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An Extended PID Type Iterative Learning Control

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Abstract: This paper presents a new iterative learning control (ILC) for discrete-time single-input single-output (SISO) linear time-invariant (LTI) systems. To establish this ILC, the input of the controlled system is modified by using a novel four-parametric algorithm. This algorithm is called the extended proportional plus integral and derivative (EPID) type, since by eliminating the fourth parameter of it one would get to the PID type ILC, therefore PID type ILC is a special case of it. The convergence of the proposed ILC is analyzed and an optimal method is presented to determine its parameters. It is shown that the given ILC has a better performance than the PID-type one. Three illustrative examples are included to demonstrate the effectiveness and the preference of the presented ILC.

Keywords: Extended PID, iterative learning control, monotonic convergence, optimal design.

1. INTRODUCTION

In automation industry we are faced with many systems that perform a certain task over a finite time duration. A sensible example of such systems is a robot manipulator that is required to repeat a same task with a high precision over a limited and constant time interval [1]. ILC [1-4] is a successful and effective method to control such systems. The main philosophy of the ILC is to measure and to records the information at the present iteration in order to use them to improve the system input at the next iteration. This is done by a mechanism which is called the learning algorithm. If the learning process to be convergence, after a number of repeated trials, the system should achieve a suitable input so that this input generates the desired output.

The general field of ILC has been shown a high interest by the scholars. As starting point one can see the relevant literature in the survey paper [5]. Various techniques in ILC design are presented such as model-based method [6], two-dimensional systems theory based technique [7], linear matrix inequalities (LMIs) and robust approach [8-9], adaptive methods [10-12] and semi-sliding window algorithm [13]. Also the issues of the stability, convergence and monotonic convergence of the various ILC algorithms have been discussed and explored [14-16].

One of the popular and effective procedures in ILC category is the usage of the optimization theory. Many researchers have been attempting to employ the optimization techniques in ILC. An ILC algorithm has been presented based on optimization techniques [17], where full convergence analysis of the algorithm with a causal representation of the algorithm is illustrated. A systematic solution for linear-type ILC is obtained in [18] by formulating the ILC design as a min–max optimization problem. The effectiveness of the optimization methods in order to obtain a good and effective ILC design is presented in [19]. A useful technique in order to increase the convergence rate of the norm optimal ILC is presented in [20]. In [21] the problem of optimal ILC of general nonlinear discrete-time plants is studied. A norm-optimal ILC is presented in [22] when tracking is only required at a subset of isolated time points along the trial duration. The possibility of applying norm-optimal ILC to a system is studied [23], when there is not any priori information about system besides the fact that the system is LTI. A multi-parameter optimal ILC algorithm is presented in [24], which uses an approximate polynomial representation of the plant inverse.

The PID (proportional plus integral and derivative) controller is highly used and is a very popular scheme in the process control industries [25]. This is mainly because of its effectiveness, simple structure, and its robustness. Therefore, many researchers are tempted to take the advantage of the PID strategy in designing the iterative learning control, which some of them can be found in [26-33]. We have already presented a PID-type ILC where its coefficients are determined in an optimal manner [34]. The aim of this paper is to present a new ILC, which is a meaningful extension of [34]. As it is explained in [34], it is imperative and interesting to notice that the P-component stabilizes the ILC system and causing monotonic convergence, and the I-term rejects the effect of non-zero initial errors and increases the convergence rate, while the D-component can reduce the effect of the inputs disturbance. Now one can easily see why the PID controller is a highly advantageous technique in the designing of ILCs.

Hence, making any modification to the PID-type ILC is a significant task. This subject is the main motivation of this paper. So far in all the presented PID-type ILCs, including [34], in order to determining the modifier term

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of the system input at the instant i th from the iteration $j+1$ th, the instant $i+1$ th from the iteration j th is considered to be the present instant. Whereas another idea is to consider the instant i th from the iteration j th as the present instant. In this paper by combining this idea and the technique in [34], a four-parametric ILC is obtained that the PID-type ILC is a special case of it. It is shown that the convergence rate of this four-parametric ILC, which is introduced as an extended PID-type ILC, is higher than the PID-type one.

The paper is organized as follows. Section 2 gives the necessary preliminaries and defines the problem. The proposed ILC is presented in Section 3. Section 4 discusses the convergence and using an optimal approach so that the controller coefficients are obtained as closed-form and explicit formula in terms of the system parameters. The performance of the presented ILC is compared by the PID-type one in Section 5. Three illustrative simulation examples are given in Section 6. Section 7 gives the conclusion.

2. NOTATIONS AND PROBLEM STATEMENT

Consider the following standard state-space equation to represent the underling discrete-time, linear, time-invariant, single-input, single-output system:

$$\begin{cases} x_j(i+1) = Ax_j(i) + Bu_j(i) + w_x(i), \\ y_j(i) = Cx_j(i) + w_y(i), \\ x_j(0) = x_0, \\ i = 0,1,\dots,M, \quad j = 0,1,\dots, \end{cases} \tag{1}$$

where $x_j \in \mathbb{R}^n$, $u_j \in \mathbb{R}$ and $y_j \in \mathbb{R}$ are the state vector, the input and the output of the system, respectively. $w_x \in \mathbb{R}^n$ and $w_y \in \mathbb{R}$ are the unknown time-varying disturbances or effects of the un-modeled dynamics of the system. A, B and C are real matrices with appropriate dimensions. x_0 is the system initial condition which is unknown. It is assumed that $CB \neq 0$, that is the relative-degree of the system is one (trivially satisfied in practice).

This system is assumed to be operating in a repetitive mode in finite discrete time interval $i \in [0, M]$, where the subscript "j" denotes the iteration or the trial number. It is assumed that the time duration of the iterations is not less than n and 4, that is $M \geq \max(n, 4)$.

The problem of ILC for system (1) is defined as follows [13,14,34]:

A reference signal $y_d(i)$ is given; present an appropriate algorithm to modify the input of the system, so that by increasing the number of operation the error between the resultant output $y_j(i)$ and reference signal $y_d(i)$ becomes as small as possible, so that the following tracking is met:

$$\lim_{j \to \infty}(y_d(i) - y_j(i)) = 0 \quad \text{for } i = 1,2,\dots,M, \tag{2}$$

that is $y(i)$ follows the $y_d(i)$ exactly on $i \in [1, M]$.

A new solution method for the ILC problem, namely EPID type ILC is to be presented in the next section. For the time being, we extend a compact formulation, which is named the "super-vectors" formulation [13,14], for model (1).

