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Robust Digital Implementation of Fuzzy Control for Uncertain Systems and Its Application to Active Magnetic Bearing System

Hwa Chang Sung, Jin Bae Park, Young Hoon Joo*, and Kuo Chi Lin

Abstract: In this paper, we propose the robust digital control for active magnetic bearing (AMB) systems. For achieving the robust stability, we deal with the uncertainties of the given system based on the Takagi-Sugeno (T-S) fuzzy model. Also, in order to solve the digital implementation for real plants, this paper presents a robust intelligent digital redesign (IDR) method. The term IDR involves converting an analog controller into an equivalent digital one in the sense of state-matching. The uncertainties in the plant dynamics is shown in the IDR condition by virtue of the pade and inverse-pade approximation method. Also, the robust stability property is preserved by the proposed method. The sufficient conditions for robust controller are obtained in terms of solutions to linear matrix inequalities (LMIs). Finally, simulation results for two AMB systems are demonstrated to visualize the feasibility of the proposed method.

Keywords: Active magnetic bearing (AMB), intelligent digital redesign (IDR), linear matrix inequalities (LMIs), Pade and inverse-Pade approximation, Takagi-Sugeno (T-S) fuzzy model.

1. INTRODUCTION

In recent year, the magnetic bearings are widespread more and more in many industrial applications such as flywheels, satellites, and high-speed turbines, etc. According to principle of producing suspension forces, we can classify the magnetic bearings as passive and active. Among them, active magnetic bearing (AMB) systems have been paid more attention. Because the AMB dramatically reduces the friction and wear, it has virtually semi-permanent life without lubrication and mechanical maintenance [1]. As a result, there are many researches to control the AMB, such as linear control [2], PID control [3], sliding mode control [4], feedback linearization [5], and adaptive control [6]. However, the dynamics of AMB has severe nonlinearities so that the control of the given system is not easy. In other words, the inherently unstable dynamics of the AMB, associated with the complexity of the rotor dynamics, makes it impossible to operate the concerned system without a proper control.

For solving these nonlinearity problems, [1] and [7] have dealt with robust control for AMB system by using the Takagi-Sugeno (T-S) fuzzy model. The main advantage of fuzzy model is to express a nonlinear AMB system by the time-varying convex combination of linear state space models using nonlinear fuzzy membership functions so that it is easy to apply the various control technique, such as output feedback control, decentralized control, H_∞ control, and so on. Especially, through the merge of the T-S fuzzy control with a digital redesign (DR) method, it is possible to use the digital devices in the control of complex dynamical systems. This control technique is called by an intelligent digital redesign (IDR) [8,9-12]. The IDR method involves converting an existing analog fuzzy controller into an equivalent digital counterpart in the sense of state-matching.

The robust IDR problem has been also studied in [13]. However, the method in [13] which is known as the local approach has two main disadvantages: first, it only considered the each subsystem in T-S fuzzy systems so that it only allowed the local state-matching; second, any stability conditions for digital control systems were not considered in their IDR procedure. To overcome these weaknesses, the global robust IDR approach has been proposed in [12]. In order to solve the robust IDR problem, they have solved the complex structural property in the procedure of discretization by using the bilinear and inverse-bilinear approximation method. However, since the robust IDR in [12] did not consider the high-order uncertain terms in discretization, it may show the poor performance according to increase of the sampling period.

Motivated by the above observations, this paper presents a novel robust control method for digital

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Hwa Chang Sung and Jin Bae Park are with the School of Electrical and Electronic Engineering, Yonsei University, Seoul 120-749, Korea (e-mails: {casfirspear, jbpark}@yonsei.ac.kr).

Young Hoon Joo is with the Department of Control and Robotics Engineering, Kunsan National University, Kunsan, Chonbuk, 573-701, Korea (e-mail: yhjoo@kunsan.ac.kr).

Kuo Chi Lin is with the Department of Mechanical, Material, and Aerospace Engineering, University of Central Florida Orlando, Florida, USA (e-mail: klin@mail.ucf.edu).

* Corresponding author.

implementation of the AMB. For achieving the robust stability, we deal with the parametric uncertainties of the concerned system in the form of the norm-bounded. Also, we derive the discretized models of the analog and digital control systems preserving the property and structure of the uncertainties by using pade and inverse-pade approximation method. Based on these discretized models, the IDR problem is to find the relevant digital fuzzy gains that minimize the norm distance between the states of the analog and the digital robust control system and stabilize the digital control system simultaneously. Its constructive conditions are provided in the linear matrix inequalities (LMIs) formats, and therefore easily tractable by the convex optimization techniques. Finally, the obtained LMIs are applied to the two AMB systems: 1) 1-dimension of freedom (DOF) AMB system; 2) 2-axis controlled vertical shaft AMB system which are constructed as the T-S fuzzy model.

This paper is organized as follows: Section 2 deals with the T-S fuzzy control scheme in nonlinear aspect. The robust control methodology based on digital controller is proposed in Section 3. The two robust simulation results of AMB system are demonstrated in Section 4. This paper concludes with Section 5.

2. PRELIMINARIES

The general T-S fuzzy model can be represented as

$$\begin{aligned} &R_i : If\ z_1(t)\ is\ about\ \Gamma_{i1}\ and \ldots and\ z_p(t)\ is\ about\ \Gamma_{ip} \\ &\quad Then\ \dot{x}_d(t) = (A_i + \Delta A_i)x_d(t) + (B_i + \Delta B_i)u_d(t), \end{aligned} \quad (1)$$

where R_i, $i \in \mathrm{I}_r = \{1, 2, \cdots, q\}$, denotes the i th fuzzy rule, $z_h(t)$, $h \in \mathrm{I}_p = \{1, 2, \cdots, p\}$, is the h th premise variable, $\Gamma_h^{\ i}$, $(i, h) \in I_r \times I_p$, is the fuzzy set of $z_h(t)$ in R_i, A_i and B_i are known constant matrices with appropriate dimensions, and ΔA_i and ΔB_i are unknown matrices with appropriate dimensions which represent the system uncertainties. The subscript 'd' means "under digital control action", and the subscript 'c' will denote "under analog control action".

Using the singleton fuzzifier, product inference engine, and center-average defuzzification, (1) is inferred as

$$\dot{x}_d(t) = \sum_{i=1}^{r} \theta_i(z(t))((A_i + \Delta A_i)x_d(t) + (B_i + \Delta B_i)u_d(t)), \quad (2)$$

where $\theta_i(z(t)) = w_i(z(t)) / \sum_{i=1}^{r} w_i(z(t))$ and $w_i(z(t)) = \prod_{h=1}^{p} \mu_{\Gamma_{ih}}(z_h(t))$, and $\mu_{\Gamma_h^{\ i}}(z_h(t)) : U_{z_h(t)} \subset \mathbb{R} \rightarrow \mathbb{R}_{[0,1]}$ is the membership function of $z_h(t)$ on the compact set $U_{z_h(t)}$.

