International Journal of Control, Automation, and Systems (2012) 10(1):61-70 http://www.springer.com/12555
DOI 10.1007/s12555-012-0107-0

Design and Implementation of a Hybrid Fuzzy Logic Controller for a Quadrotor VTOL Vehicle

Bora Erginer and Erdinç Altuğ*

Abstract: Helicopters have generated considerable interest in both the control community due to their complex dynamics, and in military community because of their advantages over regular aerial vehicles. In this paper, we present the modeling and control of a four rotor vertical take-off and landing (VTOL) unmanned air vehicle known as quadrotor aircraft. This model has been generated using Newton-Euler equations. In order to control the helicopter, classical PD (proportional derivative) and Hybrid Fuzzy PD controllers have been designed. Although fuzzy control of various dynamical systems has been presented in literature, application of this technology to quadrotor helicopter control is quite new. A quadrotor helicopter has nonlinear characteristics where classical control methods are not adequate especially when there are time delays, disturbances and nonlinear vehicle dynamics. On the other hand, Fuzzy control is nonlinear and it is thus suitable for nonlinear system control. Matlab Simulink has been used to test, analyze and compare the performance of the controllers in simulations. For the evaluation of the autonomous flight controllers, some experiments were also performed. For this purpose, an experimental test stand has been designed and manufactured. This study showed that although, both of the classical PD and the Fuzzy PD controllers can control the system properly, the Fuzzy PD controllers performed slightly better than the classical PD controllers, and have benefits such as better disturbance rejection, ease of building the controllers.

Keywords: Flight control, fuzzy control, modelling, quadrotor, UAV.

1. INTRODUCTION

Unmanned aerial vehicle (UAV) modeling and control is of recent interest in robotics and control community. This is partly due to broad applications of UAVs in many areas. They provide tremendous advantages over manned operations for applications like search and rescue, remote inspection, surveillance, as well as civilians applications such as disaster monitoring, aerial photography, and inspection. With the use of these vehicles, human pilots can now be safe from dangers of flight.

Helicopter design has been the center of attention since the beginning of the 20th century. First full-scale four rotor helicopter (quadrotor) was built by De Bothezat in 1921 [1]. Other examples are Breguet Richet helicopter, Oemnichen helicopter, Convertawings Model A and Curtis Wright VZ-7 [2,3]. At those early times due to the lacking control and sensing technologies, it was not possible to build an UAV. Advances in sensors, control technology and electronics enabled the possibility of UAVs. Currently, there are various commercial and experimental UAVs of various sizes available, and many more autonomous unmanned VTOL vehicles are being developed at universities, research centers, and by hobbists [4-7]. The studies in quadrotor UAV modeling and control increased rapidly in recent years. Some examples of these studies can be summarized as following; Hamel *et al*. modeled a quadrotor by incorporating the airframe and motor dynamics as well as aerodynamics and gyroscopic effects and controlled it separating the rigid body dynamics from the motor dynamics [8]. Altuğ *et al*. modeled a quadrotor using Euler-Newton method and worked on vision based stabilization and output tracking control [9]. Suter *et al*. also studied on image based visual servo control for quadrotors [10]. Moktari *et al*. presented a nonlinear dynamic model for a quadrotor with a state parameter control which is based on Euler angles and open-loop positions state observer [11]. Dunfied *et al*. created a neural networks controller for a quadrotor [12]. Earl *et al*. used a Kalman filter to estimate the attitude of a quadrotor [13]. In their studies, Slazar-Cruz and Escareno *et al*., used a Lagrangian model and a controller based on Lyapunov analysis using nested saturation control algorithm and designed an embedded control architecture for a quadrotor to perform autonomous hover flight [14,15]. Bouabdallah *et al*. mechanically designed, dynamically modeled and used nonlinear control techniques in their works [16]. Beji *et al*.

Manuscript received July 21, 2010; revised July 5, 2011; accepted October 12, 2011. Recommended by Editorial Board member Young Soo Suh under the direction of Editor Jae Weon Choi.

This work was supported in part by the Aviation Research and Development Project (HAGU) at the Istanbul Technical University.

Bora Erginer is with the Ford Otosan, Istanbul, Turkey (e-mail: berginer@ford.com.tr).

Erdinç Altuğ is with the Istanbul Technical University, Department of Mechanical Engineering, Inönü Cad. No:65, Gümüşsuyu, Istanbul, Turkey (e-mail: altuger@itu.edu.tr).

* Corresponding author.

presented structure and control of a quadrotor where two rotors are bidirectional [17]. Castillo *et al*., used a Lagrangian model of the quadrotor and controlled it based on Lyapunov analysis [18]. Tayebi *et al*., proposed a controller which is based upon the compensation of the Coriolis and gyroscopic torques and the use of PD2 feedback structure [19]. Finally, Lee *et al*. presented feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter [20].

The fuzzy logic control is an active field in last couple of decades, and it has been implemented on various dynamical systems [21-25]. The fuzzy logic has also been implemented in helicopter control [26-28]. In more recent papers [29,30], the researchers used a fuzzy controller for the altitude and hovering control of an unmanned helicopter. Although fuzzy control of dynamical systems has been presented extensively in literature, application of this to quadrotor helicopter control is quite new. A quadrotor helicopter is a nonlinear dynamical system which is inherently unstable and hard to control. Conventional control methods use linear theory that is suitable for linear systems only. The general approach is to take a flight envelope (such as hover or low speed flight) and linearize the model about these points. The resulting approach does not guarantee operation outside the selected flight envelope. Helicopter systems also include time delays on actuation as well as sensing elements, achieving stability by conventional feedback control is difficult. In addition, helicopters are very sensitive to the disturbances such as wind. On the other hand, Fuzzy control is nonlinear and it is thus suitable for nonlinear system control, and it can adapt to accommodate any possible environmental changes [23].