Let super-vectors $Y(j)$, $U(j)$, Y_d, W_x, W_y and $E(j)$ are defined as follows:

$$Y(j) = \begin{bmatrix} y_j(1) \\ y_j(2) \\ y_j(3) \\ \vdots \\ y_j(M) \end{bmatrix}, \; U(j) = \begin{bmatrix} u_j(0) \\ u_j(1) \\ u_j(2) \\ \vdots \\ u_j(M-1) \end{bmatrix}, \; Y_d = \begin{bmatrix} y_d(1) \\ y_d(2) \\ y_d(3) \\ \vdots \\ y_d(M) \end{bmatrix},$$
$$W_x = \begin{bmatrix} w_x(0) \\ w_x(1) \\ w_x(2) \\ \vdots \\ w_x(M-1) \end{bmatrix}, \; W_y = \begin{bmatrix} w_y(1) \\ w_y(2) \\ w_y(3) \\ \vdots \\ w_y(M) \end{bmatrix}, \; E(j) = \begin{bmatrix} e_j(1) \\ e_j(2) \\ e_j(3) \\ \vdots \\ e_j(M) \end{bmatrix}, \tag{3}$$

where

$$e_j(i) = y_d(i) - y_j(i) \quad i = 1,2,\dots,M. \tag{4}$$

Using (1) and after some manipulation one can obtain:

$$Y(j) = G_0U(j) + G_wW_x + G_xx_0 + W_y, \tag{5}$$

where G_w, G_x and G_0 are the following matrices:

$$G_w = \begin{bmatrix} C & 0 & 0 & \cdots & 0 \\ CA & C & 0 & & 0 \\ CA^2 & CA & C & & 0 \\ \vdots & & & \ddots & \vdots \\ CA^{M-1} & CA^{M-2} & \cdots & CA & C \end{bmatrix}, \; G_x = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^M \end{bmatrix},$$
$$G_o = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 & 0 \\ g_2 & g_1 & 0 & & 0 & 0 \\ g_3 & g_2 & g_1 & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ g_{M-1} & g_{M-2} & g_{M-3} & & g_1 & 0 \\ g_M & g_{M-1} & g_{M-2} & \cdots & g_2 & g_1 \end{bmatrix}, \tag{6}$$

where g_k is the standard Markov parameters of the system (1), which is defined as follows:

$$g_k = CA^{k-1}B \quad \text{for } k = 1,2,\dots,M. \tag{7}$$

It is seen that G_o is a low triangular Toeplitz matrix, which is formed by the following vector:

$$g = [g_1 \quad g_2 \quad g_3 \quad \cdots \quad g_M]^T,$$

where T denotes the transpose.

Considering (5) for the two consecutive iterations j

and $j+1$ and by subtracting them from each other one gets:

$$Y(j+1)-Y(j)=G_0U(j+1)+G_wW_x+G_xx_0+W_y \\ -G_0U(j)-G_wW_x-G_xx_0-W_y,$$

or

$$Y(j+1)=Y(j)+G_0V(j) \qquad j=0,1,..., \tag{8}$$

where

$$V(j)=U(j+1)-U(j). \tag{9}$$

Therefore, by extension the "super-vectors" formulation for model (1), the uncertain quantities $\{x_0, w_x(i), w_y(i)\}$ are eliminated from model (1), and this model is stated in the form of the relation (8) that is a dynamic equation in the repetition domain j.

From (8) one gets:

$$Y_d-Y(j+1)=Y_d-Y(j)-G_0V(j).$$

The definitions of $E(j)$, $Y(j)$ and Y_d, gives $E(j)=Y_d-Y(j)$, Thus one can rewrite the above relation such that:

$$E(j+1)=E(j)-G_0V(j) \quad j=0,1,... \tag{10}$$

As a summary of the super-vectors formulation, one can state that when there is no ILC on the system (1) the dynamics of the error super-vector $E(j)$ is governed by Eq. (10), hence this equation can be interpreted as the open-loop system dynamics in the iteration domain.

3. PROPOSED ITERATIVE LEARNING CONTROL LAW

Generally the following law is considered to update the input of system (1) [34]:

$$u_{j+1}(i)=u_j(i)+\Delta u_{j+1}(i), \\ i=0,1,...,M-1, \quad j=0,1,..., \tag{11}$$

where $\Delta u_{j+1}(i)$ is a modifier term.

The technique used, in determining $\Delta u_{j+1}(i)$ states that how to tackle the ILC problem. In the PID type ILC, $\Delta u_{j+1}(i)$ is determined as follows [26,32,34]:

$$\Delta u_{j+1}(i)=k_Pe_j(i+1)+k_I\sum_{m=1}^{i+1}e_j(m) \\ +k_D\left(e_j(i+1)-e_j(i)\right), \tag{12}$$

where $e_j(i)$, for $1\le i\le M$, is given in (4), $e_j(0)\triangleq 0$, and k_P, k_I and k_D are real constant gains (coefficients), which are called proportional, integration and derivative learning gains respectively [26,32,34].

According to (12), to extend the PID control law from the classical (non-repetitive) domain to the ILC in order to determine $\Delta u_{j+1}(i)$, the instance $i+1$ from j iteration is considered to be the present instance. Whereas there is an alternative option, since the time argument of $\Delta u_{j+1}(i)$ is i, it is reasonable to consider instance i as the present instance. That is the ILC (12) can be as follows:

$$\Delta u_{j+1}(i)=k_Pe_j(i)+k_I\sum_{m=1}^{i}e_j(m)+k_D\left(e_j(i)-e_j(i-1)\right), \\ (e_j(0)\triangleq 0,\ e_j(-1)\triangleq 0). \tag{13}$$

Now, by linearly combining (12) and (13) one can obtain a more comprehensive ILC as follows:

$$\begin{aligned}\Delta u_{j+1}(i)=&\,k_{P1}e_j(i+1)+k_{I1}\sum_{m=1}^{i+1}e_j(m)\\&+k_{D1}\left(e_j(i+1)-e_j(i)\right)\\&+k_{P2}e_j(i)+k_{I2}\sum_{m=1}^{i}e_j(m)\\&+k_{D2}\left(e_j(i)-e_j(i-1)\right).\end{aligned} \tag{14}$$

So (12) and (13) are two especial cases of (14).

According to (14), to determine $\Delta u_{j+1}(i)$ the errors $\{e_j(1),e_j(2),...,e_j(i),e_j(i+1)\}$ are used and apparently there exist six parameters (coefficients). But it can be shown that these six parameters can be interchanged to four independent parameters. For this purpose one can easily write the relation (14) in the following compact form:

$$\begin{aligned}\Delta u_{j+1}(i)=&\,k_1e_j(i+1)+k_2e_j(i)\\&+k_3e_j(i-1)+k_4\sum_{m=1}^{i-2}e_j(m),\end{aligned} \tag{15}$$

where

$$\begin{bmatrix}k_1\\k_2\\k_3\\k_4\end{bmatrix}=\begin{bmatrix}1&1&1\\0&1&-1\\0&1&0\\0&1&0\end{bmatrix}\begin{bmatrix}k_{P1}\\k_{I1}\\k_{D1}\end{bmatrix}+\begin{bmatrix}0&0&0\\1&1&1\\0&1&-1\\0&1&0\end{bmatrix}\begin{bmatrix}k_{P2}\\k_{I2}\\k_{D2}\end{bmatrix}. \tag{16}$$

The relation (15) is a lot more clear and explanatory than (14). Because the error values in deferent instant are in the form of four independent separated terms. So that the term $\sum_{m=1}^{i-2}e_j(m)$ consists of error values in the instants $\{1,2,...,i-2\}$ and each of the terms $e_j(i-1)$, $e_j(i)$ and $e_j(i+1)$ consists of the error value at the instants $i-1$, i and $i+1$ respectively. Therefore it is clearly obvious that (15) is a four-parametric relation, where its parameters are $\{k_1, k_2, k_3, k_4\}$ Since, in the PID-type ILC there exist three parameters, then (15) can be considered as an extended PID-type ILC. We do in fact expect that in a four-parametric extended PID-type ILC by zeroing one of the parameters, one would obtain the same PID-type ILC. But by zeroing none of the parameters of (15) one could not get to the PID-type ILC, which is given in (12). For this, with a little manipulation, the relation (15) can be written in the following form:

$$\Delta u_{j+1}(i) = k_P e_j(i+1) + k_I \sum_{m=1}^{i+1} e_j(m) + k_D\left(e_j(i+1) - e_j(i)\right) + k_E e_j(i-1), \tag{17}$$

where

$$\begin{bmatrix} k_P \\ k_I \\ k_D \\ k_E \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}. \tag{18}$$

Thus, the four-parametric relation (15) is transformed into another four-parametric relation in which $\{k_P, k_I, k_D, k_E\}$ are the parameters. Where by zeroing k_E one can obtain the same PID-type ILC. Therefore (17) is an EPID-type ILC.