Suppose that a digital fuzzy control

$$\begin{aligned} &R_i : If\ z_1(kT)\ is\ \Gamma_{i1}\ and \ldots and\ z_p(kT)\ is\ \Gamma_{ip} \\ &\quad Then\ u_d(t) = K_d^{\ i} x_d(kT). \end{aligned} \quad (3)$$

The defuzzified output is given by

$$u_d(t) = u_d(kT) = \sum_{i=1}^{r} \theta_i(z(kT)) K_d^{\ i} x_d(kT), \quad (4)$$

where $u_d(kT) \in \mathbb{R}^m$ is the digital control input to be determined in time interval $t \in [kT, kT + T)$, $K \in \mathbb{Z}_{\geq 0}$, and $T \in \mathbb{R}_{>0}$ is the nonpathological sampling period.

Remark 1 [14-16]**:** The analog fuzzy control system is represented in the same manner

$$\begin{aligned} \dot{x}_c(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(t))\theta_j(z(t))((A_i + \Delta A_i) \\ &+ (B_i + \Delta B_i)K_c^{\ j})x_c(t). \end{aligned} \quad (5)$$

Remark 2: In this paper, we assume that ΔA_i and ΔB_i can be described as follows:

$$[\Delta A_i \quad \Delta B_i] = D_i F_i(t)[E_{1i} \quad E_{2i}], \quad (6)$$

where D_i, E_{1i}, and E_{2i} are known real constant matrices of compatible dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and with $F_i^T(t)F_i(t) \leq I$.

3. MAIN RESULTS

In this section, we are interested in solving the robust IDR problem for the AMB system. Generally, in order to find some relevant digital control satisfying the state-matching, the IDR problem is dealt with discrete-time manner. Toward that end, we need two different models; the one is closed-loop analog control system and the other is discrete-time model of the closed-loop digitally controlled system. Motivated by above consideration, our IDR problem is formulated as follows:

Problem 1: When analog plant (5) is globally asymptotically stable, find K_d^i achieving that the closed-loop state $x_d(t)$ of (2) and (4)

$$\begin{aligned} \dot{x}_d(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(t))\theta_j(z(kT))((A_i + \Delta A_i)x_d(t) \\ &+ (B_i + \Delta B_i)K_d^{\ j} x_d(kT)) \end{aligned} \quad (5)$$

matches $x_c(t)$ of (5) for any $K \in \mathbb{Z}_{\geq 0}$, as closely as possible with guaranteed stability of (7) in some sense.

In order to solve the IDR in the discrete-time domain, it is necessary to obtain the discrete-time models of (5) and (7). However, it is not easy to obtain the exact models because 1) the structural property of the uncertainties should also be solved in the procedure of discretization; 2) the robust stabilization of the sampled-data fuzzy system should be guaranteed. Toward that end, we consider the following pade and inverse-pade approximation method.

Lemma 1: The structured uncertain matrices are shown as follows:

$$e^{(A_i+\Delta A_i)} \cong \frac{1}{2}(G_i-I)A_i^{-1}\Delta A_i(G_i+I) - \frac{T}{6}(G_i-I)A_i^{-1}\Delta A_i A_i(G_i-I), \tag{8}$$

$$\begin{aligned}\int_0^T e^{(A_i+\Delta A_i)\tau}(B_i+\Delta B_i)d\tau &= (G_i-I)(A_i+\Delta A_i)^{-1}(B_i+\Delta B_i) \\ &= (G_i-I)A_i^{-1}\Delta B_i + \frac{1}{2}(G_i-I)A_i^{-1}\Delta A_i H_i \\ &\quad - \frac{T}{6}(G_i-I)A_i^{-1}\Delta A_i A_i H_i,\end{aligned} \tag{9}$$

where $G_i = e^{A_i T}$, $H_i = (G_i-I)A_i^{-1}B_i$, $T \in \mathbb{R}_{>0}$ is the sampling period.

Proof: This proof is a trivial extension of [17,18].

Remark 3: It is well known that the pade approximation method is one of the most popular approaches to find low-order rational approximations of a high-order rational function in the field of system modeling and design [17].

In this situation, for the appropriate sampling period *T*, the discrete-time models of (5) and (7) can be represented by

$$x_c(kT+T) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT))\theta_j(z(kT))(\hat{G}_{ij}+\Delta\hat{G}_{ij})x_c(kT), \tag{10}$$

$$x_d(kT+T) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT))\theta_j(z(kT))(G_i+\Delta G_i + (H_i+\Delta H_i)K_d{}^j)x_d(kT), \tag{11}$$

where

$$\Delta\hat{G}_{ij} = \hat{\delta}_{ij}F_i(t)\hat{\varsigma}_{ij}, \tag{12}$$

$$[\Delta G_i \quad \Delta H_i] = \delta_i F_i(t)[\varsigma_{1i} \quad \varsigma_{2i}]. \tag{13}$$

Remark 4: There are several methods to obtain (10) and (11) such as general Euler approximation. Also, [12] adapted the bilinear and inverse bilinear method to reduce the structural error. To obtain more detailed models, we can use the discretization method in [8,9,11,12] together with Lemma 1. As a results, the uncertain matrices in (10) and (11) are determined by

$$\hat{G}_{ij} = e^{(A_i+B_iK_c{}^j)T},$$

$$\Delta\hat{G}_{ij} = \frac{1}{2}(\hat{G}_{ij}-I)(A_i+B_iK_c{}^j)^{-1}(\Delta A_i+\Delta B_iK_c{}^j)(\hat{G}_{ij}+I) - \frac{T}{3}(\Delta A_i+\Delta B_iK_c{}^j)(\hat{G}_{ij}+I),$$

$$\Delta G_i = \frac{1}{2}(G_i-I)A_i^{-1}\left\{\Delta A_i(G_i+I) - \frac{T}{3}\Delta A_i A_i(G_i-I)\right\},$$

$$\Delta H_i = \frac{1}{2}(G_i-I)A_i^{-1}\left(2\Delta B_i + \Delta A_i H_i - \frac{T}{3}\Delta A_i A_i H_i\right),$$

$$\hat{\delta}_{ij} = \left\{\frac{1}{2}(\hat{G}_{ij}-I)(A_i+B_iK_c{}^j)^{-1} - \frac{T}{3}\right\}D_i,$$

