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Fault Diagnosis for High Order Systems Based on Model Decomposition

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Abstract: Fault detection observer and fault estimation filter are the main tools for the model based fault diagnosis approach. The dimension of the observer gain normally depends on the system order and the system output dimension. The fault estimation filter traditionally has the same order as the monitored system. For high order systems, these methods have the potential problems such as parameter optimization and the real time implementation on-board for applications. In this paper, the system dynamical model is first decomposed into two subsystems. The first subsystem has a low order which is the same as the fault dimension. The other subsystem is not affected by the fault directly. With the new model structure, a fault detection approach is proposed where only the residual of the first subsystem is designed to be sensitive to the faults. The residual of the second subsystem is totally decoupled from the faults. Moreover, a lower order fault estimation filter (with the same dimension of the fault) design algorithm is investigated. In addition, the design of a static fault estimation matrix is presented for further improving the fault estimation precision. The effectiveness of the proposed method is demonstrated by a simulation example.

Keywords: Fault diagnosis, filter, high order, model decomposition, observer.

1. INTRODUCTION

Model based fault detection has received much attention and significant progress has been achieved, see [1-12] and the references therein. One of the particular interesting techniques among all the model based techniques is the observer based fault detection filter design. It has been shown that it is very effective in detecting sensor, actuator, and system component faults. However, finding systematic design methods for systems subjected to unknown disturbances and model uncertainties has been proven to be difficult [1,13]. Since both disturbance and faults contribute to the residual generated by the observer, it is essential to isolate their effects to the residual. A fault detection observer should be robust to the disturbance but sensitive to the faults [1]. Some recent results aiming at this goal for LTI systems are reported in [9-11,14] and the references therein.

For linear time invariant systems, fault detection observer and fault estimation filter are the main tools for the model based fault detection approach. The dimension of the observer gain normally depends on the system order and the dimension of the system output. The fault estimation filter traditionally has the same order as that of the monitored system. They are successfully used for many systems in different areas. However, most of them are for lower order systems. For high order systems, the observer-based fault detection and fault estimation method have potential problems such as parameter optimization in the design step. Normally, it takes a long time to obtain a solution and numerical problem occurs frequently. In the step of their implementation on-board for real plants, high order observers and filters exhaust the limited hardware sources (such as the computation burden) of the online monitoring device. It is very desirable to develop innovative method for designing fault detection observer and fault estimation filter for high order systems.

Aiming at designing lower order fault estimation filter, in this paper, the high order dynamical model is first decomposed into two subsystems. The first one is directly related to the faults which needs to be detected and reconstructed. It has the same order as the fault dimension. The second subsystem does not directly relate to the faults. After the model decomposition, a new observer structure is proposed, where one observer gain is designed to detect the faults in the first subsystem and the other observer gain is designed to attenuate the disturbances of the second subsystem. With the new model and with the help of the proposed observer structure, only the residual of the first subsystem is related to the fault. Furthermore, a lower order fault estimation filter (with the same dimension of the fault) design approach is investigated. We propose a fault estimation filter design algorithm in the finite frequency domain. In addition, the design of a static fault estima-

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tion matrix is presented for further improving the fault estimation precision. The effectiveness of the proposed methods is demonstrated by a simulation study.

This paper is organized as follows. The model decomposition and the problem statement are presented in Section 2. In the third section, the fault detection observer structure and its design are presented at first. After that, a fault estimation filter design in the finite frequency is presented. A simulation example is shown in Section 4. Finally, some conclusions are given in Section 5.

Notations: The notations used in this paper are quite standard. For a matrix $A\in \mathrm{R}^{m\times n}$, A', A^*, $A^{\perp}$, and A^{-1} denote its transpose, complex conjugate transpose, orthogonal complement and inverse respectively. $\lambda(A)$ stands for the eigenvalue of matrix A. $\bar{\sigma}=\sqrt{\bar{\lambda}(A'A)}$ denotes the largest singular value of A and $\underline{\sigma}=\sqrt{\underline{\lambda}(A'A)}(\sqrt{\underline{\lambda}(AA')})$ denotes the smallest singular value of A if $m<n(\mathrm{m}\geq \mathrm{n})$, where $\bar{\lambda}(A)(\underline{\lambda}(A))$ stands for the largest (smallest) eigenvalue of A. $det(A)$ denotes the determinant of matrix A. The Hermitian part of a square matrix M is denoted by $He(M):=M+M^*$. The symbol H_n stands for the set of $n\times n$ Hermitian matrices. $Re(s)$ denotes the real part of complex number s. The time mean norm of a signal vector u is defined as

$$\|u\|_{2,T}=\sqrt{\frac{1}{T}\int_t^{t+T}u'udt}\,, \tag{1}$$

where T is a large enough time constant [9].

The H_∞ norm of transfer function $G(s)$ over a finite frequency range $[\omega_1, \omega_2]$ is defined as

$$\|G(s)\|_\infty^{[\omega_1,\omega_2]}=\bar{\sigma}^{\omega\in[\omega_1,\omega_2]}(G(j\omega))\,. \tag{2}$$

The $H\_$ index of transfer function $G(s)$ over a finite frequency range $[\omega_1, \omega_2]$ is defined as

$$\|G(s)\|_-^{[\omega_1,\omega_2]}=\underline{\sigma}^{\omega\in[\omega_1,\omega_2]}(G(j\omega))\,. \tag{3}$$

2. PRELIMINARIES AND PROBLEM STATEMENT

Consider a linear dynamical system with unknown disturbances described by

$$S:\begin{cases}\dot{x}(t)=Ax(t)+B_u u(t)+B_f f(t)+B_d d(t)\\ y(t)=Cx(t)+D_u u(t),\end{cases} \tag{4}$$

where $x\in\mathscr{R}^n$ is the state vector, $d\in\mathscr{R}^{n_d}$ is the unknown input vector including modeling error, uncertain disturbances, process and measurement noises, $y\in R^{n_y}$ is the measurement vector and $f\in\mathscr{R}^{n_f}$ is the fault vector.

