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Reliable H_∞ Filter Design for a Class of Mixed-Delay Markovian Jump Systems with Stochastic Nonlinearities and Multiplicative Noises via Delay-Partitioning Method

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Abstract: This paper is concerned with the problem of reliable H_∞ filter design for a class of mixed-delay Markovian jump systems with stochastic nonlinearities and multiplicative noises. The mixed delays comprise both discrete time-varying delay and distributed delay. The stochastic nonlinearities in the form of statistical means cover several well-studied nonlinear functions. And the multiplicative disturbances are in the form of a scalar Gaussian white noise with unit variance. Furthermore, the failures of sensors are quantified by a variable varying in a given interval. A filter is designed to guarantee that the dynamics of the estimation error is asymptotically mean-square stable. Sufficient conditions for the existence of such a filter are obtained by using a new Lyapunov-Krasovskii functional and delay-partitioning method. Then a linear matrix inequality (LMI) approach for designing such a reliable H_∞ filter is presented. Finally, the effectiveness of the proposed approach is demonstrated by a numerical example.

Keywords: Delay partitioning, H_∞ filter, Markovian jump, multiplicative noise, reliable filtering, stochastic nonlinearity.

1. INTRODUCTION

Filtering problem has long been an important research topic for its theoretical and practical significance in signal processing, communication and control systems. The filtering problem can be briefly described as the design of an estimator from the measured output to estimate the state of the given system. In the last decade, filtering problems for various systems have attracted considered research interests and many important results have been reported in the literature [1-8]. Among these results, the H_∞ filter minimizes the H_∞ norm of the transfer function between the noise and the estimation error. Thus, the filter H_∞ is always used when the energy of the system noise are unknown. Therefore, H_∞ filtering approach has gained amount of research attention [9-15].

Markovian jump systems have a strong practical background, since the dynamics of systems may undergo sharp changes in their structure and parameters caused by phenomena such as component failures of repairs changing subsystem interconnections and environmental disturbance. We can model these as systems with Markovian jump parameters. Markovian jump systems have attracted great attention, for their extensive applications in manufacturing systems, power systems, economics systems, communication systems, and networked control systems. Recently, many works about filtering problem of Markovian jump systems have been reported [16-24].

Recently, the control and filtering problems for systems with multiplicative noises have received much attention since many plants may be modeled by systems with multiplicative noises, and some characteristics of nonlinear system can be approximated by models with multiplicative noises rather than by linearized models [22], and the H_∞ output-feedback control as well as passive control of discrete-time systems with state-multiplicative noises has been investigated in [23]. Therefore it is necessary to investigate the H_∞ filtering for systems with multiplicative noises.

On the other hand, it is inevitable that there exist time delays in dynamic systems due to measurement, transmission and transport lags, computational delays or unexpected inertia of system components, which have been known as main sources to degrade the performance of the control system [11]. In the last decade, significant progress has been made on the analysis and synthesis issues for systems with various types of delays [24-40]. However, to the best of the author's knowledge, the research on reliable H_∞ filtering for mixed delay systems with stochastic nonlinearities and multiplicative noises is still an open problem that deserves further investigation.

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In this paper, we focus on the reliable filtering problem against sensor failures for a class of mixed-delay Markovian jump systems with stochastic nonlinearites and multiplicative noises. The objective is to design a reliable H_∞ filter such that, in the presence of mixed delays, stochastic nonlinearities, multiplicative noises and Markovian jump parameters, the filtering error dynamics is asymptotically mean-square stable and also satisfies a prescribed H_∞ disturbance attenuation index. (I) Comparing with [10,12,15], stochastic nonlinearities, multiplicative noises, time-varying delays and possible sensor failures are considered for Markovian jumping systems, therefore the model in this paper is more general. (II) To obtain the sufficient conditions for the existence of a filter for such systems, a new Lyapunov-Krasovskii functional has been proposed and the delay-partitioning method has been employed.

Notation: The notion used through the paper is fairly standard. $\aleph^+$ stands for the set of nonnegative integers; $\Re^n$ and $\Re^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. $\mathrm{I_n}$ is the n-dimensional identity matrix. The notation $P>(\geq 0)$ means that P is real symmetric and positive definite (semi-definite). In symmetric block matrices or complex matrix expressions, we use an asterisk (*)to represent a term that is induced by symmetry and $diag\{...\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. Moreover, we may fix a probability space (Ω,ℓ,p) where, p, the probability measure, has total mass 1. $E\{x\}$ stands for the expectation of stochastic variable x. $L_2[0,+\infty)$ is the space of square integrable vectors.

2. PROBLEM FOMULATION

Consider the following mixed-delay Markovian jump system with stochastic nonlinearities and multiplicative noises:

$$\begin{cases} x(k+1)=A(r_k)x(k)+A_v(r_k)v(k)x(k), \\ \qquad +A_d(r_k)x(k-d(k))+A_l(r_k)\sum_{m=1}^{\infty}\mu_m x(k-m) \\ \qquad +B(r_k)\omega(k)+f(k), \\ y(k)=C_1(r_k)x(k)+C_2(r_k)\omega(k), \\ z(k)=C(r_k)x(k), \\ x(k)=\phi(k), -\infty<k\leq 0, \end{cases} \tag{1}$$

where $x(k)\in\Re^n$ is the state vector; $y(k)\in\Re^r$ is the signal to be estimated; $z(k)\in\Re^r$ is the output; $\omega(k)\in\Re^q$ is the disturbance input, which belongs to $L_2[0,+\infty)$; $d(k)$ denotes the time-varying delay with lower and upper bounds $\underline{d}\leq d(k)\leq\overline{d}$, $k\in\aleph^+$; $\underline{d}$, $\overline{d}$ are known positive integers, $\underline{d}$ can always be written by $\underline{d}=\tau m$ where τ and m are integers; $\phi(k)$ is the initial state of the system; and $A(r_k)$, $A_v(r_k)$, $A_d(r_k)$, $A_l(k)$, $B(r_k)$, $C_1(r_k)$, $C_2(r_k)$, $C(r_k)$ are matrix functions of the random jumping process r_k is a Markov chain taking values in a finite state space $\vartheta=\{1,...,N\}$ with transition probability matrix $p=\{\pi_{ij}\}_{N\times N}$ given by $p_r\{r_{k+1}\mid r_k\}=\pi_{ij}$, $\forall i,j\in\vartheta$, $0\leq\pi_{ij}\leq 1(i,j\in\vartheta)$ is the transition rate from i t0 j and $\sum_{j=1}^{N}\pi_{ij}=1, \forall i\in\vartheta$. $v(k)$ is a scalar Wiener process (Brownian Motion) defined on a complete probability space (Ω,ℓ,p) with

$$E\{v(k)\}=0,\ E\{v^2(k)\}=1.$$

And the constants $\mu_m\geq 0(m=1,2,..)$ satisfy the following convergence conditions:

$$\bar{u}:=\sum_{m=1}^{+\infty}\mu_m\leq\sum_{m=1}^{+\infty}m\mu_m<+\infty. \tag{2}$$

