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Stabilization of Oscillating Neural Networks with Time-Delay by Intermittent Control

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Abstract: In this paper, we study the exponential stabilization of oscillating neural networks with time-delay through the process called intermittent control. Some exponential stability criteria for the controlled neural networks are established by Lyapunov function and matrix inequality analysis technique. The present results allow us to estimate legitimately the feasible control region of control parameters. Numerical example is also given to show the effectiveness of our proposed results.

Keywords: Intermittent control, oscillating neural network, stabilization, time-delay.

1. INTRODUCTION

Intermittent control has been used for variety of purposes in such engineering fields as manufacturing, transportation and communication [1]. Up to now, the primary attention concerned about the case which activates the control only in some finite time intervals [2]. Because of its effectiveness, many researchers focused on it and obtained some crucial conclusions. See [1-5]. However, one notes that the researchers restricted that the control is periodical. Moreover, in the delayed case, it was invisible to claim that the switching time is larger than the time-delay.

In the past decades, neural networks have been applied extensively in many fields, such as pattern recognition, associative, and combinatorial optimization [6]. The stability of neural networks with time-delay is worthy investigating, see [6-8]. In this paper, we pay our attention to realizing the stability of oscillating neural networks by intermittent control. Comparing with the prior results, we do not restrict the switching time is not greater than the time-delay. In addition, the upper bounds of the convergent rate is obtained and the feasible region of control parameters is estimated.

2. PRELIMINARIES

In this paper, we consider the oscillating delayed neural networks of the form

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cg(x(t-\tau)) + u(t), \tag{1}$$

where $x \in R^n$ denotes the state vector, $x(t_0 + s) = \phi(s)$, $s \in [-\tau, 0]$, is the initial state, $A = diag(\alpha_1, \cdots, \alpha_n)$ with $\alpha_i > 0$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n}$ are the synaptic connection weight and $u(t)$ denotes the external input of system (1), $f(x(t)) = (f_1(x_1(t)), \cdots, f_n(x_n(t)))^T$ and $g(x(t)) = (g_1(x_1(t)), \cdots, g_n(x_n(t)))^T$ with $f(0) = g(0) = 0$ satisfy the Lipschitz condition, namely, there exist positive constant l_i^f, l_i^g such that $|f_i(z_1) - f_i(z_2)| \le l_i^f |z_1 - z_2|$, $|g_i(z_1) - g_i(z_2)| \le l_i^g |z_1 - z_2|$, for $i = 1, \cdots, n$ and $z_1, z_2 \in R$. For convenience, we let $L_1 = diag(l_1^f, \cdots, l_n^f)$, $L_2 = diag(l_1^g, \cdots, l_n^g)$.

In order to obtain the stability criteria of system (1) by means of intermittent feedback control, we assume that the control input $u(t)$ is of the form

$$u(t) = \begin{cases} -Kx(t), & t_k \le t < t_k + \sigma_k, \\ 0, & t_k + \sigma_k \le t < t_{k+1}, \end{cases} \tag{2}$$

where $K \in R^{n\times n}$ is the control strength matrix, $\sigma_k < t_{k+1} - t_k$, $k = 0, 1, \cdots$, denote the switching time. With control law (2), system (1) can be rewritten as follows

$$\begin{cases} \dot{x}(t) = (-A-K)x(t) + Bf(x(t)) + Cg(x(t-\tau)), \\ \qquad\qquad t_k \le t < t_k + \sigma_k, \\ \dot{x}(t) = -Ax(t) + Bf(x(t)) + Cg(x(t-\tau)), \\ \qquad\qquad t_k + \sigma_k \le t < t_{k+1}. \end{cases} \tag{3}$$

In this paper our purpose is to establish exponential stability criterion of system (3) and then determines suitable control parameters. By contrast with the prior results, one observes that the control is not periodic in system (3).

In order to establish the stability criteria in next section, we introduce the following definition, lemma

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and notations.

Definition 1: The zero solution of controlled system (3) is said to be globally exponentially stable if there exist two constants $M > 0,\ \varepsilon > 0$ such that $\|x(t)\| \le Me^{-\varepsilon(t-t_0)},\ t \ge t_0$.

