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Robust Stability Conditions for DMC Controller with Uncertain Time Delay

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Abstract: For Networked Control Systems (NCSs), the conventional Dynamic Matrix Control (DMC) may not applicable due to the unknown transmission delay. The uncertain time delay was usually converted to constant time delay by using registers. This paper addresses the stability problem for single-input-single-output (SISO) linear NCSs with uncertain time delay via DMC controller. A novel DMC controller which is effective for such NCSs has been proposed. Applying Jury's dominant coefficient lemma, the sufficient stability and stabilization conditions are presented. Finally, numerical examples are given to demonstrate the theoretical results.

Keywords: Dynamic matrix control, model predictive control, networked control systems, stability, time delay.

1. INTRODUCTION

Feedback control systems with network are usually called NCSs, which have attracted lots of attentions these years. NCSs have been widely used in large scale distributed control systems, remote control, intelligent transportation and satellite control systems.

Although with benefits of easily transporting and storage, there also exist some control issues need to be addressed, including the problems of network time delay, packet dropout, and limit of bandwidth. Lots of studies have been performed to solve these problems. Nilsson analyzed the NCSs in discrete-time domain and modeled time delays as constant, independent random or Markov chain [1]. Zhang studied modeling and robust stabilization of NCSs with time-varying delays and packet-dropout [2,3]. Bekiaris-Liberis *et al*. considered a class of general systems in strict feedback form with delayed integrators, and designed a predictor feedback stabilization controller [4]. They also designed a Lyapunov-based adaptive controller to achieve global stability for the case where the delays are of unknown length [5]. Bresch-Pietri *et al*. presented the adaptive control design for an ODE system with unknown delay value and system parameters [6]. Nicola Elia showed that the coarsest (least dense) logarithmic quantizer could quadratically stabilizes a single input linear discrete time invariant system, and could be computed by solving a special linear quadratic regulator (LQR) problem [7]. Fu studied a number of quantized feedback design problems for linear systems and considered the case where quantizers are static (memoryless). A conclusion that the classical sector bound approach was non-conservative for designing problems was derived in his work [8]. By exploring some geometric properties of the logarithmic quantizer and using the fact that the logarithmic quantizer is sector bounded and non-decreasing, Zhou presented a new approach to the stability analysis of quantized feedback control system based on Tsypkin-type Lyapunov functions [9].

Model predictive control (MPC) is also well known as moving horizon control. It is a very popular technique for the control of slow dynamic systems, such as chemical process control in paper industries, desulfurization and denitrification [10-12]. As one of the most widely used methods in MPC, DMC was first presented by Cutler and Ramaker [13]. Nowadays, traditional DMC is not applicable because of new problems such as robust stability and network induced time delay. Badgwell presented some robust stability conditions for MPC algorithms, based on Jury's dominant coefficient lemma [14]. In order to apply these results to DMC algorithm, Dai used the families of finite impulse response models to describe the uncertainty of the controlled system and derived the robust stability conditions of closed loop systems [15]. A new condition of robust stability for a set of stable, linear time-invariant plants controlled by using a simplified model predictive control algorithm (SMPC) is presented by Webber and Gupta [16]. Meanwhile, with the development of network, distributed MPC has attracted lots of attention and noticeable works include [17-24]. A fault detection and compensation scheme based on likelihood ratios (LRs) for networked predictive control systems with random network-induced time delays and clock asynchronism was presented in [22]. Random packet dropouts which deteriorate overall system's stability and

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performance was investigated in [23]. A hybrid control strategy is presented based on DMC and feedback linearization methods for designing a predictive controller of five bar linkage manipulator as a MIMO system (two inputs and two outputs) was presented in [24]. In these studies, much attention was paid to obtain improved or optimal performance in a distributed networked environment. Although significant improvement in control performance over completely decentralized MPC controllers was provided, networked induced time delays are not considered or are transformed to maximum time delay, which increased the conservativeness.

As illustrated above, the stability problem of DMC algorithm with uncertain time delay has not been fully solved so far, which decreases the possibility of applying DMC in practical control industries. To fill this gap, we study the stability of networked DMC systems while the uncertain time delay is directly considered.

This paper is organized as follows: Section 2 reviews some preliminary knowledge about finite impulse response (FIR) model, finite step response (FSR) model and SISO DMC algorithm. In section 3 we propose the main result of this study. The control system structure is presented based on the network and two sufficient stability conditions are obtained while uncertain time delay is directly handled. Numerical examples are given in Section 4 to demonstrate the theoretical results, and Section 5 concludes the paper.

Notation: We use standard notations throughout the paper. A^{T} means the transpose of matrix A. Z_+, R and R^n stand for the positive integer field, real number field and n-dimensional Euclidean space, respectively. $[A]_{M\times N}$ means matrix A with $M\times N$-dimension. $|\cdot|$ means absolute value. For a function $f(x): R\to R$, $F_{\max} = Max\ (f(x))$ means the maximum of the function $f(x)$ is $F_{\max}$, e.g., $\exists x_0 \in R,\ s.t. f(x_0) = F_{\max} \ge f(x)$ for any $x \in R$.

2. PRELIMINARIES

2.1. FIR model and FSR model family

Before discussing the control design method, we review finite impulse response (FIR) and finite step response (FSR) model firstly [14].