The function $f(k)$ describes the well-known stochastic nonlinearities that consist of $x(k)$, $x(k-d(k))$, and $\sum_{m=1}^{+\infty}\mu_m x(k-m)$, which is bounded in a statistical sense as follows:

$$\begin{aligned} &E\{f(k)\}=0, \\ &E\{f^T(k)f(k)\}=\sum_{i=1}^{q}\rho_l(r_k)\rho_l^T(r_k)(x^T(k)A_l^* x(k) \\ &\quad +x^T(k-d(k))B_l^* x(k-d(k)) \\ &\quad +\left(\sum_{m=1}^{+\infty}\mu_m x(k-m)\right)^T C_l^*\sum_{m=1}^{+\infty}\mu_m x(k-m)), \end{aligned} \tag{3}$$

where $\rho_l(l=1,...,q)$ are known column vectors, and $A_l^*, B_l^*, C_l^*(l=1,...,q)$ are known positive-definite matrices with appropriate dimensions.

Remark 1: Mixed delays are arousing increasing interest and have been intensively studied. However, almost all of the existing literature is concerned with either the discrete-delay systems [9,18,25,26] or the distributed-delay systems [27]. To the best of authors' knowledge, papers on mixed delay systems in the discrete-time setting are scarce, especially for the H_∞ filtering problem. In this paper, both discrete delay $x(k-d(k))$ and distributed delay $\sum_{m=1}^{+\infty}\mu_m x(k-m)$ have been considered simultaneously for the problem of reliable H_∞ filtering.

When the sensors experience failures, the sensor failure model, which is used to describe the measured signal sent from sensors, will be considered as

$$y^f(k)=My(k), \tag{4}$$

where the sensor fault matrix $M=diag(m_1,...,m_p)$ satisfies:

$$\begin{aligned} 0\leq\underline{M}=diag\{\underline{m}_1,...,\underline{m}_p\}\leq M \\ \leq\overline{M}=diag\{\overline{m}_1,...,\overline{m}_p\}\leq 1, \end{aligned} \tag{5}$$

there, $m_i (i = 1,..., p)$ quantify the failure of the sensors.

For simplicity, we introduce the following notation

$$M_0 = diag\{m_{01},...m_{0p}\} := \frac{\underline{M} + \bar{M}}{2} = diag\left\{\frac{\underline{m}_1 + \bar{m}_1}{2},...,\frac{\underline{m}_p + \bar{m}_p}{2}\right\}, \tag{6}$$

$$\hat{M} = diag\{\hat{m}_1,...\hat{m}_p\} := \frac{\bar{M} - \underline{M}}{2} = diag\left\{\frac{\bar{m}_1 - \underline{m}_1}{2},...,\frac{\bar{m}_p - \underline{m}_p}{2}\right\}. \tag{7}$$

Then, M can be rewritten as follows:

$$M = M_0 + \Delta = M_0 + diag\{\theta_1,...,\theta_p\}, \tag{8}$$

where $|\theta_i| \le \hat{m}_i (i = 1,..., p)$.

Consider the filter of the following structure:

$$\begin{cases} \hat{x}(k+1) = A_f(r_k)\hat{x}(k) + B_f(r_k)y^f(k), \\ \hat{z}(k) = C_f(r_k)\hat{x}(k), \end{cases} \tag{9}$$

where $A_f(r_k)$, $B_f(r_k)$, $C_f(r_k)$ are parameters to be determined. By defining $\eta(k) = [x^T(k) \quad \hat{x}^T(k)]^T$, we obtain the following filtering error system:

$$\begin{cases} \eta(k+1) = \hat{A}(r_k)\eta(k) + \hat{A}_v(r_k)v(k)\eta(k) \\ \qquad + \hat{A}_d(r_k)\eta(k-d(k)) \\ \qquad + \hat{A}_m(r_k)\sum_{m=1}^{+\infty}\mu_m\eta(k-m) \\ \qquad + \hat{B}(r_k)\omega(k) + \hat{I}f(k), \\ e(k) = \hat{C}(r_k)\eta(k), \end{cases} \tag{10}$$

where $e(k) = z(k) - \hat{z}(k)$ is the estimated error, and

$$\hat{A}(r_r) = \begin{bmatrix} A(r_k) & 0 \\ B_f(r_k)MC_1(r_k) & A_f(r_k) \end{bmatrix},$$

$$\hat{A}_v(r_r) = \begin{bmatrix} A_v(r_k) & 0 \\ 0 & 0 \end{bmatrix}, \ \hat{A}_d(r_r) = \begin{bmatrix} A_d(r_k) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\hat{A}_m(r_r) = \begin{bmatrix} A_l(r_k) & 0 \\ 0 & 0 \end{bmatrix}, \ \hat{B}(r_r) = \begin{bmatrix} B(r_k) \\ B_f(r_k)MC_2(r_k) \end{bmatrix},$$

$$\hat{I} = [I \quad 0]^T, \ \hat{C} = \begin{bmatrix} C(r_k) & -C_f(r_k) \end{bmatrix}.$$

The parameters A_f, B_f, C_f of the reliable filter (9) will be determined to make the filtering error dynamics asymptotically mean-square stable and satisfy a prescribed H_∞ disturbance attenuation index.

3. MAIN RESULTS

The following Lemmas are essential in establishing our main results.