Lemma (Halanay inequality [9]) **1:** Let w: $[t_0-\tau,\infty) \to [0,\infty)$ be a continuous function such that $\dot{w}(t) \le -aw(t) + b\overline{w}(t)$ holds for $t \ge t_0$ with $\overline{w}(t) = \sup_{-\tau\le s\le 0}(w(t+s))$. If $a > b > 0$, we have $w(t) \le \overline{w}(t_0)e^{-\varepsilon(t-t_0)}$, $t \ge t_0$, where ε > 0 satisfies equation $a - \varepsilon + be^{\varepsilon\tau} = 0$.

Throughout this paper, we use P^T to denote the transpose of square matrix P, I denotes identity matrix, $\lambda_{\max(\min)}(\bullet)$ denotes the maximum minimum) eigenvalue of the corresponding matrix. The vector (or matrix) norm is taken to be Euclidian, denoted by $\|\bullet\|$. We use $P > 0$ $(< 0,\ \le 0,\ \ge 0)$ to denote a symmetrical positive (negative, semi-negative, semi- positive)definite matrix P. $P_1 < P_2$ $(P_1 > P_2,\ P_1 \le P_2,\ P_1 \ge P_2)$ implies that P_1–P_2 is a symmetrical negative (positive, semi-negative, semi-positive). $\|\phi\|_\tau = \sup_{s\in[-\tau,0]} . \|\phi(s)\|$.

3. MAIN RESULTS

In this section, we establish sufficient conditions for exponential stability of system (3).

Theorem 1: Suppose that there exist matrix P > 0, constants $\mu_1 > 0,\ \mu_2 > 0,\ \nu_1 > 0,\ \nu_2 > 0,\ b_1 > 0,\ b_2 > 0,\ a_1 > b_1,\ a_2 > -b_2,\ q > 1,\ \varepsilon > 0$ and positive integer m such that

(i) $$\begin{cases} -A^TP - PA - K^TP - PK + \mu_1^{-1}PBB^TP \\ \quad + \mu_2^{-1}PCC^TP + \mu_1 L_1^T L_1 \le -a_1 P, \\ \mu_2 L_2^T L_2 \le b_1 P; \end{cases}$$

(ii) $$\begin{cases} -A^TP - PA + \nu_1^{-1}PBB^TP + \nu_2^{-1}PCC^TP \\ \quad + \nu_1 L_1^T L_1 \le a_2 P, \\ \nu_2 L_2^T L_2 \le b_2 P; \end{cases}$$

(iii) $\dfrac{\tau}{m} \le t_{k+1} + \sigma_{k+1} - t_k - \sigma_k \le \alpha$;

(iv) $\alpha((m+1-\varphi)\varepsilon + a_2 + b_2 q) \le r\sigma$ and $\alpha(m\varepsilon + a_2 + b_2 q) < \ln q$, where $r > 0$ satisfies $a_1 - r - b_1 e^{r\tau} = 0$, $\sigma = \inf_k(\sigma_k)$,

$$\varphi = \begin{cases} 1, & \inf_k\left(t_{k+1} - t_k - \sigma_k\right) \ge \tau, \\ 0, & \text{otherwise.} \end{cases}$$

Then the origin of (3) is globally exponentially stable by the convergence rate $\dfrac{1}{2}\varepsilon$.

Proof: We choose the Lyapunov function as follows

$$V(x(t)) = x^T(t)Px(t). \tag{4}$$

When $t \in [t_k, t_k + \sigma_k)$, we have

$$\begin{aligned} \dot{V}(x(t)) &\le x^T(t)(-A^TP - K^TP)x(t) + f^T(x(t))B^TPx(t) \\ &\quad + g^T(x(t-\tau))C^TPx(t) + x^T(t)(-PA - PK)x(t) \\ &\quad + x^T(t)PBf(x(t)) + x^T(t)PCg(x(t-\tau)) \\ &\le x^T(t)\Big(-A^TP - PA - K^TP - PK \\ &\quad + \mu_1^{-1}PBB^TP + \mu_2^{-1}PCC^TP + \mu_1 L_1^T L_1\Big)x(t) \\ &\quad + \mu_2 x^T(t-\tau)L_2^T L_2 x(t-\tau) \\ &\le -a_1 V(x(t)) + b_1 V(x(t-\tau)). \end{aligned}$$