Assumption 1: For the considered system (4), we have

$$rank(CB_f)=rank(B_f)=n_f\,, \tag{5}$$

$$rank\begin{bmatrix} sI_n-A & B_f\\ C & 0\end{bmatrix}=n+rank(B_f)\,. \tag{6}$$

Lemma 1 [15]**:** Given the system (4), we have

$$rank(CB_f)=rank(B_f)=n_f\,, \tag{7}$$

if and only if there exist nonsingular transformation matrices T and S such that

$$TAT^{-1}=\begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix},\ TB_f=\begin{bmatrix}B_{f1}\\ 0\end{bmatrix},$$

$$SCT^{-1}=\begin{bmatrix}C_1 & 0\\ 0 & C_4\end{bmatrix},\ TB_d=\begin{bmatrix}B_{d1}\\ B_{d2}\end{bmatrix},$$

$$TD_u=\begin{bmatrix}D_{u1}\\ D_{u2}\end{bmatrix},$$

where $A_1\in R^{n_f\times n_f}$, $A_4\in R^{(n-n_f)\times(n-n_f)}$, $C_1\in R^{n_f\times n_f}$, $C_4\in R^{(n_y-n_f)\times(n-n_f)}$, $rank(B_{f1})=n_f$ and C_1 is invertible.

In the following, we provide an algorithm of computing the T and S.

Applying the SVD decomposition to B_f, we obtain

$$B_f=U_{B_f}\sum\nolimits_{B_f}V_{B_f}^T,$$

where $U_{B_f}\in R^{n\times n}$ is a unitary matrix. Let $T_1=U_{B_f}^{-1}$, then we obtain

$$T_1B_f=\sum\nolimits_{B_f}V_{B_f}^T=\begin{bmatrix}B_{f1}\\ 0\end{bmatrix},$$

where $B_{f1}\in R^{n\times n_f}$ and $\mathrm{rank}(B_{f1})=n_f$. The matrix CT_1^{-1} can be partitioned as follows:

$$CT_1^{-1}=\begin{bmatrix}\tilde{C}_1 & \tilde{C}_2\end{bmatrix},$$

where $\tilde{C}_1\in\mathscr{R}^{n_y\times n_f}$. Note that

$$CB_f=(CT_1^{-1})(T_1B_f)=\tilde{C}_1B_{f1}\,.$$

By the hypothesis of Lemma 1,

$$\mathrm{rank}(B_f)=rank(CB_f)=n_f\,.$$

Hence

$$rank(\tilde{C}_1)=n_f\,.$$

Applying the SVD decomposition to $\tilde{C}_1$ yields

$$\tilde{C}_1=U_{\tilde{C}_1}\sum\nolimits_{\tilde{C}_1}V_{\tilde{C}_1}^T,$$

where

$$\sum\nolimits_{\tilde{C}_1}V_{\tilde{C}_1}^T=\begin{bmatrix}C_1\\ 0\end{bmatrix},\quad \det(C_1)\neq 0.$$

Note that $\tilde{C}_1\in\mathrm{R}^{n_y\times n_f}$. Let $S=U_{\tilde{C}_1}^{-1}$, then

$$SCT_1^{-1} = \begin{bmatrix} C_1 & C_2 \\ 0 & C_4 \end{bmatrix}.$$

Define

$$T_2^{-1} = \begin{bmatrix} I_{nf} & -C_1^{-1}C_2 \\ 0 & I_{n-nf} \end{bmatrix}$$

and we have

$$SCT^{-1} = \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix},$$

where $T = T_2 T_1$.

It can be verified that

$$T_2 = \begin{bmatrix} I_{nf} & C_1^{-1}C_2 \\ 0 & I_{n-nf} \end{bmatrix},$$

hence

$$TB_f = \begin{bmatrix} B_{f1} \\ 0 \end{bmatrix}.$$

In the new coordinates $\tilde{x} := (x_1^T, x_2^T)^{\mathrm{T}} = Tx$ and $\tilde{y} := (y_1^T, y_2^T)^{\mathrm{T}} = Sy$, the system state space description is as follows:

$$S_1: \begin{cases} \dot{x}_1 = A_1x_1 + A_2x_2 + B_{u1}u + B_{f1}f + B_{d1}d \\ y_1 = C_1x_1 + D_{u1}u, \end{cases} \tag{8}$$

$$S_2: \begin{cases} \dot{x}_2 = A_3x_1 + A_4x_2 + B_{u2}u + B_{d2}d \\ y_2 = C_4x_2 + D_{u2}u. \end{cases} \tag{9}$$

The observer used in this paper is as follows:

$$O_1: \begin{cases} \dot{\hat{x}}_1 = A_1\hat{x}_1 + A_2\hat{x}_2 + B_{u1}u + L_1(y_1 - \hat{y}_1) + L_2y_2 \\ \hat{y}_1 = C_1\hat{x}_1 + D_{u1}u, \end{cases} \tag{10}$$

$$O_2: \begin{cases} \dot{\hat{x}}_2 = A_4\hat{x}_2 + B_{u2}u + L_3y_1 + L_4(y_2 - \hat{y}_2) \\ \hat{y}_2 = C_4\hat{x}_2 + D_{u2}u. \end{cases} \tag{11}$$

The observer gain $\tilde{L}$ for the fault detection is described in the following:

$$\tilde{L} := \begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix}. \tag{12}$$

The first problem considered in this paper is that how to design the observer gain $\tilde{L}$ to achieve the following goals.

1) Observer O_2 is totally decoupled from the faults.

