International Journal of Control, Automation, and Systems (2013) 11(1):136-146 ISSN:1598-6446 eISSN:2005-4092
DOI 10.1007/s12555-011-9112-y http://www.springer.com/12555

Stability and Stabilization for Discrete-time Markovian Jump Fuzzy Systems with Time-varying Delays: Partially Known Transition Probabilities Case

Min Kook Song, Jin Bae Park*, and Young Hoon Joo

Abstract: This paper focuses on the stability analysis and the stabilization problem for a discrete-time Markovian jump fuzzy systems (MJFSs) with time-varying delays and partially known transition probabilities. These systems are made more general, by relaxing the traditional assumption in MJFSs that all the transition probabilities must be completely known. The class of MJFSs considered is described by a fuzzy model composed of two levels: a crisp level that represents the jumps and a fuzzy level that represents the system nonlinearities. Based on a stochastic Lyapunov function, stability and stabilization conditions for the MJFSs with time-varying delays are derived in both the case of completely known transition probabilities and the case of partially known transition probabilities. The derived conditions are represented in terms of linear matrix inequalities (LMIs). Finally, a numerical example is used to illustrate the effectiveness of the proposed theorem.

Keywords: Linear matrix inequality (LMI), Markovian jump fuzzy systems, probability transition matrix, time varying delays.

1. INTRODUCTION

Many practical systems have variable parameters and structures subject to random changes, which may result from abrupt phenomena such as component failures or repairs, changing of subsystem interconnections, and abrupt environmental disturbances [1]. These can be modeled as hybrid systems with two components in the state vector. The first component varies continuously and is referred to as the continuous state of the system. The second component varies discretely and is referred to as the mode assumed by the system. Markovian jump systems (MJSs) are a special class of hybrid systems in which, the random jumps in system parameters are represented by a Markov process that using values with in a finite set.

Over the past decade, some important control issues have been studied for Markovian jump linear systems (MJLSs) because of the difficulty inherent in the analysis of Markovian jump nonlinear systems (MJNLSs) [1,2]. However, if nonlinearity is not distinguished from uncertainty, the obtained results are in general conservative. Recently, a fuzzy-model-based control technique for a class of MJNLS was introduced in [3,4] and [5]. In [4], Natache developed a systematic technique to obtain a robust stochastic fuzzy controller that guarantees the $\mathcal{L}_2$ gain of the closed-loop system with respect to external inputs equal to or less than a prescribed value. Wu investigated the robust fuzzy control problem of uncertain discrete-time MJFSs without mode observations in [5]. Despite these efforts, the design of a controller for MJFSs that can handle model uncertainties has been one of the most challenging problems in recent decades.

It is well known that time-delays occur frequently in many practical systems, and they are a significant source of instability and poor performance [6,7]. The transition probabilities of the jumping process are also important yet almost all of the issues on MJFSs have been investigated assuming complete knowledge of transition probabilities. However, the likelihood of obtaining such complete knowledge is questionable, and the cost is likely to be high in most cases. To the best of the author's knowledge, the fuzzy control problem for discrete MJFSs with time-delay has not been fully investigated. This lack suggests a need for the significant and challenging investigation of more general MJFSs in which transition probabilities are partially known and time-varying delays are included.

In this paper, we are interested in the stability analysis and stabilization synthesis problems for a class of MJFSs with time-varying delays in which transition probabilities are partially known. A fuzzy controller is constructed so that the MJFSs with time-varying delays can be stabilized. An advancement of the delay-range-dependent concept is introduced here and less conservative stability and

Manuscript received August 31, 2011; revised April 19, 2012 and June 27, 2012; accepted August 27, 2012. Recommended by Editorial Board member Shengyuan Xu under the direction of Editor Hyungbo Shim.

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No.:2012014088) and the Human Resources Development program(No. 20124030200040) of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Knowledge Economy.

Min Kook Song and Jin Bae Park are with the Department of Electrical and Electronic Engineering, Yonsei University, Seodaemun-gu, Seoul 120-749, Korea (e-mails: MinKookSong@gmail.com, jbpark@yonsei.ac.kr).

Young Hoon Joo is with the Department of Control and Robotics Engineering, Kunsan National University, Kunsan, Chonbuk 573-701, Korea (e-mail: yhjoo@kunsan.ac.kr).

* Corresponding author.

stabilization conditions for the underlying systems are derived by constructing more appropriate Lyapunov functional for discrete-time MJFSs in both completely and partially known transition probabilities cases. The derived conditions are represented in terms of linear matrix inequalities (LMIs). Finally, a numerical example is presented to illustrate the effectiveness of the developed method.

Notations: $R^n := n-$ dimensional real space, $A^T :=$ Transpose of matrix A, $P \succ 0$, $(resp. P \prec 0) :=$ positive (resp., negative-definite) symmetric matrix, and $* :=$ the transposed element in symmetric positions.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider that a given probability space $(\Omega, \mathbb{F}, \mathbb{P})$ - Ω is the sample space, where $\mathbb{F}$ is the algebra of events, and $\mathbb{P}$ is the probability measure defined on $\mathbb{F}$. We consider a discrete-time MJNLS over the space $(\Omega, \mathbb{F}, \mathbb{P})$, which can be described by the following fuzzy model:

$$\begin{aligned} &R^i : \text{IF } z_1(k) \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(k) \text{ is } \Gamma_{ip} \\ &\quad \text{THEN} x(k+1) = A_{0i}(\eta(k))x(k) + B_i(\eta(k))u(k) \\ &\quad\quad + A_{1i}(\eta(k))x(k-\tau(k)), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ constitutes the state vector; $u(k) \in \mathbb{R}^m$ is the control input, R_i, $i \in \mathcal{I}_R = \{1, 2, \cdots, r\}$, is the i th fuzzy rule, $z_h(k)$, $h \in \mathcal{I}_P = \{1, 2, \cdots, p\}$, is the h th premise variable, Γ_{ih}, $(i, h) \in \mathcal{I}_R \times \mathcal{I}_P$, is the fuzzy set of $z_h(k)$ in R_i, and the system matrices of the i th rule are denoted by (A_{0i}, B_i, A_{1i}), which are assumed known and as some constant matrices of compatible dimensions. The time delay is considered to be time-varying and has lower and upper bounds, $0 < \tau_1 \leqslant \tau(k)$ which is very common in practice. The random form process $\eta(k)$ is a discrete-time Markovian process with values in a finite space state denoted by $\mathbb{T} = \{1, 2, \cdots, \mathcal{N}\}$. The set $\mathbb{T}$ comprises the operation modes of the system. The transition probabilities for the process $\eta(k)$ are defined as

$$\mathbf{Pr}(\eta(k+1) = s \mid \eta(k) = l) = \pi_{ls},$$

where π_{ls} is the transition probability from mode l at time k to mode s at time $k+1$, and where $\pi_{ls} > 0$, $\sum_{s=1}^{\mathcal{N}} \pi_{ls} = 1$, $\forall l, s \in \mathbb{T}$. The transition probabilities matrix is defined by

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1\mathcal{N}} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2\mathcal{N}} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{\mathcal{N}1} & \pi_{\mathcal{N}2} & \cdots & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}.$$

The set $\mathbb{T}$ contains $\mathbb{N}$ modes of system (1) for $\eta(k) = l \in \mathbb{T}$. In addition, the transition probabilities of the Markov chain in this paper are considered to be partially known, namely, some elements in matrix π are time-invariant but unknown. We denote

$$\mathbb{T}_{\mathbb{K}}^l = \{\text{s: if } \pi_{ls} \text{ is known}\}, \tag{2}$$

$$\mathbb{T}_{\mathbb{UK}}^l = \{\text{s: if } \pi_{ls} \text{ is unknown}\}. \tag{3}$$

If $\mathbb{T}_{\mathbb{K}}^l \neq 0$, it is described as

$$\mathbb{T}_{\mathbb{K}}^l = \{\mathbb{E}_1^l, \cdots, \mathbb{E}_m^l\}, \quad 1 \leq m \leq \mathbb{N}, \tag{4}$$

where $\mathbb{E}_m^l$ represents the m th element with the index $\mathbb{E}_m^l$ in the l th row of matrix π.

Let the mode at k be l. Using the center-average defuzzifer, product inference, and singleton fuzzifier, (1) is inferred as

$$\begin{aligned} x(k+1) = &\sum_{i=1}^{r} \theta_i(z(k)) \big(A_{0i}(\eta(k)) x(k) \\ &+ B_i(\eta(k))u(k) + A_{1i}(\eta(k))x(k-\tau(k)) \big), \end{aligned} \tag{5}$$

where

$$\theta_i(z(k)) = w_i(z(k)) / \sum_{i=1}^{r} w_i(z(k)),$$

$$w_i(z(k)) = \prod_{h=1}^{p} \mu_{\Gamma_{ih}}(z_h(k)),$$

and $\mu_{\Gamma_{ih}}(z_h(k)) : U_{z_h} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of $z_h(k)$ on the compact set U_{z_h}. Some basic properties are $\theta_i(z(k)) \geq 0$ and $\sum_{i=1}^{r} \theta_i(z(k)) = 1$. The initial condition of system (1) is given by

$$x(k) = \varphi(k), \quad k = -\tau_2, -\tau_2 + 1, \cdots, 0, \tag{6}$$

where $\varphi(\bullet)$ is a continuous vector-valued initial function.

