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RBF Neural Networks-Based Robust Adaptive Tracking Control for Switched Uncertain Nonlinear Systems

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Abstract: In this paper, a robust adaptive H∞ control scheme is presented for a class of switched uncertain nonlinear systems. Radical basis function neural networks (RBF NNs) are employed to approximate unknown nonlinear functions and uncertain terms. A robust H∞ controller is designed to enhance robustness due to the existence of the compound disturbance which consists of approximation errors of the neural networks and external disturbance. Adaptive neural updated laws and switching signals are deducted from multiple Lyapunov function approach. It is proved that with the proposed control scheme, the resulting closed-loop switched system is robustly stable and uniformly ultimately bounded (UUB) such that good capabilities of tracking performance is attained and H∞ tracking error performance index is achieved. A practical example shows the effectiveness of the proposed control scheme.

Keywords: Radical basis function neural networks, robust adaptive H∞ control, switched uncertain nonlinear systems, tracking performance.

1. INTRODUCTION

Switched systems are dynamical systems consisting of a collected of continuous-time subsystems and a switching rule that orchestrates the switching among them, which have attracted much attention in the last decades. For control analysis and synthesis of switched systems, most of the proposed results are considering the linear subsystems case, see [1-4] and references therein. A few results which are of practical interest on control synthesis of switched nonlinear systems are introduced [5-8].

It's known that stability, robustness, and performance properties of the switched nonlinear systems are affected seriously by the existence of uncertainty which is inherent in practical controlled systems. However, there are some controlled systems that are not only characterized by the unstructured uncertainties, but also represented by the terms which cannot be modeled .When the system nonlinearities are not known completely but some bounds on them are known, the nonlinearities can be approximated either by neural networks or by fuzzy systems [9-13,19,21]. Liu and Tong [9] have developed an adaptive control scheme by the universal approximation theorem of the fuzzy logic systems. The back propagation (BP) learning algorithm is presented for stable and efficient online control [10]. In addition, several other papers such as [11-13] which have been widely used in the identification and control of complex systems, are also demonstrated that the RBF NNs and multiplayer feed-forward networks can approximate arbitrary nonlinear continuous function to a given accuracy.

On the other hand, most of adaptive control methods are proposed for nonlinear systems with the condition that an accurate model of the plant is available, and an approximator is used as a tool for modeling nonlinear unknown functions [14-17]. Adaptive neural control schemes have been found to be particularly useful for the control of highly uncertain, nonlinear and complex systems [14]. However, at present, the researches on adaptive neural control for switched nonlinear systems are rare. Few attempts have been made (and are being made) to pursue this novel idea. Han, Ge and Lee [18] have presented an adaptive neural control for a class of switched triangular systems with switching jumps. They utilize discontinuous adaptive NNs control for dealing with the discrepancy between control gains, and use the classical adaptive control for handling NNs approximation errors in dealing with switching jumps. In this paper, a robust adaptive H∞ control scheme is presented for a class of switched uncertain nonlinear systems. The principal contribution described here are: (i) This work considering the existence of uncertainties is an extension of reference [7]. Also, a practical example which has good capabilities of tracking performance

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shows that the control scheme proposed in this paper is suit for practical implementation. (ii) The switching signals have been derived from the multiple switched Lyapunov function method and the adaptive neural updated laws in terms of projection approach are given. (iii) A robust H∞ compensator is introduced to enhance the robustness of switched uncertain nonlinear systems and improve the tracking error attenuation quality. Finally, the robust adaptive H∞ control scheme proposed can guarantee the resulting close-loop switched system is robustly stable such that the actual output follows the desired output and the H∞ tracking performance from the compound disturbance to the tracking error is achieved.

The rest of the paper is organized as follows. In Section 2, a class of switched uncertain nonlinear systems and the control objective are introduced. In Section 3, a control scheme of robust adaptive H∞ control using RBF NNs is presented. A numerical example is treated to illustrate the effectiveness of the design approach in Section 4. Finally, a conclusion is given in Section 5.