2) The change caused by the fault is only related to residual $r_1 = y_1 - \tilde{y}_1$.

3) The state estimation error $e_2 = x_2 - \hat{x}_2$ is bounded.

The second problem is how to design a lower order fault estimation filter F to reconstruct the fault f. The state space description of the filter F is given as follows.

$$F: \begin{cases} \dot{x}_c = A_cx_c + B_cr_1 \\ \hat{f} = C_cx_c + D_cr_1, \end{cases} \tag{13}$$

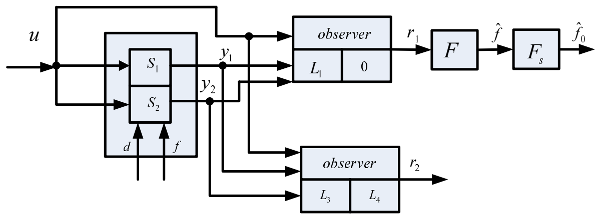

Fig. 1. Fault detection and estimation system.

where $x_c \in \mathrm{R}^{n_f}$ is the state of the fault estimation filter F, $\hat{f} \in \mathrm{R}^{n_f}$ is the output of the filter. A_c, B_c, C_c, D_c are parameter matrix of the fault estimation filter. The proposed fault detection and fault estimation framework is shown in Fig. 1. F_s is a static filter for improving the fault estimation accuracy which is stated in the next section in details.

Remark 1: In this paper, the high order system is transformed into two subsystems. The first subsystem has an order same as that of the dimension of the fault. Generally, it has a much lower order than the original system. The second subsystem is not connected to the fault directly and by choosing a special observer gain, the residual of the second observer can be totally decoupled from the faults. Furthermore, a low order fault estimation is developed by using only the residual of the first subsystem. The idea to transform the monitored system into two or more subsystems is also found in some previous fault diagnosis papers. In [2] and [6], the authors transferred the monitored system into three subsystems and furthermore, a fault detection observer and isolation approach are proposed. However, the proposed approach is quite different from the one in this paper. In [12], a similar transformation is used to separate the system into two subsystems. Nevertheless, the problem considered in the paper is the sensor fault detection for a class of nonlinear systems.

3. FAULT DETECTION OBSERVER AND FAULT ESTIMATION FILTER DESIGN

3.1. Fault detection observer design

Define $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$, $r_1 = y_1 - \hat{y}_1$ and $r_2 = y_2 - \hat{y}_2$, the derived state estimated error equations are stated as follows:

$$E_1: \begin{cases} \dot{e}_1 = A_1e_1 + A_2e_2 + B_{f1}f + B_{d1}d \\ \quad +L_1(\hat{y}_1 - y_1) + L_2y_2 \\ r_1 = C_1e_1, \end{cases} \tag{14}$$

$$E_2: \begin{cases} \dot{e}_2 = A_3x_1 + A_4e_2 + B_{d2}d - L_3y_1 \\ \quad +L_4(\hat{y}_2 - y_2) \\ r_2 = C_4e_2. \end{cases} \tag{15}$$

Notice that the state estimation error e_2 is coupled with the fault f by term A_3x_1. Let $L_3 = A_3C_1^{-1}$ (C_1 is invertible), then we have

$$L_3y_1 = A_3x_1. \tag{16}$$

Now the fault is totally decoupled from the observer O_2. For simplicity, in this paper, the observer gain L_2 is set to 0.

Remark 2: By using $L_3 y_1 = A_3 x_1$, the residual r_2 is totally decoupled from the fault f and state x_1. y_2 in (14) does not have any effect on the state estimation of system E_2. Hence, L_2 can be set to 0. The price paid for this simplification is that the pair (A_4, C_4) should be detectable. In the case that this condition cannot be met, the approach proposed in this paper does not work anymore.

The fault detection observer design problem is transferred to two sub-problems:

1) Design L_4 such that the model uncertainty and disturbance part d is attenuated as much as possible.

2) Design L_1 such that the residual r_1 is sensitive to the fault and robust to the disturbances d and the estimation error e_2.

The following lemma is necessary for designing the observer gain L_4.

Lemma 2 [16]: The pair (A_4, C_4) is detectable if and only if

$$rank\begin{bmatrix} sI_n - A & B_f \\ C & 0 \end{bmatrix} = n + n_f . \tag{17}$$

There are quite a lot approaches to obtain L_4. For example, H_∞ observer proposed in [17] and [8], pole placement method and the method proposed in [10]. For the observer gain L_1, there are also many solutions. In this paper, the observer presented in our former work [10] is applied.

3.2. Fault estimation filter design

In the light of the state estimate error equations described by

$$E_1 : \begin{cases} \dot{e}_1 = (A_1 - L_1 C_1)e_1 + A_2 e_2 + B_{f1} f + B_{d1} d_1 \\ r_1 = C_1 e_1. \end{cases} \tag{18}$$

The objective of the fault estimation filter is to optimize the parameter matrices A_c, B_c, C_c, D_c such that the fault estimation error e_f (see Fig. 2) is minimized.

Define $A_e = (A_1 - L_1 C_1)$ and $e_f = f - \hat{f}$, combine the fault estimation filter (13), then the overall dynamic equations for the fault estimation system are

$$\begin{pmatrix} \dot{e}_1 \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} A_e & 0 \\ B_c C & A_c \end{pmatrix} \begin{pmatrix} e_1 \\ x_c \end{pmatrix} + \begin{pmatrix} A_2 \\ 0 \end{pmatrix} e_2 + \begin{pmatrix} B_{d1} \\ 0 \end{pmatrix} d + \begin{pmatrix} B_{f1} \\ 0 \end{pmatrix} f,$$

$$e_f = (-D_c C - C_c)\begin{pmatrix} e_1 \\ x_c \end{pmatrix} + f.$$

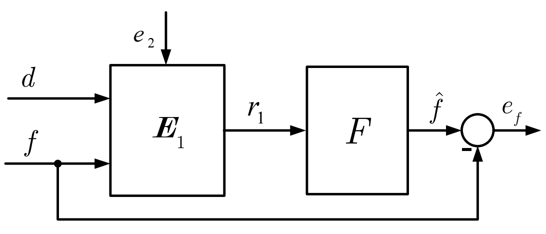

Fig. 2. The fault estimation filter.