In the following discussion, for the convenience of notations, we will denote

$$A_{0i,l}(k) = \sum_{i=1}^{r} \theta_i(z(k)) A_{0i}(\eta(k)),$$

$$A_{1i,l}(k) = \sum_{i=1}^{r} \theta_i(z(k)) A_{1i}(\eta(k)),$$

$$B_{i,l}(k) = \sum_{i=1}^{r} \theta_i(z(k)) B_i(\eta(k)).$$

Then, the purpose of this paper is to determine the feedback gains $K_j (j \in \mathcal{I}_R)$ such that the resulting closed-loop system is asymptotically stable via the mode-independent fuzzy controller

$$\begin{aligned} &R_i : \text{IF } z_1(k) \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(k) \text{ is } \Gamma_{ip} \\ &\quad \text{THEN } u(k) = K_j(\eta(k))x(k), \quad j \in \mathcal{I}_R, \end{aligned}$$

whose defuzzified output is given by

$$u(k)=\sum_{j=1}^{r}\theta_j(z(k))K_j(\eta(k))x(k) = K_{j,l}(k)x(k). \tag{7}$$

With the mode-independent fuzzy control law (7), the overall closed-loop MJFS when $\eta(k)=l\in\mathbb{T}$ can be written as

$$x(k+1)=\left(A_{0i,l}\left(k\right)+B_{i,l}(k)K_{j,l}(k)\right)x(k) + A_{1i,l}(k)x(k-\tau(k)). \tag{8}$$

Let $x(k,x_0,\eta_0)$ denote the trajectory of the state $x(k)$ from the initial state $x(0)=x_0$ and the initial mode $\eta(0)=\eta_0$. To obtain the main results for the feedback controller of the delay-dependent MJFSs, we first introduce the following definitions of stochastic stability of the Markovian jump systems, their proofs can be found in the cited references [8,9].

Definition 1: The unforced Markovian Jump Fuzzy system of (8) is said to be stochastically stable if, for any initial state x_0 and initial mode $\eta_0\in\mathbb{T}$, there exists a matrix $\mathbb{Q}=\mathbb{Q}^T>0$ satisfying

$$\lim_{N\to\infty}E\left\{\sum_{k=0}^{N}x^T(k,x_0,\eta_0)\mathbb{Q}x(k,x_0,\eta_0)\mid x_0,\eta_0\right\}<\infty, \tag{9}$$

where $E\{\bullet\}$ stands for the mathematical expectation.

Definition 2: The unforced MJFS (8) is said to be stochastically stabilizable if for any initial state x_0 and initial mode $\eta_0\in\mathbb{T}$, there exists a fuzzy controller of the form (7) such that the nominal closed-loop MJFS of (8)is stochastically stable.

In this paper, the problem under consideration is to design a fuzzy controller of the form (7) for the MJFSs with time-varying delays such that the closed-loop MJFSs (8) is stochastically stable. Further, both MJFSs for which transition probabilities are completely known and for which they are partially known are considered.

3. DELAY-RANGE DEPENDENT STABILITY AND FUZZY CONTROLLER DESIGN

3.1. Completely known transition probability case

In this section, a new delay-range dependent stochastic stability condition is developed and a bounded real lemma for MJFSs with time varying delays is established.

The following theorem presents delay-range dependent results in terms of LMIs. In addition, it gives the new stability criterion for system (1) with completely known transition probabilities, which is dependent not only on the delay upper bound τ_2, but also on the delay range $\tau_r:=\tau_2-\tau_1$.

Theorem 1: Consider the unforced MJFS (8) with $u(k)=0$ and known transition probabilities. The corresponding system is stochastically stable if there exist matrices $P_{i,l}\succ 0,\ i\in\mathcal{I}_R,\ l\in\mathbb{T},\ Q_{1i}\succ 0,\ Q_2\succ 0,\ Q_3\succ 0,\ Y_1\succ 0,\ Y_2\succ 0,\ \mathrm{M}_{ei},\ \mathrm{N}_{ei},\ \mathrm{S}_{ei},\ \mathrm{G}_{ei},\ \mathrm{e}=1,2,3,4,$ such that the following LMIs hold for each $l\in\mathbb{T}$

$$\Gamma_{itv,l}=\begin{bmatrix}\Gamma_{11} & * \\ \Gamma_{21} & \Gamma_{22}\end{bmatrix}\prec 0\quad i,g,h\in\mathcal{I}_R, \tag{10}$$

where

$$\Gamma_{11}=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14}\\ 0 & \Omega_{22} & \Omega_{23} & \Omega_{24}\\ 0 & 0 & \Omega_{33} & \Omega_{34}\\ 0 & 0 & 0 & \Omega_{44}\end{bmatrix},$$

$$\Gamma_{22}=diag\left\{-Y_2,-Y_2,-Y_2,-Y_1,-Y_1,-Y_2,-\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{g,s}\right)\right\},$$

$$\Gamma_{21}=\begin{bmatrix}\sqrt{\tau_1}M_{1i}^T & \sqrt{\tau_1}M_{2i}^T & \sqrt{\tau_1}M_{3i}^T & \sqrt{\tau_1}M_{4i}^T\\ \sqrt{\tau_r}S_{1i}^T & \sqrt{\tau_r}S_{2i}^T & \sqrt{\tau_r}S_{3i}^T & \sqrt{\tau_r}S_{4i}^T\\ \sqrt{\tau_r}N_{1i}^T & \sqrt{\tau_r}N_{2i}^T & \sqrt{\tau_r}N_{3i}^T & \sqrt{\tau_r}N_{4i}^T\\ \sqrt{\tau_r}G_{1i}^T & \sqrt{\tau_r}G_{2i}^T & \sqrt{\tau_r}G_{3i}^T & \sqrt{\tau_r}G_{4i}^T\\ \sqrt{\tau_r}\left(A_{0i,l}-I\right)Y_1 & \sqrt{\tau_r}\left(A_{1i,l}-I\right)Y_1 & 0 & 0\\ \sqrt{\tau_2}\left(A_{0i,l}-I\right)Y_2 & \sqrt{\tau_2}\left(A_{1i,l}-I\right)Y_2 & 0 & 0\\ A_{0i,l}^T\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{g,s}\right) & A_{1i,l}^T\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{g,s}\right) & 0 & 0\end{bmatrix},$$

$$\Omega_{11}=-P_{i,l}+(\tau_2-\tau_1+1)Q_{1i}+Q_2+Q_3+M_{1i}+M_{1i}^T,$$
$$\Omega_{13}=-M_{1i}+S_{1i}+M_{3i}^T,$$
$$\Omega_{14}=-N_{1i}-G_{1i}+M_{4i}^T,$$
$$\Omega_{22}=-Q_{1h}-S_{2i}+N_{2i}+G_{2i}-S_{2i}^T+N_{2i}^T+G_{2i}^T,$$
$$\Omega_{23}=-M_{2i}+S_{2i}-S_{3i}^T+N_i^T+G_{3i}^T,$$
$$\Omega_{24}=-N_{2i}-G_{2i}+S_{4i}^T+N_{4i}^T+G_{4i}^T,$$
$$\Omega_{33}=-Q_2-M_{3i}+S_{3i}-M_{3i}^T+S_{3i}^T,$$
$$\Omega_{34}=-N_{3i}-G_{3i}-M_{4i}^T+S_{4i}^T,$$
$$\Omega_{44}=-Q_3-N_{4i}-G_{4i}-N_{4i}^T-G_{4i}^T.$$

Proof: First, in order to cast our model into the framework of the Markov processes, we define a new process $\{(x_k,\eta(k),k\geq 0)\}$ by

$$x_k(o)=x(k+o).$$

Then, we can verify that $\{(x_k,\eta(k),k\geq\tau_2)$ is a Markov process with initial state $(\varphi(\bullet),\eta_0)$. Now for $k\geq\tau_2$, let mode at time k be 1, i.e., $\eta(k)=l,l\in\mathbb{T}$. Choose a stochastic Lyapunov function as

$$V(x(k),\eta(k),k)=V_1(x(k),\eta(k),k)+V_2(x(k),\eta(k),k) +V_3(x(k),\eta(k),k)+V_4(x(k),\eta(k),k) +V_5(x(k),\eta(k),k) \tag{11}$$

with

$$V_1(x(k),\eta(k),k)=x^T(k)\left(\sum_{i=1}^{r}\theta_i(z(k))P_i(\eta(k))\right)x(k),$$

$$V_2(x(k),\eta(k),k)=\sum_{o=k-\tau(k)}^{k}x^T(o)\left(\sum_{i=1}^{r}\theta_i(z(o))Q_{1i}\right)x(o),$$

$$V_3(x(k),\eta(k),k)=\sum_{o=k-\tau_1}^{k-1}x^T(o)Q_2x(o)+\sum_{o=k-\tau_2}^{k-1}x^T(o)Q_3x(o),$$

$$V_4(x(k),\eta(k),k)=\sum_{h=-\tau_2+1}^{\tau_1}\sum_{o=k+h}^{k-1}x^T(o)\left(\sum_{i=1}^{r}\theta_i(z(o))Q_{1i}\right)x(o),$$

$$V_5(x(k),\eta(k),k)=\sum_{h=-\tau_2}^{-\tau_1-1}\sum_{o=k+h}^{k-1}y^T(o)Y_1y(o)+\sum_{h=-\tau_2}^{-1}\sum_{o=k+h}^{k-1}y^T(o)Y_2y(o),$$

where $P_i(\eta(k))=P_i^T(\eta(k))\succ 0,\ Q_{i1}=Q_{i1}^T\succ 0,\ Q_2=Q_2^T\succ 0,\ Q_3=Q_3^T\succ 0,\ Y_1=Y_1^T\succ 0,$ and $Y_2=Y_2^T\succ 0$ are to be determined.

