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Adaptive Fuzzy Backstepping Dynamic Surface Control for Output-constrained Non-smooth Nonlinear Dynamic System

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Abstract: Output-constrained backstepping dynamic surface control (DSC) is proposed for the purpose of output constraint and precise output positioning of a strict feedback single-input, single-output dynamic system in the presence of deadzone and uncertainty. A symmetric barrier Lyapunov function (BLF) is employed to meet the output constraint requirement using DSC as an alternative method of backstepping control that is adopted mainly to deal with the BLF's constraint control. However, using the ordinary DSC method with the BLF limits the selection of the control gain whereas this limitation does not exist in the backstepping structure. To remove this limitation, we propose a partial backstepping DSC method in which backstepping control is added only in the first recursive DSC design step. For precise positioning, an inverse deadzone method and adaptive fuzzy system are introduced to handle unknown deadzone and unmodeled nonlinear functions. We show that the semiglobal boundedness of the overall closed-loop signals is guaranteed, the tracking error converges within the prescribed region, and precise positioning performance is ensured. The proposed control scheme is experimentally evaluated using a robot manipulator.

Keywords: Adaptive fuzzy system, backstepping dynamic surface control, deadzone, output constrained strict feedback dynamic system, robot manipulator, symmetric barrier Lyapunov function.

1. INTRODUCTION

In recent decades, constraints in control design have become important in advanced industrial robotic system and micro-devices. Many control system have constraints on their outputs, inputs, or states in the form of the physical stoppage, saturation, performance and safety specifications. During operation, violation of the constraints leads to performance degradation, hazards or system damage. The design of barrier Lyapunov function (BLF) in the Lyapunov theorem has been proposed recently for handling constraints in Brunovsky-type systems [1], strict feedback nonlinear systems [2], output feedback nonlinear systems [3], electrostatic micro-actuators [4], vessel systems [5], partial state constraints [6], switching control [7], and time-varying constraints [8]. Most of these approaches [2-8] used a backstepping control that provides a systematic procedure for designing a stabilizing controller for a nonlinear system by following a step-by-step recursive algorithm [9,10]. Another advantage of backstepping control is that it guarantees global or regional regulation and tracking properties, and also avoids the cancellation of useful nonlinearities, unlike the feedback linearization technique. However, to apply the backstepping method, a nonlinear dynamic model should be known exactly or linearly parameterized with respect to the known nonlinear function. In real situation, it is difficult to satisfy this requirement because most nonlinear systems have uncertainties. Another problem is the explosion of complexity that has greatly increased the number of controller terms for complex nonlinear systems owing to repeated differentiations of the virtual control functions. This problem leads to a considerable computational burden in real hardware implementations such as complex robotic systems. Although the backstepping method is theoretically tractable, the increasing complexity of real applications is an insurmountable obstacle that prevents its application to a multiple-state control system.

DSC [11,12] was developed for nonlinear systems to overcome the proliferation of terms associated with backstepping control technique by applying a first-order filter to the synthetic input at each step of the backstepping design procedure. Several adaptive DSC methods combined with fuzzy or neural networks have been developed [13-18]. To date, most of the constraint problems to which a BLF is applied have been approached by using only the backstepping control scheme. Here, we introduce DSC in conjunction with the BLF to address the output constraint issue among several constraint problems [2]. However, we confront an unexpected problem: the application of the conventional

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DSC design to output constraint control limits the selection of the controller design parameters related to the BLF. As we show below, the stability of the closed loop system and the tracking performance may deteriorate depending on the selection of the values for the controller gain in a conventional DSC constraint design that uses the BLF.

Hence, we suggest a convenient, effective method, the so-called 'backstepping DSC', that exploits the advantages of backstepping control in the BLF and DSC schemes. In the first design step of the DSC, we introduce partial backstepping control because the repeated differential control is generally not severe in the first design of backstepping control. The remaining controller design steps proceed by the normal DSC procedures.

In industrial applications, non-smooth nonlinearities such as friction and deadzone are encountered in most dynamic systems. The characteristics of these nonlinearities are usually poorly known, and time-varying, and they often limit system performance. Deadzone nonlinearity in the actuators also causes in the control system. We introduced the inverse deadzone method [19] to compensate for the undesirable effect of deadzone. We compensate for nonlinear uncertainty in the dynamic system by using adaptive fuzzy approximation.

Fuzzy technology is well known as an efficient tool for treating complex nonlinear processes on the basis of human experience or expert opinion [20]. Fuzzy logic has the characteristics of linguistic information and offers high-level logic control. Adaptive fuzzy controls have been applied successfully in many nonlinear control systems, and they guarantee improved performance and system stability in the Lyapunov sense [21-23].

In summarizing, the main contributions of this paper are i) developing a DSC-based output-constrained control system, partially, with the aid of backstepping control, thus ii) relaxing the limitations on choosing the control gain of the DSC caused by the output constraints and providing a more stable, simpler controller than that in a conventional DSC scheme; iii) demonstrating precise positioning by compensating for deadzone, and unknown dynamics using an inverse deadzone observer and adaptive fuzzy system; iv) ensuring the output constraint and the boundedness of closed-loop signals; and v) experimentally verifying the constraint problem using the BLF, (all of the earlier constrained control systems were verified only by simulations [1-8]).

2. PROBLEM FORMULATION

2.1. Description of the non-smooth nonlinear plant

Consider a single-input, single-output (SISO) nonlinear system in the presence of deadzone and friction whose dynamic equation is described by

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - F_{fi} + F_{di},\ (1 \le i \le n-1), \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)w - F_{fn} + F_{dn}, \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $(i = 1,\ldots,n)$ represents the states; $f_i(\bar{x}_i) \in R^i, (i = 1,\ldots,n)$ are unknown smooth nonlinear functions; $g_i(\bar{x}_i) \in R^i, (i = 1,\ldots,n)$ are known smooth functions of x; $F_{fi} \in R^i$, $(i = 1,\ldots,n)$ are non-linear frictions; and $F_{di} \in R^i$, $(i = 1,\ldots,n)$ are bounded disturbances. The output is required to remain in the set $|y| \le k_{c1}$, where k_{c1} is a positive constant.

Assumption 1: For any $k_{c1} > 0$, there exist positive constants Y_{l0}, Y_{h0}, A_0, Y_1 and Y_2 satisfying $\max\{Y_{l0}, Y_{h0}\} \le A_0 < k_{c1}$ such that the desired trajectory y_d and its time derivatives satisfy $-Y_{l0} \le y_d \le Y_{h0}$, $|\dot{y}_d| < Y_1$, $|\ddot{y}_d| < Y_2$ for all $t \ge 0$.

Assumption 2: The signs of the control gain functions, $g_i(\cdot) \in R^n$, are positive without loss of generality and there exist positive constants $0 < g_{i\min} \le g_{i\max}$ such that $g_{i\min} \le g_i \le g_{i\max}$ for $|x_1| < k_{c1}$.

The control objective for a nonlinear dynamic system is to determine a state feedback control system such that the system output x_1 can track a desired trajectory y_d while ensuring that all closed-loop signals are bounded and that the output constraint is not violated.

2.2. Barrier Lyapunov function

Definition 1 [2]**:** A BLF is a scalar function $V(x)$, defined with respect to the system $\dot{x} = f(x)$ on an open region $\mathcal{D}$ containing the origin. It is continuous, and positive definite; it has continuous first-order partial derivatives at every point of $\mathcal{D}$, has the property $V(x) \to \infty$ as x approaches the boundary of $\mathcal{D}$, and satisfies $V(x(t)) \le b\ \forall t$ along the solution of $\dot{x} = f(x)$ for $x(0) \in \mathcal{D}$ and some positive constant b.

We use a BLF candidate having the following form [1,2]:

$$V_1 = \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - S_1^2}, \tag{2}$$

where $k_{b1} = k_{c1} - A_0$ denotes the constraint on S_1, that is, $|S_1| < k_{b1}$. This BLF goes to infinity at $|S_1| = k_{b1}$. The BLF candidate is a valid Lyapunov function because V_1 is positive definite and C^1 is continuous in the set $|S_1| < k_{b1}$.

Lemma 1 [2]**:** For any positive constant k_{bl}, let $\Sigma_1 := \{S_1 \in R : -k_{b1} < S_1 < k_{b1}\} \subset R$ and $\mathrm{N} := R^l \times \Sigma_1 \subset R^{l+1}$ be open sets. Consider the system

$$\dot{\theta} = \varphi(t, \theta), \tag{3}$$

where $\theta := [\zeta\ S_1]^T \in \mathrm{N}$ is the state, and the function $\varphi : R_+ \times \mathrm{N} \to R^{l+1}$ satisfies the conditions that φ is locally Lipschitz on S_1 and φ is locally integrable on t. Suppose that there exists functions $U : R^l \to R_+$ and $V_1 : \Sigma_1 \to R_+$, which are continuously differentiable and positive definite in their respective domains, such that

$$V_1(S_1) \to \infty \quad \text{as} \quad |S_1| \to k_{b1}, \tag{4}$$

$$\gamma_1(\|\zeta\|) \le U(\zeta) \le \gamma_2(\|\zeta\|), \tag{5}$$

where γ_1 and γ_2 are class K_∞ functions. Let $V(\theta) := V_1(S_1) + U(\zeta)$, and let $S_1(0)$ belong to the set $S_1 \in (-k_{b1}, k_{b1})$. If the inequality

$$\dot{V}_1 = \frac{\partial V}{\partial \theta}\varphi \le -CV + \mu \tag{6}$$

holds in the set $\theta \in N$, and C, μ are positive constants, then S_1 remains in the open set $S_1 \in (-k_{b1}, k_{b1})$ $\forall t \in [0, \infty)$, and μ remains bounded.

Lemma 2: [3] For any positive constant k_{b1}, the following inequality holds for all S_1 in the interval $|S_1| < k_{b1}$:

$$\log \frac{k_{b1}^2}{k_{b1}^2 - S_1^2} \le \frac{S_1^2}{k_{b1}^2 - S_1^2}. \tag{7}$$

2.3. Deadzone nonlinearity

The mathematical model for deadzone nonlinearity w is described by

$$w(t) = D(u) = \begin{cases} m_r(u(t) - d_r) & \text{for } u(t) \ge d_r \\ 0 & \text{for } d_l < u(t) < d_r \\ m_l(u(t) - d_l) & \text{for } u(t) \le d_l, \end{cases} \tag{8}$$

where m_r and m_l denote the slope of the deadzone, and d_r, d_l represent the deadzone width parameters. The following practical assumptions regarding the deadzone are given for the control problem.