2. PROBLEM FORMULATION & PRELIMINARIES

Consider a class of switched uncertain nonlinear systems which can be expressed in the following form:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) \quad (1 \le i \le n-1) \\ \dot{x}_n(t) = f_{\sigma(t)}(x(t)) + \Delta f_{\sigma(t)}(x(t)) + (g_{\sigma(t)}(x(t)) \\ \qquad + \Delta g_{\sigma(t)}(x(t))) \cdot u(t) + d(t) \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $t \ge 0$, $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in R^n$ denotes the state vector of the systems, which is available. $u \in R$, $y \in R$ are the control input and the control output, respectively, and $d \in L_2[0, \infty)$ is the external disturbance. $\sigma(t): [0, +\infty) \to \Xi \overset{def}{=} \{1, 2, \cdots, N\}$ is a piecewise constant function called switching signal (or law), which takes values in the compact set Ξ. If $\sigma(t) = i$, then we say the i-th subsystem $\dot{x}_n = f_i(x) + \Delta f_i(x) + (g_i(x) + \Delta g_i(x)) \cdot u + d$ is active on and the remaining subsystems are inactive. The functions $f_i(x)$, $g_i(x): R^n \to R^n$ $(i \in \Xi)$ are both smooth functions that are known, and $\Delta f_i(x), \Delta g_i(x)$: $R^n \to R^n$ represent the uncertainties of $f_i(x)$, $g_i(x)$, respectively.

The control objective in this paper is to design a robust adaptive control scheme such that the actual output y of system (1) follows the any given bounded desired output signal while all the signals in the resulting closed-loop switched system remain bounded. To achieve the proposed control objective, we have the following assumptions.

Assumption 1: The desired signal y_d is continuously differentiable in the interval [0, ∞) and up to its n-th time derivative. Define the trajectory $Y_d = (y_d, \dot{y}_d, \cdots, y_d^{(n-1)})^T$ which is continuous and available, and $Y_d \in \Omega_d \in R^n$ with Ω_d known compact set, then the output tracking error is $e = Y_d - x = (e_0, e_1, \cdots, e_{n-1})^T = (e, \dot{e}, \cdots, e^{(n-1)})^T \in R^n$, i.e., $e_0 = y_d - x_1$, $e_1 = \dot{e} = \dot{y}_d - x_2$,…, $e_{n-1} = e^{(n-1)} = y_d^{(n-1)} - x_n$, also $e_n = e^{(n)} = y_d^{(n)} - \dot{x}_n$. Define $K = (k_1, k_2, \cdots, k_n)$, where K is Hurwitz vector.

Assumption 2: The nonlinear functions $g_i(x)$ and $g_i(x) + \Delta g_i(x)$ satisfied $|g_i(x)| \ge g_l > 0$, $|g_i(x) + \Delta g_i(x)| \ge g_L > 0$, i.e., $g_i(x), g_i(x) + \Delta g_i(x)$ can be either positive or negative, where g_l, g_L are the lower bounded of $g_i(x)$ and $g_i(x) + \Delta g_i(x)$, respectively. In many of the earlier works it is assumed that $g_i(x) > 0$, but in this paper, the assumption has been relaxed by considering $|g_i(x)| > 0$.

3. DESIGN OF ROBUST ADAPTIVE H∞ CONTROL USING RBF NNS

The purpose of this section is to develop a novel design procedure for robust adaptive H∞ control using RBF NNs for switched uncertain nonlinear systems (1).To achieve the proposed control objective in the aforementioned section, it's satisfied that tracking error converges to 0, i.e., $e \to 0$. If $f_i(x)$, $g_i(x)$, $\Delta f_i(x)$, $\Delta g_i(x)$ are both known and disturbance vector d=0, according to the feedback linearizable techniques [17,20,21], define the control law as follows:

$$u = (g_{\sigma(t)}(x) + \Delta g_{\sigma(t)}(x))^{-1} \cdot \left(y_d^{(n)} - f_{\sigma(t)}(x) - \Delta f_{\sigma(t)}(x) + \sum_{i=1}^{n-1} k_i \cdot e_i \right). \tag{2}$$

Substituting (2) into (1), we obtain:

$$\dot{x}_n = y_d^{(n)} + \sum_{i=1}^{n-1} k_i \cdot e_i \Rightarrow e_n + \sum_{i=1}^{n-1} k_i \cdot e_i = 0. \tag{3}$$

From (3) we get $\lim_{t \to \infty} e(t) = 0$, the control objective is achieved.

However, the robustness and stability are affected seriously by the existence of nonlinear systems with unmodeled dynamics, external disturbance and uncertain terms in practical control systems. If these uncertain terms and nonlinear terms are bounded, different function approximators like neural networks or fuzzy systems can be used to estimate these nonlinear functions. In control engineering, RBF NNs are usually used as a tool for modeling nonlinear functions up to a small error tolerance because of their good capabilities in function approximation. In this paper we use RBF NNs to approximate unknown functions $f_i(x)$, $g_i(x)$ and the uncertain nonlinear terms, $\Delta f_i(x)$, $\Delta g_i(x)$ considering the existence of approximation error of the neural networks $\delta_i(x)$, for solving the restraints of feedback linearizable techniques which need exact models.