$$\hat{\varsigma}_{ij} = (E_{1i}+E_{2i}K_c{}^j)(\hat{G}_{ij}+I),$$

$$\delta_i = \frac{1}{2}(G_i-I)A_i^{-1}D_i,$$

$$\varsigma_{1i} = E_{1i}\left\{(G_i+I) - \frac{T}{3}A_i(G_i-I)\right\},$$

$$\varsigma_{2i} = E_{1i}\left(H_i - \frac{T}{3}A_iH_i\right)2E_{2i}.$$

We have discussed the structural property of the uncertainties in Remark 4. In order to match trajectories of (10) and (11), we compare them, under the assumption that $x_c(0) = x_d(0)$, $x_c(kT+T) = x_d(kT+T)$. If there exists $K_d{}^i$, $i \in I_r$, then (10) and (11) satisfy

$$\hat{G}_{ij} + \Delta\hat{G}_{ij} = G_i + \Delta G_i + (H_i+\Delta H_i)K_d{}^j \tag{14}$$

for $(i,j) \in I_r \times I_r$. However, finding $K_d{}^i$, $i \in I_r$ to satisfy (14) is not theoretically solvable since $\Delta\hat{G}_{ij}$, ΔG_i, and ΔH_i are unknown and (14) for $(i,j) \in I_r \times I_r$ are usually inconsistent in the practical control engineering. In this situation, we should find $K_d{}^i$, $i \in I_r$, to closely match trajectories of (10) and (11) and to ensure the asymptotical stability of $K_d(kT)$ of (7).

Consider the following lemma and proposition which will be used in the proof of our main results:

Lemma 1 [19]**:** For any real matrices $\Lambda_1 = \Lambda_1{}^T$, Λ_2, $\Lambda_3(t)$, and Λ_4 with appropriate dimensions, the following inequality holds:

$$\Lambda_1 + \Lambda_2\Lambda_3(t)\Lambda_4 + \Lambda_4{}^T\Lambda_3{}^T(t)\Lambda_2{}^T \prec 0,$$

where $\Lambda_3(t)$ satisfies $\Lambda_3{}^T(t)\Lambda_3(t) \le I$ if and only if

$$\Lambda_1 + \begin{bmatrix}\varepsilon^{-1}\Lambda_4{}^T & \varepsilon\Lambda_2\end{bmatrix}\begin{bmatrix}\varepsilon^{-1}\Lambda_4 \\ \varepsilon\Lambda_2{}^T\end{bmatrix} \prec 0$$

for some $\varepsilon < 0$.

Proposition 1: We suppose that $x_c(kT) = x_d(kT) := x(kT)$. If there exists $K_d{}^i$, $i \in I_r$, such that

$$\left\|\hat{G}_{ij} - G_i - H_iK_c{}^j + \Delta\hat{G}_{ij} - \Delta G_i - \Delta H_iK_d{}^j\right\| \le \hat{\gamma} \tag{15}$$

for some constant $\hat{\gamma} > 0$, then (10) and (11) satisfy

$$\|x_c(kT+T) - x_d(kT+T)\| \le \hat{\gamma}\|x(kT)\|. \tag{16}$$

Proof: Subtracting (11) from (10) and taking the norms on the both sides yield

$$\|x_c(kT+T) - x_d(kT+T)\| \le \sum_{i=1}^{r}\sum_{j=1}^{r}|\,\theta_{ij}(z(kT))\,|\,\Big\|\hat{G}_{ij} - G_i - H_iK_c{}^j + \Delta\hat{G}_{ij} - \Delta G_i - \Delta H_iK_d{}^j\Big\|\|x(kT)\|$$

under the assumption of $x_c(kT)=x_d(kT):=x(kT)$. From the fact that $0\le|\theta_{ij}(z(kT))|\le 1$ for any $(i,j)\in I_r\times I_r$, it can be shown that (16) holds whenever (15) for $\hat{\gamma}>0$ is satisfied.

Remark 5: Proposition 1 indicates that, if there exists $K_d{}^i$, $i\in I_r$, such that (15) holds for a possibly small $\hat{\gamma}>0$, then $x_d(kT+T)$ of (11) closely matches $x_c(kT+T)$ of (11) under the assumption of $x_c(kT)=x_d(kT)$.

Proposition 2 [12]**:** Consider (7). There exists some constants $\eta_1>0$ such that

$$\|x_d(t)\|\le\eta_1\|x_d(kT)\|, \tag{17}$$

where $\eta_1=\sup_{(i,j)\in I_r\times I_r}(1+T\beta)e^{\alpha T}$, $\alpha=\sup_{i\in I_r}\{\|A_i\|+\|D_i\|\|E_{1i}\|\}$, and $\beta=\sup_{(i,j)\in I_r\times I_r}\{\|B_iK_d{}^j\|+\|D_i\|\|E_{2i}K_d{}^j\|\}$ for $t\in[kT,\ kT+T)$.

As shown in Propositions 1 and 2, if $x_d(kT)$ converges to the equilibrium point, then the digital trajectory $x_d(t)$ also tends to the origin. This means that, in the sufficient small sampling period, it allows us to design the stabilizing K_d^i in the discrete-time domain via (11). Indeed, [11] and [20] showed that, the nonlinear digital control system is stable if its approximate model is stable.

The main Theorem is summarized as follows:

Theorem 1: Suppose that there exist a matrix L_i, $P_i\succ 0$, $\sigma_d{}^i$, $Q_i^k\succ 0$, $Q_{ij}{}^k=(Q_{ij}{}^k)^T$ and the scalars ε_{ij} and $\hat{\varepsilon}_{ij}$ are optimal solutions to

$$\underset{P_i,\sigma_d{}^i,Q_{ij}{}^k,\varepsilon_{ij},\hat{\varepsilon}_{ij}}{\text{Minimize}}\quad \gamma \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma L & * & * & * \\ (\hat{G}_{ij}-G_i)L-H_i\sigma_d{}^j & \varepsilon_{ij}\bar{D}_{ij}-\gamma L & * & * \\ \hat{\varsigma}_{ij}L & 0 & -\varepsilon_{ij}I & * \\ -\varsigma_{1i}L-\varsigma_{2i}\sigma_d{}^j & 0 & 0 & -\varepsilon_{ij}I \end{bmatrix}\prec 0, \tag{18}$$

$$\begin{bmatrix} -4P_i+Q_{ij}{}^k & * & * & * \\ \begin{pmatrix} G_iL+H_i\sigma_d{}^j \\ +GjL+H_i\sigma_d{}^i \end{pmatrix} & \begin{pmatrix} \varepsilon_{ij}\tilde{D}_{ij}+P_k \\ -(L+L) \end{pmatrix} & * & * \\ \varsigma_{1i}L+\varsigma_{2i}\sigma_d{}^j & 0 & -\varepsilon_{ij}I & * \\ \varsigma_{1j}L+\varsigma_{2j}\sigma_d{}^i & 0 & 0 & -\varepsilon_{ij}I \end{bmatrix}\prec 0, \tag{19}$$

where $^\forall\{(i,j)\in I_r\times I_r\,|\,1\le i\le j\le r\}$, $k\in I_r$, $\bar{D}_{ij}=\hat{\delta}_{ij}\times\hat{\delta}_{ij}^T+\delta_i\delta_i^T$, $\tilde{D}_{ij}=\delta_i\delta_i^T+\delta_j\delta_j^T$. Then, $x_d(kT)$ of (7) closely matches $x_c(kT)$ of (5), and origin of (11) is globally exponentially stable. When the minimization problem subject to (18) and (19) is feasible, the digital gain is obtained by The fuzzy gains are given by $K_d{}^i=\sigma_d{}^iL^{-1}$, $i\in I_r$.