Similarly, when $t \in [t_k + \sigma_k, t_{k+1})$, we have

$$\dot{V}(x(t)) \le a_2 V(x(t)) + b_2 V(x(t-\tau)).$$

Therefore,

$$\begin{cases} \dot{V}(x(t)) \le -a_1 V(x(t)) + b_1 V(x(t-\tau)), \\ \qquad\qquad t \in [t_k, t_k + \sigma_k), \\ \dot{V}(x(t)) \le a_2 V(x(t)) + b_2 V(x(t-\tau)), \\ \qquad\qquad t \in [t_k + \sigma_k, t_{k+1}). \end{cases} \tag{5}$$

Let $T = t_0 + \sigma_0$, $V(t) = V(x(t))$, $p = a_2 + b_2 q$, from (iv), we know $e^{\alpha p}e^{\varepsilon(t_1+\sigma_1-T)} < qe^{\varepsilon(t_1+\sigma_1-T)}$, so there must exist $M \ge 1$ such that

$$\begin{aligned} \hat{V}(T) &< M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)}e^{-\alpha p} \\ &< M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)} \le q\hat{V}(T), \end{aligned} \tag{6}$$

where $\hat{V}(T) = \sup_{t\in[t_0-\tau,T]}(V(t))$.

Now we show that

$$\begin{aligned} V(t) &\le M\hat{V}(T)e^{-\varepsilon(t_l+\sigma_l-T)}, \\ &\quad t \in [t_{l-1} + \sigma_{l-1}, t_l + \sigma_l),\quad l = 1, 2, \cdots. \end{aligned} \tag{7}$$

For this aim, we first show that

$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)},\quad t \in [t_0 + \sigma_0, t_1). \tag{8}$$

From (6), one observes that

$$\begin{aligned} V(t) &\le \hat{V}(T) < M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)}e^{-\alpha p}, \\ &\quad t \in [t_0 - \tau, T]. \end{aligned}$$

If (8) does not hold, then there must exist some $t^* = \inf\{t \in (t_0 + \sigma_0, t_1) : V(t) > M\hat{V}(t)e^{-\varepsilon(t_1+\sigma_1-T)}\}$ such that $V(t^*) = M\hat{V}(t)e^{-\varepsilon(t_1+\sigma_1-T)}$, $V(t) \le M\hat{V}(t)e^{-\varepsilon(t_1+\sigma_1-T)}$, $t \in [t_0 - \tau, t^*]$, and there exists some $t^{**} \in [T, t^*)$ such that

$$V(t^{**}) = \hat{V}(T),\quad V(t) \ge \hat{V}(T),\quad t \in [t^{**}, t^*].$$

Therefore, for any $t \in (t^{**}, t^{*})$,

$$V(t+s) \le M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)} \le qV(t) \quad s \in [-\tau, 0]. \tag{9}$$

From (5) and (9), we know $\dot{V}(t) \le pV(t)$ for $t \in [t^{**}, t^{*}]$. Namely, $V(t^{*}) \le V(t^{**})e^{\alpha p}$, which implies $M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)} e^{-\alpha p} \le \hat{V}(T)$. This contradicts (6). Hence, (8) is true. Based on (5) and Lemma 1, we know $V(t)$ is decreasing in $[t_1, t_1+\sigma_1)$. Therefore,

$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_1+\sigma_1-T)}, \quad t \in [t_1, t_1+\sigma_1).$$

Namely, (7) holds for $l=1$.

