}$$

where

$$\begin{aligned}&\bar{\Psi}(t+N,t+1)=\bar{\Psi}_p(t+N-1)\cdots\bar{\Psi}_p(t+1),\\&\bar{\Psi}_p(t+i)=\mathrm{E}_\gamma\{\Psi_p(t+i)\}=\Phi[I_n-\rho K(t+i)H].\end{aligned}$$

It is obvious that $E[\tilde{w}_e(t\,|\,t+N)\tilde{w}'_e(t\,|\,t+N)]$ will be minimized precisely if we choose

$$\bar{M}_w(t\,|\,t+N)=\rho QG'\bar{\Psi}'(t+N,t+1)H'Q_e^{-1}(t+N),$$

and thus (47) is satisfied.

Remark 9: In practice, the smoother developed in Theorem 2 is a fixed-point smoother, which is with deterministic gains. The solution to this smoother is based on a deterministic-gain state estimation, which can be obtained from Lemma 2 directly. In the next, we will present a nonrecursive white-noise smoother, that is the fixed-lag smoother.

Corollary 2: Consider the system (1) and (2), the suboptimal nonrecursive input white-noise smoother is given by

$$\hat{w}_e(t\,|\,t+N)=\sum_{j=1}^{N}\bar{M}_w(t\,|\,t+j)e(t+j),$$

and the covariance matrix of the estimation error satisfies the following equation

$$P_w(t\,|\,t+N)=Q-\sum_{j=1}^{N}\bar{M}_w(t\,|\,t+j)Q_e(t+j)\bar{M}'_w(t\,|\,t+j).$$

Remark 10: We will show in the next subsection that the estimation gains and the covariance of the estimation error converge to their respective steady-state values under appropriate assumptions. Thus this new white-noise estimation possesses better property of convergence than the optimal white-noise estimation developed in Section 3.

4.3. Stationary white-noise estimator

Note from Theorem 2 that the solution to the suboptimal white-noise smoother $\hat{w}_e(t\,|\,t+N)(N>0)$ is determined by the state estimation gain of $\hat{x}_e(t\,|\,t)$. If the state estimation $\hat{x}_e(t\,|\,t)$ keeps convergent, the white-noise smoother gain of $\hat{w}_e(t\,|\,t+N)$ becomes constant. And thus the suboptimal input white-noise smoother converges to the steady-state value. In what follows, we first analyze the convergence of the suboptimal state estimation.

Lemma 3: If $(A,GQ^{\frac{1}{2}})$ is controllable, (A,H) is detectable, then there exists a $\bar{\rho}\in[0,1)$, such that the Riccati difference equation (40) for $P(t\,|\,t-1)$ converges to a unique algebraic Riccati equation for $\rho>\bar{\rho}$ and $\forall P_0\geq 0$,

$$\bar{P}=\Phi\bar{P}\Phi'-\rho\Phi\bar{P}H'(H\bar{P}H'+R)^{-1}H\bar{P}\Phi'+GQG'. \quad (52)$$

And the corresponding filter and predictor become as

$$\hat{x}_e(t\,|\,t)=\hat{x}_e(t\,|\,t-1)+\bar{K}[y(t)-\gamma_t H\hat{x}_e(t\,|\,t-1)], \quad (53)$$

$$\hat{x}_e(t+1\,|\,t)=\Phi\hat{x}_e(t\,|\,t),\hat{x}_e(0\,|\,-1)=0, \quad (54)$$

where

$$\bar{K}=\rho\bar{P}H'[\rho H\bar{P}H'+\rho R]^{-1}. \quad (55)$$

In light of the result of Lemma 3, we will obtain the stationary white-noise smoother as follows.