By employing the definitions of super-vectors $E(j)$ and $V(j)$, one can rewrite (17) in following compact form:

$$V(j) = \{(k_P + k_D)I + k_I F_1 - k_D F_2 + k_E F_3\} E(j), \tag{19}$$

where $I \in \mathbb{R}^{M\times M}$ is the identity matrix and F_1, F_2, and $F_3 \in \mathbb{R}^{M\times M}$ are defined as follows:

$$F_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & & & & \\ 0 & \cdots & 1 & 0 & 0 \end{bmatrix}. \tag{20}$$

Substituting $V(j)$ from (19) into (10) yields:

$$E(j+1) = \bar{G}_c E(j) \qquad j = 0,1,..., \tag{21}$$

where

$$\bar{G}_c = I - (k_P + k_D)G_o - k_I G_o F_1 + k_D G_o F_2 - k_E G_o F_3. \tag{22}$$

Now one can state that (21) represents the dynamics of the closed-loop system in the repetition domain when the presented EPID-type ILC is used to control system (1).

4. CONVERGENCE ANALYSIS AND OPTIMAL DESIGNE OF THE CONTROLLER PARAMETERS

The convergence concept of any given ILC is its ability to control system (1) so that for any initial input $u_0(i)$, that is for any $E(0)$, the tracking property (2) to be guaranteed, that is:

$$\lim_{j\to\infty} E(j) = 0. \tag{23}$$

A strong type of the convergence is the monotonic one, which means the better and better operation from trial to trial [34]. That is for any $E(0)$ not only the tracking property (2) holds but also we have:

$$\begin{cases} \|E(j+1)\|_\lambda < \|E(j)\|_\lambda & \text{if } E(j) \neq 0 \\ \|E(j+1)\|_\lambda = \|E(j)\|_\lambda & \text{if } E(j) = 0 \end{cases} \tag{24}$$

for $\lambda = 1, 2, \infty$ and $j = 0, 1, 2, ...,$

where $\| \ \|_\lambda$ denotes the λ-norm.

Theorem 1: The presented ILC is convergent if and only if the sum of the three coefficients k_P, k_I and k_D of (17) is chosen in the following interval:

$$|1 - g_1(k_P + k_I + k_D)| < 1. \tag{25}$$

Proof: The dynamic of the closed-loop system in the repetition domain in the PID-type ILC instead of (21) was as follows [34]:

$$E(j+1) = G_c E(j) \qquad j = 0,1,..., \tag{26}$$

where

$$G_c = I - (k_P + k_D)G_o - k_I G_o F_1 + k_D G_o F_2, \tag{27}$$

and G_o, F_1, F_2 are the same matrices which are given in (6) and (20), respectively.

It is also proven that G_c is a low triangular Toeplitz matrix which is formed by the following vector [34]:

$$g_c = [g_{c1} \quad g_{c2} \quad g_{c3} \quad \cdots \quad g_{cM}]^T, \tag{28}$$

that is

$$G_c = \begin{bmatrix} g_{c1} & 0 & 0 & \cdots & 0 & 0 \\ g_{c2} & g_{c1} & 0 & & 0 & 0 \\ g_{c3} & g_{c2} & g_{c1} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ g_{c(M-1)} & g_{c(M-2)} & g_{c(M-3)} & & g_{c1} & 0 \\ g_{cM} & g_{c(M-1)} & g_{c(M-2)} & \cdots & g_{c2} & g_{c1} \end{bmatrix}, \tag{29}$$

where [34]:

$$\begin{cases} g_{c1} = 1 - g_1(k_P + k_I + k_D), \\ g_{ci} = -g_i k_P - \left(\sum_{m=1}^{i} g_m\right) k_I - (g_i - g_{i-1})k_D, \\ \qquad \text{for } i = 2, 3, ..., M. \end{cases} \tag{30}$$

From (22) and (27) one gets:

$$\bar{G}_c = G_c - k_E G_0 F_3. \tag{31}$$

Substituting G_c, G_o and F_3 respectively from (29), (6) and (20) into (31) yields:

$$\bar{G}_c = \begin{bmatrix} \bar{g}_{c1} & 0 & 0 & \cdots & 0 & 0 \\ \bar{g}_{c2} & \bar{g}_{c1} & 0 & & 0 & 0 \\ \bar{g}_{c3} & \bar{g}_{c2} & \bar{g}_{c1} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ \bar{g}_{c(M-1)} & \bar{g}_{c(M-2)} & \bar{g}_{c(M-3)} & & \bar{g}_{c1} & 0 \\ \bar{g}_{cM} & \bar{g}_{c(M-1)} & \bar{g}_{c(M-2)} & \cdots & \bar{g}_{c2} & \bar{g}_{c1} \end{bmatrix}, \tag{32}$$

where

$$\bar{g}_{ci} = \begin{cases} g_{ci} & \text{for} \quad i = 1,2 \\ g_{ci} - k_E g_{i-2} & \text{for} \quad i = 3,\ldots,M. \end{cases} \tag{33}$$

Therefore $\bar{G}_c$ is a low triangular Toeplitz matrix which is formed by the following vector:

$$\bar{g}_c = [\bar{g}_{c1} \quad \bar{g}_{c2} \quad \bar{g}_{c3} \quad \cdots \quad \bar{g}_{cM}]^T. \tag{34}$$

From the low triangular structure of $\bar{G}_c$, the characteristic polynomial of $\bar{G}_c$ is obtained as

$$\Delta_{\bar{G}_c}(\lambda) = \det(\lambda I - \bar{G}_c) = (\lambda - \bar{g}_{c1})^M.$$

Hence all eigenvalues of $\bar{G}_c$ are equal to $\bar{g}_{c1}$.

By considering the homogeneous linear dynamical equation (21), one concludes that the presented EPID-type ILC is convergent if and only if we have

$$|\bar{g}_{c1}| < 1, \tag{35}$$

this is the same as (25).

Comment 1: According to (7) we have $g_1 = CB$, since CB is assumed to be nonzero, one can pick up numerous real numbers for k_P, k_I and k_D which they satisfy inequality (25).

The following theorem gives a sufficient condition for monotonic convergence.

Theorem 2: The presented ILC is monotonically convergent if:

$$\|\bar{g}_c\|_1 < 1. \tag{36}$$

Proof: It is similar to the proof of the presented lemma in [34].