Define the following matrices:

$$\mathscr{A} = \begin{pmatrix} A_e & 0 \\ B_c C & A_c \end{pmatrix},$$

$$\mathscr{B}_{\bar{d}} = \begin{pmatrix} A_2 & B_{d1} \\ 0 & 0 \end{pmatrix}, \quad \mathscr{B}_f = \begin{pmatrix} B_{f1} \\ 0 \end{pmatrix},$$

$$\mathscr{C} = \begin{pmatrix} -D_c C & -C_c \end{pmatrix}, \quad \mathscr{D}_f = I.$$

The transfer function from $\bar{d} = (e_2 \;\; d)'$ and f to e_f is

$$e_f = G_{e_f \bar{d}}(z)\bar{d} + G_{e_f f}(z) f, \tag{19}$$

where

$$G_{e_f \bar{d}} = \mathscr{C}(sI - \mathscr{A})^{-1}\mathscr{B}_{\bar{d}}, \tag{20}$$

$$G_{e_f f} = \mathscr{C}(sI - \mathscr{A})^{-1}\mathscr{B}_f + \mathscr{D}_f. \tag{21}$$

We use the H_∞ norm in the finite frequency domain as the performance criterion. The objective of the fault estimation filter is to minimize the estimated error e_f as much as possible such that the following criteria are satisfied:

$$\bar{\sigma}^{[\omega_{dl}, \omega_{dh}]}\left(G_{e_f \bar{d}}(j\omega)\right) < \gamma_{ed} \tag{22}$$

for attenuating the disturbances and

$$\bar{\sigma}^{[\omega_{fl}, \omega_{fh}]}(G_{e_f f}(j\omega)) < \gamma_{ef} \tag{23}$$

for the fault estimation. The frequency ranges are specified for the considered intervals $[\omega_{dl}, \omega_{dh}]$ for disturbance signal $\bar{d}$ and $[\omega_{fl}, \omega_{fh}]$ for fault signal f, respectively. An LMI solution to the frequency performance requirements stated by (22) and (23) is presented in the following lemmas. Here the techniques in [18] are applied to convert the problem to LMIs optimization problem.

Define the new variables:

$$W = \begin{pmatrix} X & (I - XY')(V')^{-1} \\ U & -UY'(V')^{-1} \end{pmatrix}, \tag{24}$$

$$Z := YX + VU,$$

$$\begin{pmatrix} M & G \\ H & L \end{pmatrix} = \begin{pmatrix} YAX & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} V & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} A_c & B_c \\ C_c & D_c \end{pmatrix}\begin{pmatrix} U & 0 \\ CX & I \end{pmatrix}. \tag{25}$$

These variables are used in the lemmas in the following.

Lemma 3: The fault estimation filter (19) meets the disturbances attenuation specification (22) if there exist matrices X, Y, Z, M, G, H, L and P_d, $Q_d \in H_n$ satisfying $Q_d > 0$, and

$$\begin{pmatrix} \Phi_{111,d} - \Phi_{112,d} & \Phi_{12,d} \\ \Phi'_{12,d} & -I \end{pmatrix} < 0, \tag{26}$$

where

$$\Phi_{111,d} := \begin{pmatrix} -Q_d & P + j\omega_{dc}Q_d & 0 \\ P - j\omega_{dc}Q_d & \omega_{dl}\omega_{dh}Q_d & 0 \\ 0 & 0 & -\gamma_{ed}^2 \end{pmatrix},$$

$$\Phi_{112,d} := He\left(\begin{pmatrix} -\mathscr{W} \\ \mathscr{E} \\ \mathscr{G}_d \end{pmatrix} R_d\right), \quad \Phi_{12,d} = \begin{pmatrix} 0 \\ \mathscr{F}_d \\ \mathscr{J}_d \end{pmatrix},$$

where R_d is a multiplier, $\omega_{dc} = (\omega_{dl} + \omega_{dh})/2$ and the other matrices are defined as follows:

$$\mathscr{W} := \begin{pmatrix} X & I \\ Z & Y \end{pmatrix},$$

$$\mathscr{E} := \begin{pmatrix} A_e X & A_e \\ M & YA_e + GC \end{pmatrix},$$

$$\mathscr{F}_d := \begin{pmatrix} B_{d1} & YB_{f1} + GD_d \end{pmatrix}, \quad \mathscr{G}_d := \begin{pmatrix} -H & -LC \end{pmatrix},$$

$$\mathscr{J}_d := -LD_d.$$

Proof: Let $\prod_d = \begin{pmatrix} I & 0 \\ 0 & \gamma_{ed}^2 I \end{pmatrix}$ and

$$\Xi_d = \begin{pmatrix} -Q_d & P_d + j\omega_{dc}Q_d \\ P_d - j\omega_d Q_d & \omega_{dl}\omega_{dh}Q_d \end{pmatrix}.$$

Using Lemma 6 in the appendix, the disturbances attenuation condition (22) is equal to (50). The condition (50) becomes

$$\Phi_{111,d} - \Phi_{112,d} + \Phi_{12,d}^{'}\Phi_{12,d} < 0\,. \quad (27)$$

Using Schur complement formulation, we have

$$\begin{pmatrix} \Phi_{111,d} - \Phi_{112,d} & \Phi_{12,d} \\ \Phi_{12,d}^{'} & -I \end{pmatrix} < 0\,. \quad (28)$$