The main contribution of this paper is the introduction of hybrid fuzzy PD controllers on quadrotor helicopter control. Although fuzzy control of various dynamical systems has been presented in literature, application of this technology to quadrotor helicopter control is quite new. Additionally, detailed model of the quadrotor helicopter is given. In this study, the performance of the fuzzy controllers are compared to classical PD controllers using Matlab simulations, and experiments. For this purpose, a custom experimental test stand is developed and manufactured for the evaluation of the quadrotor flight controllers.

The paper is organized as follows. Section 2 describes the mathematical model of a quadrotor. The controllers are presented in Section 3. The simulations supporting the objectives of the paper are presented in Section 4, followed by the experiments in Section 5. Concluding remarks are presented in Section 6.

2. MATHEMATICAL MODELLING OF THE QUADROTOR HELICOPTER

A quadrotor (Fig. 1) is an under-actuated aircraft with fixed-pitch angled four rotors. It contains four motors located on the front, back, left and right of the air frame. The rotors connected to these motors provide the necessary lift forces, and these four forces are the inputs

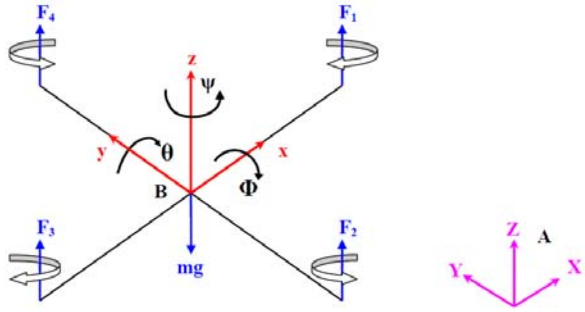

Fig. 1. The coordinate axes, the rotation directions of the rotors, the lift forces, and the Euler angle descriptions.

to the system to control six degrees of freedom; three Euler angles and three positions.

In order to move the quadrotor on the z-axis, the speed of all of the motors should be changed. Forward (backward) motion is maintained by increasing (decreasing) speed of front (rear) motor speed while decreasing (increasing) rear (front) motor speed simultaneously which means changing the pitch angle. Left and right motion is accomplished by changing roll angle by the same way, by changing the lift forces which changes the roll angle rate of the vehicle. The front and rear motors rotate counter-clockwise while other two motors rotate clockwise, so yaw command is derived by increasing (decreasing) counter-clockwise motors speed while decreasing (increasing) clockwise motor speeds. This also eliminates the need of a tail rotor.

Let us describe the mathematical model of the quadrotor using Newton-Euler equations. Consider a rigid body model of a *3D* quadrotor given in Fig. 1, where the coordinate axes, the rotation directions of the rotors, the lift forces, and the Euler angles are provided. Assume a body fixed frame (frame *B*) is located at the center of gravity of the quadrotor, and an inertial frame (frame *A*) is located on the ground. The relation of Frame *B* with respect to Frame *A* gives the helicopter's pose, which is composed of six degrees of freedom.

The *3D* position of any point with respect to axis *A* can be represented by $\vec{P} = [x \ \ y \ \ z]^T$ also representing the helicopter position. The linear velocity (V) and linear acceleration ($\dot{V}$) of the vehicle can be obtained by taking the derivative of this vector, $\vec{P}$.

A rotation matrix $R: A \rightarrow B$, where $R \in SO(3)$. A *ZYX* (Fick angles) Euler angle representation has been chosen for the representation of the rotations, which is composed of three Euler angles, (ψ, θ, ϕ), representing yaw, pitch and roll respectively:

$$RPY(\psi,\theta,\phi) = Rot(z,\psi) \cdot Rot(y,\theta) \cdot Rot(x,\phi). \tag{1}$$

This rotation matrix R can be written as following:

$$R = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}, \tag{2}$$

where S_ϕ denotes $Sin(\phi)$ and C_ϕ denotes $Cos(\phi)$.

The rotation speed of the rotors are described by Ω_i, where i corresponds to the rotor number $(i:1,2,3,4)$. The rotor spinning directions are determined to be clockwise for the first and third rotors, and counter-clockwise for second and the fourth rotor. The forces acting on the quadrotor are F_1, F_2, F_3, F_4 and mg. The lift forces generated by the rotation of the rotors will be $F_i = b\Omega_i^2$. This parameter b has been found to be constant by experiments. Then, the total lift force will be summation of the lift forces of all four rotors as $F_T = b\sum_{i=1}^{4}\Omega_i$. The acceleration obtained from the total lift is $a_F=F_T/m$. This acceleration can be represented in frame A as Ra_F. Using the force balance one can obtain relation of linear acceleration with respect to the rotation matrix and total lift force as

$$\dot{V} = -ge_z + \mathrm{Re}_z\, a_f, \tag{3}$$

where $e_z = [0 \quad 0 \quad 1]^T$.

Let $\vec{V}$ and $\vec{\omega} \in A$ represent the linear and angular velocities of the rigid body with respect to the inertial frame. Similarly, let $\vec{V}^b$ and $\vec{\omega}^b \in B$ represent the linear and angular velocities of the rigid body with respect to the body-fixed frame. Let $\vec{\zeta}$ be the vector of Euler angles, $\vec{\zeta} = [\psi,\theta,\phi]$. The body angular velocity $(\vec{\omega}^b = [p,q,r]^T)$ is related to Euler angular velocity by, $\vec{\omega}^b = unskew(R^T\dot{R})$, where *unskew()* term, represents obtaining vector $\vec{\omega}^b$ from skew symmetric matrix; $skew(\vec{\omega}^b)$. The $skew(\vec{\omega}) \in so(3)$ is the skew symmetric matrix of $\vec{\omega}$.

The Euler angle velocities to body angular velocities are mapped by, $\dot{\vec{\zeta}} = J\vec{\omega}^b$,

$$\dot{\vec{\zeta}} = \begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} 1 & S_\psi T_\theta & C_\psi T_\theta \\ 0 & C_\psi & -S_\psi \\ 0 & S_\psi / C_\theta & C_\psi / C_\theta \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \tag{4}$$

where T_ϕ denotes $\tan(\phi)$.