A linear open-loop stable SISO plant can be described either by an FIR model

$$y(k) = \sum_{i=1}^{N} h_i u(k-i) + d(k)$$

or an FSR model

$$y(k) = \sum_{i=1}^{N-1} a_i \Delta u(k-i) + a_N u(k-N) + d(k)$$

with N coefficients. At any time interval k, $y(k)$ and $u(k)$ are the output and input, respectively. $\Delta u(k) = u(k) - u(k-1)$ and $d(k)$ accounts for unmeasured disturbances or model errors. h_i denotes the impulse response while a_i denotes the step response. The two representations can be transformed into each other since

$$a_i = \sum_{j=1}^{i} h_j, h_i = a_i - a_{i-1}.$$

Hence, when an FIR model is used to predict future values of the output, we have

$$\tilde{y}(k+j) = \sum_{i=1}^{N} h_i u(k+j-i) + d(k). \tag{1}$$

Meanwhile, if an FSR model is adopt, we can obtain

$$\begin{aligned}\tilde{y}(k+j) = &\sum_{i=1}^{N-1} a_i \Delta u(k+j-i) \\ &+ a_N u(k+j-N) + d(k),\end{aligned} \tag{2}$$

where

$$d(k) = y(k) - \sum_{i=1}^{N} h_i u(k-i)$$

or

$$d(k) = y(k) - \sum_{i=1}^{N-1} a_i \Delta u(k-i) - a_N u(k-N). \tag{3}$$

Based on (1), $\tilde{y}(k+j)$ can be split into three parts as follows:

$$\begin{aligned}&\tilde{y}(k+j) = \tilde{y}_f(k+j) + \tilde{y}_P(k+j) + d(k), \\ &\tilde{y}_f(k+j) = \sum_{i=1}^{j} h_i u(k+j-i), \\ &\tilde{y}_P(k+j) = \sum_{i=j+1}^{N} h_i u(k+j-i),\end{aligned} \tag{4}$$

where $\tilde{y}_f$ and $\tilde{y}_p$ denotes the future and the past contribution, respectively.

2.2. SISO DMC algorithm based on FIR model

In order to apply FIR technique, Dai and Cheng reconstructed the DMC algorithm based on an FIR model in [15].

The aim of DMC algorithms is to compute the future control increment sequence $\{\Delta u(k), \cdots, \Delta u(k+m-1)\}$ to minimize the objective function

$$J(k) = \sum_{j=1}^{P} q_j \left(r(k) - \tilde{y}(k+j)\right)^2 + \sum_{i=1}^{m} r_i \Delta u^2(k+i-j),$$

where m is control horizon, P is predictive horizon, $r(k)$ is the set point of process output, q_j and r_i are weights of output errors and input changes. Normally we have $N > P > m$.

The predictive error is defined as

$$e(k+j) = r(k) - \tilde{y}(k+j) = \hat{e}(k+j) - \tilde{y}_f(k+j),$$

where

$$\hat{e}(k+j) = r(k) - \tilde{y}_P(k+j) - d(k). \tag{5}$$

Let

$$H=\begin{bmatrix} h_1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ h_{M-1} & \cdots & h_1 & 0 \\ h_M & \cdots & h_2 & a_1 \\ \vdots & \vdots & \vdots & \vdots \\ h_P & \cdots & h_{P-M+2} & a_{P-M+1} \end{bmatrix}_{P\times M}, \tag{6}$$

$$u=\begin{pmatrix} u(k) \\ \vdots \\ u(k+M-1) \end{pmatrix},\quad \hat{e}=\begin{pmatrix} \hat{e}(k+1) \\ \vdots \\ \hat{e}(k+P) \end{pmatrix},$$

$$e=\begin{pmatrix} e(k+1) \\ \vdots \\ e(k+P) \end{pmatrix},\quad \tilde{y}_f=\begin{pmatrix} \tilde{y}_f(k+1) \\ \vdots \\ \tilde{y}_f(k+P) \end{pmatrix},$$

$$\Delta u=\begin{pmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+M-1) \end{pmatrix},\ b=\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix},\ G=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & -1 & 1 \end{pmatrix}.$$

We have

$$\hat{y}_f=Hu,\quad e=\hat{e}-Hu,$$

$$\Delta u=Gu-bu(t-1).$$

Let $Q=diag(q_1,\cdots,q_P)$, $R=diag(r_1,\cdots,r_m)$, the objective function can be represented as

$$J(k)=(\hat{e}-Hu)^TQ(\hat{e}-Hu)+\big(Gu-bu(k-1)\big)^TR\big(Gu-bu(k-1)\big).$$

The corresponding optimal solution is

$$\begin{aligned} u&=(H^TQH+G^TRG)^{-1}(H^TQ\hat{e}+G^TRbu(k-1)) \\ &=(H^TQH+G^TRG)^{-1}H^TQ\hat{e} \\ &\quad+(H^TQH+G^TRG)^{-1}G^TRbu(k-1). \end{aligned}$$

Noting that only the first term $u(k)$ of u is employed in the DMC algorithm, we have

$$u(k)=k_e\hat{e}+k_uu(k-1), \tag{7}$$

where

$$k_e=b^T(H^TQH+G^TRG)^{-1}H^TQ=[c_1,\cdots,c_P],$$

$$k_u=b^T(H^TQH+G^TRG)^{-1}G^TRb.$$

3. MAIN RESULTS

3.1. Control system structure

Here we consider a single-loop networked control system illustrated in Fig. 1. Network exists between not only sensor and controller but also controller and actuator. The sensor is time-driven in this system while both the controller and the actuator are event-driven.

We consider a SISO system in the form of state space as following

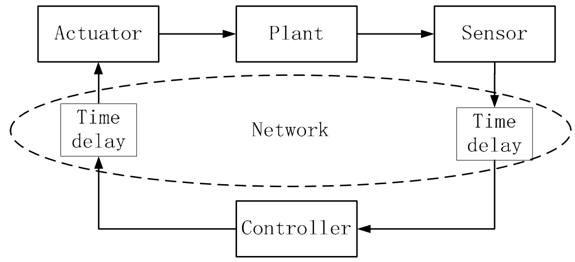

Fig. 1. Control structure with uncertain time delay in the sensor-controller and controller-actuator links.

$$\begin{cases} \dot{x}(t)=Ax(t)+Bu(t) \\ y(t)=(C+\Delta C)x(t), \end{cases} \tag{8}$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R$ is the control input and $y(t)\in R$ is the output. A, B and C are system matrices with appropriate dimensions. ΔC denotes the uncertain part of the system. It is unknown but bounded by

$$\Delta C_{\min}\le \Delta C\le \Delta C_{\max}.$$

Considering the effects of the network, time delay is introduced in the discrete form. We assume that the time delay $\tau=nh+\varepsilon$ where h is the sampling period and $\varepsilon<h$. Hence, we have the discrete form of (8)

$$\begin{cases} x(k+1)=Ax(k)+B_0u(k-n)+B_1u(k-n-1) \\ y(k)=(C+\Delta C)x(k), \end{cases} \tag{9}$$

where

$$B_0=\int_0^{h-\varepsilon_k}e^{As}Bds,\quad B_1=\int_{h-\varepsilon_k}^{h}e^{As}Bds.$$

Time delay will not only have effect on the discrete model, but on the step response and impulse response coefficient as well. Here we introduce the following lemma.