Proof: Before proceeding the state-matching condition, we consider the following nonquadratic Lyapunov function candidate

$$\begin{aligned} V_d(x(kT)) &= x_d{}^T(kT)\sum_{i=1}^r\theta_i(z(kT))P_ix_d(kT) \\ &= x_d{}^T(kT)P_zx_d(kT), \end{aligned} \tag{20}$$

where $P_i\succ 0$, $i\in I_r$ and $P_z=\sum_{i=1}^r\theta_i(z(kT))P_i$. The rate of increase of $V(x_d(kT))$ is

$$\begin{aligned} \Delta V(x_d(kT)) &= V(x_d(kT+T))-V(x_d(kT)) \\ &= x_d{}^T(kT+T)P_zx_d(kT+T) \\ &\quad -x_d{}^T(kT)P_zx_d(kT) \\ &\le \frac{1}{4}\sum_{i=1}^r\sum_{i\le j}^r\theta_i(z(kT))\theta_j(z(kT))x_d{}^T(kT)(G_i \\ &\quad +H_iK_d{}^j+G_j+H_jK_d{}^i+\Delta G_i+\Delta H_iK_d{}^j \\ &\quad +\Delta G_j+\Delta H_jK_d{}^i)^TP_z(G_i+H_iK_d{}^j+G_j \\ &\quad +H_jK_d{}^i+\Delta G_i+\Delta H_iK_d{}^j+\Delta G_j+\Delta H_i \\ &\quad \times K_d{}^j)-4P\prec 0. \end{aligned} \tag{21}$$

When the inequalities are represented as the nonquadratic case, the principal results are slightly modified as [21,22],

$$A_{ij}{}^TP_zA_{ij}-P\prec 0 \Leftrightarrow \begin{bmatrix} P_i & * \\ A_{ij}L & L+L^T-P_z \end{bmatrix}\succ 0, \tag{22}$$

where L is a matrix with appropriate dimensions. Application of the Schur complement to the foregoing inequality results in

$$\begin{bmatrix} -4P_i+Q_i^k & * \\ \begin{pmatrix} G_i+H_iK_d{}^j+G_j+H_jK_d{}^i+ \\ \Delta G_i+\Delta H_iK_d{}^j+\Delta G_j+\Delta H_jK_d{}^i \end{pmatrix}L & P_k-(L+L^T) \end{bmatrix}\prec 0, \tag{23}$$

where $(i,j,k)\in I_r\times I_r\times I_r$, $Q_{ij}{}^k\succ 0$, $Q_{ij}{}^k=(Q_{ij}{}^k)^T$. We can use (12) and (13) to show

$$\begin{aligned} &\begin{bmatrix} -4P_i+Q_i^k & * \\ (G_i+H_iK_d{}^j+G_j+H_jK_d{}^i)L & P_k-(L+L^T) \end{bmatrix} \\ &+\begin{bmatrix} 0 & 0 \\ \delta_i & \delta_i \end{bmatrix}\begin{bmatrix} F_i(t) & 0 \\ 0 & F_i(t) \end{bmatrix}\begin{bmatrix} (\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0 \\ (\varsigma_{1j}+\varsigma_{2i}K_d{}^i)L & 0 \end{bmatrix} \\ &+\begin{bmatrix} (\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0 \\ (\varsigma_{1j}+\varsigma_{2j}K_d{}^i)L & 0 \end{bmatrix}^T\begin{bmatrix} F_i(t) & 0 \\ 0 & F_i(t) \end{bmatrix}^T\begin{bmatrix} 0 & 0 \\ \delta_i & \delta_i \end{bmatrix}^T \prec 0. \end{aligned} \tag{24}$$

Since it follows from Lemma 2 that

$$\begin{bmatrix}0 & 0\\ \delta_i & \delta_i\end{bmatrix}\begin{bmatrix}F_i(t) & 0\\ 0 & F_i(t)\end{bmatrix}\begin{bmatrix}(\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0\\ (\varsigma_{1j}+\varsigma_{2j}K_d{}^i)L & 0\end{bmatrix}$$

$$+\begin{bmatrix}(\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0\\ (\varsigma_{1j}+\varsigma_{2j}K_d{}^i)L & 0\end{bmatrix}^T\begin{bmatrix}F_i(t) & 0\\ 0 & F_i(t)\end{bmatrix}^T\begin{bmatrix}0 & 0\\ \delta_i & \delta_i\end{bmatrix}^T$$

$$\le \hat{\varepsilon}_{ij}\begin{bmatrix}0 & 0\\ \delta_i & \delta_i\end{bmatrix}\begin{bmatrix}0 & 0\\ \delta_i & \delta_i\end{bmatrix}^T+\hat{\varepsilon}_{ij}{}^{-1}\begin{bmatrix}(\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0\\ (\varsigma_{1j}+\varsigma_{2j}K_d{}^i)L & 0\end{bmatrix}^T$$

$$\times\begin{bmatrix}(\varsigma_{1i}+\varsigma_{2i}K_d{}^j)L & 0\\ (\varsigma_{1j}+\varsigma_{2i}K_d{}^i)L & 0\end{bmatrix}$$

for some scalar $\hat{\varepsilon}_{ij}>0$. Denoting $\sigma_d{}^i=K_d{}^iL$ and (24) holds if

$$\begin{bmatrix}-4P_i+Q_i^k & *\\ G_iL+H_i\sigma_d{}^j+G_jL+H_j\sigma_d{}^i & \hat{\varepsilon}_{ij}\tilde{D}_{ij}+P_k-(L+L^T)\end{bmatrix}$$

$$+\begin{bmatrix}\varsigma_{1i}L+\varsigma_{2i}\sigma_d{}^j & 0\\ \varsigma_{1j}L+\varsigma_{2j}\sigma_d{}^i & 0\end{bmatrix}^T\begin{bmatrix}\hat{\varepsilon}_{ij}{}^{-1}I & 0\\ 0 & \hat{\varepsilon}_{ij}{}^{-1}I\end{bmatrix}$$

$$\times\begin{bmatrix}\varsigma_{1i}L+\varsigma_{2i}\sigma_d{}^j & 0\\ \varsigma_{1j}L+\varsigma_{2j}\sigma_d{}^i & 0\end{bmatrix}\prec 0$$

in which $\tilde{D}_{ij}=\delta_i\delta_i^T+\delta_j\delta_j{}^T$. By the Schur complement, we see that the foregoing inequality holds as (19).