When $f_i(x)$, $g_i(x)$ are known, using RBF NNs to approximate the uncertain terms $\Delta f_i(x)$, $\Delta g_i(x)$, we define as follows:

$$\begin{cases}\Delta f_i(x) = \Delta f_i(x, W_{\Delta f}) = W_{\Delta f}^T \cdot S_{\Delta fi}(x) + \delta_{\Delta f}(x) \\ \Delta g_i(x) = \Delta g_i(x, W_{\Delta g}) = W_{\Delta g}^T \cdot S_{\Delta gi}(x) + \delta_{\Delta g}(x),\end{cases} \tag{4}$$

where $i \in \Xi$, $\forall x \in \Omega$ for some compact set $\Omega \subseteq R^n$, $W_{\Delta f}$, $W_{\Delta g}$ are vectors of adjustable weights:

$$\begin{cases}W_{\Delta f} = [W_{\Delta f1}, W_{\Delta f2}, \cdots W_{\Delta fp}] \\ W_{\Delta g} = [W_{\Delta g1}, W_{\Delta g2}, \cdots W_{\Delta gq}],\end{cases} \tag{5}$$

$S_{\Delta fi}(x): x \to R^p$, $S_{\Delta gi}(x): x \to R^q$ denote vectors of Gaussian basis function, defining as:

$$\begin{aligned}&S_{\Delta fi}(x) = [S_{\Delta fi1}(x), S_{\Delta fi2}(x), \cdots, S_{\Delta fip}(x)], \\ &S_{\Delta gi}(x) = [S_{\Delta gi1}(x), S_{\Delta gi2}(x), \cdots, S_{\Delta giq}(x)],\end{aligned} \tag{6}$$

where

$$S_{\Delta fij}(x) = \exp\left[-\frac{(x-\mu_{\Delta fij})^T (x-\mu_{\Delta fij})}{\eta_{\Delta fij}^2}\right],$$

$$S_{\Delta gij}(x) = \exp\left[-\frac{(x-\mu_{\Delta gij})^T (x-\mu_{\Delta gij})}{\eta_{\Delta gij}^2}\right]$$

stand for Gaussian basis function, and $(\mu_{\Delta fij}, \eta_{\Delta fij})$, $(\mu_{\Delta gij}, \eta_{\Delta gij})$ are, respectively, center vectors and width of the hidden element for RBF NNs to approximate the functions $\Delta f_i(x)$ and $\Delta g_i(x)$.

We define the estimate value of the unknown nonlinear functions $\Delta f_i(x)$ and $\Delta g_i(x)$, respectively, as follows:

$$\Delta \hat{f}_i(x) = \hat{W}_{\Delta f}^{\mathrm{T}} \cdot S_{\Delta fi}(x), \quad \Delta \hat{g}_i(x) = \hat{W}_{\Delta g}^T \cdot S_{\Delta gi}(x), \tag{7}$$

where $\hat{W}_{\Delta f}$, $\hat{W}_{\Delta g}$ are the estimate value of weights vector $W_{\Delta f}$, $W_{\Delta g}$, then weights vector errors of RBF NNs are described by:

$$\tilde{W}_{\Delta f} = W_{\Delta f} - \hat{W}_{\Delta f}, \quad \tilde{W}_{\Delta g} = W_{\Delta g} - \hat{W}_{\Delta g}. \tag{8}$$

Respectively, the optimal weights and reconstruction approximate errors of RBF NNs are defined in the following:

$$\begin{cases}W_{\Delta f}^* := \arg\min\limits_{W_{\Delta f} \in \Omega_{\Delta f}} \{\min\limits_{i \in \Xi} \{\sup\limits_{x \in R^n} \left|\Delta f_i(x) - \Delta \hat{f}_i(x)\right|\}\} \\ W_{\Delta g}^* := \arg\min\limits_{W_{\Delta g} \in \Omega_{\Delta g}} \{\min\limits_{i \in \Xi} \{\sup\limits_{x \in R^n} \left|\Delta g_i(x) - \Delta \hat{g}_i(x)\right|\}\},\end{cases} \tag{9}$$

$$\begin{aligned}\delta_i &= (\Delta f_i(x) - \Delta \hat{f}_i(x, W_{\Delta f}^*)) \\ &\quad + (\Delta g_i(x) - \Delta \hat{g}_i(x, W_{\Delta g}^*)) \cdot u \\ &= (\tilde{W}_{\Delta f}^T \cdot S_{\Delta fi}(x) + \delta_{\Delta f}(x)) \\ &\quad + (\tilde{W}_{\Delta g}^T \cdot S_{\Delta gi}(x) + \delta_{\Delta g}(x)) \cdot u,\end{aligned} \tag{10}$$

where $\Omega_{\Delta f}$ and $\Omega_{\Delta g}$ are known compact subsets of R^p and R^q, respectively.