Theorem 3: If $(A,GQ^{\frac{1}{2}})$ is controllable, (A,C) is detectable, then there exists a $\bar{\rho}\in[0,1)$ such that for $\rho>\bar{\rho}$, $\hat{w}_e(t\,|\,t+N)$ converges to the constant-gain smoother

$$\hat{w}_e(t\,|\,t+N)=\hat{w}_e(t\,|\,t+N-1)+\bar{M}_w(N)e(t+N), \quad (56)$$

where

$$\bar{M}_w(N)=\rho QG'(\Psi'_p)^{N-1}H'[\rho H\bar{P}H'+\rho R]^{-1} \quad (57)$$

with

$$\Psi_p=\Phi[I_n-\rho\bar{K}H].$$

And the covariance matrices $P_w(t\,|\,t+N)$ converges to the constant matrix $\bar{P}_w(N)$, which satisfies

$$\bar{P}_w(N)=\bar{P}_w(N-1)-\bar{M}_w(N)[\rho H\bar{P}H'+\rho R]\bar{M}'_w(N),$$
$$\bar{P}_w(0)=Q. \quad (58)$$

5. AN ILLUSTRATIVE EXAMPLE

In this section, we present a simple numerical example to illustrate the developed theoretical results. Consider a dynamic system described in (1) and (2) with the following parameters

$$\Phi=\begin{bmatrix}0.9 & -0.1\\ 0.2 & 0.7\end{bmatrix},\quad G=\begin{bmatrix}2\\3\end{bmatrix},\quad H=\begin{bmatrix}4 & 2\end{bmatrix},$$

where $w(t)$ and $v(t)$ are white noises with zero means and covariance matrices $Q=1$ and $R=1$, respectively. The initial value x_0 and its covariance matrix are set to be

$$x_0=\begin{bmatrix}1\\1\end{bmatrix},\quad P_0=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

In this example, it is assumed that γ_t is *a prior* known to the optimal estimator design, based on which the optimal state estimator $\hat{x}(t\,|\,t)$ can be designed online by using Lemma 1. And the optimal input white noise smoother $\hat{w}(t\,|\,t+N)$ can be obtained via Theorem 1. For the suboptimal estimation design, it is assumed that only the present-time value γ_t is known to the destination. The arrival probability is set to $\Pr ob\{\gamma_t=1\}=0.9$. Then from Lemma 2, the suboptimal state filter gain can be designed offline which is independent to $\{\gamma_\tau\}_{\tau=0}^{t}$. Given this state estimator, the suboptimal white-noise smoother $\hat{w}_e(t\,|\,t+N)$ can be derived by using Theorem 2.

In this simulation, we set $N=3$. Based on one path of γ_t, the optimal 3-step smoother $\hat{w}(t\,|\,t+3)$ is plotted in Fig. 3. The tracking performance of suboptimal 3-step smoother $\hat{w}_e(t\,|\,t+3)$ is given in Fig. 4. The performance comparison is also given in this simulation. In Fig. 5, the sum of mean square errors of optimal and suboptimal white noise smoother are plotted. It is apparent that the optimal white noise smoother gives better results. The main reason is that the history path of γ_t is exactly known to the estimator. However, the drawback of the optimal estimator is that the arrival process $\{\gamma_\tau\}_{\tau=0}^{t-1}$

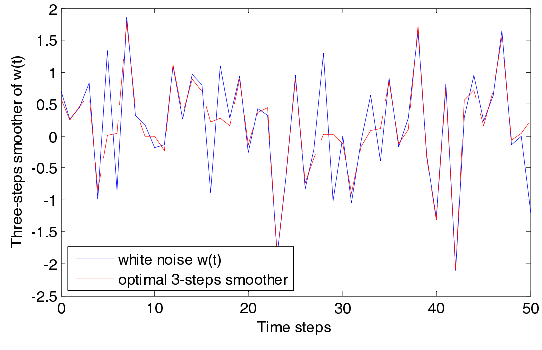

Fig. 3. The optimal 3-steps smoother of $w(t)$.

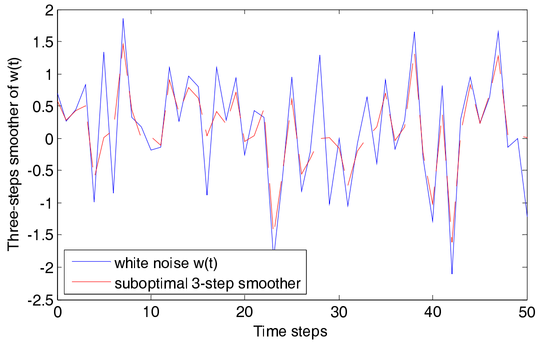

Fig. 4. The suboptimal 3-steps smoother of $w(t)$.