Next, we consider the state-matching condition in (14) to closely match trajectories of (10) and (11). In order to solve the matching problem in (14), we minimize the norm distances between $\hat{G}_{ij}+\Delta\hat{G}_{ij}$ and $G_i+\Delta G_i+H_i\times K_d{}^j+\Delta H_iK_d{}^j$ as follows:

$$\left\|\hat{G}_{ij}-G_i-H_iK_c{}^j+\Delta\hat{G}_{ij}-\Delta G_i-\Delta H_iK_d{}^j\right\|\le\hat{\gamma} \qquad (25)$$

for some $\hat{\gamma}>0$. Applying the congruence transformation with L to (25) yields the following inequality

$$L^T(\hat{G}_{ij}-G_i-H_iK_c{}^j+\Delta\hat{G}_{ij}-\Delta G_i-\Delta H_iK_d{}^j)^T$$
$$\times(\hat{G}_{ij}-G_i-H_iK_c{}^j+\Delta\hat{G}_{ij}-\Delta G_i-\Delta H_iK_d{}^j)L\le\gamma^2L^TL,$$

where some constant $\gamma>0$. Using the Schur complement, (12) and (13) yield

$$\begin{bmatrix}-\gamma L & *\\ (\hat{G}_{ij}-G_i)L-H_i\sigma_d{}^j & -\gamma L\end{bmatrix}$$

$$+\begin{bmatrix}0 & *\\ (\hat{D}_{ij}F_i(t)\hat{\varsigma}_{ij}-\delta_iF_i(t)\varsigma_{1i})L-\delta_iF_i(t)\varsigma_{2i}\sigma_d{}^j & 0\end{bmatrix}\prec 0,$$

which are equivalent to

$$\begin{bmatrix}-\gamma L & *\\ (\hat{G}_{ij}-G_i)L-H_i\sigma_d{}^j & -\gamma L\end{bmatrix}+\begin{bmatrix}0 & 0\\ \hat{D}_{ij} & \delta_i\end{bmatrix}$$

$$+\begin{bmatrix}F_i(t) & 0\\ 0 & F_i(t)\end{bmatrix}\begin{bmatrix}\hat{\varsigma}_{ij}L & 0\\ -\varsigma_{1i}L-\varsigma_{2i}\sigma_d{}^j & 0\end{bmatrix}$$

$$+\begin{bmatrix}\hat{\varsigma}_{ij}L & 0\\ -\varsigma_{1i}L-\varsigma_{2i}\sigma_d{}^j & 0\end{bmatrix}^T\begin{bmatrix}F_i(t) & 0\\ 0 & F_i(t)\end{bmatrix}^T \qquad (26)$$

$$\times\begin{bmatrix}0 & 0\\ \hat{D}_{ij} & \delta_i\end{bmatrix}^T\prec 0.$$

By Lemma 2, it can be verified that (26) holds if

$$\begin{bmatrix}-\gamma L & *\\ (\hat{G}_{ij}-G_i)L-H_i\sigma_d{}^j & \varepsilon_{ij}\bar{D}_{ij}-\gamma L\end{bmatrix}$$

$$+\begin{bmatrix}\hat{\varsigma}_{ij}L & 0\\ -\varsigma_{1i}L-\varsigma_{2i}\sigma_d{}^j & 0\end{bmatrix}\begin{bmatrix}\varepsilon_{ij}^{-1} & 0\\ 0 & \varepsilon_{ij}{}^{-1}\end{bmatrix}$$

$$\times\begin{bmatrix}\hat{\varsigma}_{ij}L & 0\\ -\varsigma_{1i}L-\varsigma_{2i}\sigma_d{}^j & 0\end{bmatrix}^T\prec 0,$$

where $\bar{D}_{ij}=\hat{\delta}_{ij}\hat{\delta}_{ij}{}^T+\delta_i\delta_i^T$. Application of the Schur complement to the foregoing inequality results in (18). This completes the proof.

Remark 6: This paper makes some contributions to the research field of IDR by considering: 1) the uncertainties in the global state-matching procedure ([8,9,11] did not consider the uncertainties); 2) the structural property of the uncertainties in the procedure of discretization which is represented in Remark 4 ([13] thought over only the structural property of the local uncertainties); 3) the robust stabilization of the sampled-data fuzzy system which is shown in Theorem 1 ([9,11] considered only the stabilization without robustness); 4) the pade approximation method which can provide an accurate approximate uncertain model (it is possible to extend the more robust result than the result in [13]).

Remark 7: Instead of the LMIs (18)-(19), we can use more relaxed condition such as non-PDC issue which is discussed in [22,23].

Remark 8 [Sampling Rate Selection]**:** It is noted that the mapping of an analog system to its corresponding discretized system can be one-to-one if a selected sampling period satisfies the sampling condition. If a sampling period that violates the sampling theorem is selected, then the satisfactory state-matching may not be achieved. Hence it is suggested to choose $T\in\mathbb{R}_{>0}$ such that

$$T<\min_{(i,j)\in I_r\times I_r}\{\pi/\zeta(\lambda((A_i+\Delta A_i)+(B_i+\Delta B_i)K_c{}^j))\}$$

to acquire an acceptable state-matching performance.

4. SIMULATION RESULTS

In this section, the two AMB systems: 1) 1-DOF AMB system; 2) 2-axis controlled vertical shaft AMB system are discussed for designing the fuzzy controller.

4.1. Case A: 1-DOF AMB system

Figure 1 shows a diagram of the simplified 1-DOF AMB system. The 2-rule based fuzzy model of the concerned system is described by [7]

$$R_1 : If\ z_1(t)\ is\ \Gamma_1,\ \ Then\ \dot{x}(t) = A_1 x(t) + B_1 u(t),$$
$$R_2 : If\ z_1(t)\ is\ \Gamma_2,\ \ Then\ \dot{x}(t) = A_2 x(t) + B_2 u(t),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{2\Phi_0}{m\mu_0 A_g} + \dfrac{1}{m\mu_0 A_g} f_1(\phi(t)) \\ 0 & & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{2\Phi_0}{m\mu_0 A_g} + \dfrac{1}{m\mu_0 A_g} f_2(\phi(t)) \\ 0 & & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1/N \end{bmatrix}$$

the control flux function $f(\phi(t))$ is a nonlinear function, N is a number of turns in coil, m is the effective mass rotor, A_g is the electromagnet pole area, and μ_0 is air permeability. Table 1 shows the specific parameter of the 1-DOF AMB system. Assuming $f(\phi(t)) \in [M_1,\ M_2]$, then it can be represented by [7]

$$f(\phi(t)) = \mu_{\Gamma_1}(f(\phi(t)))M_2 + \mu_{\Gamma_2}(f(\phi(t)))M_1, \qquad (27)$$

where

$$\mu_{\Gamma_1}(f(\phi(t))) = \frac{f(\phi(t)) - M_1}{M_2 - M_1},$$
$$\mu_{\Gamma_2}(f(\phi(t))) = \frac{M_2 - f(\phi(t))}{M_2 - M_1}.$$