Comment 2: Considering the details of the proof of the Theorem 2, which is not brought here for its similarity to the proof of the presented lemma in [34], it is concluded that if whatever one decreases $\|\bar{g}_c\|_1$ by choosing the appropriate values for the learning gains k_P, k_I, k_D and k_E then the convergence rate increases. In order to minimize $\|\bar{g}_c\|_1$ and consequently to achieve the maximum convergence rate, it is possible to use the nonlinear numerical techniques (such as optimization toolbox of the MATLAB) to calculate the learning gains k_P, k_I, k_D and k_E. However, here instead of $\|\bar{g}_c\|_1$ an upper bound of it is to be minimized for two reasons. Firstly, by numerical minimizing of $\|\bar{g}_c\|_1$ one can not to achieve a closed-form and explicit formula for the parameters k_P, k_I, k_D and k_E, while here we are interested to obtain a closed-form formula for these parameters. Secondly, a critical and important step in any numerical optimization method lies in selecting the initial values of variables. Hence, one can use the obtained values for k_P, k_I, k_D and k_E from minimizing the upper bound of $\|\bar{g}_c\|_1$ as the initial values in numerical minimizing of $\|\bar{g}_c\|_1$.

Since $\bar{g}_c$ has M components, it is easy to show that:

$$\|\bar{g}_c\|_1 < \sqrt{M}\,\|\bar{g}_c\|_2. \tag{37}$$

Therefore $\sqrt{M}\|\bar{g}_c\|_2$ is a upper bound for $\|\bar{g}_c\|_1$, hence k_P, k_I, k_D and k_E are obtained such that the following index function to be minimum:

$$\bar{\rho} = \|\bar{g}_c\|_2^2 = \bar{g}_c^T \bar{g}_c. \tag{38}$$

Using (30) and (33), one can write $\bar{g}_c$ as in the following form:

$$\bar{g}_c = \alpha - \bar{H}K, \tag{39}$$

where $\alpha \in \mathbb{R}^M$, $\bar{H} \in \mathbb{R}^{M\times 4}$ and $K \in \mathbb{R}^4$ are defined as follows:

$$\alpha = [1 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0]^T,$$

$$\bar{H} = \begin{bmatrix} g_1 & g_1 & g_1 & 0 \\ g_2 & g_1 + g_2 & g_2 - g_1 & 0 \\ g_3 & \sum_{l=1}^{3} g_l & g_3 - g_2 & g_1 \\ \vdots & \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M} g_l & g_M - g_{M-1} & g_{M-2} \end{bmatrix}, \tag{40}$$

$$K = [k_P \quad k_I \quad k_D \quad k_E]^T.$$

The following lemma is presented for $\bar{H}$.

Lemma 1: We have $\operatorname{rank}(\bar{H}) = 4$, that is $\bar{H}$ has full column rank, and hence $\bar{H}^T\bar{H}$ is invertible.

Proof: We carry out some elementary column operations on $\bar{H}$. It is known that the row or column elementary operations dose not change the rank of the matrix.

Step 1: Multiplying column 1 of $\bar{H}$ by -1 and add the result to columns 2 and 3:

$$\begin{bmatrix} g_1 & 0 & 0 & 0 \\ g_2 & g_1 & -g_1 & 0 \\ g_3 & g_1 + g_2 & -g_2 & g_1 \\ g_4 & \sum_{l=1}^{3} g_l & -g_3 & g_2 \\ \vdots & \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M-1} g_l & -g_{M-1} & g_{M-2} \end{bmatrix}. \tag{41}$$

Step 2: Adding column 2 to column 3:

$$\begin{bmatrix} g_1 & 0 & 0 & 0 \\ g_2 & g_1 & 0 & 0 \\ g_3 & g_1+g_2 & g_1 & g_1 \\ g_4 & \sum_{l=1}^{3} g_l & g_1+g_2 & g_2 \\ \vdots & \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M-1} g_l & \sum_{l=1}^{M-2} g_l & g_{M-2} \end{bmatrix}. \quad (42)$$

Step 3: Multiplying column 4 by –1:

$$\begin{bmatrix} g_1 & 0 & 0 & 0 \\ g_2 & g_1 & 0 & 0 \\ g_3 & g_1+g_2 & g_1 & -g_1 \\ g_4 & \sum_{l=1}^{3} g_l & g_1+g_2 & -g_2 \\ \vdots & \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M-1} g_l & \sum_{l=1}^{M-2} g_l & -g_{M-2} \end{bmatrix}. \quad (43)$$

Step 4: Adding column 3 to column 4:

$$\begin{bmatrix} g_1 & 0 & 0 & 0 \\ g_2 & g_1 & 0 & 0 \\ g_3 & g_1+g_2 & g_1 & 0 \\ g_4 & \sum_{l=1}^{3} g_l & g_1+g_2 & g_1 \\ \vdots & \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M-1} g_l & \sum_{l=1}^{M-2} g_l & \sum_{l=1}^{M-3} g_l \end{bmatrix}. \quad (44)$$

Since $g_1 = CB \neq 0$ the above matrix and consequently $\bar{H}$ has full column rank.

Now from (38) and (39) one gets:

$$\bar{\rho} = 1 - 2\alpha^T \bar{H}K + K^T \bar{H}^T \bar{H}K. \quad (45)$$

Obtaining the gradient (derivation) of $\bar{\rho}$ respect to K as:

$$\frac{\nabla\bar{\rho}}{\nabla K} = -2\bar{H}^T\alpha + 2\bar{H}^T\bar{H}K. \quad (46)$$

As according to Lemma 1 $\bar{H}^T\bar{H}$ is invertible, it is concluded that the equation $\frac{\nabla\bar{\rho}}{\nabla K}=0$ has a unique solution for K:

$$K^* = [k_P^* \quad k_I^* \quad k_D^* \quad k_E^*]^T = (\bar{H}^T\bar{H})^{-1}\bar{H}^T\alpha. \quad (47)$$

From (46) it is obtained:

$$\frac{\nabla^2\bar{\rho}}{\nabla K^2} = 2\bar{H}^T\bar{H}. \quad (48)$$

The symmetric matrix $\bar{H}^T\bar{H}$ is positive definite, therefore K^* makes the index $\bar{\rho}$ to become a global minimum.

By substituting K^* from (47) into (45) then the global minimum of $\bar{\rho}$ is obtained as follows:

$$\bar{\rho}^* = 1 - \alpha^T\bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T\alpha. \quad (49)$$

Comment 3: Form (47) and (49) one gets:

$$\bar{\rho}^* = 1 - \alpha^T\bar{H}K^*. \quad (50)$$

Substituting α and $\bar{H}$ from (40) into (50) yields:

$$\bar{\rho}^* = 1 - g_1(k_P^* + k_I^* + k_D^*). \quad (51)$$

From (38) and (49) it is obvious that $0 \le \bar{\rho}^* < 1$, therefore, the obtained optimal values for the controller parameters satisfy the inequality (25), which is necessary and sufficient in order to have the presented ILC to be convergent.

5. COMPARISON WITH PID-TYPE ILC

In this section it is intended to compare the performance of the presented EPID-type ILC with PID-type and show that the EPID-type is better.

In the PID-type ILC instead of inequality (36), the sufficient condition for having monotonically convergence is [34]:

$$\|g_c\|_1 < 1,$$

where g_c is the given vector in (28).