Let $\mathcal{A}$ be the weak infinitesimal generator of the random process. The weak infinitesimal operator $\mathcal{A}$ [10] of the Markov process $\{x_k,\eta(k)\}$ is given by

$$\mathcal{A}V(x_k,\eta(k)=l,k)=E\left[V(x_{k+1},\eta_{k+1}\mid x_k,\eta_k)-V(x_k,\eta_k)\right].$$

Then, for each $\eta(k)=l,\ l\in\mathbb{T},$ it can be seen that

$$\begin{aligned}\mathcal{A}V_1(x_k,\eta(k)=l,k)&=x^T(k+1)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}\sum_{i=1}^{r}\theta_i(z(k+1))P_i(\eta(k+1))\right)\\&\quad\times x(k+1)-x^T(k)\left(\sum_{i=1}^{r}\theta_i(z(k))P_i(\eta(k))\right)x(k),\end{aligned}$$

$$\begin{aligned}\mathcal{A}V_2(x_k,\eta(k)=l,k)&=x^T(k)\left(\sum_{i=1}^{r}\theta_i(z(k))Q_{1i}\right)x(k)\\&\quad-x^T(k-\tau(k))\left(\sum_{i=1}^{r}\theta_i(z(k-\tau(k)))Q_{1i}\right)x(k-\tau(k))n\\&\quad+\sum_{o=k-\tau(k+1)+1}^{k-\tau(k)}x^T(o)\left(\sum_{i=1}^{r}\theta_i(z(o))Q_{1i}\right)x(o)\\&\leqslant x^T(k)\left(\sum_{i=1}^{r}\theta_i(z(k))Q_{1i}\right)x(k)\\&\quad-x^T(k-\tau(k))\left(\sum_{i=1}^{r}\theta_i(z(k-\tau(k)))Q_{1i}\right)x(k-\tau(k))\\&\quad+\sum_{o=k-\tau_2+1}^{k-\tau_1}x^T(o)\left(\sum_{i=1}^{r}\theta_i(z(o))Q_{1i}\right)x(o),\end{aligned}$$

$$\begin{aligned}\mathcal{A}V_3(x_k,\eta(k)=l,k)&=x^T(k)Q_2x(k)-x^T(k-\tau_1)Q_2x(k-\tau_1)\\&\quad+x^T(k)Q_3x(k)-x^T(k-\tau_2)Q_3x(k-\tau_2),\end{aligned}$$

$$\begin{aligned}\mathcal{A}V_4(x_k,\eta(k)=l,k)&=(\tau_2-\tau_1)x^T(k)\left(\sum_{i=1}^{r}\theta_i(z(k))Q_{1i}\right)x(k)\\&\quad-\sum_{o=k-\tau_2+1}^{k-\tau_1}x^T(o)\left(\sum_{i=1}^{r}\theta_i(z(o))Q_{1i}\right)x(o),\end{aligned}$$

$$\begin{aligned}\mathcal{A}V_5(x_k,\eta(k)=l,k)&=(\tau_2-\tau_1)y^T(k)Y_1y(k)-\sum_{o=k-\tau_2}^{k-\tau_1-1}y^T(o)Y_1y(o)\\&\quad+\tau_2y^T(k)Y_2y(k)-\sum_{o=k-\tau_2}^{k-1}y^T(o)Y_2y(o).\end{aligned}$$

The following equations are true for any matrices M_i, N_i, S_i, and G_i, $i\in\mathcal{I}_R$, with appropriate dimensions

$$2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))M_i\times\left[x(k)-x(k-\tau_1)-\sum_{o=k-\tau_1}^{k-1}y(o)\right]=0,$$

$$2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))N_i\times\left[x(k-\tau(k))-x(k-\tau_2)-\sum_{o=k-\tau(k)}^{k-\tau(k)-1}y(o)\right]=0,$$

$$2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))S_i\times\left[x(k-\tau_1)-x(k-\tau(k))-\sum_{o=k-\tau}^{k-\tau_1-1}y(o)\right]=0,$$

$$2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))G_i\times\left[x(k-\tau_1)-x(k-\tau_2)-\sum_{o=k-\tau_2}^{k-\tau_1-1}y(o)\right]=0,$$

where

$$\zeta(k)=\left[x^T(k)\ \ x^T(k-\tau(k))\ \ x^T(k-\tau_1)\ \ x^T(k-\tau_2)\right]^T,$$

$$M_i=\left[M_{1i}^T\ \ M_{2i}^T\ \ M_{3i}^T\ \ M_{4i}^T\right]^T,$$

$$N_i=\left[N_{1i}^T\ \ N_{2i}^T\ \ N_{3i}^T\ \ N_{4i}^T\right]^T,$$

$$S_i=\left[S_{1i}^T\ \ S_{2i}^T\ \ S_{3i}^T\ \ S_{4i}^T\right]^T,$$

$$G_i=\left[G_{1i}^T\ \ G_{2i}^T\ \ G_{3i}^T\ \ G_{4i}^T\right]^T.$$

On the other hand, the following equations are also true:

$$\sum_{o=k-\tau_2}^{k-\tau_1} y^T(o)Y_1y(o) = \sum_{o=k-\tau_2}^{k-\tau(k)} y^T(o)Y_1y(o) + \sum_{o=k-\tau(k)}^{k-\tau_1} y^T(o)Y_1y(o),$$

$$\sum_{o=k-\tau_2}^{k} y^T(o)Y_2y(o) = \sum_{o=k-\tau_1}^{k} y^T(o)Y_2y(o) + \sum_{o=k-\tau(k)}^{k-\tau_1} y^T(o)Y_2y(o) + \sum_{o=k-\tau_2}^{k-\tau(k)} y^T(o)Y_2y(o).$$

Then calculating the weak infinitesimal operator of $\mathcal{A}V(x_k,\eta(k),k)$ the solution of (8) with $u(k)=0$ yields,

$$\begin{aligned}
&\mathcal{A}V(x_k,\eta(k)=l,k)\\
&\leqslant x^T(k)\Bigg(A^T_{0i,l}(k)\bigg(\sum_{s=1}^{\mathcal{N}}\pi_{ls}\sum_{i=1}^{r}\theta_i(z(k+1))P_i(\eta(k+1))\bigg)\\
&\times A_{0i,l}(k)-\sum_{i=1}^{r}\theta_i(z(k))P_i(\eta(k))\Bigg)x(k)\\
&+2x^T(k)A^T_{0i,l}(k)\bigg(\sum_{s=1}^{\mathcal{N}}\pi_{ls}\sum_{i=1}^{r}\theta_i(z(k+1))P_i(\eta(k+1))\bigg)\\
&\times A_{1i,l}(k)x(k-\tau(k))\\
&+x^T(k-\tau(k))A^T_{1i,l}(k)\bigg(\sum_{s=1}^{\mathcal{N}}\pi_{ls}\sum_{i=1}^{r}\theta_i(z(k+1))\\
&\times P_i(\eta(k+1))\bigg)A_{1i,l}(k)x(k-\tau(k))\\
&+(\tau_2-\tau_1+1)x^T(k)\bigg(\sum_{i=1}^{r}\theta_i(z(k))Q_{1i}\bigg)x(k)\\
&-x^T(k-\tau(k))\bigg(\sum_{i=1}^{r}\theta_i(z(k-\tau(k)))Q_{1i}x(k-\tau(k))\bigg)\\
&+x^T(k)Q_2x(k)-x^T(k-\tau_1)Q_2x(k-\tau_1)\\
&+x^T(k)Q_3x(k)-x^T(k-\tau_2)Q_3x(k-\tau_2)\\
&+(\tau_2-\tau_1)y^T(k)Y_1y(k)-\sum_{o=k-\tau_2}^{k-\tau(k)-1}y^T(o)Y_1y(o)\\
&-\sum_{o=k-\tau(k)}^{k-\tau_1-1}y^T(o)Y_1y(o)+\tau_2y^T(k)Y_2y(k)\\
&-\sum_{o=k-\tau_2}^{k-\tau(k)-1}y^T(o)Y_2y(o)-\sum_{o=k-\tau(k)}^{k-\tau_1-1}y^T(o)Y_2y(o)\\
&-\sum_{o=k-\tau_1}^{k-1}y^T(o)Y_2y(o)+2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))M_i\\
&\times\left[x(k)-x(k-\tau_1)-\sum_{o=k-\tau_1}^{k-1}y(o)\right]\\
&+2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))N_i\\
&\times\left[x(k-\tau(k))-x(k-\tau_2)-\sum_{o=k-\tau_2}^{k-\tau(k)-1}y(o)\right]\\
&+2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))S_i\\
&\times\left[x(k-\tau_1)-x(k-\tau(k))-\sum_{o=k-\tau(k)}^{k-\tau_1-1}y(o)\right]\\
&+2\zeta^T(k)\sum_{i=1}^{r}\theta_i(z(k))G_i\\
&\times\left[x(k-\tau(k))-x(k-\tau_2)-\sum_{o=k-\tau_2}^{k-\tau(k)-1}y(o)\right].
\end{aligned}$$