Lemma 1: For the system (9) with time delay $\tau=nh+\varepsilon$, the unit-step response coefficient $a=[a_1,a_2,\cdots,a_N]^T$ satisfies

$$a_i=\begin{cases} (C+\Delta C)A^ix_0, & i\le n \\ (C+\Delta C)(A^{n+1}x_0+B_0), & i=n+1 \\ (C+\Delta C)(A^ix_0+A^{i-n-1}B_0+A_w(B_0+B_1)), & i>n+1, \end{cases} \tag{10}$$

where $A_w=A^{i-n-2}+\cdots+A+\mathrm{I}$ and x_0 is the original state.

Meanwhile, the impulse response coefficient $h=[h_1,h_2,\cdots,h_N]^T$ satisfies

$$h_i=\begin{cases} (C+\Delta C)A^ix_0, & i\le n \\ (C+\Delta C)(A^{n+1}x_0+B_0), & i=n+1 \\ (C+\Delta C)(A^ix_0+A^{i-n-1}B_0+A^{i-n-2}B_1), & i>n+1. \end{cases} \tag{11}$$

Proof: Assume the input u is unit-step signal, which means

$$u(k)=\begin{cases}1, & k\ge 0\\ 0, & k<0.\end{cases}$$

Substituting it into (9), we have

$$\begin{aligned}
x_1 &= Ax_0+B_0u_{-n}+B_1u_{-n-1}=Ax_0\\
x_2 &= Ax_1+B_0u_{1-n}+B_1u_{1-n-1}=A^2x_0\\
&\vdots\\
x_n &= Ax_{n-1}+B_0u_{-1}+B_1u_{-2}=A^nx_0\\
x_{n+1} &= Ax_n+B_0u_0+B_1u_{-1}=A^{n+1}x_0+B_0\\
&\vdots\\
x_N &= Ax_{N-1}+B_0u_{N-1-n}+B_1u_{N-1-n-1}\\
&= A^Nx_0+A^{N-n-1}B_0+(A^{N-n-2}+\cdots+A+\mathrm{I})(B_0+B_1)\\
&= A^Nx_0+A^{N-n-1}\Gamma_0+A_w(B_0+B_1).
\end{aligned}$$

Noticing that $a_i=y_i=(C+\Delta C)x_i$ when u is unit-step signal, we have (10). And (11) can be obtained in a similar way.

The proof is completed.

Differing from [21], in which the time-varying delay was converted to the constant maximum delay by using a receiving buffer, we consider the uncertain time delay in this paper. As mentioned in Lemma 1, different time delay will lead to different step/impulse response coefficient, we use $a_{i,k}$ and $h_{i,k}$ to denote the ith step/impulse response coefficient at time interval k.

The networked-induced feedback delay is unknown but has certain upper and lower bound, denoted as $\tau^{\max}$ and $\tau^{\min}$, respectively. In this paper, we assume the time delay τ obeys the uniform distribution. We adopt the average delay τ^{ave} to predict the unknown time delay, i.e., $\tau^{\mathrm{ave}}=\frac{(\tau^{\max}+\tau^{\min})}{2}$. Hence, when we calculate step/impulse response coefficient, it changes from time-variant to time-invariant. We use $\tilde{a}_i$ and $\tilde{h}_i$ to denote the ith step/impulse response coefficient when $\tau=\tau^{\mathrm{ave}}$, and $\tilde{H}$ to replace H_k.

Meanwhile, the uncertain part of the system ΔC is also unknown. In order to get the step/impulse response coefficient, we let $\Delta C=0$ when calculating a_i and h_i. We adopt $\tilde{a}_i$ and $\tilde{h}_i$ to denote them.

Hence, we rewrite (2) and (4) as follows:

$$y(k+j)=\sum_{i=1}^{N-1}\tilde{a}_i\Delta u(k+j-i)+\tilde{a}_N u(k+j-N)+d(k),$$

$$\tilde{y}_P(t+j)=\sum_{i=j+1}^{N-1}\tilde{a}_i\Delta u(k+j-i)+\tilde{a}_N u(k+j-N).$$

And (6) should be

$$\tilde{H}=\begin{bmatrix}\tilde{h}_1 & \cdots & 0 & 0\\ \vdots & \ddots & \vdots & \vdots\\ \tilde{h}_{M-1} & \cdots & \tilde{h}_1 & 0\\ \tilde{h}_M & \cdots & \tilde{h}_2 & \tilde{a}_1\\ \vdots & \vdots & \vdots & \vdots\\ \tilde{h}_P & \cdots & \tilde{h}_{P-M+2} & \tilde{a}_{P-M+1}\end{bmatrix}_{P\times M}. \tag{12}$$

In the rest of this paper, $\tilde{a}_i$ and $\tilde{h}_i$ mean step/impulse response coefficient of the system with uncertain part $\Delta C=0$ and the time delay τ^{ave}.

Substituting (4) and (12) into (5), we have

$$\hat{e}=\begin{bmatrix}r(k)-\tilde{y}_P(k+1)-d(k)\\ \vdots\\ r(k)-\tilde{y}_P(k+P)-d(k)\end{bmatrix}
=\begin{bmatrix}r(k)-(\sum_{i=2}^{N-1}\tilde{a}_i\Delta u(k+1-i)\\ \quad+\tilde{a}_N u(k+1-N))-\sum_{i=1}^{N}(a_{i,k-i}-\tilde{a}_i)\Delta u_{k-i}\\ \vdots\\ r(k)-(\sum_{i=P+1}^{N-1}\tilde{a}_i\Delta u(k+P-i)\\ \quad+\tilde{a}_N u(k+P-N))-\sum_{i=1}^{N}(a_{i,k-i}-\tilde{a}_i)\Delta u_{k-i}\end{bmatrix}. \tag{13}$$

When substituting (13) into (7), the control system structure is obtained. The stability analysis is presented in section 3.2.