Also instead of (37) we have [34]:

$$\|g_c\|_1 < \sqrt{M}\,\|g_c\|_2.$$

For this reason instead of (38) the following index becomes to be the minimum [34]:

$$\rho = \|g_c\|_2^2 = g_c^T g_c.$$

Comparison of the performances of the PID-type and the EPID-type ILCs, is the meaning of the comparison of the obtained global minimums for ρ in [34] and $\bar{\rho}$ in this paper, respectively. The global minimum of $\bar{\rho}$ is given in (49) and similarly the global minimum of ρ is [34]:

$$\rho^* = 1 - \alpha^T H(H^T H)^{-1} H^T\alpha, \quad (52)$$

where $\alpha \in \mathbb{R}^M$ is the same given vector in (40) and $H \in \mathbb{R}^{M\times 3}$ is such that [34]:

$$H = \begin{bmatrix} g_1 & g_1 & g_1 \\ g_2 & g_1+g_2 & g_2-g_1 \\ g_3 & \sum_{l=1}^{3} g_l & g_3-g_2 \\ \vdots & \vdots & \vdots \\ g_M & \sum_{l=1}^{M} g_l & g_M-g_{M-1} \end{bmatrix}. \quad (53)$$

From Lemma 1, one gets:

$$\operatorname{rank}(H) = 3. \tag{54}$$

It is needed to prove $\bar{\rho}^* < \rho^*$. In order to do so, the following lemma is required:

Lemma 2: The scalar η which is defined as follows is a positive number:

$$\eta \triangleq L^T \left\{ I - H(H^T H)^{-1} H^T \right\} L, \tag{55}$$

where

$$L = \begin{bmatrix} 0 & 0 & g_1 & g_2 & \cdots & g_{M-2} \end{bmatrix}^T. \tag{56}$$

Proof: Considering the definitions of the matrices $\bar{H}$, H and the vector L, we get:

$$\bar{H} = [H \quad L]. \tag{57}$$

From (57) it is resulted:

$$\bar{H}^T \bar{H} = \begin{bmatrix} H^T H & H^T L \\ L^T H & L^T L \end{bmatrix}. \tag{58}$$

From Lemma 1, one concludes that the symmetric matrices $\bar{H}^T \bar{H}$ and $H^T H$ are both positive definite and hence $\det(\bar{H}^T \bar{H}) > 0$ and $\det(H^T H) > 0$.

For any invertible matrix $A \in \mathbb{R}^{n \times n}$, any vectors $x, y \in \mathbb{R}^n$, and scalar a the following identity holds [35, page 133, fact 2.14.2]:

$$\det\left(\begin{bmatrix} A & x \\ y^T & a \end{bmatrix} \right) = (a - y^T A^{-1} x) \det(A). \tag{59}$$

Applying the above formula for $\bar{H}^T \bar{H}$ results:

$$\eta = \frac{\det(\bar{H}^T \bar{H})}{\det(H^T H)}, \tag{60}$$

where η is given by (55).

Since both $\det(\bar{H}^T \bar{H})$ and $\det(H^T H)$ are positive, immediately one concludes that η is positive too.

Now the main result of this section is presented.

Theorem 3: We have $\bar{\rho}^* < \rho^*$, and hence the performance of EPID-type ILC is better than PID-type one.

Proof: Let us define:

$$\Delta\rho = \rho^* - \bar{\rho}^*. \tag{61}$$

From (49) and (52) it is obtained:

$$\Delta\rho = \alpha^T \bar{H} (\bar{H}^T \bar{H})^{-1} \bar{H}^T \alpha - \alpha^T H (H^T H)^{-1} H^T \alpha. \tag{62}$$

On the other hand from (40) and (53) we get:

$$\begin{aligned} \alpha^T H &= g_1 [1 \quad 1 \quad 1], \\ \alpha^T \bar{H} &= g_1 [1 \quad 1 \quad 1 \quad 0]. \end{aligned} \tag{63}$$

Substituting (63) into (62) yields:

$$\Delta\rho = g_1^2 \left\{ \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix} (\bar{H}^T \bar{H})^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} (H^T H)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}. \tag{64}$$

Let $(\bar{H}^T \bar{H})^{-1}$ to be partitioned in the form of four blocks as follows:

$$(\bar{H}^T \bar{H})^{-1} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_{22} \end{bmatrix}, \tag{65}$$

where $\Gamma_{11} \in \mathbb{R}^{3\times3}$, $\Gamma_{12} \in \mathbb{R}^{3\times1}$ and $\Gamma_{22} \in \mathbb{R}$.

Form (64) and (65) it is resulted:

$$\Delta\rho = g_1^2 \sum \left(\Gamma_{11} - (H^T H)^{-1} \right), \tag{66}$$

where $\sum(\Gamma_{11} - (H^T H)^{-1})$ denotes the summation of all of the components of the matrix $\Gamma_{11} - (H^T H)^{-1}$.

For four arbitrary matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$ the following identity holds [35, page 108, proposition 2.8.7] if A and $D - CA^{-1}B$ are nonsingular:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{bmatrix}. \tag{67}$$

Let us to try to use the above identity for computing $(\bar{H}^T \bar{H})^{-1}$. From (58) one gets:

$$A = H^T H, \ B = H^T L, \ C = L^T H, \ D = L^T L. \tag{68}$$

Hence:

$$D - CA^{-1}B = L^T L - L^T H (H^T H)^{-1} H^T L.$$

Here it is observed that $D - CA^{-1}B$ is a scalar, and which is the same η defined in (55). According to Lemma 2, η is a positive number, hence it is not zero. Thus $D - CA^{-1}B$ is invertible. Because $A = H^T H$ is also invertible, then one can use the formula (67) in order to calculate $(\bar{H}^T \bar{H})^{-1}$. Therefore considering (65) we obtain:

$$\begin{aligned} \Gamma_{11} &= (H^T H)^{-1} + \eta^{-1} (H^T H)^{-1} H^T L L^T H (H^T H)^{-1}, \\ \Gamma_{12} &= -\eta^{-1} (H^T H)^{-1} H^T L, \ \Gamma_{22} = \eta^{-1}. \end{aligned} \tag{69}$$

Substituting for Γ_{11} from (69) into (66) yields:

$$\Delta\rho = \frac{g_1^2}{\eta} \sum \left((H^T H)^{-1} H^T L L^T H (H^T H)^{-1} \right). \tag{70}$$

Let us to show the vector $(H^T H)^{-1} H^T L \in \mathbb{R}^3$ as:

$$\left(H^T H\right)^{-1} H^T L = \begin{bmatrix} a \\ b \\ c \end{bmatrix}. \tag{71}$$

Substituting (71) into (70) yields:

$$\Delta\rho = \frac{g_1^2}{\eta}\left(a+b+c\right)^2. \tag{72}$$

According to Lemma 2, η is positive number, hence from (72) it is resulted that $\Delta\rho$ is also positive.

Comment 4: In the relation (72), if $a + b + c = 0$ then it is resulted that $\Delta\rho = 0$. That is the performances of the PID-type and the EPID-type ILCs are identical. In fact this special case occurs when the obtained optimal value for k_E, that is k_E^*, to be zero. Obviously in this case the EPID-type ILC is reduced to the PID-type one. The following lemma clarifies this fact.

Lemma 3: We have:

$$k_E^* = -\frac{g_1}{\eta}(a+b+c), \tag{73}$$

and hence:

$$k_E^* = 0 \Leftrightarrow a+b+c = 0.$$

Proof: In taking (47) in to account, in order to determine k_E^* it is required to multiply the fourth row (last row) of $(\bar{H}^T\bar{H})^{-1}$ by the vector $\bar{H}^T\alpha = g_1[1\ 1\ 1\ 0]^T$. Then by using (65) and (69) we have:

$$k_E^* = -\frac{g_1}{\eta} L^T H(H^T H)^{-1}[1 \quad 1 \quad 1]^T. \tag{74}$$

Substituting $L^T H(H^T H)^{-1}$ from (71) into (74) results (73).