However, in many practical systems, the control accuracy is influenced seriously by the existence of external disturbance $d(t)$ and approximation errors $\delta_i(x)$ of RBF NNs. Define $\omega_i = \delta_i(x) + d$ as the compound disturbance of system (1), where $\omega_i \in L_2[0,t]$, $t \in [0,\infty)$.

When $\omega_i \neq 0$, the robust H∞ compensator is introduced to enhance the systems' robustness and improve the tracking error attenuation quality, satisfying the H∞ tracking error performance index [19,22]:

$$\begin{aligned}\int_0^t e^T \cdot Q_i \cdot e\, dt \le e^T(0) \cdot P_i \cdot e(0) + \Gamma_{\Delta f}^{-1} \tilde{W}_{\Delta f}^T(0) \tilde{W}_{\Delta f}(0) \\ + \Gamma_{\Delta g}^{-1} \tilde{W}_{\Delta g}^T(0) \tilde{W}_{\Delta g}(0)) + \gamma^2 \cdot \int_0^t \omega_i^T \omega_i dt,\end{aligned} \tag{11}$$

where Q_i are symmetrical positive semi-definite matrices; $\Gamma_{\Delta f}$ and $\Gamma_{\Delta g}$ which are both positive constants denote the adaptive gains; γ is disturbance attenuation level.

When $e(0) = 0$, $\tilde{W}_{\Delta f}(0) = 0$, $\tilde{W}_{\Delta g}(0) = 0$, the H∞ tracking error performance index (11) can be written in another form:

$$\sup_{\omega_i \in L_2[0,t]} \frac{\|e\|_{\min}}{\|\overline{\omega}\|} \le \gamma, \tag{12}$$

where

$$\begin{cases}\|e\|_{\min} = \min\left\{\int_0^t e^T \cdot Q_i \cdot e\, dt \ \ (i = 1, 2, \cdots, n)\right\} \\ \|\overline{\omega}\| = \max\left\{\int_0^t \omega_i^T \omega_i dt \ \ (i = 1, 2, \cdots, n)\right\}.\end{cases}$$

According to the feedback linearizable technique and the approximation property of RBF NNs, the robust adaptive H∞ control law is designed as follows:

$$\begin{aligned}u &= (g_i(x) + \Delta \hat{g}_i(x, W_g^*))^{-1} \\ &\quad \cdot \left(\sum_{i=1}^{n-1} k_i \cdot e_i + y_d^{(n)} - f_i(x) - \Delta \hat{f}_i(x) - u_h\right),\end{aligned} \tag{13}$$

where u_h is the control law of robust H∞ compensator.

Substituting (13) into (1), we obtain:

$$\begin{aligned}e_n + K \cdot e = \Delta \hat{f}_i(x) - \Delta f_i(x) + (\Delta \hat{g}_i(x) - \Delta g_i(x)) \\ \cdot u + u_h - d.\end{aligned} \tag{14}$$

From (14) and Assumption 1, the output tracking error dynamic equation of switched uncertain nonlinear system (1) is given by:

$$\begin{aligned}\dot{e} &= A \cdot e + B[\Delta \hat{f}_i(x) - \Delta f_i(x) + (\Delta \hat{g}_i(x) \\ &\quad - \Delta g_i(x)) \cdot u + u_h - d] \\ &= A \cdot e + B \cdot u_h - B \cdot \omega_i,\end{aligned} \tag{15}$$

where the matrix

$$A=\begin{pmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -k_1 & -k_2 & \cdots & -k_{n-1} & -k_n\end{pmatrix},\quad B=\begin{pmatrix}0\\0\\ \vdots\\1\end{pmatrix}$$

Lemma 1 [19,24]**:** For the output tracking error dynamic equation (15), given a positive constant $\gamma > 0$, if there exists matrices $P_i = P_i^T > 0,\ Q_i = Q_i^T > 0,$ and exists a constant $\xi \geq 2\gamma^2$, the necessary and sufficient condition for the stability of the resulting closed-loop system satisfies two conditions as follows: (i) matrices P_i are the positive definite solutions of Ricatti inequality $A^T P_i + P_i A + Q_i + (\gamma^{-2} - 2\xi^{-1}) \cdot P_i BB^T P_i \leq 0$; (ii) there exists a state feedback controller $u_h = -\xi^{-1} \cdot B^T \cdot P_i \cdot e$ satisfying the H∞ control performance index.