Lemma 1 (Schur Complement)**:** Given constant matrices S_1, S_2 and S_3, where $S_1 = S_1^T$, $S_2 = S_2^T$. Then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ * & -S_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ & S_1 \end{bmatrix} < 0.$$

Lemma 2 [38]**:** Let $G \in \Re^{n\times n}$ be a positive semi-definite matrix, $x_i \in \Re^n$ and constant $\alpha_i > 0 (i = 1,2,...)$. If the series concerned is convergent, then

$$\left(\sum_{i=1}^{+\infty}\alpha_i x_i\right)^T G\sum_{i=1}^{+\infty}\alpha_i x_i \le \sum_{i=1}^{+\infty}\alpha_i x_i^T G\alpha_i x_i. \tag{11}$$

In the following theorem, we will derive a sufficient condition such that system (10) is asymptotically mean square stable.

Theorem 1: Given a scalar $\gamma > 0$ and the filter parameters A_f, B_f, C_f. The filtering error system (10) is asymptotically mean-square stable with its H_∞ stable if there exist positve matrices P_i, Q_1, Q_2, S_1, S_2, R, matrices M_1, M_2, M_3, positive scalars $\delta_l (l = 1,...,q)$ satisfying the following inequality for any $i \in \vartheta$

$$\begin{bmatrix} \Pi_i & \lambda_1 M_1 & \lambda_1 M_t & \Pi_{i2} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & \Pi_{i3} \end{bmatrix} < 0, \quad t = 2,3, \tag{12}$$

$$\begin{bmatrix} -\delta_l & \rho_{li}^T \hat{I}\sum_{j\in\vartheta}\pi_{ij}P_j & \lambda_1\rho_{li}^T\hat{I}S_1 & \lambda_2\rho_{li}^T\hat{I}S_2 \\ * & -\sum_{j\in\vartheta}\pi_{ij}P_j & 0 & 0 \\ * & * & -S_1 & 0 \\ * & * & * & -S_2 \end{bmatrix} < 0, \tag{13}$$

$$l = 1,...,q,$$

where

$$\begin{aligned} \Pi_i = & -X_{P_2}^T P_i X_{P_2} + X_{Q_1}^T \bar{Q}_1 X_{Q_1} + X_{Q_2}^T \bar{Q}_2 X_{Q_2} \\ & + X_{Q_3}^T \bar{Q}_3 X_{Q_3} + X_R^T \bar{R} X_R - \gamma^2 X_\omega^T X_\omega + M_1 X_{M_1} \\ & + (M_1 X_{M_1})^T + M_2 X_{M_2} + (M_2 X_{M_2})^T \\ & + M_3 X_{M_3} + (M_3 X_{M_3})^T, \end{aligned}$$

$$\begin{aligned} \Pi_{i2} = \Big[& \left(\sum_{j\in\vartheta}\pi_{ij}P_j X_{P_{1,i}}\right)^T \quad \left(\sum_{j\in\vartheta}\pi_{ij}P_j X_{P_{6,i}}\right)^T \\ & (\lambda_1 S_1 X_{P_{4,i}})^T \quad (\lambda_2 S_2 X_{P_{4,i}})^T \quad (\hat{A}_l^* \hat{I} X_{P_2})^T \\ & (\hat{B}_l^* \hat{I} X_{P_3})^T \quad (\hat{C}_l^* \hat{I} X_{P_4})^T \quad X_{P_{5,i}}^T \Big], \end{aligned}$$

$$\Pi_{i3} = diag\left\{-\sum_{j\in\vartheta}\pi_{ij}P_j, -\sum_{j\in\vartheta}\pi_{ij}P_j, -S_1, -S_2, -\Lambda, -\Lambda, -\Lambda, -I\right\},$$

$$\lambda_1 = \sqrt{\tau}, \ \lambda_2 = \sqrt{d - \tau m}, \quad \lambda_3 = \sqrt{d - \tau m + 1},$$

$$\bar{Q}_1 = \begin{bmatrix} -Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix} -Q_2 & 0 \\ 0 & -Q_2 \end{bmatrix},$$

$$\bar{Q}_3 = \begin{bmatrix} -Q_3 & 0 \\ 0 & -Q_3 \end{bmatrix},\quad \bar{R} = \begin{bmatrix} \bar{\mu}R & 0 \\ 0 & -\frac{R}{\bar{\mu}} \end{bmatrix},$$
$$X_{P_{1,i}} = [\hat{A}_i\ 0_{2n,2mn}\ \hat{A}_{di}\ \hat{A}_{li}\ 0_{2n}\ \hat{B}_i],$$
$$X_{P_2} = [I_{2n}\ 0_{2n,2mn+6n+h}],$$
$$X_{P_3} = [0_{2n,2mn+2n}\ I_{2n}\ 0_{2n,4n+h}],$$
$$X_{P_4} = [0_{2n,2mn+6n}\ I_{2n}\ 0_{2n,h}],\quad X_{P_{4,i}} = X_{P_{1,i}} - X_{P_2},$$
$$X_{P_{5,i}} = [\hat{C}_i\ 0_{2n,2mn+6n+h}],\quad X_{P_{6,i}} = [\hat{A}_{vi}\ 0_{2n,2mn+6n+h}],$$
$$X_{Q_1} = \begin{bmatrix} I_{2mn} & 0_{2mn,8n+h} & \\ 0_{2mn,2n} & I_{2mn} & 0_{2mn,6n+h} \end{bmatrix},$$
$$X_{Q_2} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+6n+h} & \\ 0_{2n,2mn+4n} & I_{2n} & 0_{2mn,2n+h} \end{bmatrix},$$
$$X_{Q_3} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+6n+h} & \\ 0_{2n,2mn+4n} & I_{2n} & 0_{2n,2n+h} \end{bmatrix},$$
$$X_R = \begin{bmatrix} I_{2n} & 0_{2n,2mn+6n+h} & \\ 0_{2n,2mn+6n} & I_{2n} & 0_{2n,h} \end{bmatrix},$$
$$X_{M_1} = [I_{2n}\ -I_{2n}\ 0_{2n,2mn+4n+h}],$$
$$X_{M_2} = [0_{2n,2mn}\ I_{2n}\ -I_{2n}\ 0_{2n,4n+h}],$$
$$X_{M_3} = [0_{2n,2mn+2n}\ I_{2n}\ -I_{2n}\ 0_{2n,2n+h}],$$
$$X_\omega = [0_{h,2mn+8n}\ \mathrm{I}_h],$$
$$\hat{A}^* = [A_1^{*1/2},...,A_q^{*1/2}],\ \hat{B}^* = [B_1^{*1/2},...,B_q^{*1/2}],$$
$$\hat{C}^* = [C_1^{*1/2},...,C_q^{*1/2}],\ \Lambda = [\delta_1^{-1}I,...,\delta_q^{-1}].$$