The inverse deadzone technique is a useful method of compensating for deadzone effects [19]. After $u_d(t)$ is set as the signal from the controller to achieve the control objective for the plant without deadzone, the control signal $u(t)$ is generated according to the certainty equivalence deadzone inverse

$$u(t) = D^{-1}(u_d) = \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r} q + \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l}(1 - q), \tag{9}$$

where $\hat{m}_r$, $\hat{m}_l$, $\hat{d}_{mr}$ and $\hat{d}_{ml}$ are the estimated values of m_r, m_l, $m_r d_r$, and $m_l d_l$, respectively, and

$$q = \begin{cases} 1 & \text{if } u_d(t) \ge 0 \\ 0 & \text{if } u_d(t) < 0. \end{cases} \tag{10}$$

The resulting error between w and u_d is given by

$$\begin{aligned} w(t) - u_d(t) &= \left(\tilde{d}_{mr} + \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r} \tilde{m}_r \right) q \\ &+ \left(\tilde{d}_{ml} + \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l} \tilde{m}_l \right)(1 - q) + \varepsilon_d(t), \end{aligned} \tag{11}$$

where $\varepsilon_d(t)$ is known as the bounded function for all $u(t)$ [19].

2.4. Function approximation using the fuzzy system

The basic configuration of the fuzzy system consists of the fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The fuzzy inference engine maps an input linguistic vector $x = [x_1, \ldots, x_n]^T \in R^n$ to an output linguistic scalar variable $y \in R$. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules. The lth IF-THEN rules is described by

$$\begin{aligned} &R^{(l)}: \text{IF } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l, \\ &\quad \text{then } y \text{ is } \bar{y}^l,\ l = 1, 2, \cdots, M, \end{aligned} \tag{12}$$

where F_i^l, $i = 1, \ldots, n$ are fuzzy sets, $\bar{y}^l$ is the fuzzy singleton for the output of the ith rule, and M is the number of rules in the fuzzy rule base. The output of the fuzzy systems with a center-average defuzzifier, product inference, and single1ton fuzzifier is expressed as

$$y(x) = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}, \tag{13}$$

where $\mu_{F_i^l}(x_i)$ is the degree of membership of x_i in F_i^l. (13) can be rewritten as

$$y(x) = W_o^T \chi(x), \tag{14}$$

where $W_o = [\bar{y}^1, \cdots, \bar{y}^M]^T$ is an adjustable parameter vector grouping all consequence parameters, and $\chi(x) = [\chi^1, \ldots, \chi^M]^T$ is a set of fuzzy basis function defined as

$$\chi^l(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}. \tag{15}$$

It has been proven that a fuzzy logic system can uniformly approximate any nonlinear continuous function to an arbitrary degree of accuracy if enough rules are provided [20]. Thus, the fuzzy logic system performs universal approximation in the sense that, given any real continuous function $f(\cdot): R^n \to R$ on a sufficiently large compact set $\Omega_y \subset R$ and arbitrary $\varepsilon_m > 0$, there exists a fuzzy logic system $y(x)$ in the form of (14) such that

$$\sup_{x \in \Omega} |f(x) - y(x)| \le \varepsilon_m. \tag{16}$$

Then the function $f(x)$ can be expressed as

$$f(x) = W_o^{*T} \chi(x) + \varepsilon^*, \quad \forall x \in \Omega \subset R^n, \tag{17}$$

where $|\varepsilon^*| \le \varepsilon_m$, ε^* is the error of the fuzzy approximation and W_o^* is chosen as the value of W_o that minimizes the fuzzy approximation error ε^*, i.e.,

$$W_o^* = \arg \min_{W_o \in R^M} \left\{ \sup_{x \in \Omega} \left| f(x) - W_o^T \chi(x) \right| \right\}. \tag{18}$$

Because W_o^* is unknown, it will be replaced by W_o, which is the estimation of W_o^*. An adaptation law will be necessary to update the parameter W_o online to minimize the reference tracking error asymptotically. The optimal fuzzy output function can be rewritten as

$$W_o^{*T} \chi(x) = W_o^T \chi(x) + \tilde{W}_o^T \chi(x), \tag{19}$$

where $\tilde{W}_o = W_o^* - W_o$.

3. DESIGN OF CONTROL SYSTEM

In this section, the virtual control functions, adaptive laws and control law will be derived via the recursive DSC design procedures.

Assumption 3: The desired trajectory vectors are continuous and available; $[y_d, \dot{y}_d, \ddot{y}_d]^T \in \Omega_d$, and Ω_d is a known compact set, i.e., $\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le \delta_y\} \subset R^3$ where $\delta_y > 0$ is a constant.

For any initial compact set $\Omega_y^0 := \{y \in R : |y| \le k_0, k_0 > 0\} \subset R$ to which $y(0)$ belongs, another extended compact set $\Omega_y = \{y \in R : |y| \le k_{c1}, k_{c1} + A_0 + y_d(0)\} \subset R$ can be made as large as desired [3]. The fuzzy approximation is valid provided that the input variable of the fuzzy system, y, remains within the fixed Ω_y. The compact set Ω_{si} is defined as

$$\Omega_{si} = \Big\{[S_i, \lambda_{i+1}, \tilde{W}_{oi}, \tilde{\rho}_i, \tilde{m}_r, \tilde{m}_l, \tilde{d}_{mr}, \tilde{d}_{ml}]^T : \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - S_1^2} + \frac{1}{2}\sum_{i=2}^{i} S_i^2 + \frac{1}{2}\sum_{i=2}^{i-1}\lambda_{i+1}^2 + \frac{1}{2}\sum_{i=1}^{i}\frac{1}{\eta_{wi}}\tilde{W}_{oi}^T\tilde{W}_{oi} + \sum_{i=1}^{i}\frac{1}{2\eta_{\rho i}}\tilde{\rho}_i^2 + \frac{1}{2\eta_{mr}}\tilde{m}_r^2 + \frac{1}{2\eta_{ml}}\tilde{m}_l^2 + \frac{1}{2\eta_{dr}}\tilde{d}_{mr}^2 + \frac{1}{2\eta_{dl}}\tilde{d}_{ml}^2 \le \delta_i \Big\}, \tag{20}$$

where $\eta_i > 0$ and $\delta_i > 0$, for $i = 2, \ldots, n$, are constants. The state feedback control system is designed step-by-step using a DSC technique.

Step 1: We define the tracking error as the first error space as follows:

$$S_1 = x_1 - y_d. \tag{21}$$

The time derivative of (21) becomes

$$\begin{aligned}\dot{S}_1 &= f_1(x_1) + g_1(x_1)x_2 - F_{f1} + F_{d1} - \dot{y}_d \\ &= f_1'(x_1) + g_1'(x_1)x_2 + F_{u1} - \dot{y}_d \\ &\le f_1'(x_1) + g_1'(x_1)x_2 + W_{o1}^T\chi_1(x_1) \\ &\quad + \tilde{W}_{o1}^T\chi_1(x_1) + \rho_1 - \dot{y}_d,\end{aligned} \tag{22}$$

where the sign of the superscript prime denotes a known nominal value of each function; $F_{u1} = \Delta f_1(x_1) + \Delta g_1(x_1)x_1 - F_{f1} + F_{d1}$, $\Delta(\cdot)$ represents a perturbed value, $\tilde{W}_{o1} = W_{o1}^* - W_{o1}$, $F_{u1} = W_{o1}^{*T}\Theta_1 + \varepsilon_1$, and it is assumed that $|\varepsilon_1| \le \rho_1$ and ρ_1 is a positive constant. We present the hyperbolic tangent function that satisfies the following relation similar to [24]:

$$\begin{aligned}0 &\le |S_1|\hat{\rho}_1 - S_1\hat{\rho}_1\tanh\left(\frac{S_1\hat{\rho}_1}{k_{b1}^2 - S_1^2}\right) \\ &\le 0.2785(k_{b1}^2 - S_1^2) = \kappa_1,\end{aligned} \tag{23}$$

where κ_1 is a positive constant, and $\hat{\rho}_1$ is the estimate of ρ_1. We choose a virtual control law and adaptive law as follows:

$$\begin{aligned}\alpha_1 = \frac{1}{g_1'(x_1)}\Big[&-(k_{b1}^2 - S_1^2)c_1S_1 - f_1'(x_1) - W_{o1}^T\chi_1(x_1) \\ &-\hat{\rho}_1\tanh\left(\frac{S_1\hat{\rho}_1}{k_{b1}^2 - S_1^2}\right) + \dot{y}_d\Big],\end{aligned} \tag{24}$$

where $c_1 > 1$ is a design constant. We introduce the filtering virtual control z_2 and let α_1 pass through a first-order filter with a time constant τ_2 as follows:

$$\tau_2\dot{z}_2 + z_2 = \alpha_1, \quad z_2(0) = \alpha_1(0). \tag{25}$$

We set the output error of this filter as follows:

$$\lambda_2 = z_2 - \alpha_1, \tag{26}$$

and from (25), it follows that

$$\dot{z}_2 = -\frac{\lambda_2}{\tau_2}. \tag{27}$$

The time derivative of (26) becomes

$$\begin{aligned}\dot{\lambda}_2 &= \dot{z}_2 - \dot{\alpha}_1 \\ &= -\frac{\lambda_2}{\tau_2} + \xi_2(S_1, W_{o1}, \chi_1, \hat{\rho}_1, y_d, \dot{y}_d, \ddot{y}_d),\end{aligned} \tag{28}$$

where

$$\begin{aligned}\xi_2(S_1, W_{o1}, \chi_1, \hat{\rho}_1, y_d, \dot{y}_d, \ddot{y}_d) = \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_1}{\partial W_{o1}}\dot{W}_{o1} \\ + \frac{\partial\alpha_1}{\partial\chi_1}\dot{\chi}_1 + \frac{\partial\alpha_1}{\partial\hat{\rho}_1}\dot{\hat{\rho}}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d\end{aligned} \tag{29}$$

is a continuous function and has a maximum M_2 on the compact set $\Omega_d \times \Omega_{s1}$ defined in Assumption 3 and (20). From $|\lambda_2\xi_2| \le \frac{1}{2\gamma_2}\lambda_2^2\xi^2 + \frac{\gamma_2}{2}$, $\gamma_2 > 0$ and (28), it follows that

$$\begin{aligned}\lambda_2\dot{\lambda}_2 &\le -\frac{\lambda_2^2}{\tau_2} + |\lambda_2\xi_2| \le -\frac{\lambda_2^2}{\tau_2} + \frac{1}{2\gamma_2}\lambda_2^2\xi_2^2 + \frac{\gamma_2}{2} \\ &\le -\frac{\lambda_2^2}{\tau_2} + \frac{1}{2\gamma_2}\lambda_2^2M_2^2 + \frac{\gamma_2}{2}.\end{aligned} \tag{30}$$