Remark 3: Lemma 3 provides an LMI solution for the disturbance attenuation criteria (22). For the disturbance $\bar{d}$ located in the frequency interval $[\omega_{dl} + \omega_{dh}]$, the H_∞ norm in the frequency interval is minimized to smaller than γ_{ed}. Since we only optimize the parameters in the frequency interval rather than in the whole frequency domain used in the classical H_∞ norm criteria, a less conservative solution can be obtained. The physical meaning of this lemma is further explained by using Fig. 3. The H_∞ norm (the maximum singular value $\bar{\sigma}(G_{e_f\bar{d}}(j\omega))$) from $\bar{d}$ to e_f for two different fault estimation filters F_1 and F_2 are shown with bold and dash line, respectively. If the disturbance $\bar{d}$ is located in the frequency interval $[\omega_1\ \omega_2]$, then fault estimation filter F_2 is a better solution for disturbance attenuation. In the case that the disturbance $\bar{d}$ is located in the frequency interval $[\omega_3\ \omega_4]$, F_1 achieves better performance.

Lemma 4: The system (19) meets the fault estimation specification (23) if there exist matrices X, Y, Z, M, G, H, L and P_f, $Q_f \in H_n$ satisfying $Q_f > 0$, and

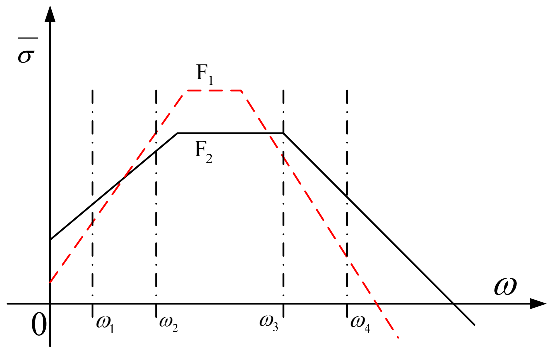

Fig. 3. The physical meaning of Lemma 3.

$$\begin{pmatrix} \Psi_{11,f} & \Psi_{12,f} \\ \Psi_{12,f}^{'} & -I \end{pmatrix} < 0, \quad (29)$$

where

$$\Psi_{11,f} := \begin{pmatrix} -Q_f & P + j\omega_{fc}Q_f & 0 \\ P - j\omega_{fc}Q_f & \omega_{fl}\omega_{fh}Q_f & 0 \\ 0 & 0 & -\gamma_{ef}^2 \end{pmatrix} - \Upsilon, \quad (30)$$

$$\Psi_{12,f} = \begin{pmatrix} 0 \\ \mathscr{F}_f \\ \mathscr{J}_f \end{pmatrix}, \quad (31)$$

where R_f is a multiplier, $\omega_{fc} = (\omega_{fl} + \omega_{fh})/2$ and

$$\Upsilon = He\left(\begin{pmatrix} -\mathscr{W} \\ \mathscr{E} \\ \mathscr{G}_f \end{pmatrix} R_f\right),$$

$$\mathscr{F}_f := \begin{pmatrix} B_{f1} & YB_e^f + GD_f \end{pmatrix}, \quad \mathscr{G}_f := \begin{pmatrix} -H & LC \end{pmatrix},$$

$$\mathscr{J}_f := I - LD_f.$$

The proof of this lemma is omitted since it is similar to that of Lemma 3.

Remark 4: The physical meaning of this lemma can be explained in a similar way as that of lemma 3. LMI (29) is a sufficient condition for the fault estimation accuracy (23), that is, it provides a solution to minimize the H_∞ norm $(\bar{\sigma}(G_{e_f f}(j\omega)))$ in the finite frequency interval $[\omega_{fl}, \omega_{fh}]$. Due to $e_f = f - \hat{f}$, a smaller $\bar{\sigma}(G_{e_f f}(j\omega))$ means that the fault estimation accuracy is higher in the considered frequency interval.

Lemmas 3 and 4 presented before only guarantee the H_∞ performance of the fault estimation filter F. Besides these two performances, the stability of the fault estimation system should be satisfied. In the following lemma, the eigenvalues of the fault estimation system are placed to the left of a vertical line $Re(s) < -\alpha(\alpha > 0)$ on the complex plane. This avoids the problem that the poles are too close to the right half-plane.

Lemma 5: Consider the system (19), the following statements are equivalent:

(i) The eigenvalues of $\mathscr{A}$ are to the left of a vertical line $Re(s) < -\alpha(\alpha > 0)$ on the complex plane.

(ii) There exist W and $P_c = P_c' > 0$ such that

$$\begin{pmatrix} 0 & P_c \\ P_c & 0 \end{pmatrix} < He\left(\begin{pmatrix} -I \\ \mathscr{A}+\alpha I \end{pmatrix} W\begin{pmatrix} -qI & pI \end{pmatrix}\right), \tag{32}$$

where p, n_f are arbitrary fixed real numbers satisfying $pq < 0$.