Proof: Define the following Lyapunov-Krasovskii functional candidate $V(k) = \sum_{i=1}^5 V_i(k)$, where

$$V_1(k) = \eta^T(K)P(r_k)\eta(k),$$
$$V_2(k) = \sum_{\alpha=k-\tau}^{-\tau m+1} \Upsilon^T(\alpha)Q_1\Upsilon(\alpha) + \sum_{\alpha=k-\bar{d}}^{k-1} \eta^T(\alpha)Q_2\eta(\alpha),$$
$$V_3(k) = \sum_{\beta=-\bar{d}+1}^{-\tau m+1}\sum_{\alpha=k+\beta}^{k-1} \eta^T(\alpha)Q_3\eta(\alpha),$$
$$V_4(k) = \sum_{\beta=-\tau}^{-1}\sum_{\alpha=k+\beta}^{k-1} \Psi^T(\alpha)S_1\Psi(\alpha) + \sum_{\beta=-\bar{d}}^{-\tau m-1}\sum_{\alpha=k+\beta}^{k-1} \eta^T(\alpha)S_2\eta(\alpha),$$
$$V_5(k) = \sum_{m=1}^{+\infty}\sum_{l=k-m}^{k-1} \eta^T(l)R\eta(l)$$

with

$$\Psi(i) = \eta(i+1) - \eta(i),$$
$$\Upsilon(i) = [\eta^T(i)\ \eta^T(i-\tau)...\eta^T(i-(m-1)\tau)].$$

For each $r_k = i \in \vartheta$, calculating the difference of $V(k)$ along system (10) and taking the mathematical expectation, then

$$E\{\Delta V(k)\} = \sum_{i=1}^5 E\{\Delta V_i(k)\} = \sum_{i=1}^5 E\{V_i(k+1) - V_i(k)\},$$

where

$$\begin{aligned} E\{\Delta V_1(k)\} = E\{\varsigma^T(k)(&X_{P_{1,i}}^T \sum_{j\in\vartheta} \pi_{ij}P_jX_{P_{1,i}} \\ &+ X_{P_{6,i}}^T \sum_{j\in\vartheta} \pi_{ij}P_jX_{P_{6,i}} \\ &+ X_{P_2}^T \sum_{l=1}^q \hat{I}A_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T)X_{P_2} \\ &+ X_{P_3}^T \sum_{l=1}^q \hat{I}B_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T)X_{P_3} \\ &+ X_{P_4}^T \sum_{l=1}^q \hat{I}C_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T)X_{P_4})\varsigma(k)\}, \end{aligned}$$
$$E\{\Delta V_2(k)\} \le E\{\varsigma^T(k)(X_{Q_1}^T\bar{Q}_1X_{Q_1} + X_{Q_2}^T\bar{Q}_2X_{Q_2})\varsigma(k)\},$$
$$E\{\Delta V_3(k)\} \le E\{\varsigma^T(k)(X_{Q_3}^T\bar{Q}_3X_{Q_3})\varsigma(k)\},$$
$$\begin{aligned} &E\{\Delta V_4(k)\} \\ &= E\{\varsigma^T(k)(\tau X_{P_{4,i}}^T S_1 X_{P_{4,i}} + (\bar{d}-\tau m)X_{P_{4,i}}^T S_2 X_{P_{4,i}} \\ &\quad + X_{P_2}^T \sum_{l=1}^q \hat{I}A_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T(\tau S_1 + (\bar{d}-\tau m)S_2))X_{P_2} \\ &\quad + X_{P_3}^T \sum_{l=1}^q \hat{I}B_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T(\tau S_1 + (\bar{d}-\tau m)S_2))X_{P_3} \\ &\quad + X_{P_4}^T \sum_{l=1}^q \hat{I}C_{li}^*\hat{I}^T tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T(\tau S_1 + (\bar{d}-\tau m)S_2))X_{P_4}) \\ &\quad \times\varsigma(k) - \sum_{\alpha=k-\tau}^{k-1}\Psi^T(\alpha)S_1\Psi(\alpha) - \sum_{\alpha=k-d(k)}^{k-\tau m-1}\Psi^T(\alpha)S_2\Psi(\alpha) \\ &\quad - \sum_{\alpha=k-\bar{d}}^{k-d(k)-1}\Psi^T(\alpha)S_2\Psi(\alpha)\}, \end{aligned}$$
$$E\{\Delta V_5(k)\} \le E\{\bar{\mu}\eta^T(k-m)R\eta(k-m) - \sum_{m=1}^{+\infty}\eta^T(k-m)R\times\eta(k-m)\}.$$

According to Lemma 3.

$$-\sum_{m=1}^{+\infty}\mu_m\eta^T(k-m)R\eta(k-m) \le \frac{1}{\bar{\mu}}(\sum_{m=1}^{+\infty}\mu_m\eta(k-m))^T R\sum_{m=1}^{+\infty}\mu_m\eta(k-m),$$

therefore

$$E\{\Delta V_5(k)\} \le E\{\varsigma^T(k)(X_R^T\bar{R}X_R)\varsigma(k)\} \tag{14}$$

with

$$\varsigma(k) = [\Upsilon^T(k)\ \eta^T(k-m)\ \eta^T(k-d(k))\ \eta^T(k-\bar{d})\ (\sum_{m=1}^{+\infty}\mu_m\eta(k-m))^T\ \omega^T(k)]^T.$$

By the definition of $\Psi(\alpha)$, for any matrices M_1, M_2 and M_3,

$$2\varsigma^T(k)M_1[\eta(k) - \eta(k-\tau) - \sum_{\alpha=k-\tau}^{k-1}\Psi(\alpha)] = 0,$$
$$2\varsigma^T(k)M_2[\eta(k-\tau m) - \eta(k-d(k)) - \sum_{\alpha=k-d(k)}^{k-1}\Psi(\alpha)] = 0,$$

$$2\varsigma^T(k)M_3[\eta(k-d(k))-\eta(k-\bar{d})-\sum_{\alpha=k-\bar{d}}^{k-d(k)}\Psi(\alpha)]=0.$$