We define the following Lyapunov function candidate to ensure that $|S_1| < k_{b1}$ with $k_{b1} = k_{c1} - A_0$ so as not to drive y out of the interval $|y| < k_{c1}$

$$V_1 = \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - S_1^2} + \frac{1}{2\eta_{w1}}\tilde{W}_{o1}^T\tilde{W}_{o1} + \frac{1}{2\eta_{\rho 1}}\tilde{\rho}_1^2 + \frac{1}{2}\lambda_2^2, \tag{31}$$

where $\tilde{\rho}_1 = \rho_1 - \hat{\rho}_1$. The time derivative of (31) considering the above results becomes

$$\begin{aligned}\dot{V}_1 \le \frac{S_1\left(f_1'(x_1) + g_1'(x_1)x_2 + W_{o1}^T\chi_1(x_1) - \dot{y}_d\right)}{k_{b1}^2 - S_1^2} + \lambda_2\dot{\lambda}_2 \\ + \frac{S_1\tilde{W}_{o1}^T\chi_1(x_1)}{k_{b1}^2 - S_1^2} + \frac{|S_1|\rho_1}{k_{b1}^2 - S_1^2} + \frac{1}{\eta_{w1}}\tilde{W}_{o1}^T\dot{\tilde{W}}_{o1} + \frac{1}{\eta_{\rho 1}}\tilde{\rho}_1\dot{\tilde{\rho}}_1\end{aligned}$$

$$\leq \frac{S_1\left(\begin{array}{l} f_1'(x_1)+g_1'(x_1)x_2+W_{o1}^T\chi_1(x_1) \\ \quad +\hat{\rho}_1\tanh\left(\dfrac{S_1\hat{\rho}_1}{k_{b1}^2-S_1^2}\right)-\dot{y}_d \end{array}\right)}{k_{b1}^2-S_1^2}$$
$$+\tilde{W}_{o1}^T\left(\frac{S_1\chi_1(x_1)}{k_{b1}^2-S_1^2}-\frac{1}{\eta_{w1}}\dot{\hat{W}}_{o1}\right)+\tilde{\rho}_1\left(\frac{|S_1|}{k_{b1}^2-S_1^2}-\frac{1}{\eta_{\rho1}}\dot{\hat{\rho}}_1\right)$$
$$-\frac{\lambda_2^2}{\tau_2}+\frac{1}{2\gamma_2}\lambda_2^2M_2^2+\frac{\gamma_2}{2}+\kappa_1. \quad (32)$$

We define the second virtual error and the adaptive laws as follows:

$$S_2=x_2-z_2, \quad (33)$$
$$\dot{\hat{W}}_{o1}=\eta_{w1}\frac{S_1\chi_1(x_1)}{k_{b1}^2-S_1^2}-\eta'_{w1}\hat{W}_{o1}, \quad (34)$$
$$\dot{\hat{\rho}}_1=\eta_{\rho1}\frac{|S_1|}{k_{b1}^2-S_1^2}-\eta'_{\rho1}\hat{\rho}_1. \quad (35)$$

We consider (33)-(35) with (24), (26), and (30) and use Young's inequalities,

$$g_1'\lambda_2S_1\leq g'_{1\max}\left(S_1^2+\frac{\lambda_2^2}{4}\right), \quad (36)$$

and

$$-\eta'_{w1}\tilde{W}_{o1}^TW_{o1}\leq-\frac{\eta'_{w1}\|\tilde{W}_{o1}\|^2}{2}+\frac{\eta'_{w1}\|W_{o1}^*\|^2}{2}, \quad (37)$$
$$-\eta'_{\rho1}\tilde{\rho}_1\hat{\rho}_1\leq-\frac{\eta'_{\rho1}\tilde{\rho}_1^2}{2}+\frac{\eta'_{\rho1}\rho_1^{*2}}{2}. \quad (38)$$

We then obtain the following expression:

$$\dot{V}_1\leq-\left(c_1-\frac{g'_{1\max}S_1}{k_{b1}^2-S_1^2}\right)S_1^2+\frac{g_1'S_1S_2}{k_{b1}^2-S_1^2}-\frac{\eta'_{w1}\|\tilde{W}_{o1}\|^2}{2}$$
$$-\frac{\eta'_{\rho1}\tilde{\rho}_1^2}{2}-\left(\frac{1}{\tau_2}-\frac{1}{2\gamma_2}M_2^2-\frac{1}{4}\frac{g'_{1\max}}{k_{b1}^2-S_1^2}\right)\lambda_2^2 \quad (39)$$
$$+\frac{\eta'_{w1}\|W_{o1}^*\|^2}{2}+\frac{\eta'_{\rho1}\rho_1^{*2}}{2}+\frac{\gamma_2}{2}+\kappa_1.$$

Remark 1: However, in (39), unlike the output constrained control system based on backstepping control [2,3], the selection range of design constant c_1 and time constant τ_2 in our DSC are limited to $c_1\geq g'_{1\max}S_1/(k_{b1}^2-S_1^2)$ and $1/\tau_2\geq M_2^2/2\gamma_2+g'_{1\max}/4(k_{b1}^2-S_1^2)$ in order to guarantee closed-loop stability. In the experimental example, we will show that a smaller design constant c_1 usually causes severe degradation of the control performance or instability, so the range from which c_1 is chosen should be limited. Furthermore, the controller structure of the DSC-based output-constrained system becomes more complex than that of the backstepping-control-based system. This problem is difficult to avoid under a conventional DSC design procedure. To solve it, we propose a hybrid control concept, in which a backstepping structure is introduced only in the first design step because the repeated differentiation, which is the main problem in backstepping control, is not serious at the first design step in most control systems.

Therefore, we redefine the following Lyapunov function candidate with the definition $S_2=x_2-\alpha_1$ instead of $S_2=x_2-z_2$ in (33):

$$V_1=\frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2-S_1^2}+\frac{1}{2\eta_{w1}}\tilde{W}_{o1}^T\tilde{W}_{o1}+\frac{1}{2\eta_{\rho1}}\tilde{\rho}_1^2. \quad (40)$$

The time derivative of (34) becomes

$$\begin{aligned}\dot{V}_1\leq&-c_1S_1^2+\frac{g_1'S_1S_2}{k_{b1}^2-S_1^2}-\frac{\eta'_{w1}\|\tilde{W}_{o1}\|^2}{2}-\frac{\eta'_{\rho1}\tilde{\rho}_1^2}{2}\\&+\frac{\eta'_{w1}\|W_{o1}^*\|^2}{2}+\frac{\eta'_{\rho1}\rho_1^{*2}}{2}+\kappa_1.\end{aligned} \quad (41)$$

In the first step, therefore, the limitation on the design parameter c_1 is simplified to $c_1>0$, as in conventional DSC systems and the time constant needs not be considered. The design procedure in the first step is the same as that of backstepping control based on an output - constrained system.

Step2: We consider the equation

$$\dot{x}_2=f_2'(\bar{x}_2)+g_2'(\bar{x}_2)x_3-F_{f2}+F_{d2}, \quad (42)$$

where $\bar{x}_2=[x_1,x_2]^T$. The time derivative of S_2 is written as

$$\begin{aligned}\dot{S}_2&=f_2'(\bar{x}_2)+g_2'(\bar{x}_2)x_3+W_{o2}^{*T}\chi_2(\bar{x}_2)+\varepsilon_2-\dot{\alpha}_1\\&\leq f_2'(\bar{x}_2)+g_2'(\bar{x}_2)x_3+W_{o2}^T\chi_2(\bar{x}_2)+\tilde{W}_{o2}^T\chi_2(\bar{x}_2)\\&\quad+\rho_2-\dot{\alpha}_1,\end{aligned} \quad (43)$$

where $F_{u2}=\Delta f_2(\bar{x}_2)+\Delta g_2(\bar{x}_2)x_3-F_{f2}+F_{d2}$, $\Delta(\cdot)$ represents a perturbed value, $\tilde{W}_{o2}=W_{o2}^*-W_{o2}$, and $F_{u2}=W_{o2}^{*T}\Theta_2+\varepsilon_2$. It is assumed that $|\varepsilon_2|\leq\rho_2$, and ρ_2 is a positive constant. We consider the following hyperbolic tangent function relation:

$$0\leq|S_2|\hat{\rho}_2-S_2\hat{\rho}_2\tanh\left(\frac{S_2\hat{\rho}_2}{\kappa_2}\right)\leq0.2785\kappa_2=\kappa_2', \quad (44)$$

where $\kappa_2>0$ is a design constant. We choose a virtual control α_2 and an adaptive law as follows:

$$\begin{aligned}\alpha_2=\frac{1}{g_2'(\bar{x}_2)}\Bigg[&-c_2S_2-f_2'(\bar{x}_2)-\frac{g_1'S_1}{k_{b1}^2-S_1^2}\\&-W_{o2}^T\chi_{o2}(\bar{x}_2)-\hat{\rho}_2\tanh\left(\frac{S_2\hat{\rho}_2}{\kappa_2}\right)+\dot{\alpha}_1\Bigg],\end{aligned} \quad (45)$$

where $c_2>0$ is a design constant and $\hat{\rho}_2$ is the estimated value of ρ_2. We introduce a new filtering virtual control z_3 and let α_2 pass through a first-order filter with a time constant τ_3 as follows:

$$\tau_3 \dot{z}_3 + z_3 = \alpha_2, \quad z_3(0) = \alpha_2(0). \tag{46}$$

By setting $\lambda_3 = z_3 - \alpha_2$, we obtain

$$\dot{z}_3 = -\frac{\lambda_3}{\tau_3}. \tag{47}$$

From (46), it follows that

$$\begin{aligned}\dot{\lambda}_3 &= \dot{z}_3 - \dot{\alpha}_2 \\ &= -\frac{\lambda_3}{\tau_3} + \xi_3(S_1, S_2, W_{o2}, \chi_2, \hat{\rho}_2, y_d, \dot{y}_d, \ddot{y}_d),\end{aligned} \tag{48}$$

where

$$\begin{aligned}&\xi_3(S_1, S_2, W_{o2}, \chi_2, y_d, \dot{y}_d, \ddot{y}_d) \\ &= \frac{\partial \alpha_2}{\partial \bar{x}_2}\dot{\bar{x}}_2 + \frac{\partial S_2}{\partial S_2}\dot{S}_2 + \frac{\partial \alpha_2}{\partial W_{o2}}\dot{W}_{o2} + \frac{\partial \alpha_2}{\partial \chi_2}\dot{\chi}_2 \\ &\quad + \frac{\partial \alpha_2}{\partial \hat{\rho}_2}\dot{\hat{\rho}}_2 + \frac{\partial \alpha_2}{\partial y_d}\dot{y}_d + \frac{\partial \alpha_2}{\partial \dot{y}_d}\ddot{y}_d\end{aligned} \tag{49}$$

is a continuous function and has a maximum M_3 on the compact set $\Omega_d \times \Omega_{s2}$ defined in Assumption 3 and (20). Using $|\lambda_3 \xi_3| \le \frac{1}{2\gamma_3}\lambda_3^2 \xi_3^2 + \frac{\gamma_3}{2}$, $\gamma_3 > 0$, we can find that

$$\begin{aligned}\lambda_3 \dot{\lambda}_3 &\le -\frac{\lambda_3^2}{\tau_3} + |\lambda_3 \xi_3| \le -\frac{\lambda_3^2}{\tau_3} + \frac{1}{2\gamma_3}\lambda_3^2 \xi_3^2 + \frac{\gamma_3}{2} \\ &\le -\frac{\lambda_3^2}{\tau_3} + \frac{1}{2\gamma_3}\lambda_3^2 M_3^2 + \frac{\gamma_3}{2}.\end{aligned} \tag{50}$$