Now, we denote

$$P_{i,l}(k)=\sum_{i=1}^{r}\theta_i(z(k))P_i(\eta(k)),$$

$$M_i(k)=\sum_{i=1}^{r}\theta_i(z(k))M_i,$$

$$N_i(k)=\sum_{i=1}^{r}\theta_i(z(k))N_i,$$

$$S_i(k)=\sum_{i=1}^{r}\theta_i(z(k))S_i,$$

$$G_i(k)=\sum_{i=1}^{r}\theta_i(z(k))G_i.$$

Therefore, we obtain

$$\begin{aligned}
&\mathcal{A}V(x(k),\eta(k))\\
&\leqslant\zeta^T(k)[\bar{\Phi}_l(k)+\bar{\Psi}_l(k)+\tau_1M_i(k)Y_2^{-1}M_i^T(k)\\
&+(\tau_2-\tau_1)S_i(k)Y_2^{-1}S_i^T(k)+(\tau_2-\tau_1)N_i(k)Y_2^{-1}N_i^T(k)\\
&+(\tau_2-\tau_1)G_i(k)Y_1^{-1}G_i^T(k)]\zeta(k)\\
&-\sum_{o=k-\tau_1}^{k-1}[\zeta^T(k)M_i(k)+y^T(o)Y_2]Y_2^{-1}\\
&\times[M_i^T(k)\zeta(k)+Y_2y(o)]\\
&-\sum_{o=k-\tau(k)}^{k-\tau_1-1}[\zeta^T(k)S_i(k)+y^T(o)Y_2]Y_2^{-1}\\
&\times[S_i(k)^T\zeta(k)+Y_2y(o)]\\
&-\sum_{o=k-\tau_2}^{k-\tau(k)-1}[\zeta^T(k)N_i(k)+y^T(o)Y_2]Y_2^{-1}\\
&\times[N_i(k)^T\zeta(k)+Y_2y(o)]\\
&-\sum_{o=k-\tau_2}^{k-\tau(k)-1}[\zeta^T(k)G_i(k)+y^T(o)Y_2]Y_2^{-1}\\
&\times[G_i(k)^T\zeta(k)+Y_2y(o)],
\end{aligned}\tag{12}$$

where

$$\bar{\Phi}_l(k)=\begin{bmatrix}\Phi_{l1}(k) & \Phi_{l2}(k) & 0 & 0\\ * & \Phi_{l3}(k) & 0 & 0\\ * & * & -Q_2 & 0\\ * & * & * & -Q_3\end{bmatrix}$$

with

$$\begin{aligned}\Phi_{l1}(k) &= A_{0i,l}^T(k)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right)A_{0i,l}(k)-P_{i,l}(k)\\&\quad+(\tau_2-\tau_1+1)Q_{1i}(k)+Q_3+Q_2\\&\quad+(\tau_2-\tau_1)(A_{0i,l}^T(k)-I)Y_1(A_{0i,l}^T(k)-I)\\&\quad+\tau_2(A_{0i,l}^T(k)-I)Y_1(A_{0i,l}^T(k)-I),\\ \Phi_{l2}(k) &= A_{0i,l}^T(k)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right)A_{1i,l}(k)\\&\quad+(\tau_2-\tau_1)(A_{0i,l}^T(k)-I)Y_1A_{1i,l}(k)\\&\quad+\tau_2(A_{0i,l}^T(k)-I)Y_2A_{1i,l}(k),\\ \Phi_{l3}(k) &= A_{1i,l}^T(k)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right)A_{1i,l}(k)\\&\quad-Q_{1i}(k-\tau(k))+(\tau_2-\tau_1)A_{1i,l}^T(k)Y_1A_{1i,l}(k)\\&\quad+\tau_2A_{1i,l}^T(k)Y_2A_{1i,l}(k).\end{aligned}$$

The following equations are true for any matrices $Y_1 \succ 0,\ Y_2 \succ 0,$ so then the last four parts in (12) are all less than 0. Thus, if

$$\begin{aligned}&\bar{\Phi}_l+\bar{\Psi}_l+\tau_1M_iY_2^{-1}M_i^T+(\tau_2-\tau_1)S_iY_2^{-1}S_i^T\\&\quad+(\tau_2-\tau_1)N_iY_2^{-1}N_i^T+(\tau_2-\tau_1)G_iY_1^{-1}G_i^T\prec 0,\end{aligned}$$

which is equivalent to (13) by Schur complements, then $\mathcal{A}V(x_k,\eta(k),k)<-\epsilon\,\|x(k)\|^2$ for a sufficiently small $\epsilon>0,$ and $x(k)\neq 0.$

$$\breve{\Gamma}_l(k)=\begin{bmatrix}\bar{\Phi}_l(k)+\bar{\Psi}_l(k) & * & * & * & *\\ \sqrt{\tau_1}M_i^T(k) & -Y_2 & * & * & *\\ \sqrt{(\tau_2-\tau_1)}S_i^T(k) & 0 & -Y_2 & * & *\\ \sqrt{(\tau_2-\tau_1)}N_i^T(k) & 0 & 0 & -Y_2 & *\\ \sqrt{(\tau_2-\tau_1)}G_i^T(k) & 0 & 0 & 0 & -Y_1\end{bmatrix}\prec 0 \tag{13}$$

where

$$\begin{aligned}\bar{\Psi}_l(k)=(&[M_i(k)\quad -S_i(k)+N_i(k)+G_i(k)\\&-M_i(k)+S_i(k)\quad -N_i(k)-G_i(k)]\\&+[M_i(k)\quad -S_i(k)+N_i(k)+G_i(k)\\&-M_i(k)+S_i(k)\quad -N_i(k)-G_i(k)]^T).\end{aligned}$$

Now, we denote

$$\Gamma_l(k)=\begin{bmatrix}\Gamma_{11}(k) & * & * & * & *\\ \Gamma_{21}(k) & \Gamma_{22}(k) & * & * & *\\ \Gamma_{31}(k) & 0 & -Y_1 & * & *\\ \Gamma_{41}(k) & 0 & 0 & -Y_2 & *\\ \Gamma_{51}(k) & 0 & 0 & 0 & \Gamma_{55}(k)\end{bmatrix}\prec 0, \tag{14}$$

$$\Gamma_{22}(k)=\operatorname{diag}\{-Y_2,-Y_2,-Y_2,-Y_1\},$$

$$\begin{aligned}\Gamma_{11}(k)&=\bar{\Psi}_l(k)\\&\quad+diag\{-P_{i,l}(k)+(\tau_2-\tau_1+1)Q_{1i}(k)+Q_2+Q_3\\&\quad-Q_{1i}\quad -Q_2\quad -Q_3\},\\ \Gamma_{21}(k)&=\Big[\sqrt{\tau_1}M_i^T(k)\quad \sqrt{(\tau_2-\tau_1)}S_i^T(k)\\&\quad \sqrt{(\tau_2-\tau_1)}N_i^T(k)\quad \sqrt{(\tau_2-\tau_1)}G_i^T(k)\Big]^T\\ \Gamma_{31}(k)&=\Big[\sqrt{(\tau_2-\tau_1)}(A_{0i,l}(k)-I)Y_1\\&\quad \sqrt{(\tau_2-\tau_1)}(A_{1i,l}(k)-I)Y_1\quad 0\quad 0\Big],\\ \Gamma_{41}(k)&=\\&\Big[\sqrt{\tau_2}(A_{0i,l}(k)-I)Y_2\quad \sqrt{\tau_2}(A_{1i,l}(k)-I)Y_2\quad 0\quad 0\Big],\\ \Gamma_{51}(k)&=\Bigg[A_{0i,l}^T(k)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right)\\&\quad A_{1i,l}^T(k)\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right)\quad 0\quad 0\Bigg].\\ \Gamma_{55}(k)&=-\left(\sum_{s=1}^{\mathcal{N}}\pi_{ls}P_{i,s}(k+1)\right),\end{aligned}$$