Remark 1: Based on Schur's lemma, above Ricatti inequality in Lemma 1 can be converted into a linear matrix inequality (LMI):

$$\begin{bmatrix} A^T P_i + P_i A + Q_i & \sqrt{\gamma^{-2} - 2\xi^{-1}} \cdot P_i B \\ \sqrt{\gamma^{-2} - 2\xi^{-1}} \cdot B^T P_i & -I \end{bmatrix} \leq 0. \qquad (16)$$

Then, the switching signal σ(t) is designed as:

$$\sigma(t) = \arg\min\left\{ \|\omega_i\|^2 < \frac{\|e\|_{\min}^2}{\gamma^2}, (i \in \Xi \overset{def}{=} 1, 2, \cdots, N) \right\}, \qquad (17)$$

where $\{t_k\}_{k=0}^{\infty} := \{t_0, t_1, \cdots, t_k, \cdots\}$ is said to be the sequence of switching times, t_0 is the initial time, and t_k is the k-th switching time. Also, for $i, j \in \Xi,$ we define the following matrix inequality:

$$\begin{bmatrix} -P_i & (\Pi_{i,j} + I)^T \cdot P_j \\ P_j \cdot (\Pi_{i,j} + I) & -P_j \end{bmatrix} < 0, \qquad (18)$$

where $\Pi_{i,j}$ are known $n \times n$ constant matrices. In general, when $\Pi_{i,i}$=0 means that there is no switched jump when a subsystem is remaining active.

To guarantee the weights of RBF NNs bounded, from Assumption 2 the adaptive neural updated laws (i.e.,the weights update laws) in terms of projection approach [24] are addressed as:

$$\dot{\hat{W}}_{\Delta f} = \begin{cases} \Gamma_{\Delta f} B^T P_i e S_{\Delta fi}(x) \\ \qquad \|W_{\Delta f}\| = M_{\Delta f} \text{ and } W_{\Delta f}{}^T \Gamma_{\Delta f} B^T P_i e S_{\Delta fi}(x) < 0 \\ \Gamma_{\Delta f} B^T P_i e S_{\Delta fi}(x) - \dfrac{W_{\Delta f}{}^T W_{\Delta f} (\Gamma_{\Delta f} B^T P_i e S_{\Delta fi}(x))}{\|W_{\Delta f}\|^2} \\ \qquad \|W_{\Delta f}\| < M_{\Delta f} \text{ or } \|W_{\Delta f}\| \\ \qquad\qquad = M_{\Delta f} \text{ and } W_{\Delta f}{}^T \Gamma_{\Delta f} B^T P_i e S_{\Delta fi}(x) \geq 0 \end{cases} \qquad (19)$$

$$\dot{\hat{W}}_{\Delta g} = \begin{cases} \Gamma_{\Delta g} B^T P_i e S_{\Delta gi}(x) u \\ \qquad \|W_{\Delta g}\| < M_{\Delta g} \text{ or } \|W_{\Delta g}\| \\ \qquad\qquad = M_{\Delta g} \text{ and } W_{\Delta g}{}^T \Gamma_{\Delta g} B^T P_i e S_{\Delta gi}(x) \geq 0 \\ \Gamma_{\Delta g} B^T P_i e S_{\Delta gi}(x) u - \dfrac{W_{\Delta g}{}^T W_{\Delta g} (\Gamma_{\Delta g} B^T P_i e S_{\Delta gi}(x))}{\|W_{\Delta g}\|^2} u \\ \qquad \|W_{\Delta g}\| = M_{\Delta g} \text{ and } W_{\Delta g}{}^T \Gamma_{\Delta g} B^T P_i e S_{\Delta gi}(x) < 0, \end{cases} \qquad (20)$$

where $\Gamma_{\Delta f}$ and $\Gamma_{\Delta g}$ which are both positive constant in general denote the adaptive gains. Also, $M_{\Delta f}$ and $M_{\Delta g}$ are positive design parameters.

According to the above analysis, for the switched uncertain nonlinear system (1), we have the following results.