We define the Lyapunov function candidate as follows:

$$V_2 = \frac{1}{2}S_2^2 + \frac{1}{2\eta_{w2}}\tilde{W}_{o2}^T \tilde{W}_{o2} + \frac{1}{2\eta_{\rho 2}}\tilde{\rho}_2^2 + \frac{1}{2}\lambda_3^2. \tag{51}$$

Differentiating (51) with respect to time with (46) and (50), we obtain

$$\begin{aligned}\dot{V}_2 &= S_2 \dot{S}_2 + \frac{1}{\eta_{w2}}\tilde{W}_{o2}^T \dot{\tilde{W}}_{o2} + \frac{1}{\eta_{\rho 2}}\tilde{\rho}_2 \dot{\tilde{\rho}}_2 + \lambda_3 \dot{\lambda}_3 \\ &\le S_2\left(f_2'(\bar{x}_2) + g_2'(\bar{x}_2)x_3 + W_{o2}^T \chi_2(\bar{x}_2) - \dot{z}_2\right) + |S_2|\rho_2 \\ &\quad + \frac{1}{\eta_{w2}}\tilde{W}_{o2}^T \dot{\tilde{W}}_{o2} + \frac{1}{\eta_{\rho 2}}\tilde{\rho}_2 \dot{\tilde{\rho}}_2 + \lambda_3 \dot{\lambda}_3 \\ &\le S_2\left(f_2'(\bar{x}_2) + g_2'(\bar{x}_2)x_3 + W_{o2}^T \chi_2(\bar{x}_2) \right. \\ &\quad \left. + \hat{\rho}_2 \tanh\left(\frac{S_2 \hat{\rho}_2}{\kappa_2}\right) - \dot{z}_2 \right) \\ &\quad + \tilde{W}_{o2}^T\left(S_2 \chi(\bar{x}_2) - \frac{1}{\eta_{w2}}\dot{\hat{W}}_{o2}\right) + \tilde{\rho}_2\left(|S_2| - \frac{1}{\eta_{\rho 2}}\dot{\hat{\rho}}_2\right) \\ &\quad - \left(\frac{1}{\tau_3} - \frac{1}{2\gamma_3}M_3^2\right)\lambda_3^2 + \frac{\gamma_3}{2} + \kappa_2'.\end{aligned} \tag{52}$$

We define the second virtual error and the adaptive laws as follows:

$$S_3 = x_3 - z_3, \tag{53}$$

$$\dot{W}_{o2} = \eta_{w2} S_2 \chi_2(\bar{x}_2) - \eta_{w2}' W_{o2}, \tag{54}$$

$$\dot{\hat{\rho}}_2 = \eta_{\rho 2}|S_2| - \eta_{\rho 2}' \hat{\rho}_2. \tag{55}$$

We consider (53)-(55) with (44), (45), and (50) and use Young's inequalities of

$$g_2' \lambda_3 S_2 \le g_{2\max}'\left(S_2^2 + \frac{\lambda_3^2}{4}\right) \tag{56}$$

and

$$-\eta_{w2}' \tilde{W}_{o2}^T W_{o2} \le -\frac{\eta_{w2}' \|\tilde{W}_{o2}\|^2}{2} + \frac{\eta_{w2}' \|W_{o2}^*\|^2}{2}, \tag{57}$$

$$-\eta_{\rho 2}' \tilde{\rho}_2 \hat{\rho}_2 \le -\frac{\eta_{\rho 2}' \tilde{\rho}_2^{\,2}}{2} + \frac{\eta_{\rho 2}' \rho_2^{*2}}{2}. \tag{58}$$

Equation (52) can then be written as follows:

$$\begin{aligned}\dot{V}_2 &\le -(c_2 - g_{2\max}')S_2^2 - \frac{g_1' S_1 S_2}{k_{b1}^2 - S_1^2} + g_2' S_2 S_3 - \frac{\eta_{w2}' \|\tilde{W}_{o2}\|^2}{2} \\ &\quad - \frac{\eta_{\rho 2}' \tilde{\rho}_2^{\,2}}{2} + \frac{\eta_{w2}' \|W_{o2}^*\|^2}{2} + \frac{\eta_{\rho 2}' \rho_2^{*2}}{2} \\ &\quad - \left(\frac{1}{\tau_3} - \frac{1}{2\gamma_3}M_3^2 - \frac{g_{2\max}'}{4}\right)\lambda_3^2 + \frac{\gamma_3}{2} + \kappa_2'.\end{aligned} \tag{59}$$

Step i: A similar design procedures is employed recursively at each step, $i = 3, \ldots, n-1$. We consider the equation

$$\begin{aligned}\dot{x}_i &= f_i'(\bar{x}_i) + g_i'(\bar{x}_i)x_{i+1} - F_{fi} + F_{ui}(\bar{x}_i) \\ &= f_i'(\bar{x}_i) + g_i'(\bar{x}_i)x_{i+1} + W_{oi}^{*T}\chi(\bar{x}_i) + \varepsilon_i,\end{aligned} \tag{60}$$

where $\bar{x}_i = [x_3, \ldots, x_i]^T$ and $|\varepsilon_i| \le \rho_i$, and ρ_i is a positive constant. The time derivative of S_i is

$$\begin{aligned}\dot{S}_i &= f_i'(\bar{x}_i) + g_i'(\bar{x}_i)x_{i+1} + W_{oi}^{*T}\chi_i(\bar{x}_i) + \varepsilon_i - \dot{z}_i \\ &\le f_i'(\bar{x}_i) + g_i'(\bar{x}_i)x_{i+1} + W_{oi}^T \chi_i(\bar{x}_i) + \tilde{W}_{oi}^T \chi_i(\bar{x}_i) \\ &\quad + \rho_i - \dot{z}_i,\end{aligned} \tag{61}$$

where $F_{ui} = \Delta f_i(\bar{x}_i) + \Delta g_i(\bar{x}_i)x_i - F_{fi} + F_{di}$, $\Delta(\cdot)$ represents a perturbed value, $\tilde{W}_{oi} = W_{oi}^* - W_{oi}$, $F_{ui} = W_{oi}^{*T}\chi_i + \varepsilon_i$, and it is assumed that $|\varepsilon_i| \le \rho_i$ and ρ_i is a positive constant. We consider the following hyperbolic tangent function relation:

$$0 \le |S_i|\hat{\rho}_i - S_i \hat{\rho}_i \tanh\left(\frac{S_i \hat{\rho}_i}{\kappa_i}\right) \le 0.2785\kappa_i = \kappa_i', \tag{62}$$

where $\kappa_i > 0$ is a design constant. We choose a virtual control α_i as follows:

$$\alpha_i = \frac{1}{g_i'(\bar{x}_i)}\Big[-c_i S_i - g_{i-1}'(\bar{x}_{i-1})S_{i-1} - f_i'(\bar{x}_i) - W_{oi}^T \chi_{oi}(\bar{x}_i) - \hat{\rho}_i \tanh\left(\frac{S_i\hat{\rho}_i}{\kappa_i}\right) + \dot{z}_i\Big], \tag{63}$$

where $c_i > 0$ is a design constant, and $\hat{\rho}_i$ is the estimated value of ρ_i. We introduce a new filtering virtual control z_{i+1} and let α_i pass through a first-order filter with a time constant τ_i as follows:

$$\begin{aligned}&\tau_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_i,\\ &z_{i+1}(0) = \alpha_i(0).\end{aligned} \tag{64}$$

By setting $\lambda_{i+1} = z_{i+1} - \alpha_i$, we obtain

$$\dot{z}_{i+1} = -\frac{\lambda_{i+1}}{\tau_{i+1}}. \tag{65}$$

From (64), it follows that

$$\begin{aligned}\dot{\lambda}_{i+1} = \dot{z}_{i+1} - \dot{\alpha}_i = -\frac{\lambda_{i+1}}{\tau_{i+1}} + \xi_{i+1}(S_1,\ldots S_i, W_{o1},\\ \ldots, W_{oi}, \chi_1,\ldots,\chi_i,\hat{\rho}_1,\ldots,\hat{\rho}_i, y_d, \dot{y}_d, \ddot{y}_d),\end{aligned} \tag{66}$$

where

$$\begin{aligned}&\xi_{i+1}(S_1,\ldots S_i, W_{o1},\ldots,W_{oi},\chi_1,\ldots,\chi_i,\hat{\rho}_1,\ldots,\hat{\rho}_i, y_d, \dot{y}_d, \ddot{y}_d)\\ &= \frac{\partial\alpha_i}{\partial\bar{x}_i}\dot{\bar{x}}_i + \frac{\partial\alpha_i}{\partial S_i}\dot{S}_i + \frac{\partial\alpha_i}{\partial W_{oi}}\dot{W}_{oi} + \frac{\partial\alpha_i}{\partial\chi_i}\dot{\chi}_i + \frac{\partial\alpha_i}{\partial\hat{\rho}_i}\dot{\hat{\rho}}_i\\ &+ \frac{\partial\alpha_i}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_i}{\partial\dot{y}_d}\ddot{y}_d\end{aligned} \tag{67}$$

is a continuous function and has a maximum M_{i+1} on the compact set $\Omega_d\times\Omega_{si}$ defined in Assumption 3 and (20). Using $|\lambda_{i+1}\xi_{i+1}| \le \lambda_{i+1}^2\xi_{i+1}^2/2\gamma_{i+1} + \gamma_{i+1}/2 \le \lambda_{i+1}^2 M_{i+1}^2/\gamma_{i+1} + \gamma_{i+1}/2,\ \gamma_{i+1} > 0,$ we find that

$$\begin{aligned}\lambda_{i+1}\dot{\lambda}_{i+1} &\le -\frac{\lambda_{i+1}^2}{\tau_{i+1}} + |\lambda_{i+1}\xi_{i+1}|\\ &\le -\frac{\lambda_{i+1}^2}{\tau_{i+1}} + \frac{1}{2\gamma_{i+1}}\lambda_{i+1}^2 M_{i+1}^2 + \frac{\gamma_{i+1}}{2}.\end{aligned} \tag{68}$$