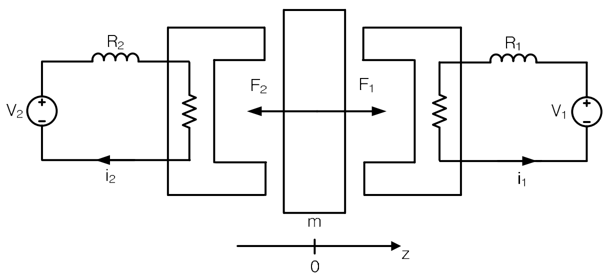

Fig. 1. Schematic diagram of a simplified 1-DOF AMB system.

Table 1. Nominal parameters of the 1-DOF AMB.

Parameter	Values
number of turns in coils N	321
effective mass rotor m	4.5 kg
electromagnet pole area A_g	137 mm
air permeability μ_0	1.25×10^{-6} H/m

During the simulation time, the system parameters are randomly varied within the bounds of 50 % of their nominal values. In this situation, we define the uncertain matrices as $\Delta A_i = D_i F_i(t) E_{1i}$, where the matrices $H =$ $[0\ 0.5\ 0]^T$ and $E_{1i} = [0\ 0.5\ \dfrac{2\Phi_0}{m\mu_0 A_g}]$. The simulations are performed in sampling period $T = 0.0004$ and we set the initial condition $x_c(0) = x_d(0) = [0.15\ \ 0\ \ -40]^T$. The well-constructed analog gains are computed by

$$K_c^{\,1} = [-2.6408\times10^6\ \ -2.0748\times10^4\ \ -2.0090\times10^6],$$
$$K_c^{\,2} = [-2.6988\times10^6\ \ -2.0979\times10^4\ \ -2.6057\times10^6].$$

Based on Theorem 1, the digital fuzzy gains are represented as

$$K_d^{\,1} = [-0.5909\times10^6\ \ -1.5001\times10^4\ \ -8.1287\times10^6],$$
$$K_d^{\,2} = [-0.5977\times10^6\ \ -1.5206\times10^4\ \ -8.2033\times10^6].$$

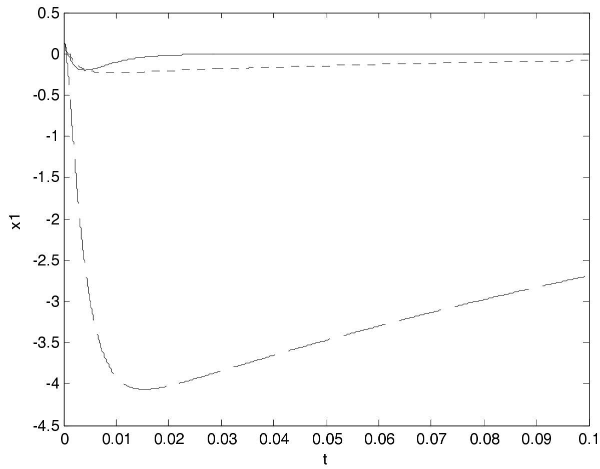

Fig. 2. State x_1 of the controlled 1-DOF AMB system with $T = 0.0004$: analog (solid), proposed (dotted), [12] (dashed).

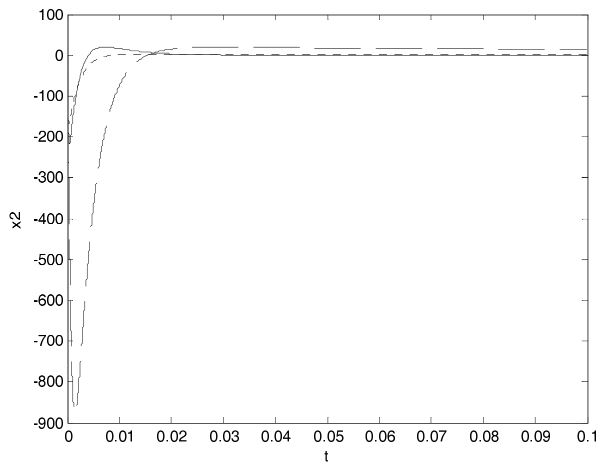

Fig. 3. State x_2 of the controlled 1-DOF AMB system with $T = 0.0004$: analog (solid), proposed (dotted), [12] (dashed).

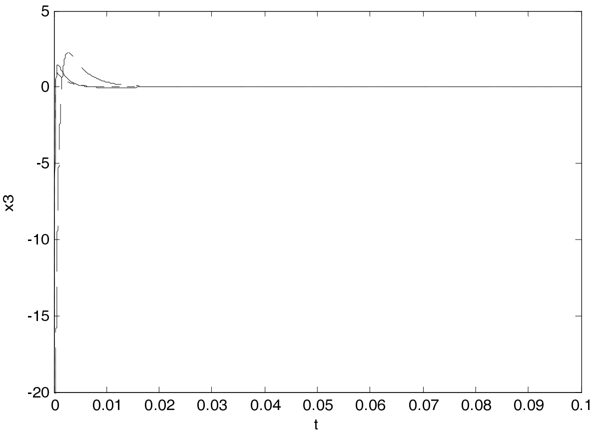

Fig. 4. State x_3 of the controlled 1-DOF AMB system with $T = 0.0004$: analog (solid), proposed (dotted), [12] (dashed).

Other available IDR method in [12] is also simulated at the same sampling period. As shown in Figs. 2-4, the state trajectories of the proposed method are identical to those of the original analog control system. However, the compared method which was used the bilinear approximation has poorer responses than our approach.

4.2. Case B: 2-axis Controlled Vertical Shaft AMB System

The AMB system employed in this research is a two-axis controlled vertical shaft magnetic bearing with a symmetric structure that has been studied previously in [6,24,25]. An outline of this system is depicted in Fig. 5.