Remark 1: Using the time-invariant time delay instead of time-variant time delay τ to calculate a_i and h_i has two benefits. Firstly, it will reduce the computational burden greatly since there is no need to update H at each sample time k. Secondly, it solves the problem that the controller need to know the future time delay in order to calculate the amount of control. But using time-invariant time delay will lead to extra error which will affect stabilization. Our work is to find the maximum time delay in which the system is asymptotically stable.

Remark 2: The uncertain part ΔC will affect the step/impulse responses of the system. But it is not able to eliminate its effect and get the exact coefficient because ΔC is unknown. To deal with this problem, we let $\Delta C=0$ to compute a_i and h_i. The difference between the real step/impulse response coefficient and the one we used can be considered as model mismatch. Hence, this becomes a robust stability problem associated with time delay.

3.2. Stability analysis

Lemma 2: Let a_k^{τ} denotes the step response coefficient of a system S with time delay τ at time interval k, if the following three conditions are satisfied

1) $[a_k]$ is monotone increasing and asymptotic stable;
2) The time delay τ is bounded by $\tau^{\min}\le\tau\le\tau^{\max}$;
3) Two functions $f(x):R\to R$ and $g(x):R\to R$ are given, with the constraint that $F_{\min}\le f(x)\le F_{\max}$ and $G_{\min}\le g(x)\le G_{\max}$ for all x.

Then the following two conclusions exist

(a) $a_k^{\tau_1}\le a_k^{\tau_2}$,

where τ_1 and τ_2 satisfy $\tau_1\ge\tau_2$.

(b) $F^* + G^* = \underset{(f(\cdot),g(\cdot))}{Max}\left(\left|f(\cdot)+g(\cdot)\right|\right)$,

where $(F^*, G^*) \in \{(F_{\min}, G_{\min}), (F_{\max}, G_{\max})\}$.

Proof: (a) As proved in Lemma 1, we have $a_k^{\tau_1} = a^0_{k-\frac{\tau_1}{h}}$ and $a_k^{\tau_2} = a^0_{k-\frac{\tau_2}{h}}$. Since $\tau_1 \ge \tau_2$, we can obtain that $k-\frac{\tau_1}{h} \le k-\frac{\tau_2}{h}$. Considering the condition 1), we can conclude that $a^0_{k-\frac{\tau_1}{h}} \le a^0_{k-\frac{\tau_2}{h}}$. Hence we have $a_k^{\tau_1} \le a_k^{\tau_2}$.

(b) Assume that there exists (F_0, G_0) which is neither equal to $(F_{\min}, G_{\min})$ nor $(F_{\max}, G_{\max})$ and satisfies

$$F_0 + G_0 = \underset{(f(\cdot),g(\cdot))}{Max}\left(\left|f(\cdot)+g(\cdot)\right|\right). \tag{14}$$

Thus we have the following two inequalities:

$$\left|F_0 + G_0\right| > \left|F_{\min} + G_{\min}\right|, \tag{15}$$

$$\left|F_0 + G_0\right| > \left|F_{\max} + G_{\max}\right|. \tag{16}$$

If $F_0 + G_0 \ge 0$, we have

$F_{\max} + G_{\max} > F_0 + G_0 \ge 0.$

Equation (16) can be rewritten as

$$F_0 + G_0 > F_{\max} + G_{\max}. \tag{17}$$

As (17) is contradicted with the conditions, $F_0 + G_0$ should be less than zero. Hence we have $F_{\min} + G_{\min} < F_0 + G_0 < 0$, and (15) can be rewritten as

$$F_0 + G_0 < F_{\min} + G_{\min}. \tag{18}$$

As (18) is also contradicted with the conditions, (F_0, G_0) satisfies (14) does not exist. Hence, we can obtain Lemma 2(b).

The proof is completed.

Lemma 3 [25] (Jury's dominant coefficient lemma)**:** Consider a characteristic equation $A(z^{-1}) = \sum_{i=0}^{\infty} a_i z^{-j}$. If condition $\sum_{i=0}^{\infty}|a_i| < 1$ is satisfied, then the discrete system described by the characteristic equation is stable.

Theorem 1: The system (9) with $\Delta C = 0$ is stable if the following conditions are satisfied

$$\Psi = \sum_{i=1}^{N+1}\left|\Lambda_i \Upsilon_i^{\tau_1,\tau_2}\right| < 1 \tag{19}$$

for $\forall \Upsilon_i^{\tau_1,\tau_2} \in \{\Upsilon_i^{\tau^{\min},\tau^{\max}}, \Upsilon_i^{\tau^{\max},\tau^{\min}}\}$, $i = 1, 2, \cdots, N+1$, where

$$\Lambda_i = \begin{bmatrix}\sigma_{1,i} & \sigma_{2,i} & k_r\sigma_{3,i} & -k_r\sigma_{4,i} & -\sigma_{5,i} & \sigma_{6,i} & -\sigma_{7,i}\end{bmatrix},$$

$$\Upsilon_i^{\tau_1,\tau_2} = \begin{bmatrix}\sum_{j=1}^{P} c_j\hat{a}_{i-1+j} & -\sum_{j=1}^{P} c_j\hat{a}_{i+j} & \partial_{i-1}^{\tau_1} & \partial_i^{\tau_2} & a_N c_{N-i} & k_u & a_N c_1\end{bmatrix}^T,$$

$$k_r = \sum_{i=1}^{P} c_i, \quad \partial_j^{\tau} = a_j^{\tau} - \tilde{a}_j,$$

$$\sigma_{1,i} = \begin{cases}1, & i \in [2, N-1]\\ 0, & else,\end{cases} \quad \sigma_{2,i} = \begin{cases}1, & i \in [1, N-2]\\ 0, & else,\end{cases}$$

$$\sigma_{3,i} = \begin{cases}1, & i \in [2, N+1]\\ 0, & else,\end{cases} \quad \sigma_{4,i} = \begin{cases}1, & i \in [1, N]\\ 0, & else,\end{cases}$$

$$\sigma_{5,i} = \begin{cases}1, & i \in [N-P, N-2]\\ 0, & else,\end{cases} \quad \sigma_{6,i} = \begin{cases}1, & i = 1\\ 0, & else,\end{cases}$$

$$\sigma_{7,i} = \begin{cases}1, & i = N-1\\ 0, & else,\end{cases} \quad \hat{a}_i = \begin{cases}\tilde{a}_i, & i \in [2, N-1]\\ 0, & \text{else.}\end{cases}$$