(iii) There exist X, Y, Z, M, G and $\mathscr{P}_t = \mathscr{P}_t' > 0$ such that

$$\begin{pmatrix} 0 & \mathscr{P}_t \\ \mathscr{P}_t & 0 \end{pmatrix} < He\left(\begin{pmatrix} -I \\ \mathbb{A} \end{pmatrix} \mathscr{W}\begin{pmatrix} -qI & pI \end{pmatrix}\right), \tag{33}$$

where

$$\mathbb{A} = \begin{pmatrix} (A_e + \alpha I)X & A_e \\ M + \alpha Z & Y(A_e + \alpha I) + GC_e \end{pmatrix},$$

$$\mathscr{W} = \begin{pmatrix} X & I \\ Z & Y \end{pmatrix}.$$

Proof: Using the fact that $\lambda(\mathscr{A} + \alpha I) = \lambda(\mathscr{A}) + \alpha$ and Lyapunov inequality, $Re(\lambda(\mathscr{A})) < -\alpha$ is equivalence to

$$(\mathscr{A} + \alpha I)P_c + P_c(\mathscr{A} + \alpha I)' < 0, \tag{34}$$

where $P_c = P_c' > 0$, that is

$$\begin{pmatrix} \mathscr{A}+\alpha I & I \end{pmatrix}\begin{pmatrix} 0 & P_c \\ P_c & 0 \end{pmatrix}\begin{pmatrix} (\mathscr{A}+\alpha I)' \\ I \end{pmatrix} < 0. \tag{35}$$

Notice that

$$\begin{pmatrix} pI & qI \end{pmatrix}\begin{pmatrix} 0 & P_c \\ P_c & 0 \end{pmatrix}\begin{pmatrix} pI \\ qI \end{pmatrix} = 2pqP_c < 0. \tag{36}$$

Using the following facts

$$\begin{pmatrix} pI & qI \end{pmatrix}\begin{pmatrix} qI \\ -pI \end{pmatrix} = 0, \tag{37}$$

$$\begin{pmatrix} \mathscr{A}+\alpha I & I \end{pmatrix}\begin{pmatrix} I \\ -(\mathscr{A}+\alpha I) \end{pmatrix} = 0, \tag{38}$$

and Lemma 7 in the appendix, one obtains

$$\begin{pmatrix} 0 & P_c \\ P_c & 0 \end{pmatrix} < He\left(\begin{pmatrix} -I \\ \mathscr{A}+\alpha I \end{pmatrix} W\begin{pmatrix} -qI & pI \end{pmatrix}\right). \tag{39}$$

The equivalence between (i) and (ii) is proven. A similar result can be found in [19].

Define nonsingular matrix

$$\Gamma = \begin{pmatrix} I & 0 \\ Y & V \end{pmatrix}. \tag{40}$$

Multiply $diag(\Gamma, \Gamma)$ to the left side of the inequality (32) and $diag(\Gamma', \Gamma')$ to the right side. We obtain inequality (33).

Now the existence of the fault estimation filter is discussed here. The transfer function from f to residual r_1 is

$$G_{r_1 f}(s) = C_1\left(sI - (A_1 - L_1C_1)\right)^{-1} B_{f1}.$$

The transfer function of the fault estimation filter is

$$G_{\hat{f}r_1}(s) = C_c(sI - A_c)B_c + D_c.$$

Notice that the fault can be precisely estimated if the $G_{r_1\hat{f}}(s)$ is the inverse of $G_{r_1 f}(s)$, at least when the disturbance disappears totally.

Define a rational transfer function $G_F(s) = C_c(sI - A_c)B_c$, then $G_{\hat{f}r_1}(s) = G_F(s) + D_c$. In the case that $G_{\hat{f}r_1}(s)$ is the inverse of $G_{r_1 f}(s)$, we have

$$\left(G_F(s) + D_c\right)G_{r_1 f}(s) = I. \tag{41}$$

It is clearly seen that the above equation is undetermined. We need more constraints to achieve a solution for the filter.

To achieve a feasible solution, a small direct fault transfer matrix $D_f = \beta C_1(-A + L_1C_1)^{-1}B_{f1}$ (β is a small constant) is added into the state estimation error equation (42). That is

$$\begin{aligned} \dot{e}_1 &= (A_1 - L_1C_1)e_1 + A_2e_2 + B_{f1}f + B_{d1}d, \\ r_1 &= C_1e_1 + D_f f. \end{aligned} \tag{42}$$

Now we have

$$G_{e_f f} = \mathscr{C}(sI - \mathscr{A})^{-1}\mathscr{B}_f + \left(I - D_cD_f\right). \tag{43}$$

To minimize $\bar{\sigma}^{[\omega_{fl},\omega_{fh}]}(G_{e_f f}(j\omega))$, D_c is optimized to minimize $\bar{\sigma}(I - D_cD_f)$ which results in $D_c \to D_f^{-1}$. The achieved D_c decouples the different faults at static state since

$$D_cG_{fr_1}(0) = \beta D_cD_f.$$

Define the obtained filter as $\bar{F}(s)$. The fault estimation filter $F(s)$ can be yield by using the following equation:

$$F(s)G_{r_1 f} = \bar{F}(s)\left(G_{fr_1}(s) + D_f\right), \tag{44}$$

that is

$$F(s) = \bar{F}(s)(G_{r_1 f}(s) + D_f)(G_{r_1 f}(s))^{-1}. \tag{45}$$

Notice that $D_c = \beta C_1(-A + L_1C_1)^{-1}B_{f1}$ (β is a small constant), at the low frequency domain, one yields $F(s) \approx \bar{F}(s)$. Now we present the filter design algorithm in the following.

Algorithm 1 (Fault estimation filter design algorithm)**:** Given β, L_1 and D_f, choosing multipliers R_d and R_f, the fault estimation filter design can be obtained by solving the following optimization problem in terms of LMIs:

$$\min \alpha_d \gamma_{ed} + \alpha_f \gamma_{ef}$$
$$s.t.$$
$$P_t > 0,\ Q_d > 0,\ Q_f > 0,\ (26),(29),(33),$$

where α_d and α_f are two weighting constants.