Due to the angular rotations ω, angular momentum of $L_{xyz}=I\omega$ will be formed. The I term in this equation is a *3x3* diagonal matrix whose elements are moment of inertia I_x, I_y and I_z of x, y and z axes respectively. The time rate change of momentum corresponds to torque (T_B) formed by the angular velocities of the helicopter:

$$T_B = \dot{L} = \omega \times I\omega + I\dot{\omega}. \tag{5}$$

The gyroscopic torque formed by rotation of the helicopter and its rotors around their axes can be written as [19]:

$$T_G = \sum_{i=1}^{4} J(\omega \times e_z)\Omega_i(-1)^i, \tag{6}$$

where J is inertia of a single rotor around its axis.

The lift forces of each rotor also produces torques acting on the helicopter. The torques along x, y and z axes can be given as:

$$T_A = \begin{bmatrix} lb(\Omega_4^2 - \Omega_2^2) \\ lb(\Omega_3^2 - \Omega_1^2) \\ d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{bmatrix}, \tag{7}$$

where l is difference between rotor center to quadrotor center, d is the drag coefficient. One can write the torque balance as

$$T_G + T_B = T_A. \tag{8}$$

Substituting the torque equations given above in (8), and solving for $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ leads to

$$\begin{aligned} \ddot{\phi} &= \dot{\psi}\dot{\theta}\frac{I_x - I_z}{I_x} - \frac{J}{I_x}\dot{\theta}(-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \\ &\quad + \frac{l}{I_x}b(\Omega_4^2 - \Omega_2^2), \end{aligned} \tag{9}$$

$$\begin{aligned} \ddot{\theta} &= \dot{\psi}\dot{\phi}\frac{I_x - I_z}{I_y} + \frac{J}{I_y}\dot{\phi}(-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \\ &\quad + \frac{l}{I_y}b(\Omega_3^2 - \Omega_1^2), \end{aligned} \tag{10}$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi}\frac{I_x - I_y}{I_z} - \frac{d}{I_y}(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2). \tag{11}$$

Expanding (3) leads to

$$\ddot{x} = (C_\psi S_\theta S_\phi + S_\psi C_\phi)\frac{b}{m}\sum_{i=1}^{4}\Omega_i^2, \tag{12}$$

$$\ddot{y} = (S_\psi S_\theta C_\phi - C_\psi S_\phi)\frac{b}{m}\sum_{i=1}^{4}\Omega_i^2, \tag{13}$$

$$\ddot{z} = -g + (C_\theta C_\phi)\frac{b}{m}\sum_{i=1}^{4}\Omega_i^2. \tag{14}$$

Equations (9) to (14) represent the full mathematical model of the quadrotor helicopter, with rotor speeds Ω_i as the inputs.

3. CONTROLLER DESIGN

In this section two controllers for the stabilization and guidance of the quadrotor helicopter will be represented; the Proportional Derivative (PD) controller, and the Hybrid Fuzzy PD Controller.

Consider the equations of motion of the quadrotor given with (9) to (14). Let us choose inputs as

$$\begin{aligned} U_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2), \\ U_2 &= b(\Omega_4^2 - \Omega_2^2), \\ U_3 &= b(\Omega_3^2 - \Omega_1^2), \\ U_4 &= d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2), \end{aligned} \tag{15}$$

where U_1 controls the motion along z-axis, U_2 controls motion along y-axis (roll angle), U_3 controls the motion along the x-axis (pitch angle) and U_4 controls rotation along the z-axis (yaw angle). The designed controllers should set values to U_i parameters which determines the four rotor speed parameters Ω_i by (15). A low level controller is still needed to keep this speed constant for each motor.

3.1. Proportional derivative (PD) controller

The first approach is to use a PD controller. A PD controller is useful since it can be obtained from the dynamical model and also it stabilizes the system exponentially. Altitude control is performed by (14) as

$$U_1 = \frac{mg}{C_\theta C_\phi} + \frac{m(K_p(z - z_d) + K_{d_z}(\dot{z} - \dot{z}_d))}{C_\theta C_\phi}, \tag{16}$$

where K_p is the proportional and K_d is the derivative control coefficient, z_d corresponds to the desired altitude.

On the other hand, the Euler angles of the quadrotor are controlled with PD controllers as given in (17). These controllers will be used for hover control, as well as to control the heading, x-motion and y-motion of the vehicle.

$$\begin{aligned} U_2 &= K_{p\phi}(\phi_d - \phi) + K_{d\phi}(\dot{\phi}_d - \dot{\phi}), \\ U_3 &= K_{p\theta}(\theta_d - \theta) + K_{d\theta}(\dot{\theta}_d - \dot{\theta}), \\ U_4 &= K_{p\psi}(\psi_d - \psi) + K_{d\psi}(\dot{\psi}_d - \dot{\psi}), \end{aligned} \tag{17}$$

where ϕ_d, θ_d, ψ_d are the desired roll, pitch and yaw angles. For hover, the desired roll and pitch angles are taken as zero. The $\dot{\phi}_d$, $\dot{\theta}_d$, $\dot{\psi}_d$ are the desired derivatives of the roll, pitch and yaw angles.

To control the motions along x-axis and y-axis, the θ and ϕ angles and their derivatives should be controlled. The desired values of the roll and pitch values and the desired roll and pitch velocities will depend on the desired values of the x and y locations.