6. NUMERICAL EXAMPLES

In order to illustrate the effectiveness of the presented ILC, three examples are given in this section.

Example 1: The first example is considered as similar to the example of [34], which is a position servo control system. Let us consider a DC motor that its armature is supplied by a constant current source. Its field winding is supplied by an adjustable voltage source, so that, this voltage controls the rotational angle of the motor, as shown in Fig. 1. The motor rotates a mechanical load.

In this case the dynamics of the motor is modeled by the following state space equations [34]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bv_f(t) \\ y(t) = Cx(t) \end{cases} \quad t \ge 0,$$

where

$$x(t) = \left[i_f(t) \quad \omega(t) \quad \theta(t)\right]^T, \quad y(t) = \theta(t),$$

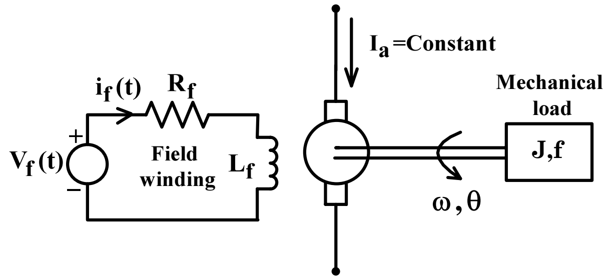

Fig. 1. DC motor with constant armature current.

and

$$A = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ \frac{k_m}{\mathrm{J}} & -\frac{f}{\mathrm{J}} & 0 \\ 0 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{bmatrix},\ C = [0 \quad 0 \quad 1].$$

The parameters and the signals of this model are as follows. R_f, L_f are the field winding resistance and the inductance respectively, k_m is the motor torque ratio, J and f are momentum of inertia and the friction ratio of the mechanical load respectively, $v_f(t)$ and $i_f(t)$ are respectively the field winding source voltage and current, $\omega(t)$ and $\theta(t)$ are the motor shaft rotational speed and angle respectively.

It is desired to control the motor so that its output follows periodically a given desired signal $y_d(t)$ in the time interval $[0,\ t_f]$. In order to determine the motor input voltage according to the EPID-type ILC, firstly the model of motor should be discretized. For this, let us choose sampling period $T = 0.01$ sec and motor parameters to be:

$$R_f = 15\ \Omega,\quad L_f = 1.25\ \mathrm{H},\quad k_m = 120\ \frac{\mathrm{Nm}}{\mathrm{A}},$$

$$f = 0.8\ \frac{\mathrm{Nms}}{\mathrm{rad}},\quad \mathrm{J} = 7.5\ \frac{\mathrm{Nms}^2}{\mathrm{rad}},\quad t_f = 12\ \mathrm{sec}.$$

The obtained discrete mode is as follows:

$$\begin{cases} x_j(i+1) = A_D x_j(i) + B_D V_{fj}(i) \\ y_j(i) = C_D x_j(i) \\ i = 0,1,\ldots,1200,\ \ j = 0,1,\ldots, \end{cases}$$

where j denotes the iteration number and

$$A_D = \begin{bmatrix} 0.8869 & 0 & 0 \\ 0.1507 & 0.9989 & 0 \\ 0.0008 & 0.0100 & 1 \end{bmatrix},$$

$$B_D = \begin{bmatrix} 0 \\ 0.0097 \\ 0.0093 \end{bmatrix},\quad C_D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T.$$

The desired output trajectory is chosen to be a parabolic signal as follows:

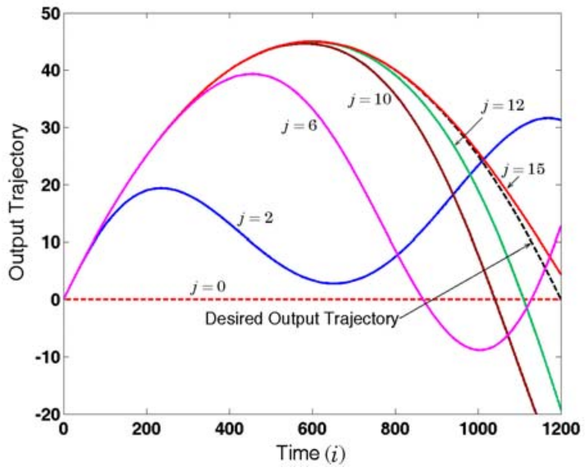

Fig. 2. The desired output trajectory and the motor output (rotational angle) in iterations j = 0,2,6,10, 12,15.

$$y_d(t) = 1.25t(t_f - t) \quad 0 < t \le t_f, \quad t_f = 12 \text{ sec},$$

that is:

$$y_d(i) = \frac{1}{8000} i(M - i) \quad 1 \le i \le M = \frac{t_f}{T} = 1200.$$

All initial conditions as well as the motor input voltage at the iteration j = 0 are chosen to be zero.

Fig. 2 shows the obtained trajectories for the motor rotational angle (the motor output) for some iterations and the desired output trajectory. As can be seen from this figure by increasing the iterations number, the motor output is rapidly converged to the given desired output trajectory.

Figs. 3-5 show all the three norms 1, 2 and ∞, for $E(j)$ versus the iteration number j. These figures indicate that the convergence is monotonic in the sense of all three norms 1, 2 and ∞.

For purposes of comparing, the obtained norms from the PID-type ILC, are also included in Figs. 3-5. It is observed that in the sense of all three norms the convergence rate of the EPID-type ILC is faster, which confirms the theoretical results of Theorem 3. Particularly Fig. 5 shows that the PID-type ILC convergence is not monotonic in the sense of ∞ norm whereas in EPID-type ILC the convergence is monotonic.

Example 2: As for the performance comparison of the presented method in this paper with the method of [14], the second example is selected from [14]. The system is stable oscillatory with the following transfer function [14]:

$$G(z) = \frac{z - 0.8}{(z - 0.5)(z + 0.6)}.$$

By taking the Z inverse transform from the $G(z)$, the Markov parameters of the system are obtained as follows:

$$g_k = -\frac{3}{11}(0.5)^{k-1} + \frac{14}{11}(-0.6)^{k-1}.$$

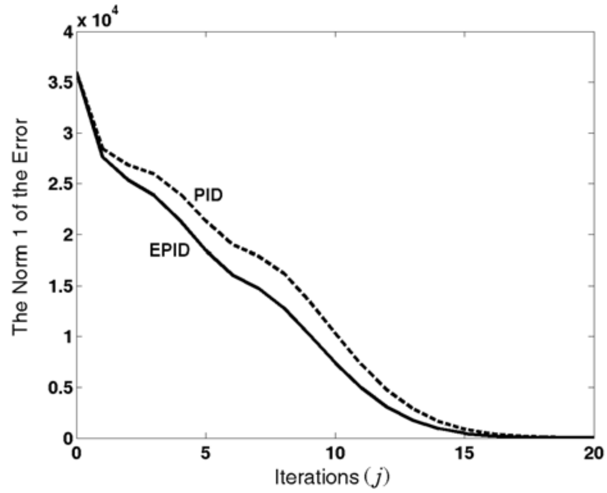

Fig. 3. The norm 1 of the error vector $E(j)$ with respect to j.