The dynamic equation of AMB system is represented as follows [1] and [25]:

$$\dot{x}_1(t) = x_2(t),$$

$$\dot{x}_2(t) = \frac{k}{m}\left(\frac{(i_b + u(t))^2}{(G - \beta x_1(t))^2} - \frac{(i_b - u(t))^2}{(G + \beta x_1(t))^2}\right) + w(t),$$

where $i_p(t)$ is the control current in amperes, $x_1(t)$ is the rotor displacement measured in meters, k is the force constant, β is the sensitivity of air gap to shaft displacement, i_b is the bias current, G is the nominal air gap, and m is the mass of the rotor. Table 2 shows the specific parameters of studied AMB system.

Above AMB fuzzy system has two nonlinear terms ν_1 and ν_2 which can be obtained by

$$\nu_1 = \frac{1}{(G - \beta x_1(t))^2 (G + \beta x_1(t))^2},$$

$$\nu_2 = \frac{G2 + \beta^2 x_1^2(t)}{(G - \beta x_1(t))^2 (G + \beta x_1(t))^2}.$$

Assuming that $\nu_1 \in [M_{11},\ M_{12}]$ and $\nu_2 \in [M_{21},\ M_{22}]$ and endowing the following membership function:

$$\mu_{\Gamma_{11}} = \frac{\nu_1(t) - M_{12}}{M_{12} - M_{11}},\quad \mu_{\Gamma_{21}} = \frac{-\nu_1(t) + M_{11}}{M_{12} - M_{11}},$$

$$\mu_{\Gamma_{12}} = \frac{\nu_2(t) - M_{22}}{M_{22} - M_{21}},\quad \mu_{\Gamma_{21}} = \frac{-\nu_2(t) + M_{21}}{M_{22} - M_{21}}.$$

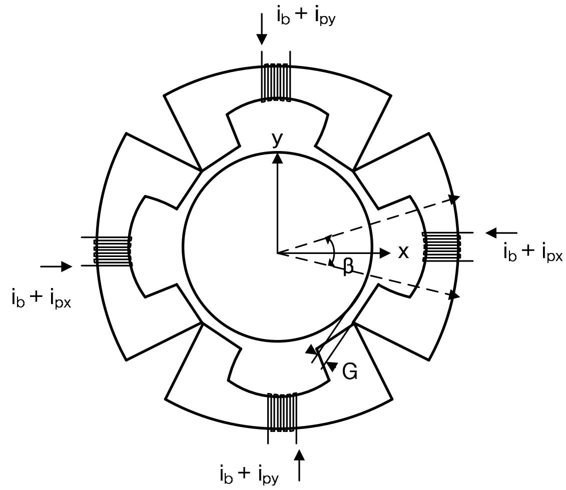

Fig. 5. 2-axis controlled vertical shaft AMB system.

Table 2. Nominal parameters of the 2-axis AMB system.

Parameter	Values
rotor mass m	4.5532 kg
Force constant k	$5.3379\times10^{-6}\ N(m^2/A^2)$
Sensitivity of air gap β	0.974
bias current i_b	$0.3A$
Nominal air gap G	$5.88\times10^{-4}m$

Casting the above system into a T-S fuzzy system, the nonlinear terms should be expressed as convex combinations. The fuzzy model for AMB system is constructed as following 4-rule fuzzy models [1]:

R_1 : *If* $z_1(t)$ *is* Γ_{11} *and* $z_2(t)$ *is* Γ_{12},
Then $\dot{x}(t) = A_1 x(t) + B_1 u(t)$

R_2 : *If* $z_1(t)$ *is* Γ_{11} *and* $z_2(t)$ *is* Γ_{22},
Then $\dot{x}(t) = A_2 x(t) + B_2 u(t)$

R_3 : *If* $z_1(t)$ *is* Γ_{21} *and* $z_2(t)$ *is* Γ_{12},
Then $\dot{x}(t) = A_3 x(t) + B_3 u(t)$

R_4 : *If* $z_1(t)$ *is* Γ_{21} *and* $z_2(t)$ *is* Γ_{22},
Then $\dot{x}(t) = A_4 x(t) + B_4 u(t)$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{4ki_b^2 G\beta}{m} M_{11} & 0 \end{bmatrix},\quad B_1 = \begin{bmatrix} 0 \\ \frac{2ki_b}{m} M_{12} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{4ki_b^2 G\beta}{m} M_{11} & 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 \\ \frac{2ki_b}{m} M_{22} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ \frac{4ki_b^2 G\beta}{m} M_{21} & 0 \end{bmatrix},\quad B_3 = \begin{bmatrix} 0 \\ \frac{2ki_b}{m} M_{12} \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 1 \\ \frac{4ki_b^2 G\beta}{m} M_{21} & 0 \end{bmatrix},\quad B_4 = \begin{bmatrix} 0 \\ \frac{2ki_b}{m} M_{22} \end{bmatrix}.$$

In order to analyze the influence of uncertainties, we set the variation of all system parameters as 30 % of their nominal values. That is, the uncertain matrix $\Delta A_i = D_i F_i(t) E_{1i}$ and $\Delta B_i = D_i F_i(t) E_{2i}$ can be represented as

$$D_1 = D_2 = D_3 = D_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.3 & 0.3 \end{bmatrix},$$

$$E_{11} = E_{12} = \begin{bmatrix} 0 & 0 \\ M_{11}\dfrac{4ki_b^2 G\beta}{m} & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_{13} = E_{14} = \begin{bmatrix} 0 & 0 \\ M_{12}\dfrac{4ki_b^2 G\beta}{m} & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_{21} = E_{23} = \begin{bmatrix} 0 \\ 0 \\ M_{21}\dfrac{2ki_b}{m} \end{bmatrix},$$

$$E_{21} = E_{23} = \begin{bmatrix} 0 \\ 0 \\ M_{22}\dfrac{2ki_b}{m} \end{bmatrix},$$

where $i \in I_4$ and we set $x_c(0) = x_d(0) = [2.54\times10^{-6}\ 0]^T$. Based on the Theorem 1, the gain matrices for T = 0.0005 are obtained as

$$K_d^{\,1} = [-23508.256 \quad -384.335],$$
$$K_d^{\,2} = [-23506.804 \quad -384.997],$$
$$K_d^{\,3} = [-23508.071 \quad -384.164],$$
$$K_d^{\,4} = [-23506.544 \quad -384.817],$$

which guarantee the asymptotic stability of the closed-loop system. As shown in Fig. 6, we observe that the state trajectory of the proposed method is almost identical to that of the original analog system. However, the other method in [12] does not give us the satisfactory state-matching performance. The simulation results for relatively large sampling period T = 0.0005 are also shown in Figs. 7. and 8. From those figures, we can recognize the proposed method is quite successful, even in the presence of parametric uncertainties for complex nonlinear systems. To sum up the two simulation results, we set the performance measures which is defined as

$$\mathrm{P} = \sum_{i=1}^{r}\left(\int_0^{t_f} \left|x_c(t) - x_d(t)\right| dt\right),$$

where t_f is the final time. As shown in Table 3, the proposed method shows better performance than the conventional method.