Motion along the y-axis is related to the roll angle. In order to define a relation for desired values of roll and its derivative, let us use (13). Rearrange terms as

$$\begin{aligned} \ddot{y} &= -K_{py}(y - y_d) - K_d(\dot{y} - \dot{y}_{dy}) \\ &= (S_\psi S_\theta C_\phi - C_\psi S_\phi)\frac{1}{m}U_1 \end{aligned} \tag{18}$$

assume that rotation around y-axis, θ, is zero, no motion present along x-axis, the yaw angle, ψ is also zero. Then we can obtain,

$$Sin\phi_d = \frac{(K_{py}(y - y_d) + K_{dy}(\dot{y} - \dot{y}_d))m}{U_1}. \tag{19}$$

The desired roll angle can be obtained by taking the *arcsin* of the above equation. Let us combine U_1 and m terms with the control constants to obtain,

$$\phi_d = Arc\sin(K_{py}(y - y_d) + K_{dy}(\dot{y} - \dot{y}_d)). \tag{20}$$

Taking the derivative of this equality and ignoring higher derivatives, one can obtain the relation for the derivative of the desired roll angle as,

$$\dot{\phi}_d = \frac{K_{py}m(\dot{y} - \dot{y}_d)}{\sqrt{\Delta_\phi}}, \tag{21}$$

$$\begin{aligned} \Delta_\phi &= 1 - K_{py}^2 m^2 (y - y_d)^2 - 2K_p K_{dy} m^2 \\ &\quad + (y - y_d)(\dot{y} - \dot{y}_d) - K_{dy}^2 m^2 (\dot{y} - \dot{y}_d)^2. \end{aligned} \tag{22}$$

We can repeat the same procedure with (12) to obtain the desired values for the pitch angle and the desired pitch angle derivative. The desired pitch angle will be

$$\theta_d = Arc\sin\left(K_{px}(x - x_d) + K_{dx}(\dot{x} - \dot{x}_d)\right). \tag{23}$$

The desired pitch angles derivative will be

$$\dot{\theta}_d = \frac{K_{px}m(\dot{x} - \dot{x}_d)}{\sqrt{\Delta_\theta}}, \tag{24}$$

$$\begin{aligned} \Delta_\theta &= 1 - K_{px}^2 m^2 (x - x_d)^2 - 2K_p K_{dx} m^2 \\ &\quad + (x - x_d)(\dot{x} - \dot{x}_d) - K_{dx}^2 m^2 (\dot{x} - \dot{x}_d)^2. \end{aligned} \tag{25}$$

Desired pitch and roll angles, and the desired roll and pitch angles' derivatives are used in (17) to guide the vehicle along x and y-axes.

3.2. Hybrid fuzzy PD controller

The idea of the Fuzzy PD controllers is that these kind of controllers can be used independent of the model. Since the Fuzzy controllers depend on simple rules, they are much easier to understand and to implement. The designed autonomous navigation controller consists of six fuzzy logic modules for the control of altitude, yaw angle, roll angle, pitch angle, and motion along x and y axes using the error and the rate of change of these errors. The error in altitude is $(z - z_d)$ and the rate of change of altitude error is $(\dot{z} - \dot{z}_d)$. The error in yaw is $(\psi - \psi_d)$ and the rate of change of this error is $(\dot{\psi} - \dot{\psi}_d)$. The error along x-axis and y-axis are given as $(x - x_d)$ and $(y - y_d)$. The rate of change of error along x and y axes are given as $(\dot{x} - \dot{x}_d)$ and $(\dot{y} - \dot{y}_d)$, respectively.

For altitude and yaw control the desired values are given by the operator. The desired values for roll, pitch and their derivatives are obtained from (20) to (25), and they will not be obtained by the fuzzy controllers due to the complex nature of the equations. These valued will be obtained by classical PD controllers. Fig. 2 represents the controllers, their inputs and outputs. Fig. 2 also shows that the desired values for $\dot{\phi}$ and $\dot{\theta}$ are obtained by the PD controllers.

The membership functions of the error and the rate of change of error have been chosen identical. The width of the fuzzy sets used for the controllers are same and they are [-1, 1]. The width difference between the sets were accomplished using a scale factor.

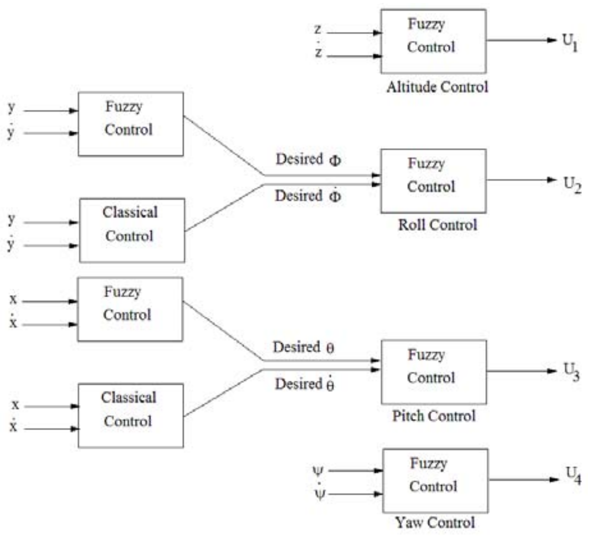

Fig. 2. The fuzzy controllers, inputs and outputs of the system.

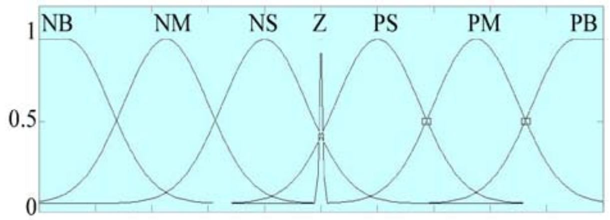

Fig. 3. The controller output fuzzy set and the membership functions for the fuzzy altitude controller.

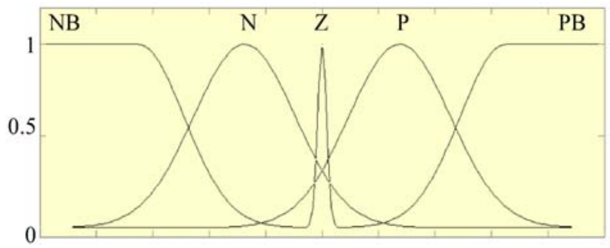

Fig. 4. The error and error rate fuzzy set and the membership functions.

The inputs of error and the rate of change of error are used by the rules to generate the output sets as shown in Fig. 3.