Now, some simple manipulations give

$$\begin{aligned}\Gamma_l(k)&=\sum_{i=1}^{r}\sum_{g=1}^{r}\sum_{h=1}^{r}\theta_i(z(k))\theta_g(z(k+1))\\&\quad\times\theta_h(z(k-\tau(k)))\Gamma_{igh,l}.\end{aligned} \tag{15}$$

By the Schur complements, it follows from (10) that, for each $l\in\mathbb{T}$

$$\Gamma_{igh,l}\leqslant 0,\quad i,g,h\in\mathcal{I}_R. \tag{16}$$

Then, from (15), we have that

$$\mathcal{A}V(x_k,\eta(k),k)\leqslant 0. \tag{17}$$

Noting (13), we have

$$\mathcal{A}V(x_k,\eta(k),k)\leqslant -\alpha_1\zeta^T(k)\zeta(k), \tag{18}$$

where $\alpha_1=\min(\lambda_{\min}(-\Gamma_l(k)))>0.$ By Dynkin's formula, we have for each $\eta(k)=l\in\mathbb{T},$

$$\begin{aligned}&E[V(x(k),\eta(k),k)-V(\phi,\eta_0)]\\&\quad=E\left\{\sum_{k=0}^{k}\mathcal{A}V(x(k),\eta(k),k)\right\}\\&\quad\leqslant-\alpha_1E\left\{\sum_{k=0}^{k}\zeta^T(k)\zeta(k)\right\}.\end{aligned} \tag{19}$$

From (19), we obtain that $T\geqslant 1,$

$$\begin{aligned}E\left\{\sum_{k=0}^{k}\zeta^T(k)\zeta(k)\right\}&\leqslant\frac{1}{\alpha_1}\{E[V(\zeta(k),0)]\\&\quad-E[V(\zeta(k+1),k+1)]\}\\&\leqslant\frac{1}{\alpha_1}\{E[V(x(k),0)]\},\end{aligned}$$

which implies that

$$E\left\{\sum_{k=1}^{k} x^T(k)x(k)\right\} \leqslant E\left\{\sum_{k=1}^{k} \zeta^T(k)\zeta(k)\right\} \leqslant \frac{1}{\alpha_1}\left\{E\left[V(\zeta(k),0)\right]\right\} \leqslant \infty.$$

Thus, the system is stochastically stable, based on Definition 1.

Now let us consider the stabilizing controller design. From the above development, it can be seen that the system with time-varying delays and completely known-transition probabilities is simply a general case of MJFSs.

Theorem 2: Consider the MJFSs (8) with completely known probabilities (2). There exists a controller (7) such that the resulting closed-loop systems are stochastically stable if there exist matrices $X_{i,l} \succ 0,\ i \in \mathcal{I}_R,\ l \in \mathbb{T}$, $R_{1i} \succ 0,\ R_2 \succ 0,\ R_3 \succ 0,\ T_1 \succ 0,\ T_2 \succ 0,\ Z > 0,\ W_{i,l}$, $\bar{M}_{ei},\ \bar{N}_{ei},\ \bar{S}_{ei},\ \bar{G}_{ei},\ e = 1,2,3,4,\ i \in \mathcal{I}_R$ such that

$$\Lambda_{ijgh,l} = \begin{bmatrix} \Lambda_{11} & * & * & * & * \\ \Lambda_{21} & \Lambda_{22} & * & * & * \\ \Lambda_{31} & 0 & \Lambda_{33} & * & * \\ \Lambda_{41} & 0 & 0 & \Lambda_{44} & * \\ \Lambda_{51} & 0 & 0 & 0 & \Lambda_{55} \end{bmatrix} \prec 0, \quad (20)$$

$$1 \le i \le j \le r, g, h \in \mathcal{I}_R$$

where

$$\Lambda_{11} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} \\ 0 & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} \\ 0 & 0 & \tilde{\Omega}_{33} & \tilde{\Omega}_{34} \\ 0 & 0 & 0 & \tilde{\Omega}_{44} \end{bmatrix},$$

$$\Lambda_{12} = \left[\sqrt{\tau_1}\bar{M}_i^T \quad \sqrt{\tau_r}\bar{S}_i^T \quad \sqrt{\tau_r}\bar{N}_i^T \quad \sqrt{\tau_r}\bar{G}_i^T\right]^T,$$

$$\Lambda_{22} = diag\{-T_2 \quad -T_2 \quad -T_2 \quad -T_1\},$$

$$\Lambda_{33} = T_1 - Z - Z^T,$$

$$\Lambda_{44} = T_2 - Z - Z^T,$$

$$\Lambda_{55} = diag\{X_{g,1}, X_{g,2}, \cdots, X_{g,\mathcal{N}}\},$$

$$\Lambda_{31} = \Big[\sqrt{\tau_r}(A_{0i,l}Z + B_{i,l}W_{j,l} - Z) + \sqrt{\tau_r}(A_{0j,l}Z + B_{j,l}W_{i,l} - Z) \quad \sqrt{\tau_2}A_{1i,l}Z \quad 0 \quad 0\Big],$$

$$\Lambda_{41} = \Big[\sqrt{\tau_2}(A_{0i,l}Z + B_{i,l}W_{j,l} - Z) + \sqrt{\tau_2}(A_{0i,l}Z + B_{j,l}W_{i,l} - Z) \quad \sqrt{\tau_2}A_{1i,l}Z \quad 0 \quad 0\Big],$$

$$\Lambda_{51} = \begin{bmatrix} \sqrt{\pi_{l1}}((A_{0i,l}Z + B_{i,l}W_{j,l}) + (A_{0j,l}Z + B_{j,l}W_{i,l})) & \sqrt{\pi_{l1}}A_{1i,l}Z & 0 & 0 \\ \sqrt{\pi_{l2}}((A_{0i,l}Z + B_{i,l}W_{j,l}) + (A_{0j,l}Z + B_{j,l}W_{i,l})) & \sqrt{\pi_{l2}}A_{1i,l}Z & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{\pi_{l\mathcal{N}}}((A_{0i,l}Z + B_{i,l}W_{j,l}) + (A_{0j,l}Z + B_{j,l}W_{i,l})) & \sqrt{\pi_{l\mathcal{N}}}A_{1i,l}Z & 0 & 0 \end{bmatrix}.$$

$$\tilde{\Omega}_{11} = X_{i,l} - Z - Z^T + (\tau_2 - \tau_1 + 1)R_{1i} + R_2 + R_3 + \bar{M}_{1i} + \bar{M}_{1i}^T,$$

$$\tilde{\Omega}_{12} = -\bar{S}_{1i} + \bar{N}_{1i} + \bar{G}_{1i} + \bar{M}_{2i}^T,$$

$$\tilde{\Omega}_{13} = -\bar{M}_{1i} + \bar{S}_{1i} + \bar{M}_{3i}^T,$$

$$\tilde{\Omega}_{14} = -\bar{N}_{1i} - \bar{G}_{1i} + \bar{M}_{4i}^T,$$

$$\tilde{\Omega}_{22} = -R_{1h} - \bar{S}_{2i} + \bar{N}_{2i} + \bar{G}_{2i} - \bar{S}_{2i}^T + \bar{N}_{2i}^T + \bar{G}_{2i}^T,$$

$$\tilde{\Omega}_{23} = -\bar{M}_{2i} + \bar{S}_{2i} - \bar{S}_{3i}^T + \bar{N}_{3i}^T + \bar{G}_{3i}^T,$$

$$\tilde{\Omega}_{24} = -\bar{N}_{2i} - \bar{G}_{2i} + \bar{S}_{4i}^T + \bar{N}_{4i}^T + \bar{G}_{4i}^T,$$

$$\tilde{\Omega}_{33} = -R_2 - \bar{M}_{3i} + \bar{S}_{3i} - \bar{M}_{3i}^T + \bar{S}_{3i}^T,$$

$$\tilde{\Omega}_{34} = -\bar{N}_{3i} - \bar{G}_{3i} - \bar{M}_{4i}^T + \bar{S}_{4i}^T,$$

$$\tilde{\Omega}_{44} = -R_3 - \bar{N}_{4i} - \bar{G}_{4i} - \bar{N}_{4i}^T - \bar{G}_{4i}^T.$$

Furthermore, the desired state-feedback controller is given in the form of (7) with fuzzy controller gains as follows

$$K_{i,l} = W_{i,l}Z^{-1}, \quad i \in \mathcal{I}_R, \quad l \in \mathbb{T}. \quad (21)$$