Proof: Substituting (13) into (7), we have

$$\begin{aligned} u(k) &= k_e\hat{e} + k_u u(k-1) \\ &= [c_1, \cdots, c_P] \begin{bmatrix} r_k - \left(\sum_{i=2}^{N-1}\tilde{a}_i\Delta u_{k+1-i} + a_N u_{k+1-N}\right) \\ -\sum_{i=1}^{N}\left(a_i^{\tau} - \tilde{a}_i\right)\Delta u_{k-i} \\ \vdots \\ r_k - \left(\sum_{i=P+1}^{N-1}\tilde{a}_i\Delta u_{k+P-i} + a_N u_{k+P-N}\right) \\ -\sum_{i=1}^{N}\left(a_i^{\tau} - \tilde{a}_i\right)\Delta u_{k-i} \end{bmatrix} + k_u u_{k-1}. \end{aligned} \tag{20}$$

Follows from (20), we have

$$\begin{aligned} u(k) &= \sum_{i=1}^{P} c_i r_k - [c_1, \cdots, c_P] \begin{bmatrix} \sum_{i=2}^{N-1}\tilde{a}_i\Delta u_{k+1-i} + a_N u_{k+1-N} \\ +\sum_{i=1}^{N}\left(a_i^{\tau} - \tilde{a}_i\right)\Delta u_{k-i} \\ \vdots \\ \sum_{i=P+1}^{N-1}\tilde{a}_i\Delta u_{k+P-i} + a_N u_{k+P-N} \\ +\sum_{i=1}^{N}\left(a_i^{\tau} - \tilde{a}_i\right)\Delta u_{k-i} \end{bmatrix} + k_u u_{k-1} \\ &= \sum_{i=1}^{P} c_i r_k - \sum_{i=1}^{N-2}\sum_{j=1}^{P} c_j\hat{a}_{i+j}\Delta u_{k-i} - \sum_{i=1}^{P} a_N c_i u_{k+i-N} \\ &\quad - \sum_{i=1}^{P} c_i \sum_{j=1}^{N}\left(a_j^{\tau} - \tilde{a}_j\right)\Delta u_{k-j} + k_u u_{k-1} \\ &= W - \sum_{i=1}^{N-2}\sum_{j=1}^{P} c_j\hat{a}_{i+j}\Delta u_{k-i} - \sum_{i=1}^{P} a_N c_i u_{k+i-N} \\ &\quad - k_r\sum_{j=1}^{N}\left(a_j^{\tau} - \tilde{a}_j\right)\Delta u_{k-j} + k_u u_{k-1}, \end{aligned} \tag{21}$$

where

$$W=\sum_{i=1}^{P}c_i r_k.$$

Noticing that $\Delta u_{k-i}=u_{k-i}-u_{k-i-1}$, (21) can be rewritten as follows

$$\begin{aligned}u(k)=&W-\sum_{i=1}^{N-2}\sum_{j=1}^{P}c_i\hat{a}_{i+j}\left(u_{k-i}-u_{k-i-1}\right)\\&-\sum_{i=1}^{P}a_N c_i u_{k+i-N}+k_u u_{k-1}\\&-k_r\sum_{j=1}^{N}\left(a_j^\tau-\tilde{a}_j\right)\left(u_{k-j}-u_{k-j-1}\right).\end{aligned}\tag{22}$$

Reforming (22), we have

$$\begin{aligned}u(k)=&W-\sum_{i=1}^{N-2}\sum_{j=1}^{P}c_j\hat{a}_{i+j}\left(u_{k-i}-u_{k-i-1}\right)-\sum_{i=1}^{P}a_N c_i u_{k+i-N}\\&-k_r\sum_{i=1}^{N}\partial_i^\tau\left(u_{k-i}-u_{k-i-1}\right)+k_u u_{k-1}\\=&W+\left(-\sum_{i=1}^{P}c_i\hat{a}_{1+i}-k_r\partial_1^\tau+k_u\right)u_{k-1}\\&+\sum_{i=2}^{N-P-1}\sum_{j=1}^{P}c_j\left(\hat{a}_{i-1+j}-\hat{a}_{i+j}\right)+k_r\left(\partial_{i-1}^\tau-\partial_i^\tau\right)u_{k-i}\\&+\sum_{i=N-P}^{N-2}\sum_{j=1}^{P}(c_j\left(\hat{a}_{i-1+j}-\hat{a}_{i+j}\right)+k_r\left(\partial_{i-1}^\tau-\partial_i^\tau\right)\\&-a_N c_{N-i})u_{k-i}\\&+\left(\sum_{j=1}^{P}c_j\hat{a}_{N-2+j}-a_N k_{e1}+k_r\left(\partial_{N-2}^\tau-\partial_{N-1}^\tau\right)\right)\\&\quad u_{k-N+1}+k_r\left(\partial_{N-1}^\tau-\partial_N^\tau\right)u_{k-N}+k_r\partial_N^\tau u_{k-N-1}\\=&W+\sum_{i=1}^{N+1}\left(\sum_{j=1}^{P}c_j\left(\sigma_{1,i}\hat{a}_{i-1+j}-\sigma_{2,i}\hat{a}_{i+j}\right)\right.\\&+k_r\left(\sigma_{3,i}\partial_{i-1}^{\tau_1}-\sigma_{4,i}\partial_i^{\tau_2}\right)-\sigma_{5,i}a_N c_{N-i}\\&\left.+\sigma_{6,i}k_u-\sigma_{7,i}a_N c_1\right)u_{k-i}\\=&W+\sum_{i=1}^{N+1}\left(\Lambda_i\Upsilon_i^{\tau_1,\tau_2}\right)u_{k-i}.\end{aligned}\tag{23}$$