There are five membership functions for each input set as Negative Big (NB), Negative (N), Zero (Z), Positive (P) and Positive Big (PB). There are seven membership sets for the controller outputs as Negative Big (NB), Negative Mean (NM), Negative Small (NS), Zero (Z), Positive Small (PS) Positive Mean (PM) and Positive Big (PB). The fuzzy sets used for the output, and the errors are shown in Figs. 3 and 4, respectively.

The membership functions used for the input and output are Gaussian,

$$\mu_{A_i} = e^{\frac{(c_i-x)^2}{2\sigma_i^2}} . \qquad (26)$$

Table 1. The values of the input and output membership functions.

Input Fuzzy Sets		
Membership Functions	c_i	σ_i
NB	-∞, -0.7	∞, 0.18
N	-0.275	0.18
Z	0	0.015
P	-0.275	0.18
PB	0.7, ∞	0.18, ∞
Output Fuzzy Sets		
Membership Functions	c_i	σ_i
NB	-∞, -0.9	∞, 0.15
NM	-0.55	0.15
NS	-0.2	0.15
Z	0	0.006
PS	0.2	0.15
PM	0.55	0.15
PB	0.9, ∞	0.15, ∞

Table 2. The table of rules.

		Error				
		NB	**N**	**Z**	**P**	**PB**
Error Rate	**NB**	NB	NB	NB	NM	NS
	N	NB	NM	NS	Z	PS
	Z	NM	NS	Z	PS	PM
	P	NS	Z	PS	PM	PB
	PB	Z	PS	PM	PB	PB

Table 1 presents the values of the membership functions used for the input and output fuzzy sets. The NB and PB functions are formed by two Gaussian type membership functions.

The rules presented at Table 2 can be read as follows. If the error is A and the rate of change of error is B, then the output is C. For the case when the error is Negative (N) and the rate of change of error is Positive Big (PB) then the output will be Positive Small (PS). The output is fuzzy in fuzzy logic controller.

We can not provide these fuzzy outputs to a dynamical system as control inputs directly. Defuzzification process is needed to convert these fuzzy outputs to numbers that can represent the fuzzy output. The control signal should be continuous, any variation in input should not produce big changes in output signal. The defuzzification algorithm should be clear and the process to determine the output signal should be identified clearly. Also, the defuzzification should be logical, should have high membership degree and it should correspond to the approximately middle of the graph. It should be simple so that it can work in real-time to perform the calculations. For the reasons discussed above we have selected mean-area defuzzification method for the controllers. In this method, the control output has been selected as the value of the line that is cutting the graph formed by the membership functions into half. If required, a normalization can be performed before supplying this signal to the system.

4. SIMULATIONS

The mathematical dynamical model of the quadrotor vehicle as well as the controllers have been developed in Matlab Simulink for simulation. The goal of this analysis is to test how well the controllers can stabilize and guide the helicopter for different starting and goal positions. Firstly, helicopter is commanded to change its position and yaw angle while keeping the pitch and roll angles constant. In the second simulation, a random error has been implemented to simulate the sensor noise. Both of the controllers are run on these two scenarios and the results are compared.

In simulation 1, quadrotor starts with 0° pitch and roll, 20° yaw angle on (0,0,0) position and it is expected to arrive at (3,4,2) position while keeping pitch and roll angles zero and driving yaw angle to zero (Fig. 5). The PD controllers and Fuzzy PD controllers performed properly and helicopter reaches the desired coordinates in 7.1 s. According to the graph, the motion along x, y and z axes is similar for both controllers. The control effort on yaw motion is better with fuzzy PD control based on settling time. The proportional and derivative control coefficients are presented in Table 3.

In order to improve the altitude control two different fuzzy controllers are used. The first controller is used when the altitude (z) is above 0.05 meters, and the second controller is used when the altitude (z) is below 0.05 meters, which enabled more precise altitude control near $z=0$.

Table 3. The control coefficients used in simulation 1.

PD Controller			Fuzzy PD Controller		
	K_p	K_d		K_p	K_d
z	2	2	z > 0.05	1	1
Pitch	3048.9	15	z < 0.05	20	20
Roll	31.8	10	Pitch	10	3
Yaw	0.008	0.01	Roll	10	0.1
ϕ_d	0.05	0.05	Yaw	7	1
θ_d	0.45	1	ϕ_d	0.4	0.2
$\dot{\phi}_d$	1	1	θ_d	0.7	0.4
$\dot{\theta}_d$	1	15			

The controllers are expected to work with noisy sensor readings in real-world conditions. In the second simulation, random noise of 0° mean and 0.001° variance has been added to roll, pitch and yaw states to simulate estimation errors. In this analysis, helicopter starts at (0, 0, 0, 0, 0, 40) and it is expected to arrive the pose (2, 2, 4, 0, 0, 0). Fig. 6 shows the results of the simulation, in which the helicopter was able to stabilize itself and achieve the desired position. It is evident from the plots that the helicopter goes through fluctuations along the roll, pitch, and yaw angles but at the end of the simulation the helicopter was able to reach the desired