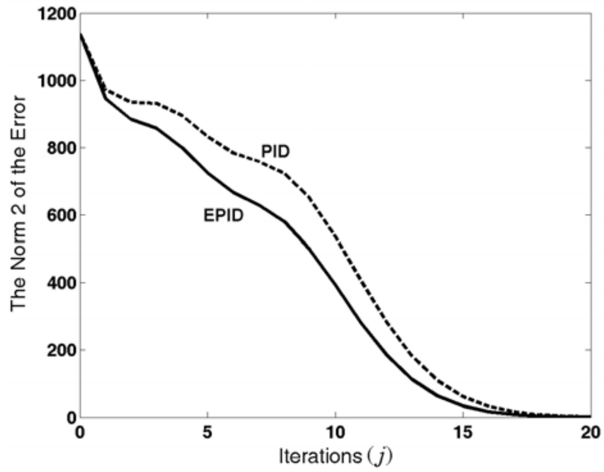

Fig. 4. The norm 2 of the error vector $E(j)$ with respect to j.

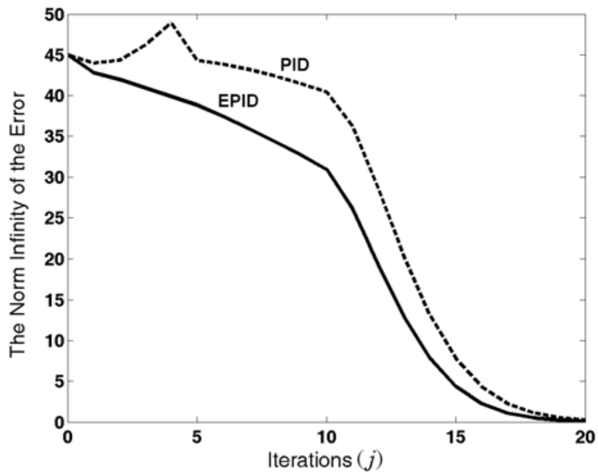

Fig. 5. The norm ∞ of the error vector $E(j)$ with respect to j.

Just as in [14], let M = 60 and the desired trajectory is a triangle (ramp), having the maximum height of 1, given by:

$$y_d(i) = \begin{cases} \dfrac{i}{30} & 1 \le i \le 30 \\ \dfrac{(60-i)}{30} & 31 \le i \le 60. \end{cases}$$

The desired output trajectory and the obtained output for the system from EPID method are given in Fig. 6. This figure demonstrates that the convergence rate is drastically fast.

Since in [14] the 2-norm of $E(j)$, that is the root mean square error, is given with respect to j, in here also this is done in order to be able to compare the results. Fig. 7 shows the obtained 2-norm of $E(j)$ versus the iteration number j from the presented method as well as the method of [14]. By comparing the two graphs in Fig. 7 clearly one can see that the convergence rate of the presented ILC in this paper is drastically faster than the method which is given in [14]. The learning coefficient in [14] is time-variant, whereas the method of this paper benefits from the time-invariant learning gains. It is clear that the implementation of an ILC having time-invariant learning gains is a lot easier than an ILC with time-variant gains.

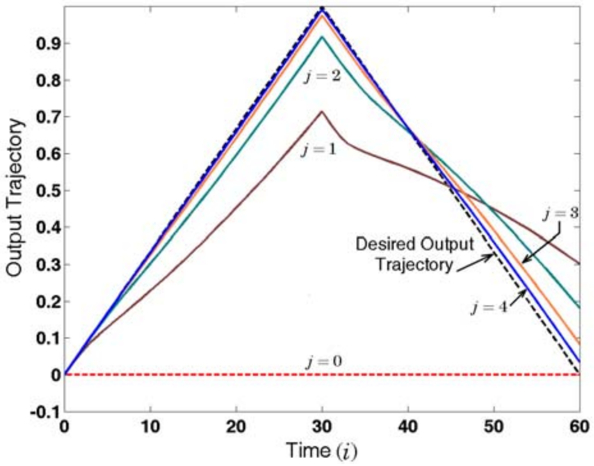

Fig. 6. The desired output trajectory and the system output in iterations $j = 0$ up to 4.

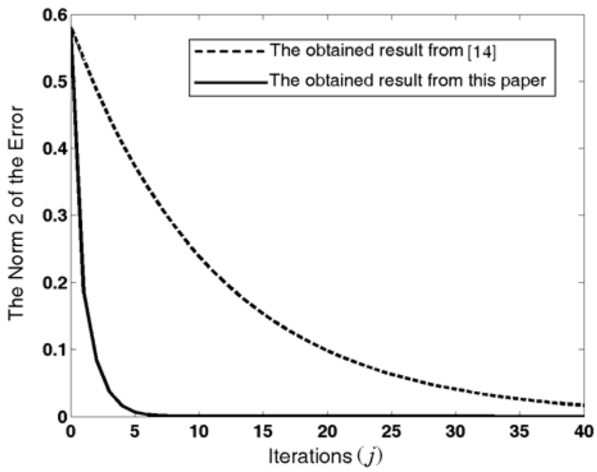

Fig. 7. The norm 2 of the error vector $E(j)$ with respect to j (solid line is for this paper and dash line is for [14]).

Table 1. The norm 2 of the error vector $E(j)$ values for the first 10 iterations.

Iteration number	The obtained results from this paper	The obtained results from [24]
$j=0$	79.0569	79.0569
$j=1$	0.0076	2.0750
$j=2$	8.4118×10^{-7}	0.8450
$j=3$	1.0837×10^{-10}	0.5409
$j=4$	1.6490×10^{-14}	0.3993
$j=5$	1.5299×10^{-18}	0.3160
$j=6$	8.4487×10^{-23}	0.2621
$j=7$	2.0102×10^{-27}	0.2230
$j=8$	5.9616×10^{-34}	0.1929
$j=9$	9.9219×10^{-41}	0.1692
$j=10$	1.2225×10^{-47}	0.1498

Example 3: The third example is chosen from recently published paper [24]. The system has the following transfer function:

$$G(z) = \frac{0.02771z - 0.02713}{z^2 - 1.958z + 0.9589}.$$

For this system the Markov parameters are obtained as follows:

$$g_k = 0.0283(0.97923)^k \left\{\cos(21.91\times10^{-3}k) + 0.0186\sin(21.91\times10^{-3}k)\right\}.$$

The reference signal is $y_d(i) = 5\sin(0.5\pi i)$ over the time interval $1 \le i \le 14$.

Since in [24] the 2-norm of $E(j)$ is tabulated in term of the repetitions number, in here also this is done in order to be able to compare the results. Table 1 shows the 2-norm of $E(j)$ in the first ten iterations. It is observed that the convergence rate of the presented method in this paper is a lot faster. Whereas in [24] a large number of learning coefficients are used but in the presented technique in this paper there are just four learning coefficients.

7. CONCLUSION

This paper presented a novel iterative learning control approach for trajectory tracking. To establish this ILC, a linear combination of the previously presented [34] and a new PID-type ILC was considered. It was shown that the presented ILC has four independent learning gains where by zeroing its fourth learning gain, one achieves the PID-type ILC. Consequently, it is merited to be called an extended PID-type ILC.

The convergence and the monotonic convergence of the given ILC was analyzed and a norm-optimization based method was developed to determine its learning gains, and these gains were obtained as the explicit closed-form formulas in terms of the Markov parameters of the system. The performance of the presented EPID-type ILC was compared with the PID-type ILC and it was mathematically proven that its performance is better, that is its convergence rate is faster than the PID-type. Finally, by some examples the effectiveness and the preference of the presented ILC were illustrated.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123-140, 1984.