Once the matrices X, Y, Z, M, G, H, L are solved, the fault estimation filter can be recovered by using (25). Notice that the matrices U and V in (24) are unknown. We can make a decomposition (such as QR or SVD) of $(Z - YX)$ to obtain V and U ($VU = Z - XY$). Then the fault estimation filter matrices can be recovered by:

$$D_c = L, \tag{46}$$
$$C_c = (H - D_c CX)U^{-1}, \tag{47}$$
$$B_c = V^{-1}G, \tag{48}$$
$$A_c = V^{-1}(M - YAX - VB_c CX)U^{-1}. \tag{49}$$

3.3. Design of the static matric F_s

The proposed fault estimation filter reaches an accuracy to some extend. However, it is found that, in some cases, the static precision is not good enough. To improve the static estimation performance, a constant matrix F_s is designed in the following.

$$F = (G_{\hat{f}\eta_1}(0)G_{\eta_1 f}(0))^{-1},$$

where

$$G_{\eta_1 f}(0) = C_1(-A_1 + L_1C_1)^{-1}B_{f1},$$
$$G_{\hat{f}\eta_1}(0) = C_c(-A_c)^{-1}B_c + D_c.$$

4. SIMULATION

Example 1: Consider the longitudinal dynamics of the VTOL aircraft with a state space model

$$\dot{x} = Ax + B_u u + B_d d + B_f f,$$
$$y = Cx.$$

The system parameters are given as follows:

$$A = \begin{pmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1.0000 & 0 \end{pmatrix},$$

$$B_u = \begin{pmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{pmatrix},\quad B_d = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$B_f = B_u,\quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}.$$

The transformation matrices T and S are given as follows:

$$T = \begin{pmatrix} -0.0409 & -0.5361 & 0.8436 & 0.1025 \\ 0.0557 & -0.8484 & -0.5403 & -0.4629 \\ 0.9823 & 0.1118 & 0.1505 & 0 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix},$$

$$S = \begin{pmatrix} -0.0644 & -0.5163 & 0.8040 & 0.2877 \\ 0.0914 & -0.6320 & -0.1269 & -0.7590 \\ 0.8250 & 0.3423 & 0.3748 & -0.2484 \\ 0.5539 & -0.4656 & -0.4438 & 0.5286 \end{pmatrix}.$$

The observer gains achieved:

$$L_1 = \begin{pmatrix} -4.2547 & 0.0415 \\ 3.4283 & -2.0069 \end{pmatrix},\ L_3 = \begin{pmatrix} 0.2161 & -0.0133 \\ 0.8040 & -0.1269 \end{pmatrix},$$

$$L_4 = \begin{pmatrix} 3565 & 1688 \\ -1.50 & 2.40 \end{pmatrix}.$$

The fault estimation filter matrices:

$$A_c = \begin{pmatrix} -50.8661 & -0.0397 \\ 0.0458 & -50.9297 \end{pmatrix},$$

$$B_c = \begin{pmatrix} -220.58 & 12.16 \\ 0.89 & 202.23 \end{pmatrix} \times 10^3,$$

$$C_c = \begin{pmatrix} -1.647 & -1.707 \\ -0.005 & -1.869 \end{pmatrix} \times 10^{-2},$$

$$D_c = \begin{pmatrix} -7.2599 & 7.3121 \\ 0.0052 & 7.5694 \end{pmatrix}.$$

Finally, the static gain matrix is given as follows:

$$F_s = \begin{pmatrix} 1.0196 & -0.0003 \\ -0.0003 & 1.0198 \end{pmatrix}.$$

The simulation result for the first actuator fault is shown in Fig. 4. The disturbances are simulated by filtered white noise. The actuator fault is $f(t) = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}$ since

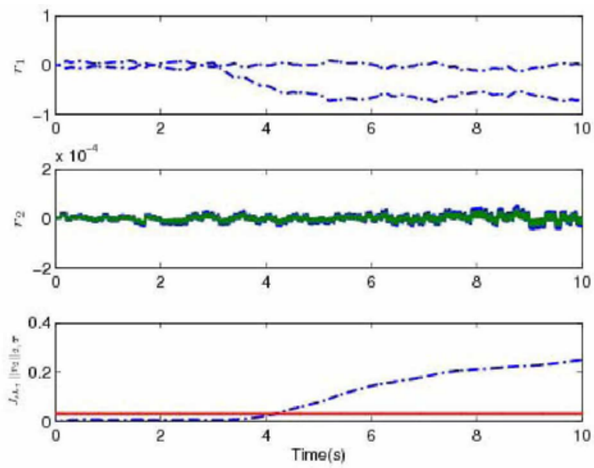

Fig. 4. The evolution of the norm mean of the residuals r_1 and r_2 when the first actuator occurs a fault 3s after the simulation starts.

and $f = 0$, elsewhere. It can be seen that r_2 does not have any clear change when the fault occurs since it is decoupled from the faults. The residual r_1 is sensitive to the fault and the small fault is detected just 1s after the fault occurs. In Fig. 5, the case for the fault occurring in the second actuator $f(t) = \begin{pmatrix} 0.0 \\ 0.1 \end{pmatrix}$ is shown.

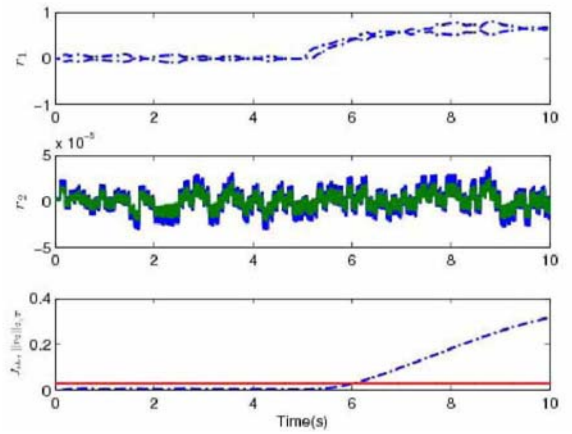

Fig. 5. The evolution of the norm mean of the residuals r_1 and r_2 when the second actuator occurs a fault 5s after the simulation starts.