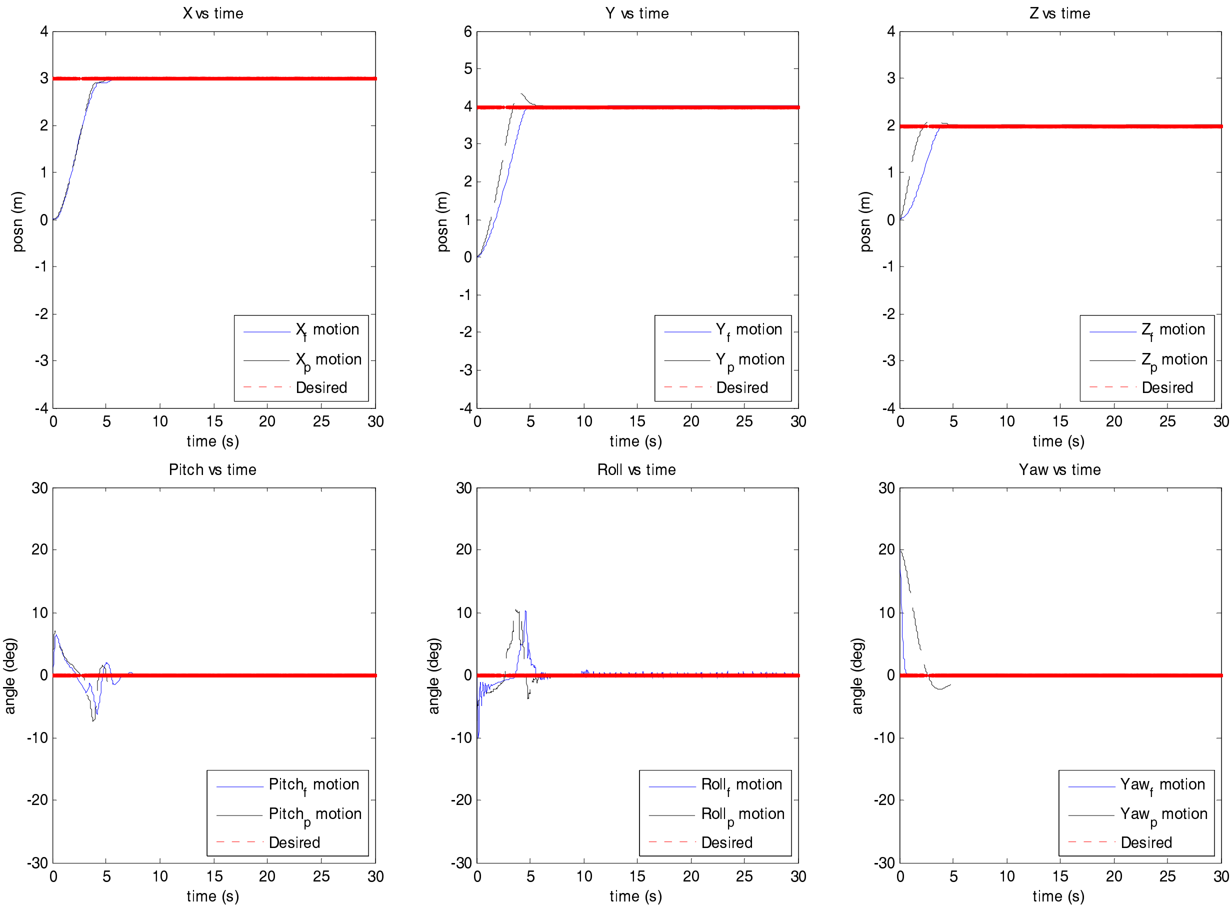

Fig. 5. The control of the quadrotor using PD (dashed line) and Fuzzy PD (solid line) controllers.

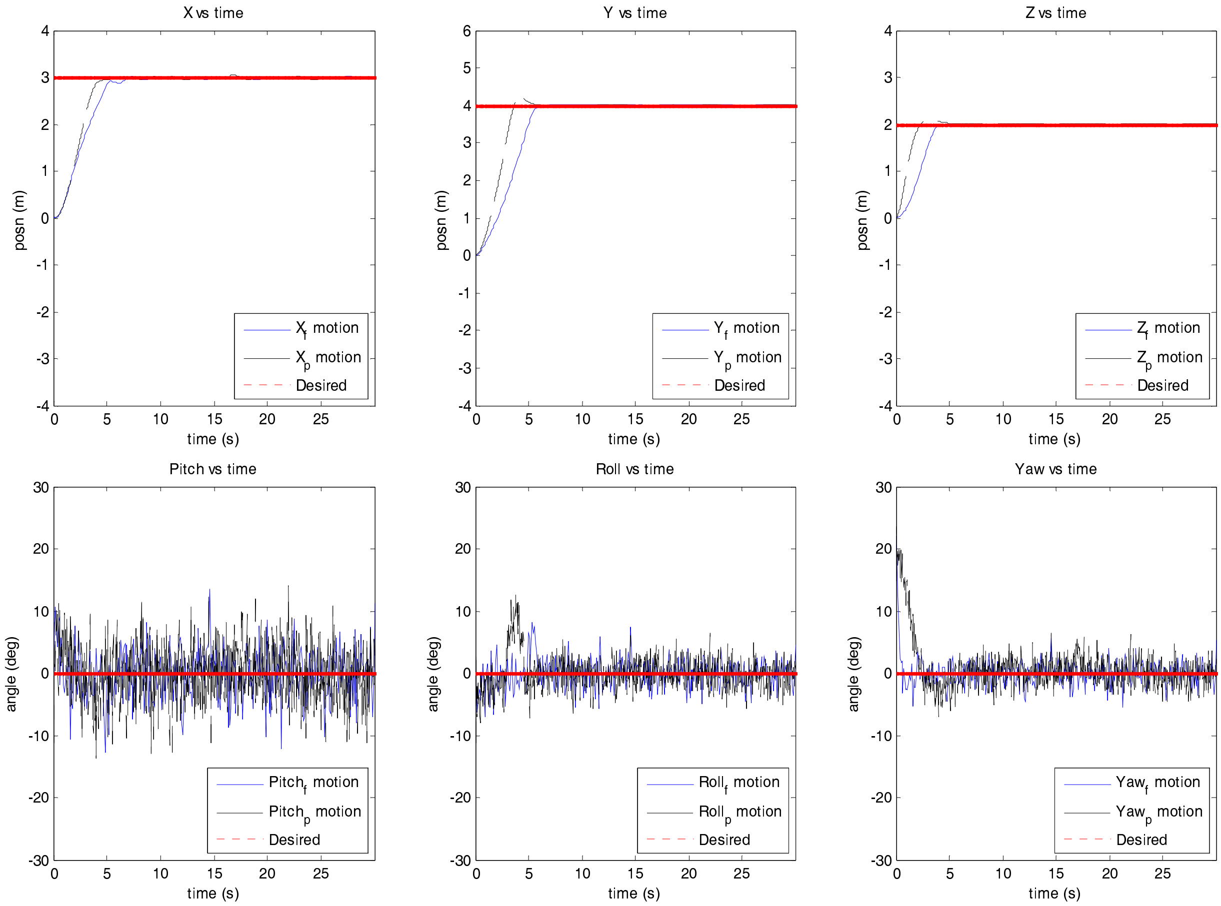

Fig. 6. Helicopter position and Euler angles during the second simulation, where PD (dashed line) and Hybrid Fuzzy PD controllers (solid line) are used to control the helicopter.