Proof: Note that, for each $i \in \mathcal{I}_R$, and $l \in \mathbb{T}$,

$$(X_{i,l} - Z)^T X_{i,l}^{-1}(X_{i,l} - Z) = X_{i,l} - Z - Z^T + Z^T X_{i,l}^{-1} Z \prec 0, \quad (22)$$

which implies

$$-Z^T X_{i,l}^{-1} Z \prec X_{i,l} - Z - Z^T. \quad (23)$$

Similarly, it is clear that

$$-ZY_1^{-1}Z^T \prec Y_1 - Z - Z^T, \quad (24)$$

$$-ZY_2^{-1}Z^T \prec Y_2 - Z - Z^T. \quad (25)$$

By using the Schur complement again ($\mathbb{N}$ times), then we show that 10) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & * & * \\ \bar{\Gamma}_{21} & \bar{\Gamma}_{22} & * \\ \bar{\Gamma}_{31} & 0 & \bar{\Gamma}_{33} \end{bmatrix} \prec 0, \quad (26)$$

where Γ_{11} is defined in Theorem 1 and

$$\bar{\Gamma}_{22} = \mathrm{diag}\{-Y_2, -Y_2, -Y_2, -Y_1, -Y_1, -Y_2\}, \quad (27)$$

$$\bar{\Gamma}_{31} = \begin{bmatrix} \sqrt{\pi_{l1}}A_{0i,l}^T & \sqrt{\pi_{l1}}A_{1i,l}^T & 0 & 0 \\ \sqrt{\pi_{l2}}A_{0i,l}^T & \sqrt{\pi_{l2}}A_{1i,l}^T & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{\pi_{l\mathbb{N}}}A_{0i,l}^T & \sqrt{\pi_{l\mathbb{N}}}A_{1i,l}^T & 0 & 0 \end{bmatrix}, \quad (28)$$

$$\bar{\Gamma}_{33} = \text{diag}\left\{-P_{g,1}, -P_{g,2}, \cdots, -P_{g,\mathbb{N}}\right\}.$$

Consider the systems with the control input (7), and replace $A_{0i,l}$ in (27) $A_{0i,l} + B_{i,l}K_{j,l}$,

$$\hat{\Gamma}_{ijtv} = \begin{bmatrix} \Gamma_{11} & * & * \\ \hat{\Gamma}_{21} & \bar{\Gamma}_{22} & * \\ \hat{\Gamma}_{31} & 0 & \bar{\Gamma}_{33} \end{bmatrix} \prec 0, \tag{29}$$

where

$$\hat{\Gamma}_{31} = \begin{bmatrix} \sqrt{\pi_{l1}}\left(A_{0i,l} + B_{i,l}K_{j,l}\right) & \sqrt{\pi_{l1}}A_{1i,l}^T & 0 & 0 \\ \sqrt{\pi_{l2}}\left(A_{0i,l} + B_{i,l}K_{j,l}\right) & \sqrt{\pi_{l2}}A_{1i,l}^T & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{\pi_{l\mathbb{N}}}\left(A_{0i,l} + B_{i,l}K_{j,l}\right) & \sqrt{\pi_{l\mathbb{N}}}A_{1i,l}^T & 0 & 0 \end{bmatrix}.$$

Then, from (20) and (23)-(25), it is clear that

$$\bar{\Lambda}_{ijgh,l} \leqslant \Lambda_{ijgh,l}, \tag{30}$$

where

$$\bar{\Lambda}_{ijgh,l} = \begin{bmatrix} \bar{\Lambda}_{11} & * & * & * & * \\ \bar{\Lambda}_{21} & \bar{\Lambda}_{22} & * & * & * \\ \bar{\Lambda}_{31} & 0 & -Z^TY_1^{-1}Z & * & * \\ \bar{\Lambda}_{41} & 0 & 0 & -Z^TY_2^{-1}Z & * \\ \bar{\Lambda}_{51} & 0 & 0 & 0 & \bar{\Lambda}_{55} \end{bmatrix}$$

with Λ_{11}, Λ_{12}, Λ_{22}, Λ_{31}, Λ_{41}, Λ_{51}, and Λ_{55} defined in (21).

From (20) and (31), we obtain that, for each $l \in \mathbb{T}$

$$\bar{\Lambda}_{ijgh,l} \prec 0. \tag{31}$$

Now we denote,

$$P_{i,l} = X_{i,l}^{-1},\quad Q_{1h} = Z^{-T}R_{1h}Z^{-1},\quad Q_2 = Z^{-T}R_2Z^{-1},$$
$$M_i = Z^{-T}\bar{M}_iZ^{-1},\quad N_i = Z^{-T}\bar{N}_iZ^{-1},\quad G_i = Z^{-T}\bar{G}_iZ^{-1},$$
$$S_i = Z^{-T}\bar{S}_iZ^{-1},\quad K_{i,l} = W_{i,l}Z^{-1},\quad T_1 = Z^{-T}Y_1Z^{-1},$$
$$T_2 = Z^{-T}Y_2Z^{-1},$$

Pre-and post-multiplying the LMI in (30) by

$$\bar{T} = \text{diag}\left\{Z, Z, Z, Z, Z, Z, Z, Z, T_1^{-1}, T_2^{-1}, X_{i,1}, \cdots, X_{i,\mathbb{N}}\right\},$$

and using the Schur complements, it is clear that

$$\bar{\Lambda}_{ijtv} = \bar{T}^T\hat{\Gamma}_{ijgh}\bar{T}. \tag{32}$$

Therefore, if (20) holds, we obtain the condition (10) in Theorem 1. We conclude that the underlying system is stochastically stable with the fuzzy controller gains in (21). This completes the proof.

3.2. Partially known transition probability case

The following theorem represents the sufficient conditions for the stochastic stability of a system (8) with partially known transition probabilities.

Theorem 3: Consider the unforced MJFSs with partially known transition probabilities (3). The corresponding systems is stochastically stable if there exist matrices $P_{i,s} \succ 0,\ i \in \mathcal{I}_R,\ l \in \mathbb{T},\ P_1 \succ 0,\ Q_{1i} \succ 0,\ Q_2 \succ 0,\ Q_3 \succ 0,$ $Y_1 \succ 0,\ Y_2 \succ 0,\ M_{ei},\ N_{ei},\ S_{ei},\ G_{ei},\ e = 1,2,3,4,\ i \in \mathcal{I}_R$ such that the following LMIs hold for each $l, s \in \mathbb{T}$

$$\begin{bmatrix} \pi_{\mathbb{K}}^l\Xi_{2l} & \sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s}\Xi_{1l} \\ * & -\sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s} \end{bmatrix} \prec 0,\quad i, g, h \in \mathcal{I}_R, \tag{33}$$

$$\begin{bmatrix} \Xi_{2l} & P_{i,s}\Xi_{1l} \\ * & -P_{i,s} \end{bmatrix} \prec 0,\quad i, g, h \in \mathcal{I}_R, \tag{34}$$

where

$$\Xi_{1l} = \begin{bmatrix} A_{0i,l} & A_{1i,l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Xi_{2l} = \begin{bmatrix} \Gamma_{11} & * \\ \bar{\Gamma}_{21} & \bar{\Gamma}_{22} \end{bmatrix}, \tag{35}$$
$$\bar{\Gamma}_{21} = \begin{bmatrix} & \Gamma_{21} & & \\ \sqrt{\tau_r}\left(A_{0i,l} - I\right)Y_1 & \sqrt{\tau_r}\left(A_{1i,l} - I\right)Y_1 & 0 & 0 \\ \sqrt{\tau_2}\left(A_{0i,l} - I\right)Y_2 & \sqrt{\tau_2}\left(A_{1i,l} - I\right)Y_2 & 0 & 0 \end{bmatrix},$$

Γ_{11} is defined in Theorem 1 and $\bar{\Gamma}_{22}$ are defined in (27), with $\pi_{\mathbb{K}}^l := \sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}$.

Proof: First of all, we know that the unforced MJFSs are stochastically stable if (10) holds. Note that (10) can be written as

$$\Gamma_{igh,l} \equiv \begin{bmatrix} \pi_{\mathbb{K}}^l\Xi_{2l} & \sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s}\Xi_{1l} \\ * & -\sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s} \end{bmatrix} + \sum_{s\in\mathbb{T}_{\mathbb{UK}}^l}\pi_{ls}\begin{bmatrix} \Xi_{2l} & P_{i,s}\Xi_{1l} \\ * & -P_{i,s} \end{bmatrix}. \tag{36}$$

Therefore, if

$$\begin{bmatrix} \pi_{\mathbb{K}}^l\Xi_{2l} & \sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s}\Xi_{1l} \\ * & -\sum_{s\in\mathbb{T}_{\mathbb{K}}^l}\pi_{ls}P_{i,s} \end{bmatrix} \prec 0, \tag{37}$$

$$\begin{bmatrix} \Xi_{2l} & P_{i,s}\Xi_{1l} \\ * & -P_{i,s} \end{bmatrix} \prec 0, \tag{38}$$

then $\Gamma_{igh,l} < 0$. There fore, the system is stochastically stable under partially known transition probabilities, which is concluded from the clear fact hat no knowledge on π_{ls}, $\forall s \in \mathbb{T}^l_{\mathbb{UK}}$ is required in (37) or (38). Thus, for $\pi^l_{\mathbb{K}} \neq 0$ and $\pi^l_{\mathbb{K}} = 0$, respectively, one can readily obtain (36), since if $\pi^l_{\mathbb{K}} = 0$, the conditions (37) and (38) will reduce to (36). This completes the proof.