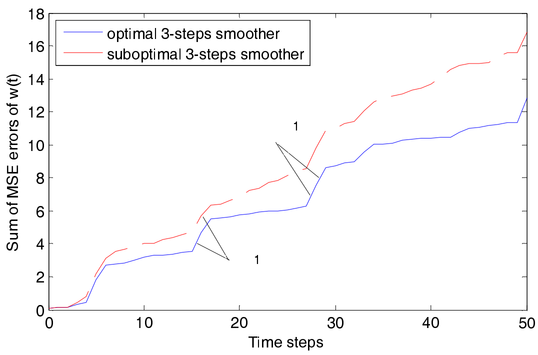

Fig. 5. Performance comparison of the proposed suboptimal and optimal smoother

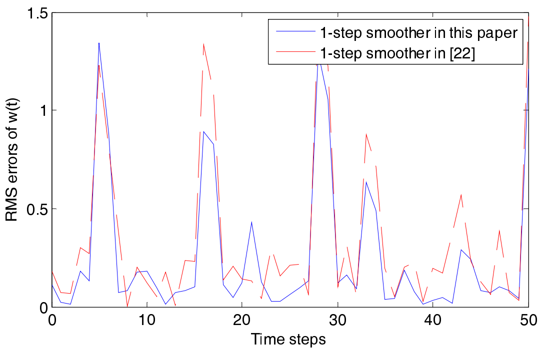

Fig. 6. Root mean square errors.

must be stored and can only be designed online. The suboptimal estimator only need the present arrival information γ_t, and the filter gain is independent to $\{\gamma_\tau\}_{\tau=0}^{t}$ and can be designed offline. Due to the packet loss, the sum of the mean square errors exist some obvious increases, see the locality labeled by 1 in Fig. 5.

The proposed suboptimal estimator is also compared to the one in [22] with the same parameters. Fig. 6 gives the root mean square errors, while the estimation error variances are given in Fig. 7. It shows that the estimation error variance is less than the one proposed in Ref. [22], and the ratio is about 85.6% at the final step with $\rho = 0.9$. The estimation error variance (final step) versus observation arrival rate ρ is also given in Fig. 8. From Fig. 8, we can conclude that with the same parameters the suboptimal white noise estimators proposed in this paper have a better estimation accuracy when $\rho < 1$, and have the same estimation performance when $\rho = 1$.

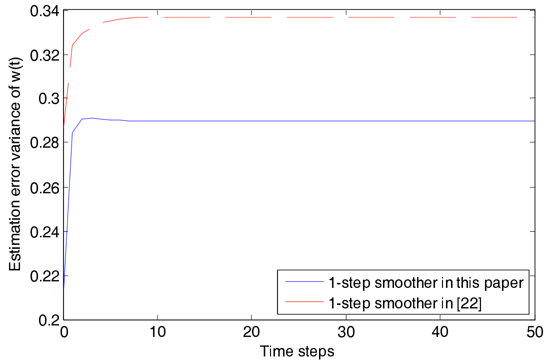

Fig. 7. Estimation error variances with $\rho = 0.9$.

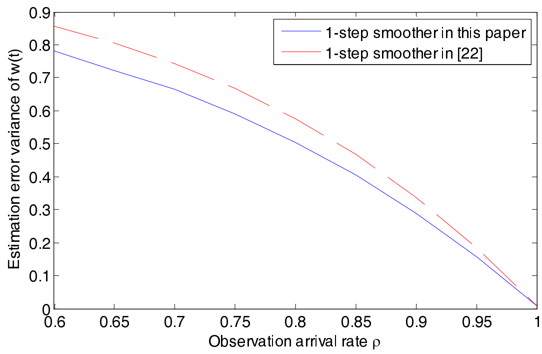

Fig. 8. Estimation error variances (final step) versus observation arrival rate ρ.

6. CONCLUSIONS

In this paper, we have studied the optimal and suboptimal input white noise estimation for networked systems with random packet losses. An optimal input white-noise estimator, with time-varying and stochastic gains has been presented via innovation analysis method and stochastic Kalman filtering result. Also, a suboptimal input white-noise estimator with deterministic gains has been proposed under a new performance index. The estimator gains were obtained with a new suboptimal state estimator. It has been shown that the suboptimal white-noise estimator converges to a steady-state one under natural assumptions.

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