Noticing that (23) is a characteristic equation, by applying Lemma 3, the discrete system is stable if the sum of the absolute value of all coefficients is less than 1. Thus, to discuss the stability of (23) we should investigate

$$\Psi=\sum_{i=1}^{N+1}\left|\Lambda_i\Upsilon_i^{\tau_1,\tau_2}\right|=\sum_{i=1}^{N+1}\Psi_i.\tag{24}$$

To find the maximum of Ψ, we need to investigate c_i, $\hat{a}_i$, k_r, k_u, $\sigma_{k,i}$ and ∂_i^τ. Since c_i, $\hat{a}_i$, k_r and k_u are determined by system matrix and controller matrix, meanwhile, $\sigma_{k,i}\in\{0,1\}$ are switching functions and are determined by i only. Hence, Ψ_i is affected only by ∂_i^τ. Thus, we should investigate $\sigma_{3,i}\partial_{i-1}^{\tau_1}-\sigma_{4,i}\partial_i^{\tau_2}$ to find the maximum of Ψ_i. For simplicity, we use $f(\tau_1)$ to denote $\sigma_{3,i}\partial_{i-1}^{\tau_1}$ and $g(\tau_2)$ to denote $-\sigma_{4,i}\partial_i^{\tau_2}$. $f_{\min}$, $g_{\min}$, $f_{\max}$, $g_{\max}$ stands for the minimum of $f(\tau_1)$ and $g(\tau_2)$, the maximum of $f(\tau_1)$ and $g(\tau_2)$, respectively.

By applying Lemma 2(b), we have

$$\left(f^*,g^*\right)=\arg\underset{(f(\tau_1),g(\tau_2))}{Max}\left(\left|f\left(\tau_1\right)+g\left(\tau_2\right)\right|\right),$$

where $(f^*,g^*)\in\{(f_{\min},g_{\min}),(f_{\max},g_{\max})\}$

As $\partial_i^\tau=a_i^\tau-\tilde{a}_i$, by applying Lemma 2(a), we have

$$\tau^{\min}=\arg\underset{\tau}{Max}\left(a_i^\tau-\tilde{a}_i\right),$$

$$\tau^{\max}=\arg\underset{\tau}{Min}\left(a_i^\tau-\tilde{a}_i\right).$$

As $\sigma_{k,i}$ are non-negative, we have

$$\left(\tau_1^*,\tau_2^*\right)=\arg\underset{(\tau_1,\tau_2)}{Max}\left(\left|f\left(\tau_1\right)+g\left(\tau_2\right)\right|\right),$$

where $(\tau_1^*,\tau_2^*)\in\{(\tau^{\min},\tau^{\max}),(\tau^{\max},\tau^{\min})\}$.

Noticing that $\underset{(\tau_1,\tau_2)}{Max}\left(\left|f(\tau_1)+g(\tau_2)\right|\right)=\underset{(\tau_1,\tau_2)}{Max}(\Psi_i)$, we have

$$(\Upsilon_i^{\tau_1^*,\tau_2^*})=\arg\underset{\Upsilon}{Max}(\Psi_i),$$

where $\Upsilon_i^{\tau_1^*,\tau_2^*}\in\{\Upsilon_i^{\tau^{\min},\tau^{\max}},\Upsilon_i^{\tau^{\max},\tau^{\min}}\}$.

Since $\underset{\Upsilon}{Max}(\Psi)=\sum_{i=1}^{N+1}\underset{\Upsilon}{Max}(\Psi_i)$, we have (19).

The proof is completed.

In order to give the robust stability conditions, we firstly give the following definition.

Definition 2: For a given bounded uncertain part ΔC, we use the family of plants π to denote all the possibly plants satisfy (9) and the maximum and minimum step response coefficient are denoted as $\overline{a}_i$ and $\underline{a}_i$. In other words, for any plant belongs to π with step response coefficient a_i, the following result exists:

$$\underline{a}_i\le a_i\le\overline{a}_i.\tag{25}$$

Theorem 2: The system (9) with $\Delta C\neq 0$ is stable if the following conditions are satisfied

$$\Psi=\sum_{i=1}^{N+1}\left|\Lambda_i\Upsilon_i^{\tau_1,\tau_2}\right|<1\tag{26}$$

for $\forall\Upsilon_i^{\tau_1,\tau_2}\in\{\overline{\Upsilon}_i^{\tau^{\min},\tau^{\max}},\underline{\Upsilon}_i^{\tau^{\max},\tau^{\min}}\}$, $i=1,2,\cdots,N+1$,

where

$$\overline{\Upsilon}_{i,j}^{\tau_1,\tau_2}=\begin{bmatrix}\sum_{j=1}^{P}c_j\hat{a}_{i-1+j} & -\sum_{j=1}^{P}c_j\hat{a}_{i+j} & \overline{\partial}_{i-1}^{\tau_1}\\ \underline{\partial}_i^{\tau_2} & a_N c_{N-i} & k_u & a_N c_1\end{bmatrix}^T,$$

$$\Upsilon_{i,j}^{\tau_1,\tau_2} = \left[\sum_{j=1}^{P} c_j \hat{a}_{i-1+j} \quad -\sum_{j=1}^{P} c_j \hat{a}_{i+j} \quad \underline{\partial}_{i-1}^{\tau_1} \right.$$
$$\left. \overline{\partial}_i^{\tau_2} \quad a_N c_{N-i} \quad k_u \quad a_N c_1 \right]^T,$$

$$\overline{\partial}_j^{\tau} = \overline{a}_j^{\tau} - \tilde{a}_j, \quad \underline{\partial}_j^{\tau} = \underline{a}_j^{\tau} - \tilde{a}_j,$$

and other parameters are the same within Theorem 1.