From (12)

$$tr(\hat{I}\rho_{li}\rho_{li}^T\hat{I}^T(\sum_{j\in\vartheta}\pi_{ij}P_j+\tau S_1+(\bar{d}-\tau m)S_2) \le \delta_l(l=1,...,q).$$

To analyze the H_∞ performance of the filtering error system (10), consider the following index:

$$\begin{aligned} J_N &= E\{\sum_{k=0}^{N}(e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k))\} \\ &= E\{\sum_{k=0}^{N}(e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k) \\ &\quad +V(k+1)-V(k))\}-V(N+1) \\ &\le E\{\sum_{k=0}^{N}(e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(k))\}. \end{aligned} \quad (15)$$

Then

$$\begin{aligned} & E\{\sum_{k=0}^{N}(e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(k))\} \\ &\le E\{\varsigma^T(k)\{\frac{d(k)-\tau m}{\bar{d}-\tau m}(\Pi_i+X_{P_{1,i}}^T\sum_{j\in\vartheta}\pi_{ij}P_jX_{P_{1,i}} \\ &\quad +\tau X_{P_{4,i}}^T S_1 X_{P_{4,i}}+(\bar{d}-\tau m)X_{P_{4,i}}^T S_2 X_{P_{4,i}}+X_{P_{5,i}}^T X_{P_{5,i}} \\ &\quad +X_{P_{6,i}}^T\sum_{j\in\vartheta}\pi_{ij}P_jX_{P_{6,i}}+X_{P_2}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{A}_{li}^*\hat{I}^TX_{P_2}))\} \\ &\quad +X_{P_3}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{B}_{li}^*\hat{I}^TX_{P_3}+X_{P_4}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{C}_{li}^*\hat{I}^TX_{P_4} \\ &\quad +\tau M_1S_1^{-1}M_1^T+(\bar{d}-\tau m)M_2S_2^{-1}M_2^T) \\ &\quad +\frac{d(k)-\tau m}{\bar{d}-\tau m}(\Pi_i+X_{P_{1,i}}^T\sum_{j\in\vartheta}\pi_{ij}P_jX_{P_{1,i}} \\ &\quad +\tau X_{P_{4,i}}^T S_1 X_{P_{4,i}}+(\bar{d}-\tau m)X_{P_{4,i}}^T S_2 X_{P_{4,i}}+X_{P_{5,i}}^T X_{P_{5,i}} \\ &\quad +X_{P_{6,i}}^T\sum_{j\in\vartheta}\pi_{ij}P_jX_{P_{6,i}}+X_{P_2}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{A}_{li}^*\hat{I}^TX_{P_2}))\} \\ &\quad +X_{P_3}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{B}_{li}^*\hat{I}^TX_{P_3}+X_{P_4}^T\sum_{l=1}^{q}\psi_l\hat{I}\hat{C}_{li}^*\hat{I}^TX_{P_4} \\ &\quad +\tau M_1S_1^{-1}M_1^T+(\bar{d}-\tau m)M_3S_2^{-1}M_3^T)\}\varsigma(k)\}. \end{aligned} \quad (16)$$

According to Lemma 2, it follows from (13) that

$$E\left\{\sum_{k=0}^{N}(e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(k))\right\}<0,$$

which implies $J_N < 0$. When $\omega(k)\equiv 0$, by (13) and Lemma 2

$$E\{\Delta V(k)\}<0.$$

As discussed in [25], inequality (12) holds. This completes the proof.

To solve the reliable H_∞ filtering problem by the LMI technique, the stability conditions (13) in Theorem 1 have to be inverted into LMI forms. Using the numerical convex optimization algorithm [41] to solve the modified LMI conditions, a reliable H_∞ filter can be obtained. In the following conditions, we will try to find a possible way for the solution of (13).

Remark 2: The asymptotically stability conditions for the filtering error system (10) with a prescribed H_∞ performance level have been obtained in Theorem 1 via the delay-partitioning method. The condition can be checked by solving a set of LMIs. As reported that delay-partitioning approach is effective in reducing the possible conservatism, at the cost of increasing the computation burden, therefore, the partitioning number m should be properly chosen.

Theorem 2: Given a scalar $\gamma>0$ and the filter parameters A_f, B_f, C_f The filtering error system (10) is asymptotically mean-square stable with its H_∞ stable if there exist positive matrices P_i, Q_1, Q_2, S_1, S_2, R, matrices M_1, M_2, M_3, Θ_i, positive scalars $\delta_l(l=1,...,q)$ satisfying the following inequality for any $i\in\vartheta$

$$\Xi_{ti}=\begin{bmatrix} \Pi_i & \lambda_1 M_1 & \lambda_1 M_t & \hat{\Pi}_{i2} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & \hat{\Pi}_{i3} \end{bmatrix}<0, t=2,3, \quad (17)$$

$$\Xi_{li}=\begin{bmatrix} -\delta_l & \rho_{li}^T\hat{I}\Theta_i & \lambda_1\rho_{li}^T\hat{I}\Theta_i & \lambda_2\rho_{li}^T\hat{I}\Theta_i \\ * & \Theta_{p,i} & 0 & 0 \\ * & * & \Theta_{S_1} & 0 \\ * & * & * & \Theta_{S_2} \end{bmatrix}<0, \quad (18)$$

$$l=1,...,q,$$

where

$$\Theta_{p,i}=\sum_{j\in\vartheta}\pi_{ij}P_j-\Theta_i-\Theta_i^T,$$
$$\Theta_{S_1,i}=S_1-\Theta_i-\Theta_i^T,$$
$$\Theta_{S_2,i}=S_2-\Theta_i-\Theta_i^T,$$
$$\hat{\Pi}_{i2}=\Big[(\Theta_iX_{P_{1,i}})^T\ (\Theta_iX_{P_{6,i}})^T\ (\lambda_1\Theta_iX_{P_{4,i}})^T\ (\lambda_2\Theta_iX_{P_{4,i}})^T$$
$$(\hat{A}_l^*\hat{I}X_{P_2})^T\ (\hat{B}_l^*\hat{I}X_{P_3})^T\ (\hat{C}_l^*\hat{I}X_{P_4})^T\ X_{P_{5,i}}^T\Big],$$
$$\hat{\Pi}_{i3}=diag\{\Theta_{p,i}\Theta_{p,i},\Theta_{S_1,i},\Theta_{S_2,i},-\Lambda,-\Lambda,-\Lambda,-I\}.$$