Now, we assume that (7) holds for $l = 1, 2, \cdots, \bar{l}$ $(\bar{l} \ge 1)$, i.e.,

$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_l+\sigma_l-T)}, \quad t \in [t_{l-1}+\sigma_{l-1}, t_l+\sigma_l).$$

Note that (see Remark 1)

$$\bar{V}(t_{\bar{l}}) \le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}-m+\varphi}+\sigma_{\bar{l}-m+\varphi}-T)}.$$

Then from Lemma 1 and (5), we have

$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}-m+\varphi}+\sigma_{\bar{l}-m+\varphi}-T)} e^{-r(t-t_{\bar{l}})}. \tag{10}$$

Then from (iv) and (10),

$$\begin{aligned} V(t_{\bar{l}}+\sigma_{\bar{l}}) &\le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}-m+\varphi}+\sigma_{\bar{l}-m+\varphi}-T)} e^{-r\sigma_{\bar{l}}} \\ &\le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)} e^{-\alpha p}. \end{aligned} \tag{11}$$

Next, we prove that

$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)}, \quad t \in [t_{\bar{l}}+\sigma_{\bar{l}}, t_{\bar{l}+1}). \tag{12}$$

If (12) is not true, there must exist some $t^{***} = \inf\{t \in [t_{\bar{l}}+\sigma_{\bar{l}}, t_{\bar{l}+1}) : V(t) > M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)}\}$ such that

$$V(t^{***}) = M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)},$$
$$V(t) \le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)}, \quad t \in [t_{\bar{l}}+\sigma_{\bar{l}}, t_{\bar{l}+1}).$$

Based on (11), there must exist some $t^{****} \in [t_{\bar{l}}+\sigma_{\bar{l}}, t^{***})$ such that

$$V(t^{****}) = M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)} e^{-\alpha p},$$
$$V(t^{****}) \le V(t) \le V(t^{***}), \quad t \in [t^{****}, t^{***}].$$

From condition (iv), we know that $t+s \in [t_{\bar{l}-m}+\sigma_{\bar{l}-m}, t^{***}]$ (see Remark 1) for $t \in [t^{****}, t^{***}]$ and $s \in [-\tau, 0]$. Therefore

$$\begin{aligned} V(t+s) &\le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}-m+1}+\sigma_{\bar{l}-m+1}-T)} \\ &\le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)} \\ &\quad \times e^{\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-t_{\bar{l}-m+1}+\sigma_{\bar{l}-m+1})} \\ &\le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)} e^{-\alpha p} e^{(m\varepsilon+p)\alpha} \\ &\le qV(t^{****}) \le qV(t). \end{aligned}$$

Similarly, we have $\dot{V}(t) \le pV(t)$, $t \in [t^{****}, t^{***}]$. This implies that

$$\begin{aligned} V(t^{***}) &\le V(t^{****})e^{p(t^{***}-t^{****})} \\ &< M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)} = V(t^{***}), \end{aligned}$$

which is a contradiction. So (12) is true. Similarly as before, we have $V(t) \le M\hat{V}(T)e^{-\varepsilon(t_{\bar{l}+1}+\sigma_{\bar{l}+1}-T)}$, $t \in [t_{\bar{l}+1}, t_{\bar{l}+1}+\sigma_{\bar{l}+1})$, namely, (7) also holds for $l = \bar{l}+1$. Thus, by mathematical induction, (7) is true.

For $t \in [t_k+\sigma_k, t_{k+1}+\sigma_{k+1})$, $k = 0, 1, \cdots,$ we have

$$\begin{aligned} V(t) &\le M\hat{V}(T)e^{-\varepsilon(t_{k+1}+\sigma_{k+1}-T)} \\ &\le M\bar{V}(t_0)e^{-\varepsilon(t-t_0)}e^{\varepsilon(T-t_0)} \\ &\le M\bar{V}(t_0)e^{\varepsilon\sigma_0}e^{-\varepsilon(t-t_0)}. \end{aligned} \tag{13}$$

For $t \in [t_0, t_0+\sigma_0)$, we have

$$\begin{aligned} V(t) &\le \bar{V}(t_0) \le e^{\varepsilon\sigma_0}\bar{V}(t_0)e^{-\varepsilon(t-t_0)} \\ &\le Me^{\varepsilon\sigma_0}\bar{V}(t_0)e^{-\varepsilon(t-t_0)}. \end{aligned} \tag{14}$$

Based on (13) and (14), we know

$$V(t) \le Me^{\varepsilon\sigma_0}\bar{V}(t_0)e^{-\varepsilon(t-t_0)}, \quad t \in [t_0, \infty).$$

Therefore, for $t \in [t_0, \infty)$, we have

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)Me^{\varepsilon\sigma_0}}{\lambda_{\min}(P)}} \|\phi\|_\tau e^{-\frac{1}{2}\varepsilon(t-t_0)}.$$

The proof is complete.