Theorem 1: Supposed that Assumptions 1, 2, and Lemma 1 hold for the system (1). The proposed robust adaptive H∞ control law (13), adaptive neural laws (19) and (20) , together with switching signal (17) guarantee that the resulting closed-loop switched system is robustly stable and uniformly ultimately bounded (UUB) while the actual output follows the desired output signal and H∞ tracking error performance index is achieved.

Proof: Consider the multiple Lyapunov function candidate:

$$\begin{aligned} V = &\sum_i^n \theta_i(t) \cdot e^T P_i e + \Gamma_{\Delta f}{}^{-1} \cdot \tilde{W}_{\Delta f}{}^T \cdot \tilde{W}_{\Delta f} \\ &+ \Gamma_{\Delta g}{}^{-1} \cdot \tilde{W}_{\Delta g}{}^T \cdot \tilde{W}_{\Delta g}, \end{aligned} \qquad (21)$$

where the characteristic function:

$$\theta_i(t) := \begin{cases} 1 & t \in \Omega_i \\ 0 & t \notin \Omega_i, \end{cases}$$

$\Omega_i = \{t \mid$ the ith subsystem is active at time instant t$\}$.

For $t \in (t_{k-1}, t_k] \in \Omega_{r_k}\ (r_k \in \Xi),$ and $t \in (t_k, t_{k+1}] \in \Omega_{r_{k+1}}$, from (18) and (21), we obtain:

$$\begin{aligned} \Delta V(t) &= V(t_{k+1}) - V(t_k) \\ &= e^T(t_{k+1}) P_{r_{k+1}} e(t_{k+1}) - e^T(t_k) P_{r_k} e(t_k) \\ &= e^T(t_k) \cdot [(\Pi_{r_k, r_{k+1}} + I)^T \cdot P_{r_{k+1}} \\ &\quad \cdot (\Pi_{r_k, r_{k+1}} + I) - P_{r_k}] \cdot e(t_k) \\ &< 0. \end{aligned} \qquad (22)$$

Then, for $\forall t \in (t_{k-1}, t_k] \in \Omega_{r_k}$, taking the time derivative of V along the solutions of (13), (17), (19) and (20) yields:

$$\begin{aligned} \dot{V} &= \dot{e}^T P_{r_k} e + e^T P_{r_k} \dot{e} + \Gamma_{\Delta f}{}^{-1} \cdot \tilde{W}_{\Delta f}{}^T \cdot \dot{\tilde{W}}_{\Delta f} \\ &\quad + \Gamma_{\Delta g}{}^{-1} \cdot \tilde{W}_{\Delta g}{}^T \cdot \dot{\tilde{W}}_{\Delta g} \\ &= \dot{e}^T [A^T P_{r_k} + P_{r_k} A - 2\xi^{-1} P_{r_k} BB^T P_{r_k}] e \\ &\quad + 2\omega_{r_k}{}^T B^T P_{r_k} e \end{aligned} \qquad (23)$$

$$
\begin{aligned}
&+2\tilde{W}_{\Delta f}{}^{T}[B^{T}P_{r_k}eS_{\Delta fi}(x)+\Gamma_{\Delta f}{}^{-1}\dot{\tilde{W}}_{\Delta f}] \\
&+2\tilde{W}_{\Delta g}{}^{T}[B^{T}P_{r_k}eS_{\Delta gi}(x)u+\Gamma_{\Delta g}{}^{-1}\dot{\tilde{W}}_{\Delta g}] \\
&=-e^{T}Q_{r_k}e+\gamma^{2}\omega_{r_k}{}^{T}\omega_{r_k} \\
&\quad+(\gamma^{-1}B^{T}P_{r_k}e-\gamma\omega_{r_k})^{T}(\gamma^{-1}B^{T}P_{r_k}e-\gamma\omega_{r_k}) \\
&\le\gamma^{2}\omega_{r_k}{}^{T}\omega_{r_k}-e^{T}Q_{r_k}e \\
&\le\gamma^{2}\left\|\omega_{r_k}\right\|^{2}-\left\|e\right\|_{\min}{}^{2}
\end{aligned}
$$

From (17), (22) and (23), we have $\dot{V}(t)<0$, i.e., the resulting closed-loop switched system is stable. Then, integrating the above inequality from 0 to t yields:

$$
\begin{aligned}
&\int_0^t \dot{V}dt \le \gamma^2 \cdot \int_0^t \omega_{r_k}{}^T \omega_{r_k} dt - \int_0^t e^T Q_{r_k} e dt \\
&\Rightarrow V(t)-V(0) \le \gamma^2 \left\|\omega_{r_k}\right\|^2 - \left\|e\right\|_{\min}{}^2 .
\end{aligned}
\tag{24}
$$

From (24) and Barbalat's Lemma [24,25], we get: $\lim_{t\to\infty} e(t)=0.$ Therefore, it implies that the tracking error e is UUB. From Assumptions 1, 2, and (13), the control law u is bounded. Furthermore, controller (13) does not depend on the system model, which implies that the controller is strongly robust with respect to the unknown nonlinear functions and bounded disturbance.