Proof: Similar with the proof of Theorem 1, we should investigate (24). The difference lies on the factor of ∂_i^τ. In Theorem 1, ∂_i^τ is determined by time delay only; while in Theorem 2, ∂_i^τ is affected by both time delay and uncertain part ΔC. As $\partial_i^\tau = a_i^\tau - \tilde{a}_i$ and due to (25), we have

$$\underline{a}_i - \tilde{a}_i \le a_i - \tilde{a}_i \le \overline{a}_i - \tilde{a}_i. \quad (27)$$

Due to Lemma 2(a), we have

$$\underline{a}_i^{\tau^{\max}} \le \underline{a}_i^{\tau} \le \overline{a}_i^{\tau} \le \overline{a}_i^{\tau^{\min}}. \quad (28)$$

Substituting (28) into (27) we have

$$\underline{a}_i^{\tau^{\max}} - \tilde{a}_i \le a_i^{\tau} - \tilde{a}_i \le \overline{a}_i^{\tau^{\min}} - \tilde{a}_i,$$

which equals to

$$\underline{\partial}_i^{\tau^{\max}} \le \partial_i^{\tau} \le \overline{\partial}_i^{\tau^{\min}}.$$

Similar with the proof of Theorem 1, by applying Lemma 2 and Lemma 3 we could obtain Theorem 2, which ends the proof of Theorem 2.

Remark 3: The effects of time delay and model mismatch on system stability are essentially the same. The exact step/impulse response coefficient is unable to be obtained due to the existence of τ and ΔC. Theorem 1 only considers the effect of τ, while in Theorem 2, we present the relationship among stability, time delay and model mismatch by Definition 2. Since the two factors are coupled with each other in Theorem 2, we can only get the conclusion that a less ΔC will lead to a larger maximum time delay.

4. NUMERICAL EXAMPLES

4.1. Example 1 ($\Delta C = 0$)

We firstly consider the following SISO system with $\Delta C = 0$

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.4522 & -0.3283 \\ 0.25 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(k) \\ y(k+1) = [0.1103 \quad 0.1886] x(k). \end{cases} \quad (29)$$

The sampling period h is 0.5 second. The parameters of DMC are set as followed: $N = 80$, $P = 12$, $m = 5$, $Q = 2I_P$, $R = 0.2I_m$. All the simulation programs were implemented in the MATLAB R2011a with TRUETIME 1.5. The simulation structure is shown by Fig. 2.

There are five nodes in the structure. Node 4 is a sensor. It is time-driven and will send the measurement value y to node 1. Node 1 is an interference block. It is used to generate and simulate uncertain time delay τ_{sc}. After that, node 1 will send y to node 3, the controller. When the input u is generated, it will be sent to node 5. Node 5 is another interference block, in which τ_{ca} is generated and simulated. Finally, u will be sent to Node 2, the actuator.

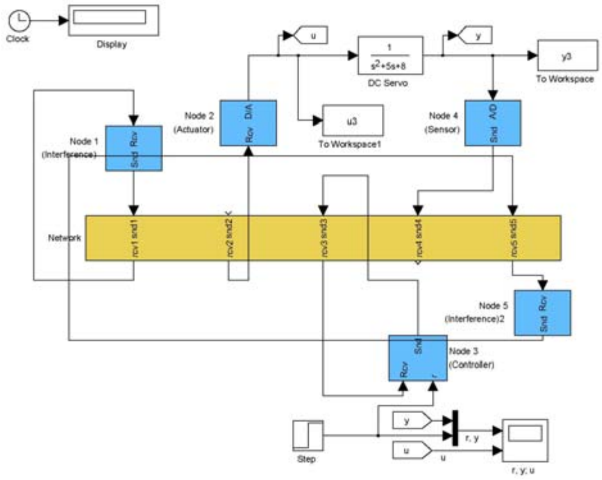

Fig. 2. Simulation structure based on TrueTime 1.5 and Matlab R2011a.

Assuming the desired output $r(k)$ is a step function and $0 \le \tau_{ca} \le 5$, $\tau_{sc} = 0$. Applying Theorem 1 we can easily get $\Psi = 0.7245 < 1$. Thus the closed-loop system with uncertain time delay which is less than 5 seconds is asymptotically stable. Meanwhile, by using Theorem 1 in [21], we can also conclude the closed-loop system can be asymptotically stable if the maximum delay method is used. The comparison between uncertain delay method in this paper and the maximum delay method in [21] is drawn in Figs. 3 to 5.

From the above three figures, we can conclude that our method can achieve a better control performance than the maximum delay method which was presented in [21].

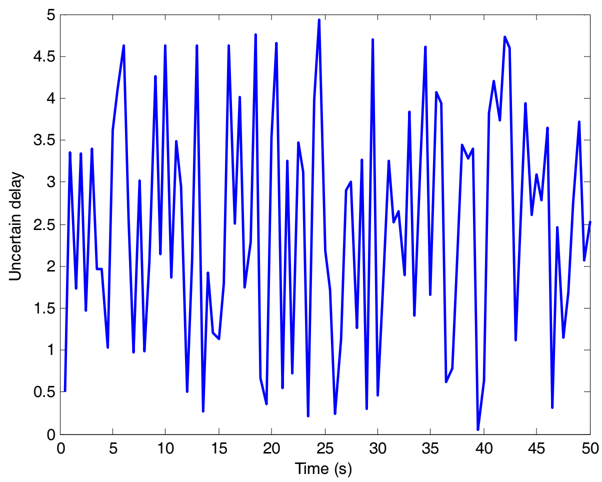

Fig. 3. Uncertain delays.

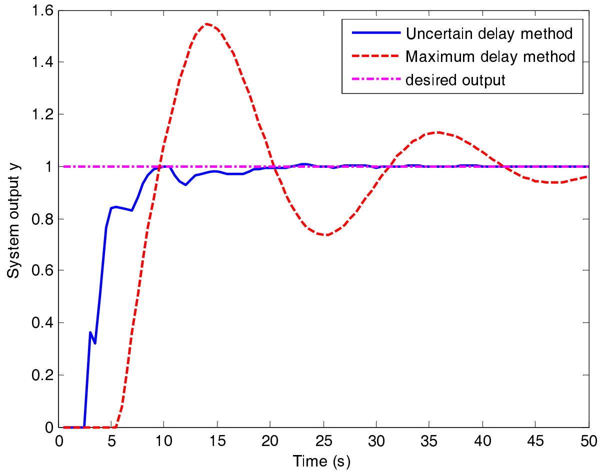

Fig. 4. The output trajectories.