Proof: Consider that

$$\sum_{j\in\vartheta}\pi_{ij}P_j-\Theta_i-\Theta_i^T\ge\Theta_i\left(\sum_{j\in\vartheta}\pi_{ij}P_j\right)^{-1}\Theta_i^T,$$
$$S_1-\Theta_i-\Theta_i^T\ge\Theta_i(S_1)^{-1}\Theta_i^T,$$
$$S_2-\Theta_i-\Theta_i^T\ge\Theta_i(S_2)^{-1}\Theta_i^T,$$

then

$$\begin{bmatrix} \Pi_i & \lambda_1 M_1 & \lambda_1 M_t & \hat{\Pi}_{i2} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & \vec{\Pi}_{i3} \end{bmatrix}<0, \quad t=2,3, \quad (19)$$

where

$$\bar{\Pi}_{i3} = diag\{-\Theta_i(\sum_{j\in\vartheta}\pi_{ij}P_j)^{-1}\Theta_i^T, -\Theta_i(\sum_{j\in\vartheta}\pi_{ij}P_j)^{-1}\Theta_i^T, -\Theta_i(S_1)^{-1}\Theta_i^T, -\Theta_i(S_2)^{-1}\Theta_i^T, -\Lambda, -\Lambda, -\Lambda, -I\}.$$

And define

$$\Psi_i = diag\{I, I, I, \sum_{j\in\vartheta}\pi_{ij}P_j\Theta_i^{-T}, \sum_{j\in\vartheta}\pi_{ij}P_j\Theta_i^{-T}, I, I, I, I\}.$$

Pre- and Post-multiply (19) by Ψ_i and Ψ_i^T respectively, it is direct to drive inequality (13). By the same way, it is easy to obtain that (17) can imply (12). The proof is completed.

Remark 3: By a variable Θ_i, we can eliminate the coupling between the Lyapunov matrices and the filtering error system matrices. Furthermore, this variable does not require any structural constraint such as symmetry, and provide potentially less conservative results.

Theorem 3: Given a scalar $\gamma > 0$ and the filter parameters A_f, B_f, C_f The filtering error system (10) is asymptotically mean-square stable with its H_∞ stable if there exist positive matrices P_i, Q_1, Q_2, S_1, S_2, R, matrices, M_1, M_2, M_3, Θ_i, positive scalars $\delta_l (l = 1,...,q)$ satisfying the following inequality for any $i \in \vartheta$

$$\bar{\Xi}_{ti} = \begin{bmatrix} \Pi_i & \lambda_1 M_1 & \lambda_1 M_t & \hat{\Pi}_{i2} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & \hat{\Pi}'_{i3} \end{bmatrix} < 0, \quad t = 2,3, \tag{20}$$

$$\bar{\Xi}_{li} = \begin{bmatrix} -\delta_l & \rho_{li}^T \hat{I}\Theta_i & \lambda_1\rho_{li}^T \hat{I}\Theta_i & \lambda_2\rho_{li}^T \hat{I}\Theta_i \\ * & \bar{\Theta}_{p,i} & 0 & 0 \\ * & * & \Theta_{S_1} & 0 \\ * & * & * & \Theta_{S_2} \end{bmatrix} < 0, \quad l = 1,...,q \tag{21}$$

hold, where

$$\hat{\Pi}'_{i3} = diag\{\bar{\Theta}_{p,i}\bar{\Theta}_{p,i}, \bar{\Theta}_{S_1,i}, \bar{\Theta}_{S_2,i}, -\Lambda, -\Lambda, -\Lambda, -I\},$$

$$\bar{\Theta}_{p,i} = \Lambda_j - \Theta_i - \Theta_i^T,$$

$$\Lambda_j = \left(\sum_{j\in\vartheta}\pi_{ij}\right)^{-1}\sum_{j\in\vartheta}\pi_{ij}P_j.$$

Proof: For any $i \in \vartheta$, Ξ_{ti} in (17) can be rewritten as

$$\Xi_{ti} = \sum_{j\in\vartheta}\pi_{ij}\bar{\Xi}_{ti}|_{\Lambda_j = (\sum_{j\in\vartheta}\pi_{ij})^{-1}\sum_{j\in\vartheta}\pi_{ij}P_j}, \quad t = 2,3 \tag{22}$$

Then $\Xi_{ti} < 0$, $t = 2,3$. By the same way, $\Xi_{li} < 0$ from (21). Therefore, the filtering error system (10) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ. The proof is completed.

In the following part, the problem of reliable H_∞ filter design will be solved.

Theorem 4: Given a scalar $\gamma > 0$ and the filter parameters A_f, B_f, C_f The filtering error system (10) is asymptotically mean-square stable with its H_∞ stable if there exist positive matrices P_{1i}, P_{2i}, P_{3i}, Q_1, Q_2, S_{11}, S_{12}, S_{13}, S_{21}, S_{22}, S_{23}, R, matrices M_1, M_2, M_3, Θ_{1i}, Θ_{2i}, Θ_{3i}, positive scalars $\delta_l (l = 1,...,q)$ satisfying the following inequality for any $i \in \vartheta$

$$\tilde{\Xi}_{ti} = \begin{bmatrix} \Pi_i & \lambda_1 M_1 & \lambda_1 M_t & \tilde{\Pi}_{i2} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & \tilde{\Pi}_{i3} \end{bmatrix} < 0, \quad t = 2,3, \tag{23}$$

$$\tilde{\Xi}_{li} = \begin{bmatrix} -\delta_l & \Sigma_{3i} & \lambda_1\Sigma_{3i} & \lambda_2\Sigma_{3i} \\ * & \Sigma_{4i} & 0 & 0 \\ * & * & \Sigma_{5i} & 0 \\ * & * & * & \Sigma_{6i} \end{bmatrix} < 0, \quad l = 1,...,q \tag{24}$$

hold, where

$$\tilde{\Pi}_{i2} = \begin{bmatrix} \Sigma_{7i}^T & \Sigma_{7i}^T & \lambda_1\Sigma_{8i}^T & \lambda_2\Sigma_{8i}^T & \Sigma_{9i}^T & \Sigma_{10i}^T & \Sigma_{11i}^T & \Sigma_{12i}^T \end{bmatrix},$$