We define the Lyapunov function candidate as follows:

$$V_i = \frac{1}{2}S_i^2 + \frac{1}{2\eta_{wi}}\tilde{W}_{oi}^T\tilde{W}_{oi} + \frac{1}{2\eta_{\rho i}}\tilde{\rho}_i^2 + \frac{1}{2}\lambda_{i+1}^2. \tag{69}$$

Differentiating (69) with respect to time with (61), (62), (63) and (68), we obtain

$$\begin{aligned}\dot{V}_i &= S_i\dot{S}_i + \frac{1}{\eta_{wi}}\tilde{W}_{oi}^T\dot{\tilde{W}}_{oi} + \frac{1}{\eta_{\rho i}}\tilde{\rho}_i\dot{\tilde{\rho}}_i + \lambda_{i+1}\dot{\lambda}_{i+1}\\ &\le S_i\Bigg(f_i'(\bar{x}_i) + g_i'(\bar{x}_i)x_{i+1} + W_{oi}^T\chi_i(\bar{x}_i)\\ &\quad + \hat{\rho}_i\tanh\left(\frac{S_i\hat{\rho}_i}{\kappa_i}\right) - \dot{z}_i\Bigg)\\ &\quad + \tilde{W}_{oi}^T\left(S_i\chi(\bar{x}_i) - \frac{1}{\eta_{wi}}\dot{\hat{W}}_{oi}\right) + \tilde{\rho}_i\left(|S_i| - \frac{1}{\eta_{\rho i}}\dot{\hat{\rho}}_i\right)\\ &\quad - \left(\frac{1}{\tau_{i+1}} - \frac{1}{2\gamma_{i+1}}M_{i+1}^2\right)\lambda_{i+1}^2 + \frac{\gamma_{i+1}}{2} + \kappa_i'.\end{aligned} \tag{70}$$

We define the second virtual error and the adaptive laws as follows:

$$S_{i+1} = x_{i+1} - z_{i+1}, \tag{71}$$

$$\dot{W}_{oi} = \eta_{wi}S_i\chi_i(\bar{x}_i) - \eta_{wi}'W_{oi}, \tag{72}$$

$$\dot{\hat{\rho}}_i = \eta_{\rho i}|S_i| - \eta_{\rho i}'\hat{\rho}_i. \tag{73}$$

We consider (71)-(73) and use Young's inequalities of

$$g_i'\lambda_{i+1}S_i \le g_{imax}'\left(S_i^2 + \frac{\lambda_{i+1}^2}{4}\right), \tag{74}$$

and

$$-\eta_{wi}'\tilde{W}_{oi}^T W_{oi} \le -\frac{\eta_{wi}'\|\tilde{W}_{oi}\|^2}{2} + \frac{\eta_{wi}'\|W_{oi}^*\|^2}{2}, \tag{75}$$

$$-\eta_{\rho i}'\tilde{\rho}_i\hat{\rho}_i \le -\frac{\eta_{\rho i}'\tilde{\rho}_i^2}{2} + \frac{\eta_{\rho i}'\rho_i^{*2}}{2}. \tag{76}$$

Equation (70) can then be written as follows:

$$\begin{aligned}\dot{V}_i &\le -(c_i - g_{i\max}')S_i^2 - g_i'S_iS_{i-1} + g_i'S_iS_{i+1}\\ &\quad - \frac{\eta_{wi}'\|\tilde{W}_{oi}\|^2}{2} - \frac{\eta_{\rho i}'\tilde{\rho}_i^2}{2} + \frac{\eta_{wi}'\|W_{oi}^*\|^2}{2} + \frac{\eta_{\rho i}'\rho_i^{*2}}{2}\\ &\quad - \left(\frac{1}{\tau_{i+1}} - \frac{1}{2\gamma_{i+1}}M_{i+1}^2 - \frac{g_{i\max}'}{4}\right)\lambda_{i+1}^2 + \frac{\gamma_{i+1}}{2} + \kappa_i'.\end{aligned} \tag{77}$$

Step n: The final control law will be derived in this step. We consider the following equation:

$$\begin{aligned}\dot{x}_n &= f_n'(\bar{x}_n) + g_n'(\bar{x}_n)w + F_{un}\\ &= f_n'(\bar{x}_n) + g_n'(\bar{x}_n)w + W_{on}^{*T}\chi_n(\bar{x}_n) + \varepsilon_n.\end{aligned} \tag{78}$$

We consider the nth error surface as follows:

$$S_n = x_n - z_n. \tag{79}$$

The time derivative of (79) can then be written as

$$\begin{aligned}\dot{S}_n &\le f_n'(\bar{x}_n) + g_n'(\bar{x}_n)u_d + W_{on}^T\chi_n(\bar{x}_n) + \tilde{W}_{on}^T\chi_n(\bar{x}_n)\\ &\quad + \hat{\rho}_n - \dot{z}_n + g_n'(\bar{x}_n)\left(\tilde{d}_{mr} + \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r}\tilde{m}_r\right)q\\ &\quad + g_n'(\bar{x}_n)\left(\tilde{d}_{ml} + \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l}\tilde{m}_l\right)(1-q) - \tilde{\rho}_n,\end{aligned} \tag{80}$$

where $|\varepsilon_n + F_{dn} + \varepsilon_d| \le \rho_n$, ρ_n is a positive constant, and $\hat{\rho}_n$ is the estimated value of ρ_n. We specify the control law and adaptive laws as follows:

$$u_d = \frac{1}{g_n'(\bar{x}_n)}\left(-c_nS_n - g_{n-1}'(\bar{x}_{n-1})S_{n-1} - f_n'(\bar{x}_n)\right)$$

$$-W_{on}^T \chi_n(\bar{x}_n) - \hat{\rho}_{dn} + \dot{z}_n \big), \tag{81}$$

$$\dot{W}_{on} = \eta_{wn} S_n \chi_n(\bar{x}_n) - \eta'_{wn} W_{on}, \tag{82}$$

$$\dot{\hat{\rho}}_n = \eta_n S_n - \eta'_n \hat{\rho}_n, \tag{83}$$

$$\dot{\hat{m}}_r = \eta_{mr} S_n g'_n \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r} q - \eta'_{mr} \hat{m}_r, \tag{84}$$

$$\dot{\hat{m}}_l = \eta_{ml} S_n g'_n \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l} (1-q) - \eta'_{ml} \hat{m}_l, \tag{85}$$

$$\dot{\hat{d}}_{mr} = \eta_{dr} S_n g'_n(\bar{x}_n) - \eta'_{dr} \hat{d}_{mr}, \tag{86}$$

$$\hat{d}_{ml} = \eta_{dl} S_n g'_n(\bar{x}_n) - \eta_{dl} \hat{d}_{ml}, \tag{87}$$

where $c_n > 0$ and $\eta_{(\cdot)} > 0$ are design constants. We define the Lyapunov function candidate as follows:

$$\begin{aligned} V_n &= \frac{1}{2} S_n^2 + \frac{1}{2\eta_{wn}} \tilde{W}_{on}^T \tilde{W}_{on} + \frac{1}{2\eta_n} \tilde{\rho}_n^2 + \frac{1}{2\eta_{mr}} \tilde{m}_r^2 \\ &\quad + \frac{1}{2\eta_{ml}} \tilde{m}_l^2 + \frac{1}{2\eta_{dr}} \tilde{d}_{mr}^2 + \frac{1}{2\eta_{dl}} \tilde{d}_{ml}^2. \end{aligned} \tag{88}$$

The time derivative of (88) can be written as follows:

$$\begin{aligned} \dot{V}_n &= S_n \left(f'_n(\bar{x}_n) + g'_n(\bar{x}_n) u_d + W_{on}^T \chi_n(\bar{x}_n) + \hat{\rho}_n - \dot{z}_n \right) \\ &\quad + \tilde{W}_{on}^T \left(S_n \chi_n(\bar{x}_n) - \frac{1}{\eta_{wn}} \dot{W}_{on} \right) + \tilde{\rho}_n \left(S_n - \frac{1}{\eta_{dn}} \dot{\hat{\rho}}_n \right) \\ &\quad + \tilde{m}_r \left(g'_n(\bar{x}_n) S_n \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r} q - \frac{1}{\eta_{mr}} \dot{\hat{m}}_r \right) \\ &\quad + \tilde{m}_l \left(g'_n(\bar{x}_n) S_n \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l} (1-q) - \frac{1}{\eta_{ml}} \dot{\hat{m}}_l \right) \\ &\quad + \tilde{d}_{mr} \left(g'_n(\bar{x}_n) S_n q - \frac{1}{\eta_{dr}} \dot{\hat{d}}_{mr} \right) \\ &\quad + \tilde{d}_{ml} \left(g'_n(\bar{x}_n) S_n (1-q) - \frac{1}{\eta_{dl}} \dot{\hat{d}}_{ml} \right). \end{aligned} \tag{89}$$

We consider (77)-(85) and the following relations

$$-\eta'_{wn} \tilde{W}_{on}^T W_{on} \le -\frac{\eta'_{wn} \left\| \tilde{W}_{on} \right\|^2}{2} + \frac{\eta'_{wn} \left\| W_{on}^* \right\|^2}{2}, \tag{90}$$

$$-\eta'_{(\cdot)} \tilde{\Xi} \hat{\Xi} \le -\frac{\eta'_{(\cdot)} \tilde{\Xi}^2}{2} + \frac{\eta'_{(\cdot)} \Xi^{*2}}{2}. \tag{91}$$

Equation (85) can then be rearranged as follows:

$$\begin{aligned} \dot{V}_n &\le -c_n S_n^2 - g'_{n-1}(\bar{x}_{n-1}) S_{n-1} S_n - \frac{\eta'_{wn} \left\| \tilde{W}_{on} \right\|^2}{2} - \frac{\eta'_{\rho i} \tilde{\rho}_i^2}{2} \\ &\quad - \frac{\eta'_{mr} \tilde{m}_r^2}{2} - \frac{\eta'_{ml} \tilde{m}_l^2}{2} - \frac{\eta'_{dr} \tilde{d}_{mr}^2}{2} - \frac{\eta'_{dl} \tilde{d}_{ml}^2}{2} + \frac{\eta'_{wn} \left\| W_{on}^* \right\|^2}{2} \\ &\quad + \frac{\eta'_{\rho i} \rho_i^{*2}}{2} + \frac{\eta'_{mr} m_r^{*2}}{2} + \frac{\eta'_{ml} m_l^{*2}}{2} + \frac{\eta'_{dr} d_{mr}^{*2}}{2} + \frac{\eta'_{dl} d_{ml}^{*2}}{2}. \end{aligned} \tag{92}$$

4. STABILITY ANALYSIS

Theorem 1: Under Assumptions 1-3, we consider a closed-loop output-constrained strict feedback system consisting of the plant (1), the virtual control function (63), the adaptive laws (72), (73), and (84)-(87), and the control law (81). If the initial conditions are such that $\bar{S}(0) \in \Omega_{s0} := \{\bar{S}_n \in R^n : |S_1| < k_{b1}\}$, where $\bar{S} = [S_1, S_2, \ldots, S_n]^T$, and for any initial bounded compact set Ω_y^0, to which $y(0)$ also belongs, then the following properties hold:

i) The output $y(t)$ remains in the set Ω_y; that is, the output constraint is never violated.

ii) The closed-loop signals are semiglobally uniformly ultimately bounded overall.

iii) The output tracking error is smaller than the prescribed error bound and the size of the tracking can be arbitrarily decreased by the appropriate selection of the design parameters.