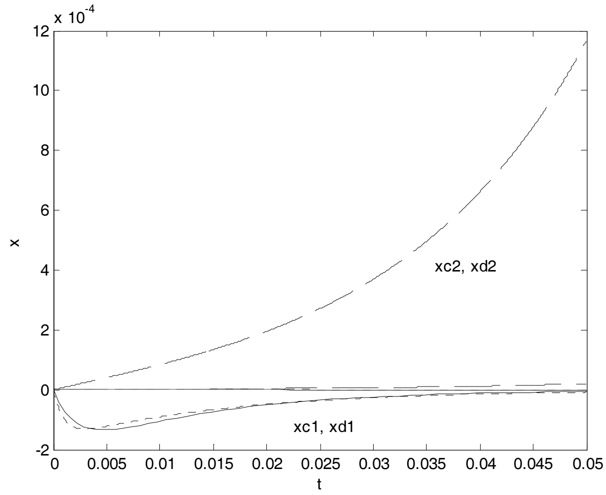

Fig. 6. Time responses of the 2-axis AMB system with T = 0.0002: analog (solid), proposed (dotted), [12] (dashed).

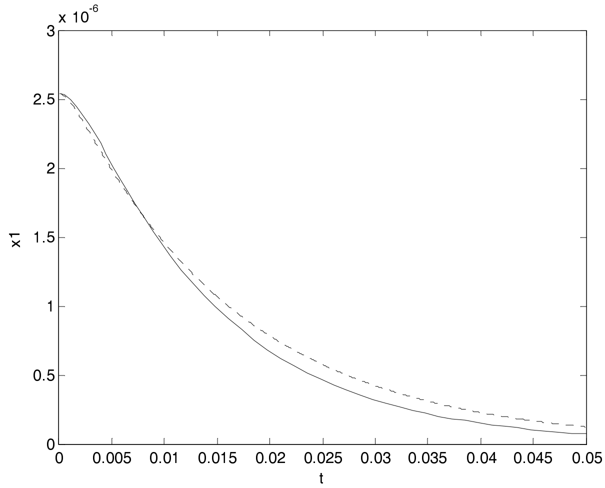

Fig. 7. Time responses x_1 by the proposed method with T = 0.0005: analog control (solid) and proposed (dotted).

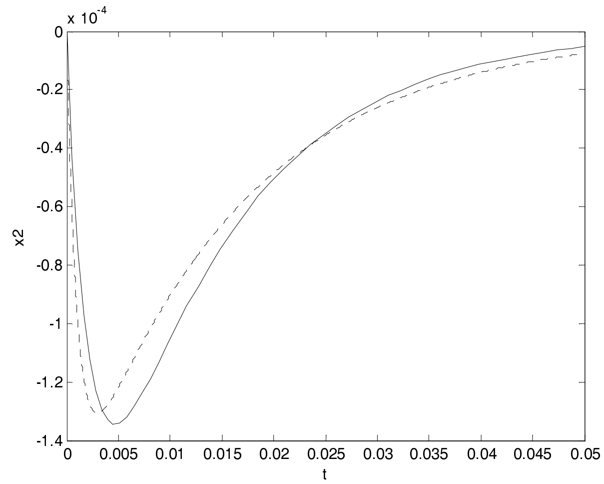

Fig. 8. Time responses x_2 by the proposed method with T = 0.0005: analog control (solid) and proposed (dotted).

Table 3. Comparison of the IDR performance P.

Method		[12]	Ours
Case A	$T = 0.0004$	2.5587	0.4602
	$T = 0.001$	4.3966	1.5046
Case B	$T = 0.0002$	9.0393×10^{-5}	1.7284×10^{-6}
	$T = 0.0005$	Unstable	2.0548×10^{-5}

5. CONCLUSIONS

This paper has presented the robust digital control for AMB systems which is composed as uncertain nonlinear system. We have investigated the parametric uncertainties of the concerned system based on the T-S fuzzy model so that we have achieved the robust stability. Also, in order to solve the digital implementation for real plants, we have presented the robust IDR method. The uncertainties in the plant dynamics is shown in the IDR condition by virtue of the pade and inverse-pade approximation method. The sufficient conditions for robust stabilizing controller designs have been given in terms of solutions to a set of LMIs. Through the simulation results of the two HMB systems, we have demonstrated the superiority of the proposed IDR method.

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Hwa Chang Sung received his B.S., M.S. and Ph.D. degrees from Yonsei University, Seoul, Korea, in 2005, 2007 and 2012, respectively, in electrical and electronic engineering. His current research interests include robust fuzzy control, wind power control, magnetic bearing system, and intelligent digital redesign.

Jin Bae Park received his B.E. degree in electrical engineering from Yonsei University, Seoul, Korea, and his M.S. and Ph.D. degrees in electrical engineering from Kansas State University, Manhattan, in 1977, 1985, and 1990, respectively. Since 1992, he has been with the Department of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea, where he is currently a professor. His research interests include robust control and filtering, nonlinear control, intelligent mobile robot, fuzzy logic control, neural networks, and genetic algorithms. He served as Editor-in-Chief for the International Journal of Control, Automation, and Systems (IJCAS) (2006-2010) and is serving as the Vice-President for Institute of Control, Robotics, and Systems (ICROS) (2009-present).

Young Hoon Joo received his B.S., M.S., and Ph.D. degrees in electrical engineering from Yonsei University, Seoul, Korea, in 1982, 1984, and 1995, respectively. He worked with Samsung Electronics Company, Seoul, Korea, from 1986 to 1995, as a project manager. He was with the University of Houston, Houston, TX, from 1998 to 1999, as a visiting professor in the Department of Electrical and Computer Engineering. He is currently a professor in the Department of Control and Robotics Engineering, Kunsan National University, Korea. His major interest is mainly in the field of intelligent robot, intelligent control, and human-robot interaction. He served as President for Korea Institute of Intelligent Systems (KIIS) (2008-2009) and is serving as Editor for the International Journal of Control, Automation, and Systems (IJCAS) (2008-present) and is serving as the Vice-President for the Korean Institute of Electrical Engineers (KIEE) (2011-present).

Kuo Chi Lin received his M.S. and Ph.D. degrees in aerospace engineering from University of Michigan, Ann Arbor, Michigan, USA, in 1986, 1990, respectively. He has been a joint faculty between Institute for Simulation and Training (IST) and Department of Mechanical, Materials and Aerospace Engineering (MMAE) since joining University of Central Florida (UCF) in 1990. He is currently the Associate Chair of MMAE and an Associate Faculty in Florida Space Institute (FSI) at UCF. His major interest is mainly in the fields of renewable energy, rotating machinery, modeling and simulation, and small satellite.