Remark 1: When $l-m<0$, we set $t_{l-m} = t_0-\tau$, $\sigma_{l-m} = 0$; furthermore, for $t \in [t_0-\tau, T]$, $V(t) \le \hat{V}(T) \le M\hat{V}(T)e^{-\varepsilon(t_0+\sigma_0-T)}$, therefore, the above proof also stands.

Remark 2: In Theorem 1, a general criterion ensuring the globally exponential stability of the controlled system (3) is established. The conditions which Γ, σ_k, τ and κ should satisfy are obtained. One could choose appropriate undetermined parameters to guarantee the proposed results are less conservative. In [2,4,5], the researchers restricted intermittent control is periodical and switching time is larger than time-delay for stabilization or synchronization of chaotic systems with time-delay. Yet, in our result we remove these restrictions. It is also suitable to use Theorem 1 to establish synchronization criterion for chaotic delayed neural networks because the form of synchronization error system is similar to system (1).

For computational purposes, we now restrict $\Gamma = t_{k+1} - t_k$, $\sigma = \sigma_k$ for $k = 0,1,\cdots$, $K = \kappa I$, then the control law (2) can be rewritten as follows

$$u(t) = \begin{cases} -\kappa x(t), & k\Gamma \le t < k\Gamma + \sigma, \\ 0, & k\Gamma + \sigma \le t < (k+1)\Gamma. \end{cases} \tag{15}$$

If choose $P=I$, based on Theorem 1, we can obtain the following corollary.

Corollary 1: Suppose that there exist constants $\mu_1 > 0$, $\mu_2 > 0$, $\nu_1 > 0$, $\nu_2 > 0$, $b_1 > 0$, $b_2 > 0$, $a_1 > b_1$, $a_2 > -b_2$, $q > 1$, $\varepsilon > 0$ and positive integer m such that

(i) $\begin{cases} -A^T - A - 2\kappa I + \mu_1^{-1} BB^T \\ \quad + \mu_2^{-1} CC^T + \mu_1 L_1^T L_1 \le -a_1 I, \\ \mu_2 L_2^T L_2 \le b_1 I; \end{cases}$

(ii) $\begin{cases} -A^T - A + \nu_1^{-1} BB^T \\ \quad + \nu_2^{-1} CC^T + \nu_1 L_1^T L_1 \le a_2 I, \\ \nu_2 L_2^T L_2 \le b_2 I; \end{cases}$

(iii) $\Gamma \ge \dfrac{\tau}{m}$;

(iv) $\Gamma((m+1-\varphi)\varepsilon + a_2 + b_2 q) \le r\sigma$ and $\Gamma(m\varepsilon + a_2 + b_2 q) < \ln q$, where $r > 0$ satisfies $a_1 - r - b_1 e^{r\tau} = 0$, $\varphi = \begin{cases} 1, & \Gamma - \sigma \ge \tau, \\ 0, & \text{otherwise}. \end{cases}$

Then the origin of system (3) is globally exponentially stable with the convergence rate $\frac{1}{2}\varepsilon$ under control law (15).

Remark 3: For convenience to use Corollary 1, one could let $\ln q = r\sigma$. If there exists some ε such that

$$0 < \varepsilon < \frac{1}{m+1-\varphi}\left(\frac{1}{\Gamma}\ln q - a_2 - b_2 q\right).$$

Then the two inequalities in condition (iv) are satisfied.

Let $\varepsilon(q) = \dfrac{1}{m+1-\varphi}\left(\dfrac{1}{\Gamma}\ln q - a_2 - b_2 q\right)$, then we have

$$\dot{\varepsilon}(q) = \frac{1}{m+1-\varphi}\left(\frac{1}{\Gamma q} - b_2\right),$$

$$\ddot{\varepsilon}(q) = -\frac{1}{(m+1-\varphi)\Gamma q^2} < 0.$$

By virtue of extreme value theorem, we know that there exists a maximal value of $\varepsilon(q)$. It is easy to obtained that $\varepsilon_{\max} = -\dfrac{1}{(m+1-\varphi)\Gamma}(\ln(\Gamma b_2) + a_2\Gamma + 1)$ when $q = \dfrac{1}{\Gamma b_2}$. Namely, $\varepsilon_{\max}$ is an upper bound of ε.