Table 4. The mean and standard deviation of the Euler angles for simulation 2.

	Fuzzy Cont.	PD Cont.	% Change
Pitch mean	$7.4\ 10^{-4}$	$-1.4\ 10^{-3}$	47.1
Pitch std.	0.073	0.076	3.9
Roll mean	$2.9\ 10^{-4}$	$1.9\ 10^{-3}$	84.7
Roll std.	0.044	0.046	4.3
Yaw mean	0.0083	0.0483	82.8
Yaw std.	0.0811	0.17	52.2

goal position successfully.

Based on the simulation analysis, both of the controllers were robust enough to control the helicopter even when there are errors in the estimates, but the hybrid fuzzy PD controller performed 3.9 to 84.7 % better as presented in Table 4.

5. EXPERIMENTS

To perform secure and reliable experiments, a custom experimental test stand has been designed and manufactured to test the helicopter flight controllers and the UAV systems (Fig. 7). The test stand allows the helicopter to perform yaw motions freely, allows up to 2 meters of altitude, as well as up to ±20° roll and pitch motion. The test stand can be used for various sized model helicopters and it has a security panel around it. A

Fig. 7. The custom manufactured experimental test stand.

Draganflyer-III quadrotor model helicopter has been installed to the test stand as well as an IMU to read the helicopter states (Fig. 8).

In this study, an IMU has been used to obtain the acceleration, velocity information of the helicopter. The angles are obtained by integrating the velocity information. This helicopter can be controlled with a radio transmitter. The generated control signals on the

Fig. 8. Quadrotor on the test stand.

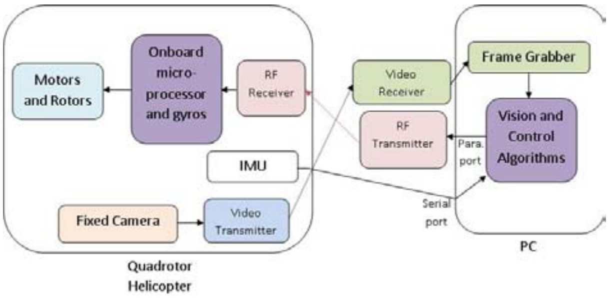

Fig. 9. The experimental system block diagram.

computer are transmitted to the helicopter through a transmitter that is connected to the parallel port.

The helicopter experimental block diagram is shown in Fig. 9. The helicopter consists of one cameras, one IMU, one wireless video transmitter (to transmit video), one RF receiver (to receive control commands), an on-board controller including gyros and motor controllers. The vision system has been developed for future studies and it has not been used for this study. The proposed controllers are used to generate the appropriate control commands which are then transmitted to the helicopter with a wireless RF transmitter. All of the coding has been implemented in Matlab. Due to the limitations of the wired connection and the thrust capacity of the helicopter only yaw control has been tested in experiments.

Various experiments were performed to compare the controllers, and the effectiveness of the system. In experiment 1, PD controller controls the yaw angle while the helicopter start at 0° and ends at the goal yaw angle of 50°. Fig. 10 present the results of this experiment as the helicopter performs the desired motion successfully. The accelerations, angular velocities and the controller signals are presented in Fig. 11. In this experiment the proportional constant was selected as 5, and the derivative constant has been selected as 1.

In experiment 2, the Fuzzy PD controller has been used to control the yaw motion of the helicopter. Helicopter starts at 0° and ends at the goal yaw angle of 30°. Fig. 12 present the results of this experiment as the helicopter performs the desired motion successfully. In this experiment the proportional constant was selected as

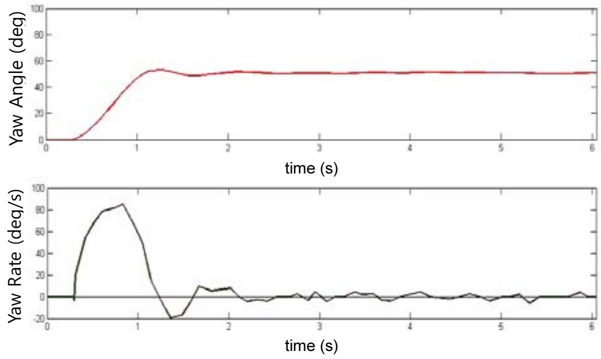

Fig. 10. Control of the yaw angle with PD controllers. Yaw v.s. time (top), Yaw Rate v.s. time (below).

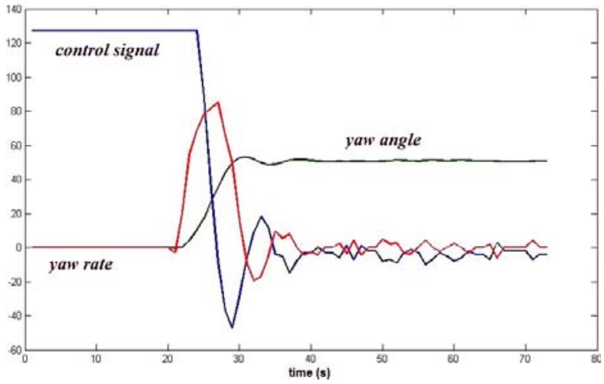

Fig. 11. Controller signal, angular speed, and the yaw angle during the experiment.