$\because V(t)>0, when\ \ V(0)=0,$ we obtain:

$$
\gamma > \sup_{\omega_{r_k}\in L_2[0,t]} \frac{\| e \|_{\min}}{\| \omega_{r_k} \|}. \tag{25}
$$

So far, the proof of Theorem 1 and the controller design have been completed.

Remark 2: The conditions (17)-(20) reflect the conservativeness of adaptive control in switched systems. For improving accuracy, small $\Gamma_{\Delta f}$ and $\Gamma_{\Delta g}$, leading to small $M_{\Delta f}$ and $M_{\Delta g}$, are desired. However, (17) shows that the property of the switching signals and the gain must be taken into account in selecting γ. Thus, given a switching signal, arbitrary control accuracy cannot be achieved for switched systems. Instead, the smaller $M_{\Delta f}$ and $M_{\Delta g}$ are, the better the achievable control accuracy is.

Remark 3: The control law u_h of robust H∞ compensator is used to improve the robustness of the controller in the presence of the reconstruction approximate errors of RBF NNs. For the (19) and (20), it is not recommended to use very large positive design parameters $M_{\Delta f}$ and $M_{\Delta g}$ because this may lead to a high-gain control and increase the bandwidth of the closed-loop system. In addition, large adaptation gain also results in a high-gain control. Therefore, in practical applications the parameters should be adjusted carefully for achieving suitable performance and control action.

4. DESIGN EXAMPLE

The performance of the proposed control scheme is demonstrated through simulation results. In this section, consider switched uncertain nonlinear systems (1) of two inverted pendulum systems [24,25]:

$$
\Sigma_1 : \begin{cases}
f_1(x) = \dfrac{g\sin x_1 - \dfrac{m_1 l x_2^2 \cos x_1 \sin x_1}{m_{c1}+m_1}}{l\left(\dfrac{4}{3} - \dfrac{m_1 \cos x_1}{m_{c1}+m_1}\right)}, \\
\Delta f_1(x) = \mu_1 \cos(0.5t), \\
g_1(x) = \dfrac{\dfrac{\cos x_1}{m_{c1}+m_1}}{l\left(\dfrac{4}{3} - \dfrac{m_1 \cos x_1}{m_{c1}+m_1}\right)}, \\
\Delta g_1(x) = \rho_1 \cdot \sin(0.5t), \\
d_1 = 0.2e^{-0.1t}\sin(20t),
\end{cases}
$$

$$
\Sigma_2 : \begin{cases}
f_2(x) = \dfrac{g\sin x_2 - \dfrac{m_2 l x_2^2 \cos x_1 \sin x_1}{m_{c2}+m_2}}{l\left(\dfrac{4}{3} - \dfrac{m_2 \cos x_1}{m_{c2}+m_2}\right)}, \\
\Delta f_2(x) = \mu_2 \cos(0.5t), \\
g_2(x) = \dfrac{\dfrac{\cos x_1}{m_{c2}+m_2}}{l\left(\dfrac{4}{3} - \dfrac{m_2 \cos x_1}{m_{c2}+m_2}\right)}, \\
\Delta g_2(x) = \rho_2 \cdot \sin(0.5t), \\
d_2 = 0.3e^{-0.1t}\sin(20t).
\end{cases}
$$