Proof: i) We define the following Lyapunov function candidate:

$$\begin{aligned} V &= \sum_{i=1}^{n} V_i = \frac{1}{2} \log \frac{k_{b1}^2}{k_{b1}^2 - S_1^2} + \frac{1}{2} \sum_{i=2}^{n} S_i^2 \\ &\quad + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{\eta_{wi}} \tilde{W}_{oi}^T \tilde{W}_{oi} + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{\eta_{\rho i}} \tilde{\rho}^2 + \frac{1}{2} \sum_{i=2}^{n-1} \lambda_{i+1}^2 \\ &\quad + \frac{1}{2\eta_{mr}} \tilde{m}_r^2 + \frac{1}{2\eta_{ml}} \tilde{m}_l^2 + \frac{1}{2\eta_{dr}} \tilde{d}_{mr}^2 + \frac{1}{2\eta_{dl}} \tilde{d}_{ml}^2. \end{aligned} \tag{93}$$

By using the time derivative of V and some manipulations, (93) can be rearranged as:

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} \dot{V}_i \le -\frac{c_1 S_1^2}{k_{b1}^2 - S_1^2} (k_{b1}^2 - S_1^2) \\ &\quad - \sum_{i=2}^{n-1} \left(c_i - g'_{i\max} \right) S_i^2 - c_n s_n^2 - \sum_{i=1}^{n} \frac{\eta'_{wi} \tilde{W}_{oi}^T \tilde{W}_{oi}}{2} \\ &\quad - \sum_{i=1}^{n} \frac{\eta'_{\rho i} \tilde{\rho}_i^2}{2} - \frac{\eta'_{mr} \tilde{m}_r^2}{2} - \frac{\eta'_{ml} \tilde{m}_l^2}{2} - \frac{\eta'_{br} \tilde{d}_{mr}^2}{2} - \frac{\eta'_{bl} \tilde{d}_{ml}^2}{2} \\ &\quad - \sum_{i=2}^{n-1} \left(\frac{1}{\tau_{i+1}} - \frac{1}{2\gamma_{i+1}} M_{i+1}^2 - \frac{g'_{i\max}}{4} \right) \lambda_{i+1}^2 + \sum_{i=1}^{n} \frac{\eta'_{wi} \left\| W_{oi}^* \right\|^2}{2} \\ &\quad + \sum_{i=1}^{n} \frac{\eta'_{\rho i} \rho_i^{*2}}{2} + \frac{\eta'_{mr} m_r^{*2}}{2} + \frac{\eta'_{ml} m_l^{*2}}{2} + \frac{\eta'_{dr} d_{mr}^{*2}}{2} + \frac{\eta'_{dl} d_{ml}^{*2}}{2} \\ &\quad + \sum_{i=2}^{n-1} \left(\frac{\gamma_{i+1}}{2} + \kappa'_i \right) + \kappa_1. \end{aligned} \tag{94}$$

From Lemma 2, $-\frac{S_1^2}{k_{b1}^2 - S_1^2} \le -\log \frac{k_{b1}^2}{k_{b1}^2 - S_1^2}$ in the set $|z_1| < k_{b1}$. Hence, (94) can be represented as follows:

$$\dot{V} \le -c_1 (k_{b1}^2 - S_1^2) \log \frac{k_{b1}^2}{k_{b1}^2 - S_1^2} - \sum_{i=2}^{n-1} \left(c_i - g'_{i\max} \right) S_i^2 - c_n s_n^2$$

$$-\sum_{i=1}^{n}\frac{\eta'_{wi}\tilde{W}_{oi}^{T}\tilde{W}_{oi}}{2}-\sum_{i=1}^{n}\frac{\eta'_{\rho i}\tilde{\rho}_i^2}{2}-\frac{\eta'_{mr}\tilde{m}_r^2}{2}-\frac{\eta'_{ml}\tilde{m}_l^2}{2}$$
$$-\frac{\eta'_{dr}\tilde{d}_{mr}^2}{2}-\frac{\eta'_{dl}\tilde{d}_{ml}^2}{2}+\mu, \tag{95}$$

where

$$\mu=\sum_{i=1}^{n}\frac{\eta'_{wi}\left\|W_{oim}^{*}\right\|^2}{2}+\sum_{i=1}^{n}\frac{\eta'_{\rho i}\rho_i^{*2}}{2}+\frac{\eta'_{mr}m_r^{*2}}{2}+\frac{\eta'_{ml}m_l^{*2}}{2}$$
$$+\frac{\eta'_{dr}d_{mr}^{*2}}{2}+\frac{\eta'_{dl}d_{ml}^{*2}}{2}+\sum_{i=2}^{n-1}\left(\frac{\gamma_{i+1}}{2}+\kappa'_i\right)+\kappa_1,$$
$$\left\|W_{oi}^{*}\right\|\le W_{oim}^{*},$$

and the positive definite matrix $\mathbf{Q}$ is described as

$$\mathbf{Q}=\begin{bmatrix} c_1(k_{b1}^2-S_1^2) & 0 & \cdots & 0 \\ 0 & c_2-g'_{2\max} & \cdots & 0 \\ 0 & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_n \end{bmatrix}.$$

We select $C=\min[2\lambda_{\min}(\mathbf{Q}),2(1/\tau_{i+1}-M_{i+1}^2/2\gamma_{i+1}-g'_{i\max}/4)\,\eta'_{wi},\eta'_{\rho i},\eta'_{pi},\eta'_{si},\eta'_{mr},\eta'_{ml},\eta'_{dr},\eta'_{dl}]$, c_1, c_i, c_n, and τ_i such that $c_1(k_{b1}^2-S_1^2)\ge C$, $c_i>g'_{i\max}+C$, $(2\le i\le n-1)$, $c_n\ge C$, and $(1/\tau_{i+1}\ge M_{i+1}^2/2\gamma_{i+1}+g'_{i\min}/4+C)$, $(2\le i\le n-1)$.

Then, we can obtain

$$\dot{V}\le -CV+\mu, \tag{96}$$

in the set $|z_1|<k_{b1}$. In the relation $\dot{\theta}=\varphi(t,\theta)$ in Lemma 1, $\theta=[\bar{S}^T,\tilde{\bar{W}}_o^T,\tilde{\bar{\rho}}^T,\tilde{m}_r,\tilde{m}_l,\tilde{d}_{mr},\tilde{d}_{ml}]^T$. $\varphi(t,\theta)$ satisfies the conditions of Lemma 1 for $\theta\in\Omega=\{\bar{S}^T,\tilde{\bar{W}}_o^T,\tilde{\bar{\rho}}^T,\tilde{m}_r,\tilde{m}_l,\tilde{d}_{mr},\tilde{d}_{ml}:|S_1|<k_{b1}\}$. From $S_1(0)=y(0)-y_d(0)$, $y(0)\le k_0$ and $k_0+A_0+|y_d(0)|$ in the definition of Ω_y^0 and Ω_y, it is shown that $|S_1(0)|<k_{b1}$ and Ω is an invariant set. Thus, $|S_1|<k_{b1}$, $\forall t>0$ from Lemma 1 and (96) and we can show that $|y(t)|\le|S_1(t)|+|y_d(t)|<k_{b1}+A_0=k_{c1}$, $\forall t>0$. Hence, we conclude that $y(t)\in\Omega_y$, $\forall t>0$.

ii) If $V=p$ and $C=\mu/p$, then $\dot{V}\le 0$. That is, if $V(0)\le p$, then $V(t)\le p,\forall t\ge 0$. This implies that $V(t)\le p$ is an invariant. Multiplying (96) by e^{Ct} yields

$$\frac{d}{dt}(Ve^{Ct})\le \mu e^{Ct}. \tag{97}$$

Integrating (97) over [0, t] yields

$$0\le V\le\left(V(0)-\frac{\mu}{C}\right)e^{-Ct}+\frac{\mu}{C}. \tag{98}$$

Therefore, this means that all the error signals are semiglobally, uniformly ultimately bounded.

iii) From (93) and (98), we can further represent the Lyapunov function as

$$\frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2-S_1^2}=\left(V(0)-\frac{\mu}{C}\right)e^{-Ct}+\frac{\mu}{C}. \tag{99}$$

Then, exponential computing of (99) yields

$$\frac{k_{b1}^2}{k_{b1}^2-S_1^2}=\mathrm{e}^{2\left((V(0)-\mu/C)e^{-Ct}+\mu/C\right)}. \tag{100}$$

Multiplying both sides of (100) by $(k_{b1}^2-S_1^2)>0$ and applying algebraic manipulations lead to the inequality:

$$|S_1(t)|\le k_{b1}^2\sqrt{1-\mathrm{e}^{-2\left((V(0)-\mu/C)e^{-Ct}+\mu/C\right)}}. \tag{101}$$

As $t\to\infty$, $|S_1|\le k_{b1}^2\sqrt{1-\mathrm{e}^{-2\mu/C}}$, thus,

$$|y-y_d|\le k_{b1}^2\sqrt{1-\mathrm{e}^{-2\mu/C}} \quad \text{as} \quad t\to\infty. \tag{102}$$

Therefore, the output tracking error is smaller than the prescribed error bound, and the size of $y-y_d$ can be arbitrarily regulated to small values by controlling the design parameters.