[2] Z. Bien and J.-X. Xu, *Iterative Learning Control: Analysis, Design, Integration and Applications*, Kluwer Academic Press, Boston 1998.

[3] Y. Q. Chen and C. Wen, *Iterative Learning Control: Convergence, Robustness and Applications*, vol. LNCIS-248 of Lecture Notes series on Control and Information Science, Springer-Verlag, London, 1999.

[4] J.-X. Xu and Y. Tan, *Linear and Nonlinear Iterative Learning Control*, Springer-Verlag, New York, 2003.

[5] H. S. Ahn, Y. Q. Chen, and K. L. Moore, "Iterative learning control: brief survey and categorization," *IEEE Transactions on Systems, Man, and Cybernetics -Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1099-1121, 2007.

[6] J. H. Lee, K. S. Lee, and W. C. Kim, "Model-based iterative learning control with a quadratic criterion for time-varying linear systems," *Automatica*, vol. 36, no. 5, pp. 641-657, 2000.

[7] T. Al-Towaim, A. D. Barton, P. L. Lewin, E. Rogers, and D. H. Owens, "Iterative learning control - 2D control systems from theory to application," *International Journal of Control*, vol. 77, no. 9, pp. 877-893, 2004.

[8] D. H. Nguyen and D. Banjerdpongchai, "An LMI approach for robust iterative learning control with quadratic performance criterion," *Journal of Process Control*, vol. 19, no. 6, pp. 1054-1060, 2009.

[9] W. Chen, J. M. Li, and J. Li, "Practical adaptive iterative learning control framework based on robust adaptive approach," *Asian Journal of Control*, vol. 13, no. 1, pp. 85-93, 2011.

[10] A. Madady, "A self-tuning iterative learning controller for time variant systems," *Asian Journal of Control*, vol. 10, no. 6, pp. 666-677, 2008.

[11] W. Chen, L. Jiao, R. Li, and J. Li, "Adaptive backstepping fuzzy control for nonlinearly parameterized systems with periodic disturbances," *IEEE Trans. on Fuzzy Systems*, vol. 18, no. 4, pp. 674-685, 2010.

[12] W. Chen and Z. Zhang, "Nonlinear adaptive learning control for unknown time-varying parameters and unknown time-varying delays," *Asian Journal of Control*, vol. 13, no. 6, pp. 903-913, 2011.

[13] A. Madady and H.-R. Reza-Alikhani, "A guaranteed monotonically convergent iterative learning control," *Asian Journal of Control*, vol. 14, no. 5, pp. 1299-1316, 2012.

[14] K. L. Moore, Y. Q. Chen, and V. Bahl, "Monotonically convergent iterative learning control for linear discrete-time systems," *Automatica*, vol. 41, no. 9, pp. 1529-1537, 2005.

[15] T.-Y. Doh and J. R. Ryoo, "Feedback-based iterative learning control for MIMO LTI systems," *International Journal of Control, Automation, and Systems*, vol. 6, no. 2, pp. 269-277, 2008.

[16] X. Bu, Z. Hou, and F. Yu, "Stability of first and high order iterative learning control with data dropouts," *International Journal of Control, Automation, and Systems*, vol. 9, no. 5, pp. 843-849, 2011.

[17] N. Amann, D. H. Owens, and E. Rogers, "Iterative learning control for discrete time systems using optimal feedback and feedforward actions," *Proc. of the 34th IEEE Conf. on Decision and Control*, pp. 1696-1701, New Orleans, LA, USA, December 1995.

[18] J.-X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, no. 11, pp. 1867-1880, 2002.

[19] D. H. Owens and J. Hatonen, "Iterative learning control - an optimization paradigm," *Annual Reviews in Control*, vol. 29, no. 1, pp. 57-70, 2005.

[20] B. Chu and D. H. Owens, "Accelerated norm-optimal iterative learning control algorithms using successive projection," *International Journal of Control*, vol. 82, no. 8, pp. 1469-1484, 2009.

[21] X. Hong-Wei, "Optimal iterative learning control for nonlinear discrete-time systems," *Advanced Materials Research*, vol. 320, pp. 605-609, 2011.

[22] D. H. Owens, C. T. Freeman, and T. V. Dinh, "Norm-optimal iterative learning control with intermediate point weighting: theory, algorithms, and experimental evaluation," *IEEE Trans. on Control Systems Technology*, vol. 21, no. 3, pp. 999-1007, 2013.

[23] P. Janssens, G. Pipeleers, and J. Swevers, "A data-driven constrained norm-optimal iterative learning control framework for LTI systems," *IEEE Trans. on Control Systems Technology*, vol. 21, no. 2, pp. 546-551, 2013.

[24] D. H. Owens, B. Chu, and M. Songjun, "Parameter-optimal iterative learning control using polynomial representations of the inverse plant," *International Journal of Control*, vol. 85, no. 5, pp. 533-544, 2012.

[25] K. J. Aström and T. Hägglund, *Advanced PID Controller*, Instrument Society of America, NC, USA, 2006.

[26] K.-H. Park, Z. Bien, and D.-H. Hwang, "A study on the robustness of a PID-type iterative learning controller against initial state error," *International Journal of Systems Science*, vol. 30, no. 1, pp. 49-59, 1999.

[27] J. D. Ratcliffe, J. J. Hatonen, P. L. Lewin, E. Rogers, T. J. Harte, and D. H. Owens, "P-type iterative learning control for systems that contain resonance," *International Journal of Adaptive Control and Signal Processing*, vol. 19, no. 10, pp. 769-796,

2005.
[28] Z. Baolin, T. Gongyou, and Z. Shi, "PD-type iterative learning control for nonlinear time delay system with external disturbance," *Journal of Systems Engineering and Electronics*, vol. 17, no. 3, pp. 600-605, 2006.
[29] G.-P. Zhou and J. Luo, "Robustness of nonlinear systems with open-closed-loop PD-type iterative learning control," *Systems Engineering and Electronics*, vol. 29, no. 11, pp. 1931-1934, 2007.
[30] J. Xu, Q. Zhao, and X. Wang, "A PID-IMP type iterative learning controller," *Proc. of the 7th World Congress on Intelligent Control and Automation*, pp. 785-789, Chongqing, China, June 25-27, 2008.
[31] A. Luo, X. Xu, L. Fang, H. Fang, J. Wu, and C. Wu, "Feedback-feedforward PI-type iterative learning control strategy for hybrid active power filter with injection circuit," *IEEE Trans. on Industrial Electronics*, vol. 57, no. 11, pp. 3767-3779, 2010.
[32] F. Ma and C. Li, "Open-closed-loop PID-type iterative learning control for linear systems with initial state error," *Journal of Vibration and Control*, vol. 17, no. 12, pp. 1791-1797, 2011.
[33] X. Ruan, Z. Z. Bien, and Q. Wang, "Convergence characteristics of proportional-type iterative learning control in the sense of Lebesgue-*p* norm," *IET Control Theory and Application*, vol. 6, no. 5, pp. 707-714, 2012.
[34] A. Madady, "PID type iterative learning control with optimal gains," *International Journal of Control, Automation, and Systems*, vol. 6, no. 2, pp. 194-203, 2008.
[35] D. S. Bernstein, *Matrix Mathematics: Theory, Facts, and Formulas*, 2nd ed., Princeton University Press, 2009.

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