$$\tilde{\Pi}_{i3} = diag\{\Sigma_{4i}^T, \Sigma_{4i}^T, \Sigma_{5i}^T, \Sigma_{6i}^T, -\Lambda, -\Lambda, -\Lambda, -I\},$$

$$\Sigma_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{13} \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} S_{21} & S_{22} \\ * & S_{23} \end{bmatrix},$$

$$P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ * & P_{i3} \end{bmatrix}, \quad \Sigma_{3i} = \begin{bmatrix} \rho_{li}^T\Theta_{1i} & \rho_{li}^T\Theta_{3i} \end{bmatrix},$$

$$\Sigma_{4i} = \begin{bmatrix} \Lambda_{1j} - \Theta_{1i} - \Theta_{1i}^T & \Lambda_{2j} - \Theta_{2i} - \Theta_{2i}^T \\ * & \Lambda_{3j} - \Theta_{3i} - \Theta_{3i}^T \end{bmatrix},$$

$$\Sigma_{5i} = \begin{bmatrix} S_{11} - \Theta_{1i} - \Theta_{1i}^T & S_{12} - \Theta_{2i} - \Theta_{2i}^T \\ * & S_{13} - \Theta_{3i} - \Theta_{3i}^T \end{bmatrix},$$

$$\Sigma_{6i} = \begin{bmatrix} S_{21} - \Theta_{1i} - \Theta_{1i}^T & S_{22} - \Theta_{2i} - \Theta_{2i}^T \\ * & S_{23} - \Theta_{3i} - \Theta_{3i}^T \end{bmatrix},$$

$$\Sigma_{7i} = \begin{bmatrix} \Theta_{1i}^T A_i + \hat{B}_i M C_{1i} & \hat{A}_i & 0_{n,2mn} & \Theta_{1i}^T A_{di} & 0_{n,3n} & \Theta_{1i}^T A_{li} & 0_{n,3n} & \Theta_{1i}^T B_i + \hat{B}_i M C_{1i} \\ \Theta_{3i}^T A_i + \hat{B}_i M C_{1i} & \hat{A}_i & 0_{n,2mn} & \Theta_{3i}^T A_{di} & 0_{n,3n} & \Theta_{3i}^T A_{li} & 0_{n,3n} & \Theta_{3i}^T B_i + \hat{B}_i M C_{1i} \end{bmatrix},$$

$$\Sigma_{8i} = \begin{bmatrix} \Theta_{1i}^T A_i - \Theta_{1i}^T + \hat{B}_i M C_{1i} & \hat{A}_i - \Theta_{2i}^T & 0_{n,2mn} & \Theta_{1i}^T A_{di} & 0_{n,3n} & \Theta_{1i}^T A_{li} & 0_{n,3n} & \Theta_{1i}^T B_i + \hat{B}_i M C_{1i} \\ \Theta_{3i}^T A_i - \Theta_{3i}^T + \hat{B}_i M C_{1i} & \hat{A}_i - \Theta_{2i}^T & 0_{n,2mn} & \Theta_{3i}^T A_{di} & 0_{n,3n} & \Theta_{1i}^T A_{li} & 0_{n,3n} & \Theta_{3i}^T B_i + \hat{B}_i M C_{1i} \end{bmatrix},$$

$$\Sigma_{9i} = \begin{bmatrix} \delta_1 A_{li}^{*1/2} & \dots & 0_{n,2mn+7n+h} \\ \dots & \dots & \dots \\ \delta_q A_{li}^{*1/2} & \dots & 0_{n,2mn+7n+h} \end{bmatrix},$$

$$\Sigma_{10i} = \begin{bmatrix} 0_{n,2mn+2n} & \delta_1 B^{*1/2}_{li} & \dots & 0_{n,5n+h} \\ \dots & \dots & \dots & \dots \\ 0_{n,2mn+2n} & \delta_q B^{*1/2}_{li} & \dots & 0_{n,5n+h} \end{bmatrix},$$

$$\Sigma_{11i} = \begin{bmatrix} 0_{n,2mn+6n} & \delta_1 C^{*1/2}_{li} & \dots & 0_{n,n+h} \\ \dots & \dots & \dots & \dots \\ 0_{n,2mn+6n} & \delta_q C^{*1/2}_{li} & \dots & 0_{n,n+h} \end{bmatrix},$$

$$\Sigma_{12i} = \begin{bmatrix} C_i & -\hat{C}_i & 0_{n,2mn+6n+h} \end{bmatrix},$$

$$\Lambda_{fi} = \left(\sum\nolimits_{j\in\vartheta} \pi_{ij}\right)^{-1} \sum\nolimits_{j\in\vartheta} \pi_{ij} P_{fj}, \quad f = 1,2,3.$$

And the parameters of the desired filter are given as

$$A_{fi} = \Theta_{2i}^{-T} \hat{A}_i, \quad B_{fi} = \Theta_{2i}^{-T} \hat{B}_i, \quad C_{fi} = \hat{C}_i. \qquad (25)$$

Proof: Partition Θ_i, P_i, S_1, S_2, as

$$\Theta_i = \begin{bmatrix} \Theta_{1i} & \Theta_{2i} \\ * & \Theta_{3i} \end{bmatrix}, \quad P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix},$$

$$S_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{13} \end{bmatrix}, \quad S_2 = \begin{bmatrix} S_{21} & S_{22} \\ * & S_{23} \end{bmatrix}.$$

Substituting (25), (26) into (20) and (21), it is easy to obtain LMIs (23) and (24). This proof is completed.

In Theorems 1-4, the asymptotically stability conditions of the filtering error system (10) and an H_∞ filter based on the method are obtained with known sensor failure paramter and disturbance lever γ.

Remark 4: As an important topic, the stability of nonlinearity stochastic systems has been widely investigated [17,24,28,34,36,37,39,40]. However, there are few works about the mixed time-delay systems with Markovian jump. In this regard, for the mixed time-delay systems with Markovian jump considered in this paper, people can further reduce the possible conservatism of the main results by making an effort to construct more general Lyapunov functionals, which leaves an interesting research issue for further investigation.