5. EXPERIMENTAL EVALUATION

In this section, the application of the proposed output-constrained control scheme is described and experimentally evaluated. A single manipulator of the Scorbot robot system in the presence of deadzone and friction in the joints is chosen for the experimental evaluation. A photograph and detailed description of the Scorbot robot control system are presented in [25]. We consider only the second upper arm as the control application among the four links of the Scorbot robot manipulator. The dynamic equation for the second single link is

$$\begin{aligned} &J\ddot{q}+G(q)+F_f=D(\tau),\\ &\tau=NK_t i,\\ &L_m\frac{di}{dt}+R_m i+K_b\dot{q}=V,\\ &F_f=\sigma_0 z+\sigma_1\dot{z}+\sigma_2\dot{q},\\ &\dot{z}=\dot{q}-\frac{\sigma_0|\dot{q}|}{h(\dot{q})}z, \end{aligned} \tag{103}$$

where $J=mL^2/3$; q is the angular position of the link; $G(q)=mL\cos q$; the mass of the link $m=3.59$ kg; the length of the link $L=0.41$ m; $g=9.806\,\mathrm{m/s^2}$; the inductance of the motor $L_m=0.6292mH$ the resistance of the motor $R_m=0.8294\Omega$; the torque constant $K_t=0.0182$ Nm/A; the back emf constant $K_b=0.0182$ Vsec/rad; the gear ratio of reduction gear $n=64$; the bristle stiffness $\sigma_0=2300$ Nm/rad; the pre-sliding damping $\sigma_1=10$ Nm/rad/sec; $h(\dot{q})=1/[F_c+(F_s-F_c)\exp(-(\dot{q}/\dot{q}_s)^2]$; the Coulomb friction torque $F_c=0.61$ Nm; the stiction level $F_s=0.68$ Nm; the viscous friction coefficient $F_v=0.63$ Nm; and the Stribeck velocity $\dot{q}_s=0.00063$ rad/sec. The assumed initial values of the deadzone parameters are $m_r = 1$, $m_l = 1$, $d_r = 0.35$ and $d_l = -0.35$. We define the state variables as $x_1 = q$, $x_2 = \dot{q}$ and $x_3 = i$.

Then, the state equations are written as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{1}{J} mL\cos x_1 - \frac{1}{J} F_f + \frac{NK_t}{J} x_3 + \frac{u_\Delta}{J} \\ &= f_2'(x_1) + g_2 x_3 + F_{u2}, \\ \dot{x}_3 &= \frac{R_m}{L_m} x_3 - \frac{K_b}{L_m} x_2 + \frac{1}{L_m} u_d \\ &= f_3'(x_2, x_3) + g_3 u_d + F_{u3}, \end{aligned} \tag{104}$$

where $u_\Delta = Aq + B(1-q) + \varepsilon_d$; $A = \tilde{d}_{mr} + \frac{u_d(t) + \hat{d}_{mr}}{\hat{m}_r} \tilde{m}_r$; $B = \tilde{d}_{ml} + \frac{u_d(t) + \hat{d}_{ml}}{\hat{m}_l} \tilde{m}_l$; $f_2(x_1) = -mL\cos x_1 / J + u_\Delta / J + \Delta f_2$; $f_3(x_2, x_3) = R_m / L_m x_3 - K_b / L_m x_2$; $F_{u2} = -F_f / J$; $g_2 = NK_t / J$; $g_3 = 1/L_m$; $u_d = V$, and $F_{u3} = \Delta f_3$. The fuzzy membership functions for F_{u2} are chosen as

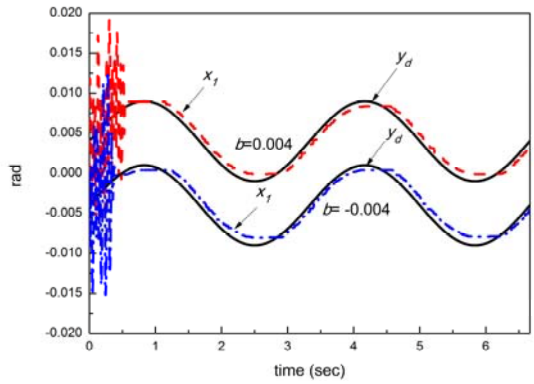
$$\begin{aligned} \mu_{F_1^1} &= \frac{1}{1 + \exp[(S_1 - 1\times 10^{-3})^2]}, \\ \mu_{F_1^j} &= \exp[-0.1(S_1 - m_j)^2], \\ \mu_{F_1^{11}} &= \frac{1}{1 + \exp[(S_1 + 1\times 10^{-3})^2]}, \end{aligned} \tag{105}$$

where $m_j = -1\times 10^{-3} + (j-1)\times 0.2\times 10^{-3}$, for $j = 2, \ldots, 10$. The second membership functions corresponding to F_{u3} are the same as the first membership functions except that the input variable is $\dot{S}_1$ instead of S_1. The desired trajectory of the manipulator is $y_d(t) = b + R\sin(\omega \times t)$, where $-0.004\,\text{rad} \le b \le 0.004$ rad, $R = 0.005$ rad and $\omega = 1.8849$ rad/sec subject to the output constraint. $|x_1| < 0.014$ rad. Because $|y_d| \le A_0 = 0.009$ rad, we have $k_{b1} = 0.014\,\text{rad} - 0.009\,\text{rad} = 0.005\,\text{rad}$. The initial conditions are $x_1(0) = 0$ and $x_2(0) = 0$. The design parameters of the controller are $c_1 = 500$, $c_2 = 20$, $c_3 = 100$, $\tau_2 = \tau_3 = 0.1$, $\eta_{w2} = \eta_{w3} = 0.2$, $\eta'_{w2} = \eta'_{w3} = 0.001$, $\eta_{mr} = \eta_{ml} = 0.5$, $\eta'_{mr} = \eta'_{ml} = 0.01$, $\eta_{dr} = \eta_{dl} = 0.25$, and $\eta'_{dr} = \eta'_{dl} = 0.01$.

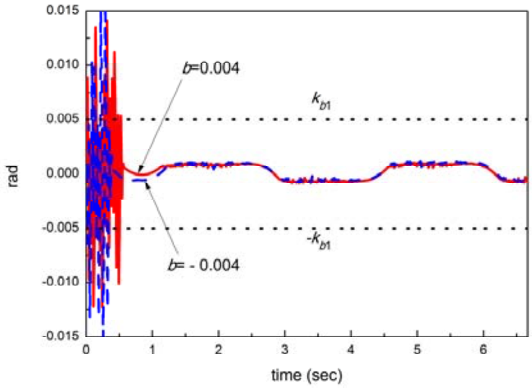
We designed four controllers to evaluate the performance of the proposed control system:

1) a quadratic Lyapunov function (QLF) based nominal DSC system (QLF-DSC), 2) a BLF-based backstepping DSC system (BLF-BDSC), 3) a BLF-based DSC system with deadzone compensation (BLF-DSC-D), and 4) a BLF-based BDSC system with adaptive fuzzy compensation of deadzones and uncertainty (FBLF-BDSC-D). The designed controllers were generated using a computer and implemented in the Matlab RTI system using an MF624 Humusoft board [26]. The control signal was transferred to the DC servo motor of the Scorbot robot through the servo drive. The output position of the link angle was measured by a rotary encoder. The selected sample frequency was 1 kHz.

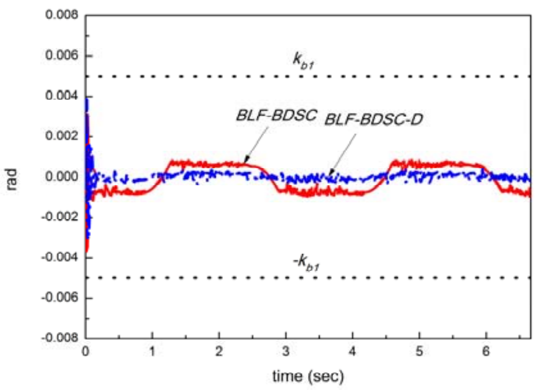
The experimentally observed responses of QLF-DSC, BLF-BDSC, and BLF-BDSC-D to the command input with $b = \pm 0.004$ are shown in Fig. 1. In Fig. 1(a) and (b), the QLF-DSC system exhibits higher over- and undershoot in the transient stage, and the constraint $-0.005 \le S_1 \le 0.005$ is violated. Fig. 1(c) shows that the tracking errors of the BLF-BDSC and BDSC-D systems remain within the range of the constraint, $|S_1| \le k_{b1} =$ 0.005. The tracking errors for BLF-BDSC-D are smaller than those of BLF-BDSC; thus, the deadzone nonlinearity is effectively compensated by the inverse deadzone compensator.



(a) Position tracking outputs of QLF-DSC system.

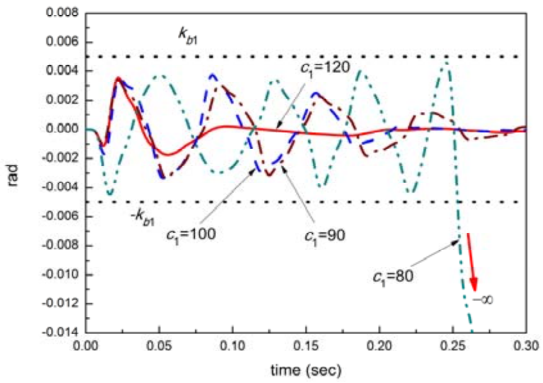


(b) Position tracking errors of QLF-DSC system.

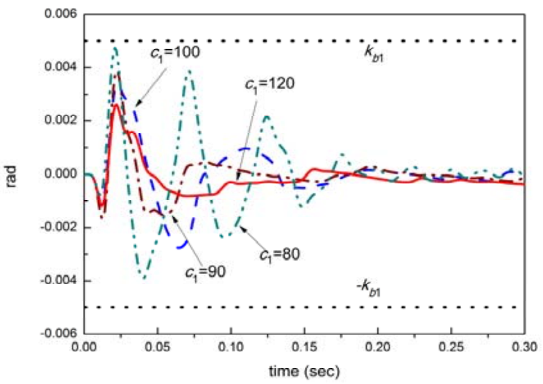


(c) Position tracking errors of BLF-BDSC and BLF-BDSC-D systems.

Fig. 1. Output tracking performance for output-constrained QLF-DSC, BLF-BDSC and BLF-BDSC-D systems.



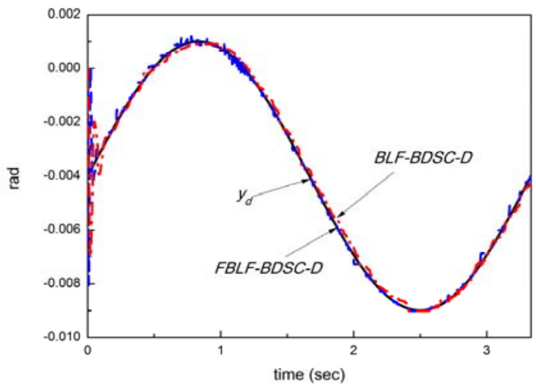
(a) Position tracking errors of BLF-DSC-D.