Now, we give the stabilization conditions of the system with partially known transition probabilities as generalized results.

Theorem 4: Consider the system (8) with partially known transition probabilities (3). There exists a controller (7) such that the resulting closed-loop system is stochastically stable if there exists matrices $X_{i,s} > 0$, $i \in \mathcal{I}_R$, $l \in \mathbb{T}$, $X_l > 0$, $\bar{R}_{1i} > 0$, $\bar{R}_2 > 0$, $\bar{R}_3 > 0$, $\bar{T}_1 > 0$, $T_2 > 0$, $Z > 0$, $W_{i,l}$, $\bar{M}_{lm}$, $\bar{N}_{lm}$, $\bar{S}_{lm}$, $\bar{G}_{lm}$, $m = 1,2,3,4$, $\forall l \in \mathbb{T}$, such that

$$\begin{bmatrix} \pi^l_{\mathbb{K}}\bar{\bar{\Xi}}_{2l} & \sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s}\bar{\bar{\Xi}}_{1l} \\ * & -\sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s} \end{bmatrix} \prec 0, \quad i,j,g,h \in \mathcal{I}_R, \tag{39}$$

$$\begin{bmatrix} \bar{\bar{\Xi}}_{2l} & X_{i,s}\bar{\bar{\Xi}}_{1l} \\ * & -P_{i,s} \end{bmatrix} \prec 0, \quad i,j,g,h \in \mathcal{I}_R. \tag{40}$$

Furthermore, a desired state-feedback controller is given in the form of (7) with the fuzzy controller gains as follows

$$K_{i,l} = W_{i,l}Z^{-1}, \quad i \in \mathcal{I}_R, \quad l \in \mathbb{T}. \tag{41}$$

Proof: Note that (20) can be written as

$$\Gamma_{ijgh,l} \equiv \begin{bmatrix} \pi^l_{\mathbb{K}}\bar{\bar{\Xi}}_{2l} & \sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s}\bar{\bar{\Xi}}_{1l} \\ * & -\sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s} \end{bmatrix} + \sum_{s\in\mathbb{T}^l_{\mathbb{UK}}} \pi_{ls} \begin{bmatrix} \bar{\bar{\Xi}}_{2l} & X_{i,s}\bar{\bar{\Xi}}_{1l} \\ * & -X_{i,s} \end{bmatrix}, \tag{42}$$

where

$$\bar{\bar{\Xi}}_{1l} = \begin{bmatrix} A_{0i,l}Z + B_{i,l}W_{i,l} & A_{1i,l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\bar{\Xi}}_{2l} = \begin{bmatrix} \Gamma_{11} & * \\ \hat{\Gamma}_{21} & \bar{\Gamma}_{22} \end{bmatrix},$$

$$\hat{\Gamma}_{21} = \begin{bmatrix} & \Gamma_{21} & & \\ \sqrt{\tau_r}\left(A_{0i,l} + B_{i,l}K_{j,l} - I\right)Y & \sqrt{\tau_r}\left(A_{1i,l} - I\right)Y_1 & 0 & 0 \\ \sqrt{\tau_2}\left(A_{0i,l} + B_{i,l}K_{j,l} - I\right)Y & \sqrt{\tau_2}\left(A_{1i,l} - I\right)Y_2 & 0 & 0 \end{bmatrix}.$$

Therefore, if

$$\begin{bmatrix} \pi^l_{\mathbb{K}}\hat{\Gamma}_{2l} & \sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s}\hat{\Gamma}_{1l} \\ * & -\sum_{s\in\mathbb{T}^l_{\mathbb{K}}} \pi_{ls}X_{i,s} \end{bmatrix} \prec 0, \tag{43}$$

$$\begin{bmatrix} \hat{\Gamma}_{2l} & X_{i,s}\hat{\Gamma}_{1l} \\ * & -X_{i,s} \end{bmatrix} \prec 0, \tag{44}$$

then $\Lambda_{ijgh} \prec 0$. Note that if $\pi^l_{\mathbb{K}} = 0$, then (20) is equivalent to (44). Therefore, if (40) holds, we obtain the condition (20) in Theorem 2. We conclude that the underlying system is stochastically stable with the fuzzy controller gains in (41). This completes the proof.

4. SIMULATIONS

In this section, a numerical example is presented to verify the proposed design approach of the fuzzy controller developed in the previous section. Consider the following discrete-time MJNLSs with time-varying delays:

$$\begin{aligned} x_1(k+1) &= -x_1^2(k) + a_1x_1(k)x_1(k-\tau(k)) \\ &\quad - 0.3x_1(k)x_2(k-\tau(k)) + 0.1u_1(k), \end{aligned} \tag{45}$$

$$\begin{aligned} x_1(k+1) &= -x_1^2(k) + a_1x_1(k)x_1(k-\tau(k)) \\ &\quad - 0.3x_1(k)x_2(k-\tau(k)) + 0.1u_1(k). \end{aligned} \tag{46}$$

The nonlinear system switches between the two modes, and the transition probability matrix is given by

$$\pi = \begin{bmatrix} 0.75 & 0.25 \\ 0.30 & 0.70 \end{bmatrix}. \tag{47}$$

Similar to previous results, we assume that $x_1(k) \in [-2,2]$ and set the membership functions $h_1(x_1(k))$ and $h_2(x_1(k))$ as obtained as follows:

$$h_1(x_1(k)) = \frac{1}{2}\left(1 - 0.5x_1(k)\right), \quad h_2(x_1(k)) = \frac{1}{2}\left(1 + 0.5x_1(k)\right).$$

Then, we represent the MJNLS in (49)-(50) as the following T-S fuzzy model:

R_i : IF $\mathrm{z_1(k)}$ is Γ_{i1} and$\cdots$and $\mathrm{z_p(k)}$ is Γ_{ip}

$$\begin{aligned} \text{THEN } x(k+1) &= A_{0i}(\eta(k))x(k) \\ &\quad + A_{1i}(\eta(k))x(k-\tau(k)) \\ &\quad + B_i(\eta(k))u(k), \end{aligned}$$

where

$$A_{01}(1) = \begin{bmatrix} 0.5 & 0.3 \\ 0.1 & 1.0 \end{bmatrix}, \quad A_{01}(2) = \begin{bmatrix} 0.5 & 0.4 \\ 0.1 & 0.06 \end{bmatrix},$$

$$A_{02}(1) = \begin{bmatrix} -0.5 & 0.3 \\ 0.1 & 1.0 \end{bmatrix}, \quad A_{02}(2) = \begin{bmatrix} -0.5 & 0.4 \\ 0.1 & 1.06 \end{bmatrix},$$

$$A_{11}(1) = \begin{bmatrix} -0.05 & 0.1 \\ 0 & -0.05 \end{bmatrix}, \quad A_{11}(2) = \begin{bmatrix} -0.07 & 0.1 \\ 0 & 0.05 \end{bmatrix},$$

$$A_{12}(1) = \begin{bmatrix} 0.05 & -0.1 \\ 0 & -0.05 \end{bmatrix}, \quad A_{12}(2) = \begin{bmatrix} 0.07 & -0.1 \\ 0 & -0.05 \end{bmatrix},$$

$$B_1(1) = B_1(2) = B_2(1) = B_2(2) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.7 \end{bmatrix}.$$

Using Theorem 2, it can be found that the LMIs of (34) have feasible solutions. We can obtain the following state-feedback gain matrices:

$$\begin{aligned} K_1(1) &= 10^5 \times \begin{bmatrix} 1.5909 & -0.8926 \\ -1.2542 & 6.2052 \end{bmatrix}, \\ K_1(2) &= 10^5 \times \begin{bmatrix} 1.3252 & -0.2501 \\ 0.1958 & -3.3820 \end{bmatrix}, \\ K_2(1) &= 10^5 \times \begin{bmatrix} -1.5909 & 0.8926 \\ 1.2542 & -6.2052 \end{bmatrix}, \\ K_2(2) &= 10^3 \times \begin{bmatrix} -1.3239 & 0.2491 \\ -0.1964 & 3.3803 \end{bmatrix}. \end{aligned} \tag{48}$$

Now, we apply the designed fuzzy static controller in the form of (7) to the nonlinear systems in (49)-(50). The state responses of the resulting closed-loop system are shown in Fig. 1. These results show that the designed fuzzy static feedback controller can effectively stabilize the MJFSs with time-varying delays in (49)-(50).

Table 1. Modes of the parameters a_1, a_2 and b.