Remark 4: In Corollary 1, one could determine parameters a_2, b_2, q, ν_1, ν_2 according to condition (ii), the second inequality of condition (iv) and Remark 3. In addition, on the basis of condition (i), the first inequality and the equality in condition (iv), parameters a_1, b_1, μ_1, μ_2 could be determined by choosing appropriate feedback strength κ.

Corollary 2: If condition (iii) in Corollary 1 hold and there exist constants $r > 0$ and $\varepsilon > 0$ such that

$$\begin{aligned} &(\underline{a} + \kappa) - l_{\max}\sqrt{\lambda_{\max}(BB^T)} - e^{0.5r\tau} l_{\max}\sqrt{\lambda_{\max}(CC^T)} \\ &\quad - 0.5r = 0, \end{aligned} \tag{16}$$

$$\underline{a} - l_{\max}\sqrt{\lambda_{\max}(BB^T)} < l_{\max}\sqrt{\lambda_{\max}(CC^T)} < 1,$$

$$\Gamma\left((m+1-\varphi)\varepsilon - 2\underline{a} + 2l_{\max}\sqrt{\lambda_{\max}(BB^T)} + \Gamma + \frac{1}{\Gamma}\right) \le r\sigma,$$

$$\Gamma\left(m\varepsilon - 2\underline{a} + 2l_{\max}\sqrt{\lambda_{\max}(BB^T)} + \Gamma + \frac{1}{\Gamma}\right) < \ln\left(\frac{1}{l_{\max}^2 \lambda_{\max}(CC^T)}\right),$$

where $\underline{a} = \min\limits_{1\le i\le n}(\alpha_i)$,

$$l_{\max} = \max\left(\sqrt{\lambda_{\max}(L_1^T L_1)}, \sqrt{\lambda_{\max}(L_2^T L_2)}\right)$$

and φ is defined in according with Corollary 1.

Then the origin of system (1) is globally exponentially stable under the control law (15).

Remark 5: According to the proof of Corollary 2, we know $q = \dfrac{1}{\Gamma b_2}$, then

$$\begin{aligned} \varepsilon_{\max} = \frac{1}{m+1-\varphi}\Bigg(&\frac{1}{\Gamma}\ln\frac{1}{l_{\max}^2\lambda_{\max}(CC^T)} \\ &+ 2\underline{a} - 2l_{\max}\sqrt{\lambda_{\max}(BB^T)} - \Gamma - \frac{1}{\Gamma}\Bigg). \end{aligned}$$

Assume that (16) is always true, given the period Γ, based on Corollary 2 we can estimate the feasible region D of the control parameters (κ, s). Let $s^* = \dfrac{1}{r}((m+1-\varphi)\varepsilon_{\max} - 2\underline{a} + 2l_{\max}\sqrt{\lambda_{\max}(BB^T)} + \Gamma + \dfrac{1}{\Gamma})$ where $s = \dfrac{\sigma}{\Gamma}$. Then for precise value of r, one can obtain a pair (κ, s) from Corollary 2, by which one then obtains a curve of the function $s^* = s^*(\kappa)$. The region above this curve is the estimated feasible region D of the control parameters (κ, s). The estimated region D can be written as follows

$$\begin{aligned} D = \Big\{&(\kappa, s) \in R^2 \mid r > 0, s > s^*, \ \kappa = l_{\max} \\ &\left(\sqrt{\lambda_{\max}(BB^T)} + e^{0.5r\tau}\sqrt{\lambda_{\max}(CC^T)}\right) + 0.5r - \underline{a}\Big\}. \end{aligned}$$

4. NUMERICAL EXAMPLES

We take oscillating BAM neural network with time-delay as numerical example to show the effectiveness of the proposed results.

Example 1: Consider the following system which is described by

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -0.1x_1(t) + 0.4\sin x_2(t-2), \\ \dfrac{dx_2(t)}{dt} = -0.1x_2(t) + 0.3\sin x_1(t-2). \end{cases} \tag{17}$$

System (17) is oscillating, as shown in Fig. 1.