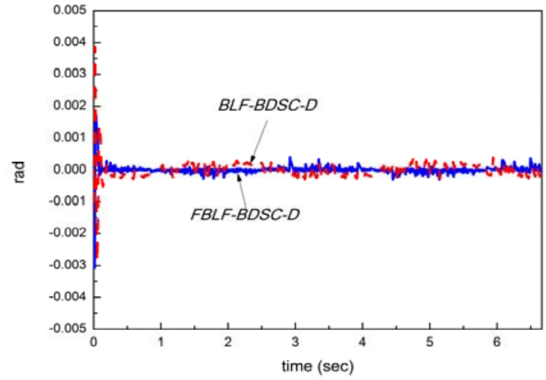
(b) Position tracking errors of BLF-BDSC-D.

Fig. 2. The transient response variations according to the selection of control gain c_1.

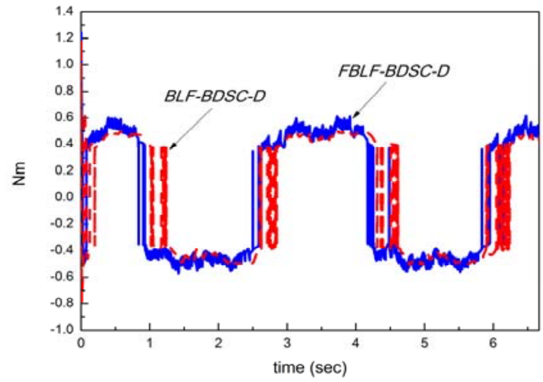
In Fig. 2(a), as mentioned in Remark 1, the tracking performance of the BLF-DSC-D system is very sensitive to the decrease in the control gains c_1 and even becomes unstable at a certain gain value. On the other hand, in Fig. 2(b), the transient performance of the proposed BLF-BDSC-D system is more stable than that of the nominal DSC-based control system. Finally, Fig. 3(a) illustrates the tracking response of BLF-BDSC-D and FBLF-BDSC-D. Fig. 3(b) and Table 1 show that the magnitude of the position tracking error of the proposed FBLF-BDSC-D system is much smaller than that of the other control systems owing to compensation for the nonlinear deadzone, friction and uncertainty by the adaptive fuzzy



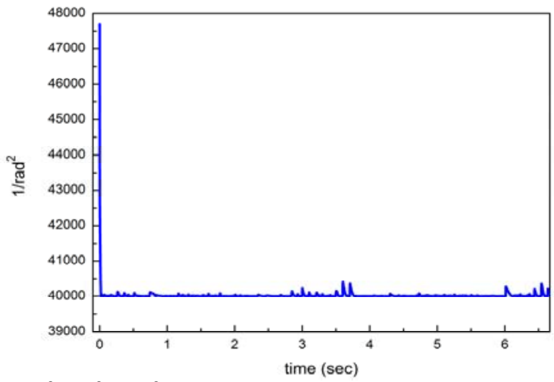
(a) Position tracking response for the command input at $b = -0.004$ rad.



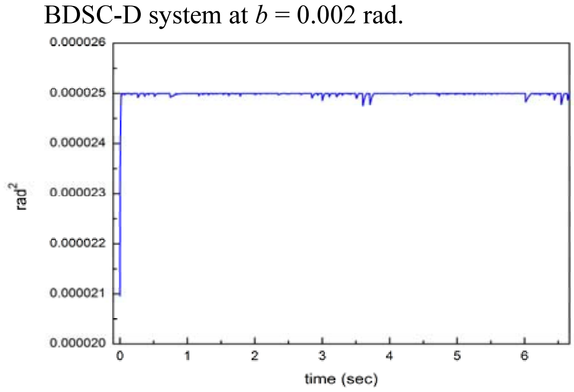
(b) Position tracking error for the command input at $b = -0.004$ rad.



(c) Control inputs.



(d) $S_1^2/(k_{b1}^2 - S_1^2)$ for the command input of FBLF-BDSC-D system at $b = 0.002$ rad.



(e) $k_{b1}^2 - S_1^2$ for the command input of FBLF-BDSC -D system at $b = 0.002$ rad.

Fig. 3. Output tracking performance for output-constrained BLF-BDSC-D and FBLF-BDSC-D systems.

Table 1. The RMS position tracking errors.

	BDSC	BDSC-D	FBLF-BDSC-D
rad	6.60×10^{-4}	2.53×10^{-4}	2.09×10^{-4}
	100 %	38 %	31 %

system. The control inputs are given in Fig. 3(c). Figs. 3(d) and (e) confirm experimentally that $S_1^2/(k_{b1}^2-S_1^2)$ and $k_{b1}^2-S_1^2$ always have positive values. The experimental results for several bias command values, –0.004 rad $\le b \le 0.004$ rad, and the adaptive estimated results of the deadzone and uncertainties are not presented because of space limitations.

6. CONCLUSION

In this paper, a backstepping DSC control scheme was developed to provide greatly enhanced position tracking performance and to guarantee output constraint of a strict feedback SISO nonlinear dynamic system in the presence of deadzone and uncertainty. The backstepping control was partially combined with a recursive DSC design procedure to improve the stability of the closed loop of a conventional DSC-based output-constraint system. The deadzone and uncertainties in each recursive step of the backstepping design were compensated by the inverse deadzone method and adaptive fuzzy system. Using the Lyapunov stability theorem, we proved that all closed-loop signals are bounded and the tracking error converges within a prescribed level. As a design example, a Scorbot robot manipulator in the presence of joint friction and deadzone was chosen. We obtained favorable position tracking performance as well as output constraint from the proposed control scheme by effective compensation for deadzone, friction, and uncertainty.

REFERENCES

[1] K. Ngo and Z. Jiang, "Integrator backstepping using barrier functions for systems with multiple state constraints," *Proc. of the 44th IEEE Conf. Decision and Contr. Eur. Contr. Conf.*, pp. 8306-8312, 2005.

[2] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918-927, 2009.

[3] B. B. Ren, S. S. Ge, K. P. Tee, and T. H. Lee, "Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function," *IEEE Trans. Neural Networks*, vol. 21, no. 8, pp. 1339-1345, 2010.

[4] K. P. Tee, S. S. Ge, and E. H. Tay, "Adaptive control of electrostatic microactuator with bidirectional drive," *IEEE Trans. Contr. Syst. Technol.*, vol. 17, no. 2, pp. 340-352, 2009.

[5] B. V. E. How, S. S. Ge, and Y. S. Choo, "Control of coupled vessel, crane, cable, and payload dynamics for subsea installation operation," *IEEE Trans. Contr. Sys. Tech.*, vol. 19, no. 1, pp. 208-220, 2011.

[6] K. P. Tee and S. S. Ge, "Control of nonlinear systems with partial state constraints using a barrier Lyapunov function," *International Journal of Control*, vol. 84, no. 12, pp. 2009-2023, 2010.

[7] B. Niu and J. Zhao, "Tracking control for output-constrained nonlinear switched systems with a barrier Lyapunov function," *International Journal of Science*, iFirst, pp. 1- 8, 2012.

[8] K. P. Tee, B. Ren, and S. S. Ge, "Control of nonlinear systems with time-varying output constraints," *Automatica*, in press, 2012.

[9] M. Kristic, L. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

[10] Y. Zhang, B. Fidan, and P. A. Ioannou, "Backstepping control of linear time-varying systems with known and unknown parameters," *IEEE Trans. Autom. Contr.*, vol. 48, no. 11, pp. 1908-1925, 2003.

[11] S. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems," *IEEE Trans. Autom. Contr.*, vol. 45, no. 10, pp. 1893-1899, 2000.

[12] J. K. Hedrick and P. P. Yip, "Multiple sliding surface control: theory and application," *Trans. ASME*, vol. 122, pp. 586-593, 2000.

[13] D. Wang and J. Huang, "Neural network-based adaptive dynamic surface control for a class of unknown nonlinear systems in strict-feedback form," *IEEE Trans. Neural Networks*, vol. 16, no. 1, pp. 195-202, 2005.

[14] S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive dynamic surface control of flexible-joint robots using self-recurrent wavelet neural networks," *IEEE Trans. Sys. Man and Cyber., Part B: Cyber.*, vol. 36, no. 6, pp. 1342-1354, 2006.

[15] S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive dynamic surface control for disturbance attenuation of nonlinear systems," *International Journal of Control, Automation, and Systems*, vol. 7, no. 6, pp. 882-887, 2009.

[16] W. S. Chen, "Adaptive backstepping dynamic surface control for systems with periodic disturbances using neural networks," *IET Contr. Theo. & Appl.*, vol. 3, no. 10, pp. 1383-1394, 2009.

[17] S. Tong, G. Feng, and S. X. Ding, "Observer-based adaptive fuzzy dynamic surface control for a class of non-linear systems," *IET Contr. Theo. & Appl.*, vol. 5, no. 12, pp. 1426-1438, 2011.

[18] X. Zhang, Y. Lin, and J. Mao, "A robust adaptive dynamic surface control for a class of nonlinear systems with unknown Prandtl–Ishilinskii hysteresis," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 13, pp. 1541-1561, 2011.

[19] G. Tao and P. V. Kokotovic, "Adaptive control of plants with unknown dead-zones," *IEEE Trans. Autom., Contr.*, vol. 39, no. 1, pp. 59-68, 1994.

[20] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall, Englewood Cliffs, New York, 1995.

[21] Y. C. Chang, "Robust tracking control of nonlinear MIMO systems via fuzzy approaches," *Automatica*, vol. 36, no. 10, pp. 1535-1545, 2000.

[22] E. Kim, "Output feedback tracking control of robot

manipulators with model uncertainty via adaptive fuzzy logic," *IEEE Trans. Fuzzy Syst*., vol. 12, no. 3, pp. 368-378, 2004.
[23] J. P. Hwang and E. Kim, "Robust tracking control of an electrically driven robot: adaptive fuzzy logic approach," *IEEE Trans. Fuzzy Syst*., vol. 14, no. 2, pp. 232-246, 2006.
[24] M. M. Polycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica*, vol. 32, no. 3, pp. 423-427, 1996.
[25] S. I. Han and J. M. Lee, "Friction and uncertainty compensation of robot manipulator using optimal recurrent cerebellar model articulation controller and elasto-plastic friction observer," *IET Control Theory & Appl*., vol. 5, no. 18, pp. 2120-2041, 2011.
[26] Humusoft Comp. *MF 624 Multifunction I/O Card Manual*, Czech Republic, 2006.

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