Mode I	a_1	a_2	b
1	0.3	0.1	1.0
2	0.4	0.12	1.06

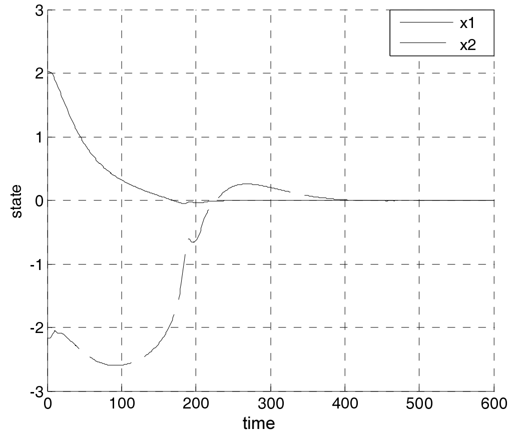

Fig. 1. State response of the closed-loop system with completely known transition probabilities.

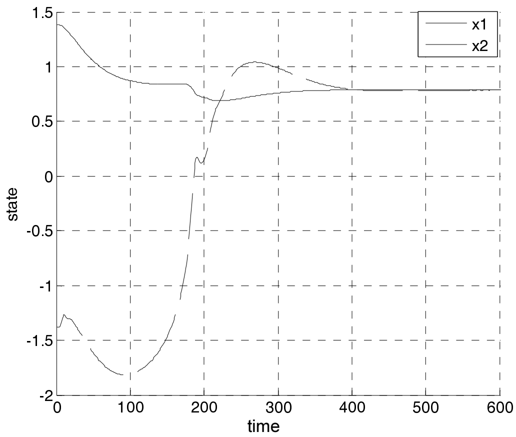

Fig. 2. State response of the closed-loop system with p known transition probabilities.

Now, we assign fixed values to the unknown elements in the partially known transition probability matrix given by

$$\pi = \begin{bmatrix} 0.75 & 0.25 \\ ? & ? \end{bmatrix},$$

where "?" represents unknown variables.

Analogous to the partially known case, an admissible controller can be solved by (40) and (41) in Theorem4 with the following gains:

$$\begin{aligned} K_1(1) &= 10^5 \times \begin{bmatrix} 1.6305 & -0.8415 \\ -1.3473 & 6.3156 \end{bmatrix}, \\ K_1(2) &= 10^3 \times \begin{bmatrix} 1.4781 & -0.2410 \\ 0.2384 & -3.1854 \end{bmatrix}, \\ K_2(1) &= 10^5 \times \begin{bmatrix} -1.6742 & 0.9741 \\ 1.3215 & -6.3140 \end{bmatrix}, \\ K_2(2) &= 10^3 \times \begin{bmatrix} -1.2412 & 0.2974 \\ -0.1541 & 3.5412 \end{bmatrix}. \end{aligned} \tag{49}$$

The state responses of the resulting closed-loop system are shown in Fig. 2. Despite the partly unknown transition probabilities, the designed controllers are feasible and effective in ensuring the resulting closed-loop systems are stable.

5. CONCLUSIONS

The stability analysis and stabilization problem for a class of discrete-time MJFS with time varying delays in which transition probabilities are partially known have been investigated in this paper. Based on a stochastic Lyapunov function, stability and stabilization conditions for the MJFS with time-varying delays have been derived in both completely known transition probabilities and partially known transition probabilities cases. The derived conditions have been represented in terms of LMIs. Finally, the effectiveness of the proposed design method has been demonstrated numerically by simulation results.

REFERENCES

[1] Y. Ji and H. J. Chizeck, "Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control," *IEEE Trans. on Automatic Control*, vol. 35, no 7, pp. 777-788, 1990.

[2] J. Xiong, J. Lam, H. Gao, and D. W. Ho, "On robust stabilization of Markovian jump systems with uncertain switching probabilities," *Automatica*, vol. 41, no 5, pp. 897-903, 2005.

[3] N. S. D. Arrifano and V. A. Oliveira, "State feedback fuzzy-model based control for Markovian jump nonlinear systems," *Controle Automacao: Revista da Sociedade Brasileira de Automacao*, vol. 15, no. 3, pp. 279-290, 2004.

[4] S. Natache, N. S. D. Arrifano, and V. A. Oliverira, "Robust H_∞ fuzzy control approach for a class of Markovian jump nonlinear systems," *IEEE Trans.*

on Fuzzy Systems, vol. 13, no. 6, pp. 738-754, 2006.

[5] H. N. Wu and K. Y. Cai, "Robust fuzzy control for uncertain discrete-time nonlinear Markovian jump systems without mode observations," *Information Science*, vol. 177, no. 6, pp. 1509-1522, 2007.

[6] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 2, pp. 200-211, 2000.

[7] Y. Y. Cao and P. M. Frank, "Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models," *Fuzzy Sets and Systems*, vol. 124, no. 2, pp. 213-229, 2001.

[8] E. K. Boukas, *Stochastic Switching Systems: Analysis and Design*, Birkhauser, Basel, Berlin, 2005.

[9] O. L. V. Costa, M. D. Fragoso, and R. P. Marques, *Discrete-time Markovian Jump Linear Systems*, Springer-Verlag, London, 2005.

[10] H. J. Kushner, *Stochastic Stability and Control*, Academic, New York, 1967.

[11] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMIcontrol Toolbox*, Math Works, Natick, MA, 1995.

[12] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," *IEEE Trans. on Automatic Control*, vol. 47, no. 11, pp. 1883-1887, 2002.

[13] J. Daafouz, P. Riedinger, and C. Iung, "Transition probability bounds for the stochastic stability robustness of continuous and discrete-time Markovian jump linear systems," *Automatica*, vol. 42. no. 12, pp. 2159-2168, 2006.

[14] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Trans. on Fuzzy Systems*, vol. 2, no. 2, pp. 119-134, 1994.

[15] B. Ding, H. Sun, and P. Yang, "Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form," *Automatica*, vol. 42, no. 3, pp. 503-508, 2006.

[16] B. Chen and Xi. Liu, "Fuzzy Guaranteed cost control for nonlinear systems with time-varying delay," *IEEE Trans. on Fuzzy Systems*, vol. 13, no. 2, pp. 238-249, 2005.

[17] T. M. Guerra and L. Vermeiren, "LMI-based relaxed non quadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form," *Automatica*, vol. 40, no. 5, pp. 823-829, 2004.

[18] Y. Zhang, S. Xu, Y. Zou, and J. Lu, "Delay-dependent robust stabilization for uncertain discrete-time fuzzy Markovian jump systems with mode-dependent time delays," *Fuzzy Sets and Systems*, vol. 164, no. 1, pp. 66-81, 2011.

[19] Y. He, Q. G. Wang, L. Xie, and C. Lin, "Further improvement of free-weighting matrices technique for systems with time-varying delay," *IEEE Trans. on Automatic Control*, vol. 52, no. 2, pp. 293-298, 2007.

[20] H. Gao, Z. Fei, J. Lam, and B. Du, "Further results on exponential estimates of Markovian jump systems with mode-dependent time-varying delays," *IEEE Trans. on Automatic Control*, vol. 56, no. 1, pp. 223-229, 2011.

[21] S. Wen, Z. Zeng, and T. Huang, "Reliable H_∞ filter design for a class of mixed-delay Markovian jump systems with stochastic nonlinearities and multiplicative noises via delay-partitioning method," *International Journal of Control and Automation System*, vol. 10, no. 4, pp. 711-720, 2012.

[22] N. Chaibi and E. H. Tissir, "Delay dependent robust stability of singular systems with time-varying delay," *International Journal of Control and Automation System*, vol. 10, no. 3, pp. 632-638, 2012.

Min Kook Song received his B.S. and M.S. degrees in Electrical and Electronic Engineering from Yonsei University, Seoul, Korea, in 2006 and 2007, respectively. He is currently working toward a Ph.D. degree in Electrical and Electronic Engineering at the Department of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea. His research interests include fuzzy control and filtering, nonlinear control, networked control system, neural networks, and genetic algorithms.

Jin Bae Park received his B.E. degree in Electrical Engineering from Yonsei University, Seoul, Korea, and his M.S. and Ph.D. degrees in Electrical Engineering from Kansas State University, Manhattan, in 1977, 1985, and 1990, respectively. Since 1992, he has been with the Department of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea, where he is currently a professor. His research interests include robust control and filtering, nonlinear control, intelligent mobile robot, fuzzy logic control, neural networks, and genetic algorithms. He served as Editor-in-Chief of the International Journal of Control, Automation, and Systems (IJCAS) (2006-2010) and has been serving as the President of the Institute of Control, Robotics, and Systems (ICROS) in 2013.

Young Hoon Joo received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Yonsei University, Seoul, Korea, in 1982, 1984, and 1995, respectively. He worked with Samsung Electronics Company, Seoul, Korea, from 1986 to 1995, as a project manager. He was with the University of Houston, Houston, TX, from 1998 to 1999, as a visiting professor in the Department of Electrical and Computer Engineering. He is currently a professor in the Department of Control and Robotics Engineering, Kunsan National University, Korea. His major interest is mainly in the field of intelligent robot, intelligent control, and human-robot interaction. He served as President for Korea Institute of Intelligent Systems (KIIS) (2008-2009) and is serving as Editor for the International Journal of Control, Automation, and Systems (IJCAS) (2008-present) and is serving as the Vice-President for the Korean Institute of Electrical Engineers (KIEE) (2012-present).