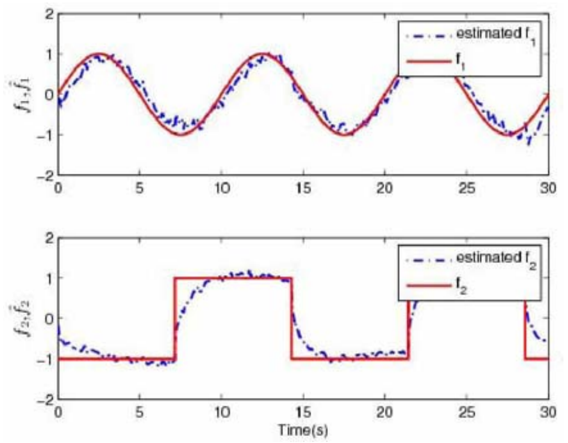

Fig. 6. The fault estimation result by the designed filter F.

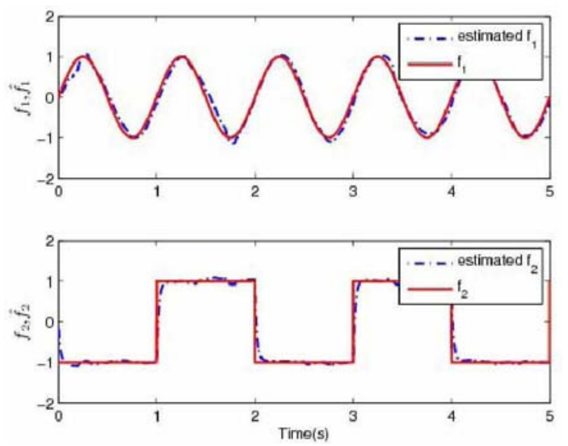

Fig. 7. The fault estimation results improved by the static matrix F_s.

The fault signal and estimated fault by the filter F and F_s are shown in Figs. 6 and 7, respectively. It can be seen that the fault estimation filter achieves a good estimation performance.

5. CONCLUSIONS

Based on a model decomposition algorithm proposed in this paper, a novel fault detection system is designed such that only the residuals with the same dimension as the dimension of the faults are sensitive to the faults. The rest residuals are totally decoupled from the faults. A lower order fault estimation filter design approach is proposed. In addition, the design of a static fault estimation matrix is presented for further improving the fault estimation precision. The effectiveness of the proposed methods are validated by a simulation example.

APPENDIX

Lemma 6: The considered system in (19) meets the specifications:

$$(G_k(j\omega) \quad I)\Pi(G_k(j\omega) \quad I)^* < 0,$$

where $G_k(j\omega)$ can be replaced by any $G_{e_f d}(j\omega)$ or $G_{e_f f}(j\omega)$, if there exist matrices X, Y, Z, M, G, H, $L \in \mathscr{R}^{n\times n}$ and $P_k, Q_k \in H_n$ satisfying $Q_k > 0$ and

$$\begin{pmatrix} \Xi_k & 0 \\ 0 & 0 \end{pmatrix} + \mathscr{H}_k \Pi_k \mathscr{H}_k^{'} < He(\mathscr{L}_k R),$$

where $R \in \mathscr{R}^{(2n)\times(2n+n_y)}$ is a multiplier need to be chosen, Ξ_k is defined as follows

$$\Xi = \begin{pmatrix} -Q_k & P_k + j\omega_c Q_k \\ P_k - j\omega_c Q_k & -\omega_1\omega_2 Q_k \end{pmatrix}, \quad \omega_c = \frac{(\omega_1 + \omega_2)}{2}$$

and

$$\mathscr{H}_k := \begin{pmatrix} 0 & 0 \\ \mathscr{F}_k & 0 \\ \mathscr{J}_k & I \end{pmatrix}, \quad \mathscr{L}_k := \begin{pmatrix} -\mathscr{W} \\ \mathscr{E} \\ \mathscr{G} \end{pmatrix}, \quad \mathscr{W} := \begin{pmatrix} X & I \\ Z & Y \end{pmatrix},$$

$$\mathscr{E} := \begin{pmatrix} A_e X & A_e \\ M & YA_e + GC \end{pmatrix}, \quad \mathscr{G} := \begin{pmatrix} -H & -LC \end{pmatrix},$$

where the lower index k can be replaced by d for disturbance attenuating cases and the corresponding matrices are as follows:

$$\Pi_f = \begin{pmatrix} \beta^2 I & 0 \\ 0 & I \end{pmatrix}, \quad \mathscr{F}_d = \begin{pmatrix} B_e^d & YB_e^d + GD_d \end{pmatrix}, \quad \mathscr{J}_d := -LD_d$$

and the lower index k can be replaced by f or attenuating the fault f to the estimated error e_f case:

$$\Pi_d = \begin{pmatrix} I & 0 \\ 0 & -\gamma^2 I \end{pmatrix}, \quad \mathscr{F}_f := \begin{pmatrix} B_e^f & YB_e^f + GD_f \end{pmatrix}, \quad \mathscr{J}_f := I - LD_f.$$

Proof: This lemma is a corollary of Theorem 2 in [18].

Lemma 7 [20]**:** Let matrices $B \in \mathscr{C}^{n\times m}$, $C \in \mathscr{C}^{k\times n}$ and $Q = Q^* \in \mathscr{C}^{n\times n}$ be given. Then the following statements are equivalent:

(i) There exists a matrix X satisfying

$$BXC+(BXC)^{*}+Q<0.$$

(ii) The following two conditions hold

$$B^{\perp}QB^{\perp *}<0, \tag{50}$$

$$C^{*\perp}QC^{*\perp *}<0. \tag{51}$$

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