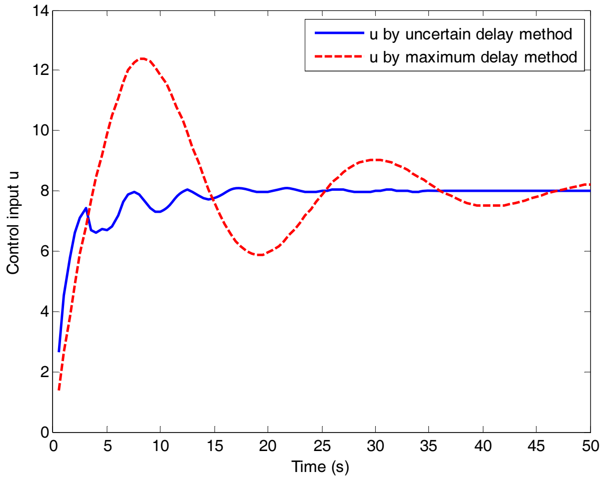

Fig. 5. The control input trajectories.

4.2. Example 2 ($\Delta C \neq 0$)

Here we consider a third-order SISO plant-family with $\Delta C \neq 0$

$$\begin{cases} x(k+1) = \begin{bmatrix} 2.1594 & -0.7735 & 0.3679 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} x(k) \\ \qquad\qquad + \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} u(k) \\ y(k) = (1 \pm \kappa)[0.3297 \quad -0.2455 \quad 0.1809] x(k). \end{cases} \tag{30}$$

The sampling period h is 0.2 second. The parameters of DMC are set as followed: $N = 80$, $P = 12$, $m = 5$, $Q = 2I_P$, $R = 0.2I_m$. The different upper bound of time delay with different uncertainty ε are given in Table 1.

We assume that uncertainty $\kappa = 0.1$ and the maximum delay is 2.2 s. Figs. 6 and 7 show the output trajectories of the maximum delay method and our method, respectively. It is found that under the maximum delay method, the closed-loop system cannot be asymptotically stable. However, by applying the approach proposed in this paper, the closed-loop system is asymptotically stable, which is illustrated in Fig. 7.

Table 1. Upper bound of time delay.

Method	$\kappa = 0.05$	$\kappa = 0.1$	$\kappa = 0.3$
Uncertain delay	$\tau = 2.8$	$\tau = 2.6$	$\tau = 2.2$
Maximum delay [21]	$\tau = 1.8$	$\tau = 1.8$	$\tau = 1.6$

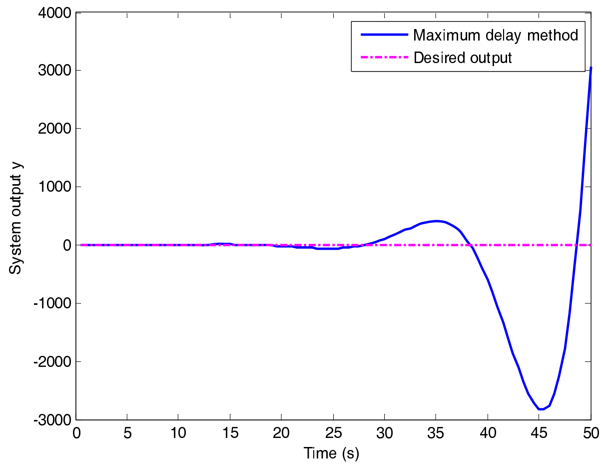

Fig. 6. The output trajectories with $\tau = 2.2$ s and $\kappa = 0.1$.

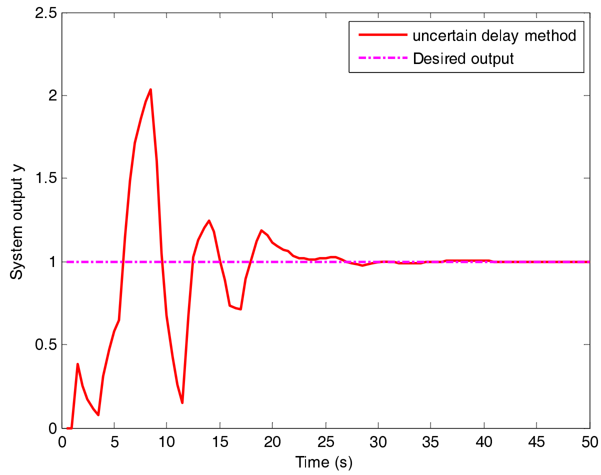

Fig. 7. The output trajectories with $\tau = 2.2$ s and $\kappa = 0.1$.

5. CONCLUSION

This study mainly discusses the stability problems of NCS with DMC controller. Introducing networks into control systems will bring lots of new problems such as time delay. Design methods and stability analysis in a NCS are very challenging issues.

The main contribution of this study is to obtain some simple stability criteria for a DMC controller to control networked processes with time delay. Uncertain time delay is considered, which is different from previous methods in which the time-varying delay is converted to constant maximum delay by using a receiving buffer. Receiving buffer will help to simplify the analysis but obviously increase the conservatism. Robust stability conditions are further presented.

Our future work might be the extension of the proposed theory in the multiple-in-multiple-out (MIMO) cases. Meanwhile, the networked MPC problem for nonlinear processes needs to be further investigated.

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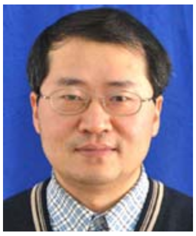

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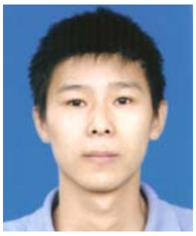

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