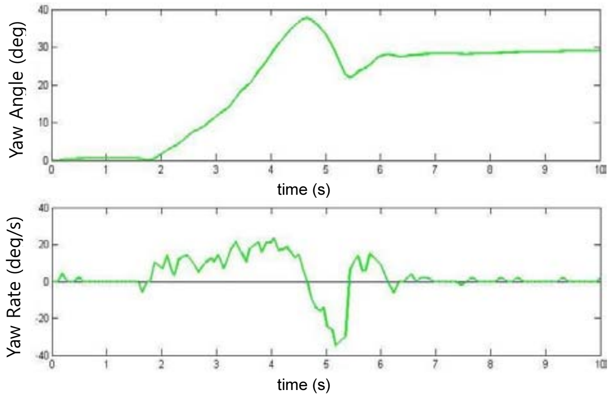

Fig. 12. Yaw angle control experiment results using Fuzzy PD controller. Yaw v.s. time (top), Rate of change of Yaw v.s. time (below).

0.01, and the derivative constant has been selected as 0.004. The control output has been normalized.

The PD and the Fuzzy PD controllers are compared using the following experiment by setting an arbitrary desired yaw angle of 50°. The result of this experiment is presented in Fig. 13. Although, both of the controllers were able to stabilize the helicopter and reach the desired angle accurately, the performance of the Fuzzy PD controller is slightly better then the classical PD controller in terms of settling time and disturbance rejection.

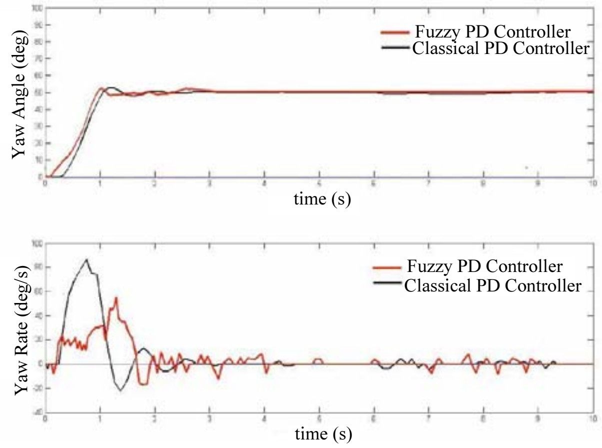

Fig. 13. Comparison of the performance of the two controllers. Yaw v.s. time (top), Yaw rate v.s. time (below).

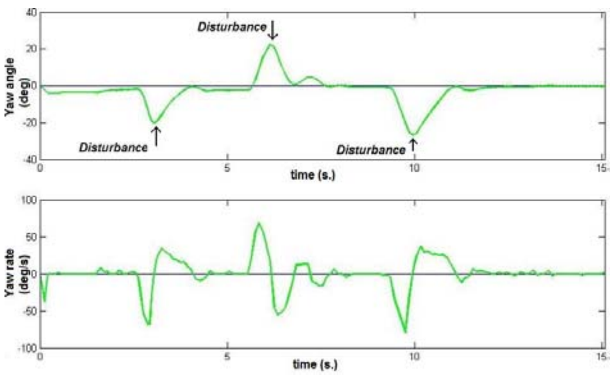

Fig. 14. Yaw angle control with Fuzzy PD controller while the helicopter subjecting to disturbance. Yaw v.s. time (top), Yaw rate v.s. time (below).

The robustness of the Fuzzy PD controller has further been tested in the following experiment, where random disturbances are applied to the helicopter at 3, 6 and 10 seconds of the experiment. The Fuzzy PD controller stabilized the helicopter rapidly even with disturbance applied (Fig. 14).

6. CONCLUSION

In this paper, modelling and hybrid fuzzy PD control of a four-rotor helicopter have been presented. Although fuzzy control of dynamical systems has been presented extensively in literature, application of this technology to quadrotor helicopter control is quite new. In modelling of the helicopter, we considered the gyroscopic effects and generated the equations with Newton-Euler equations. For the control of the helicopter, classical and Fuzzy PD controllers have been developed and presented. Various simulations were performed to test and compare the controllers. When comparing these two controller we should note that the selection of the membership functions and the rules of the fuzzy controller are based on the experience of the designer, similarly for the classical PD controller the coefficients are based on a trial and error approach.

Table 5. Comparing the results of the controllers.

	PD Cont.	Fuzzy PD Cont.
Steady State Error	Good	Good
Tracking Performance	Good	Good
Disturbance Rejection	Good	Better
Requiring Modelling	Yes	No
Computation Time	Small	Moderate
Ease of Tuning	Moderate	Simple and verbal

The real behavior of helicopter usually differs from the expected ones that has developed through simulations. One reason for that is, it is not possible to model the system exactly as the real helicopter due to the assumptions or unmeasured quantities as well as the effect of the disturbances. Therefore one can not guarantee the control of the helicopter using PD controllers by analyzing the simulation results only. On the other hand, for Fuzzy controllers, the control language is more verbal then numerical, which is parallel to what a pilot thinks as he/she controls the vehicle. Also, the dependence of the Fuzzy controller on the helicopter model is much less then the the classical controllers used. The Fuzzy control is nonlinear and it is thus more suitable for nonlinear system control. Experiments performed on the test stand as well as the simulations show that both of the classical PD and the Fuzzy PD controllers can be used to stabilize and guide a quadrotor. The control performance of the Fuzzy PD controller was slightly better then the classical PD controller in simulations and experiments. The biggest advantage of the hybrid fuzzy PD controller is the robustness against noise, and its ease for implementation. For the classical PD control, successful control depends on careful system modelling and coefficient selection. The comparison of the controllers are summarized in Table 5.

Our future studies will involve developing an on-board micro-controller to eliminate the need for a remote processing computer that is limiting the mobility of the UAV system.

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Bora Erginer received his B.S. degree in Department of Mechanical Engineering from Yıldız Technical University in 2003, and his M.S. degree in mechanical engineering in Istanbul Technical University, in 2007. His research interests include control, sensors, and modeling.

Erdinç Altuğ received his B.S. degree in Department of Mechanical Engineering from Middle East Technical University in 1996, an M.S. degree from Carnegie Mellon University, and a Ph.D. degree from the University of Pennsylvania in 2003. His research interests include robotics, control, and unmanned systems.