Suppose that the trajectory-planning problem for a weight-lifting operation is considered and this inverted pendulum system suffers from uncertainties and exogenous disturbances. The design of control objective is the actual output $y = x_1$ follows the desired output signal $y_d = 0.5\cdot\sin(1.5\cdot t)+\sin(0.5\cdot t).$ According to the design procedures in Section 3, we select: the mass of the pendulum $m_c = (m_{c1}, m_{c2}) = (1, 2),$ the mass of the pendulum, the mass of the vehicle $m = (m_1, m_2) = (0.1,$ $0.2)$, the length of the pendulum $l = 0.5$ m, $\mu = (\mu_1, \mu_2)$ $= (0.3, 0.5),$ $\xi = 1.05,$ $\rho = (\rho_1, \rho_2) = (0.2, 0.2),$ $\gamma = 0.75,$ $\Gamma_{\Delta f} = 0.11,$ $\Gamma_{\Delta g} = 0.04,$ $\Pi_{1,2} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},$ $\Pi_{2,1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$ Also, we choose $K=(k_1, k_2) = (3, 2)$ which is Hurwitz vector, so from (15) we get matrix A=[0 1; –3 –2], B=[0 1]^T. Choosing that the parameter matrices Q_1 and Q_2 are taken as diagonal matrices with diagonal elements 3 and 5, respectively, i.e., $Q_1 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix},$ $Q_2 = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix},$ from (16) and (18), we have: $P_1 = \begin{pmatrix} 88.24 & 4.63 \\ 4.63 & 30.43 \end{pmatrix},$ $P_2 = \begin{pmatrix} 7.75 & 1.16 \\ 1.16 & 2.30 \end{pmatrix}.$ The switching signal is designed from

(17). The initial values of state vectors is $x(0)=[1\ \ 1]^T$, and the initial weights values of RBF NNs are chosen randomly between 0 and 1. The number of hidden units for the RBF NNs is taken as 32.

The simulation results are shown in Figs. 1 and 2. From Fig. 1 which denotes the output error tracking performance, the tracking objective is well obtained. So the satisfactory tracking performance is obtained, and the tracking error performance $e=[e_0\ \ e_1]^T=[y_d-y\ \ \dot{y}_d-x_2]^T$ is well-achieved in Fig. 2. It is observed that using the proposed control scheme in this paper there has good capabilities of tracking performance.

Remark 4: In general, the larger the number of neurons is, the smaller the approximation accuracy is achieved. As we know, the number of hidden units in the neural networks is a very important parameter of the proposed control scheme. Namely, the larger it is the more complexities the controller will contain. Thus, the design of the controller and the error indirectly determines the number of neurons. A point to note is that the direct determination of the number is still an open problem in control. Therefore, how to select the optimal number of hidden units remains an open research problem for the switched nonlinear systems, the results in [26] could be very useful for this problem in the future work.

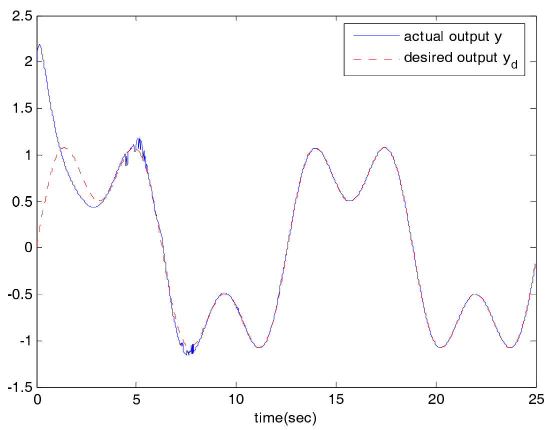

Fig. 1. Tracking performance.

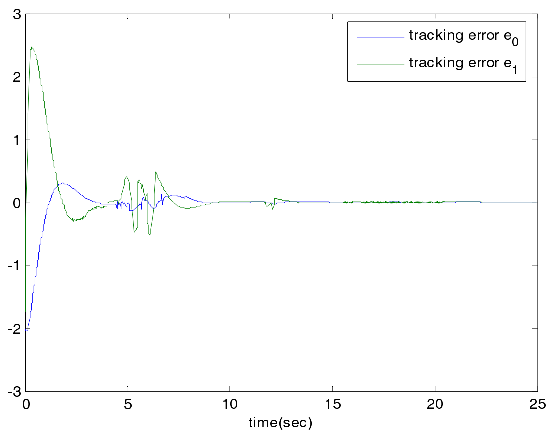

Fig. 2. Tracking error.

5. CONCLUSION

In this paper, a robust adaptive control scheme including the control laws, the adaptive neural updated laws and the switching signals has been presented for a class of switched uncertain nonlinear systems. To overcome the influences of nonlinear uncertainty and external disturbance, RBF NNs have been utilized to approximate unknown functions and uncertain nonlinear terms. A robust H∞ controller mainly using feedback linearizable technique has been designed to enhance robustness. Moreover, the resulting closed-loop switched system is robustly stable such that the actual output of follows the desired output signal and the H∞ tracking performance from the compound disturbance to the tracking error has been achieved. Finally, simulation results show the satisfactory performance of the control scheme.

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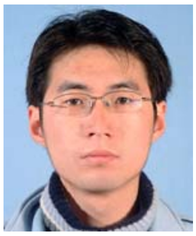

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