4. NUMERICAL EXAMPLE

In this section, a numerical example is used to demonstrate the effectiveness of the proposed reliable H_∞ filter for a class of discrete-time mixed delay systems with nonlinearities and stochastic noises. Consider system (1) and the reliable filter (9) with the parameters as follows:

$$A_1 = A_3 = \begin{bmatrix} -0.5 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \quad A_2 = A_4 = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.2 \end{bmatrix},$$

$$A_{d1} = A_{d2} = A_{d3} = A_{d4} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_{l1} = A_{l2} = A_{l3} = A_{l4} = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T,$$

$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T,$$

$$C_{11} = C_{12} = C_{13} = C_{14} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

$$C_{21} = C_{22} = C_{23} = C_{24} = 0.01,$$

$$\mu_m = 3^{-(3+m)}, \quad d(k) = 1.5 + \frac{1+(-1)^k}{2},$$

$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \begin{bmatrix} 0.003 & 0.003 \end{bmatrix}^T,$$

$$A_1^* = A_2^* = A_3^* = A_4^* = B_1^* = B_2^* = B_3^* = B_4^* = C_1^* = C_2^* = C_3^* = C_4^* = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

and the transition probability matrix as shown in Fig. 1 is given by

$$\vartheta = \begin{bmatrix} 0.1 & 0.1 & 0.4 & 0.4 \\ 0.2 & 0.1 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.4 \end{bmatrix}.$$

Then the following condition satisfying (2) holds.

$$\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m = \frac{1}{54} < \sum_{m=1}^{+\infty} m\mu_m = \frac{1}{36} < +\infty.$$

And it is easy to verify that

$$\underline{d} = 1.5, \quad \bar{d} = 2.5.$$

And $v_i(k)\,(i = 1,\dots,4)$ represent the mutually uncorrelated white noise sequences with unity covariances. The sensor fault matrix M is assumed to satisfy $0.6 \le M \le 0.8$. Then we can obtain that $M_0 = 0.7$, $\hat{M} = 0.1$. And let $m = 1$, $\omega(k) = \exp(-k/10) \times n(k)$, $n(k)$ is uniformly distributed over $[-0.5, 0.5]$.

With the above parameters and by using Matlab LMI Toolbox, we can solve LMIs (20) and (21), and obtain the filter parameters

$$A_f = \begin{bmatrix} -0.2740 & -0.1941 \\ 0.0010 & -0.1401 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.8156 \\ -0.3146 \end{bmatrix},$$

$$C_f = \begin{bmatrix} 0.8957 & 0.0054 \end{bmatrix},$$

and $\gamma^* = 0.2925$. With these parameters, according to Theorem 4, the filter error system (10) is asymptotically mean-square stable. The simulation results are shown in Figs. 2 and 3. Fig. 2 shows estimation of $z(k)$ and Fig. 3 shows the estimated error $e(k)$ which verify that the expected system performance requirements are achieved well.

Remark 5: It is obvious that, as m becomes larger, it will greatly decrease the conservatism of the conditions in main result, however, this will increase the computation burden, so how to choose proportional partitioning number m leaves for a further study.

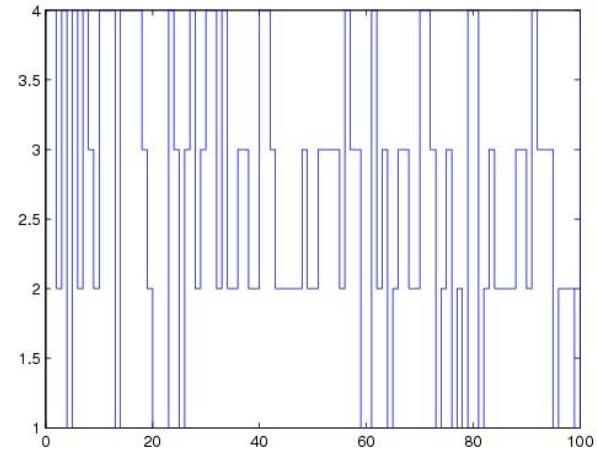

Fig. 1. The transition of Markovian chain p.

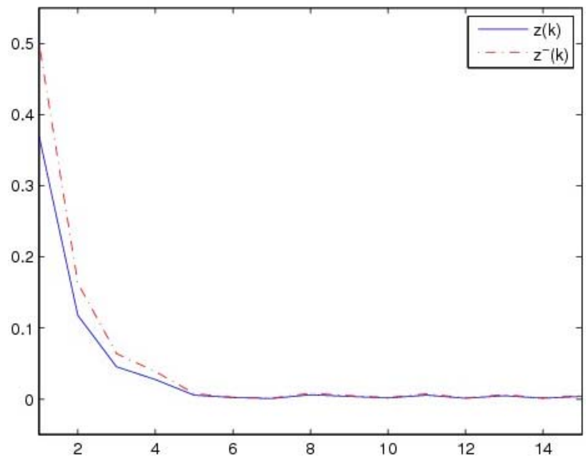

Fig. 2. Output $z(k)$, and estimated output $\hat{z}(k)$.

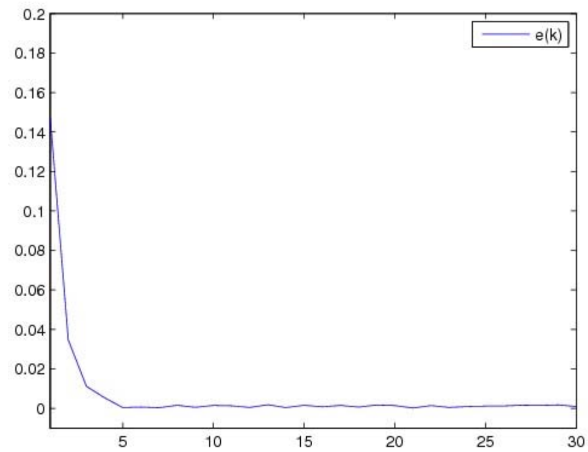

Fig. 3. The estimated error $e(k)$

5. CONCLUSION

In this paper, the problem of the reliable H_∞ filtering for a class of mixed-delay Markovian jump systems with stochastic nonlinearities and multiplicative noises has been investigated. A new Lyapunov-Krasovskii functional and delay-partitioning technique have been used to design the filter, such that the filtering error system is asymptotically mean-square stable. And the filter parameters can be obtained by solving certain LMIs. An illustrative example has been used to show the effectiveness of the proposed method.

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