Based on the Corollary 2, choosing $\Gamma=1$, we make the curves of $s^*=s^*(\kappa)$ and the feasible D of the control parameters (κ, s), as shown in Fig. 2. For the purpose of numerical simulation, we choose $\Gamma=1$, $s=0.8$ and $\kappa=6$, which are included in the feasible region. Fig. 3 shows the time response curves with different initial conditions.

Noting that if control period in this examples is less than the time delay, one could not establish the stabilization criterion by virtue of the theoretical results in [2,4,5]. This example shows that oscillating neural networks with time delay can be stabilized via intermittent control.

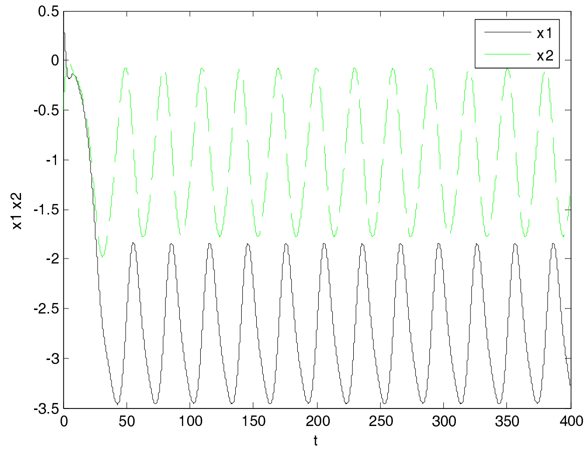

Fig. 1. The oscillating curves of system (17) with initial condition x_1=0.5, x_2=–0.5.

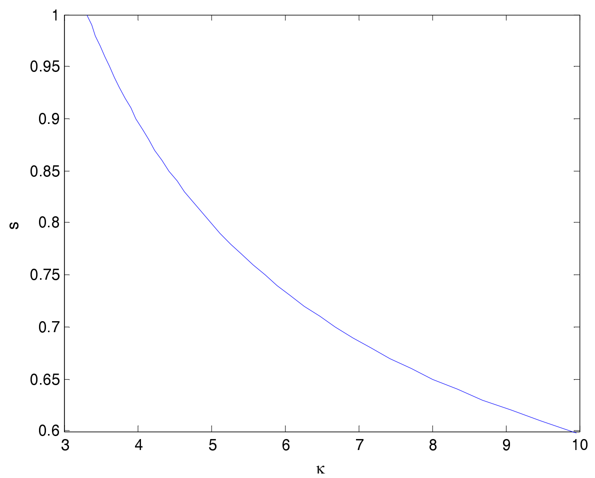

Fig. 2. Curves of the function $s^*=s^*(\kappa)$ for different Γ=1.

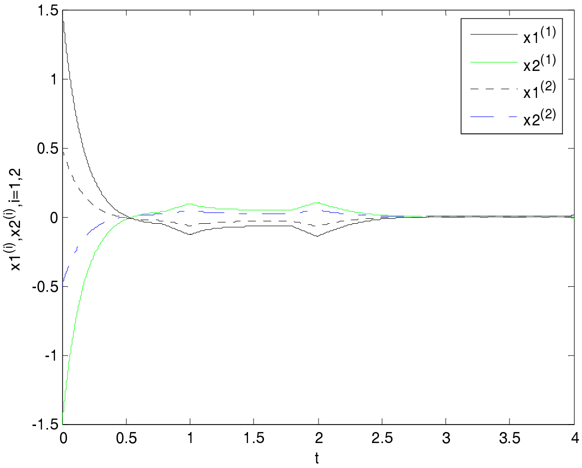

Fig. 3. Time response curves of system (17) for different initial conditions with intermittent control.

5. CONCLUSIONS

In this paper, we have investigated the exponential stability of oscillating neural networks with time-delay via intermittent control. Some tractable stability criteria have been established for controlled neural networks. For the case of periodically intermittent control, our results are also effective when the period is smaller than the time delay. The numerical examples have been introduced to show the effectiveness of proposed theoretical results.

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