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Design of Stabilizing Control for Synchronous Machines via Polynomial Modelling and Linear Matrix Inequalities Approach

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Abstract: This paper deals with the design and evaluation of a nonlinear state feedback controller to improve the global asymptotic stabilization and transient performance of synchronous machines. The nonlinear Park's model is developed around the working point on a third order polynomial system. An innovative technique is used to design a nonlinear polynomial controller, based on the Lyapunov's direct method and Linear Matrix Inequalities (LMIs) approach. The control laws are derived from the resolution of a sufficient LMI stabilization condition. The proposed polynomial control has been tested numerically on a generator infinite-bus power system and the simulations results show an excellent damping of the system oscillations over a wide range of operating conditions whilst retaining good voltage control.

Keywords: Global asymptotic stabilization, linear matrix inequalities (LMI), nonlinear state feedback control, polynomial approach, power system stabiliser, synchronous machine modelling.

1. INTRODUCTION

Control system design and analysis methods are very useful to be applied in real time development. A literature survey on power system control shows that there has been a great deal of interest in synchronous machine modelling along with its controlling equipments [1-6]. The considered electrical power generator is a single machine connected to an infinite bus through a transmission network and which is used almost exclusively in power system as a source of electrical energy [7,8]. In the past, a lot of attempts have been made to design controllers for synchronous generators [9,10]. However, the most of known works have considered the linearized model of the electrical machine [11] and less attention has been devoted to nonlinear control solutions which are more reassuring in the case of a more aggressive perturbation [12]. The mathematical models of synchronous generators are developed and simulated based on Park's transformation which is a widely used transformation in the modelling and analysis [13,14]. Such transformation leads to nonlinear models which can be turned out to be difficult to apply for the synthesis of a performant controller. An idea to reduce the complexity of this model, but without loosing the nonlinear character of the process, one can develop the Park's model in polynomial series with an order greater than one, which yields a nonlinear polynomial model.

In this paper, we consider the description of such electrical process by polynomial approximation of the nonlinear functions based on the Kronecker product and the Kronecker power of the state vector [15-20]. The 7th order state space model of the synchronous machine is developed around the operating point on a third order polynomial system to improve the performances of the considered power system controlling, and particularly in the sense of the widening of the stability domain around the operating point. Furthermore, in this present work, we make use of recent results on nonlinear polynomial system stabilization [21], to derive a polynomial stabilizer of a synchronous machine connected to an infinite busbar. This approach has the advantage to lead to a closed loop system, which stability is guaranteed in a large domain around the operating point. This enlargement of the stability domain is due to the accurate polynomial development of the original nonlinear system. The proposed technique is associated to Linear Matrix Inequalities (LMIs) principle [22-24] for the research of a quadratic stabilizing polynomial controller. Thus, important algebraic manipulations have been elaborated and implemented to lead to the derived results.

This paper is organized as follows: Section 2 reviews the synchronous machine modelling. The design of polynomial feedback stabilizing controller is expressed in Section 3. Section 4 presents some simulation results. Finally, some conclusions and future works are presented in the last section.

2. SYNCHRONOUS MACHINE MODELLING

2.1. Nonlinear Park's model of a synchronous machine

A great simplification in the mathematical description of the synchronous machine is obtained using the Park's

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transformation which is widely used in the analysis and control of electrical machines [25,8]. The variables used to describe a synchronous generator are the following:

- i_d, i_q : stator currents in d and q axis circuits respectively;
- i_f : field current;
- i_D, i_Q : damper currents in d and q axis circuits respectively;
- ω : rotor speed;
- δ : torque angle;

The control variables are:

- V_f : field voltage;
- C_m: mechanical torque;

The output variables are considered as:

- i_f : the field current;
- i_s: the stator current;
- P_e: the electrical power of the machine which can be expressed as:

$$P_e = \sqrt{3}V_\infty(cos(\delta-\alpha)i_q - sin(\delta-\alpha)i_d), \tag{1}$$

where V_∞ is the infinite-busbar voltage and α is the angle of infinite-busbar voltage.

The parameters involved in modelling of the synchronous machine are listed as follows:

r_s : stator resistance;
r_f : field resistance;
r : armature resistance;
r_D : damper resistance;
L_{sd} : direct-axis stator inductance;
L_{sq} : quadrature-axis stator inductance;
L_f : self inductance of field winding;
L_D : self inductance of damper windings along d-axis;
L_Q : self inductance of damper windings along q-axis;
M_F : mutual inductance between stator and field windings;
M_D : mutual inductance between stator and d-axis damper windings;
M_Q : mutual inductance between stator and q-axis damper windings;
M_R : mutual inductance between field and d-axis damper windings;
H : inertia constant;
D : mechanical damper constant;
ω_r : the synchronous speed;
k : constant $\left(\sqrt{\frac{3}{2}}\right)$.

All the above quantities are considered in p.u except time (seconds), torque angle (radians), the angular speed (rad/s) and the inertia constant H (seconds). A detailed representation of synchronous machine system is shown in Fig. 1.

The derived model, called Park's model, is characterized by the following state vector:

$$X = [i_d \quad i_f \quad i_D \quad i_q \quad i_Q \quad \omega \quad \delta]^T \tag{2}$$

and described by the following state space equation [8].

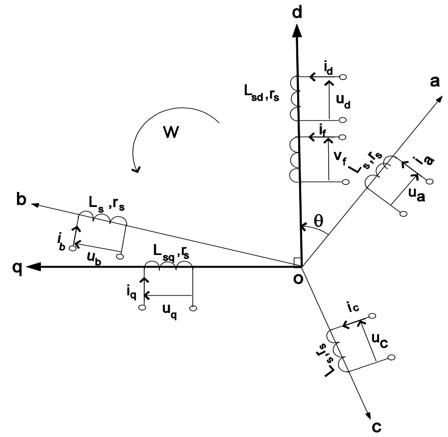

Fig. 1. Pictorial representation of a synchronous machine.

We denote by

$$a_1 = -\frac{L_d i_q}{3\tau_j},\ a_2 = -\frac{kM_f i_q}{3\tau_j},\ a_3 = -\frac{kM_D i_q}{3\tau_j},$$
$$a_4 = \frac{L_d i_d}{3\tau_j},\ a_5 = -\frac{kM_Q i_d}{3\tau_j} \text{ and } a_6 = -\frac{D}{3\tau_j},$$

$$\dot{X} = \begin{bmatrix} & & & & & & 0 \\ & & & & & & 0 \\ & & -L^{-1}(R+\omega N) & & & & 0 \\ & & & & & & 0 \\ & & & & & & 0 \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} & & & & & 0 \\ & & & & & 0 \\ & & -L^{-1} & & & 0 \\ & & & & & 0 \\ & & & & & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{3}V_\infty sin(\delta-\alpha) \\ -V_f \\ 0 \\ \sqrt{3}V_\infty cos(\delta-\alpha) \\ 0 \\ \frac{C_m}{\tau_j} \\ -1 \end{bmatrix}, \tag{3}$$

where

$$R = \begin{bmatrix} r & 0 & 0 & 0 & 0 \\ 0 & r_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$N = \frac{1}{\omega_r}\begin{bmatrix} 0 & 0 & 0 & L_q & kM_q \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -L_d & -kM_f & -kM_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

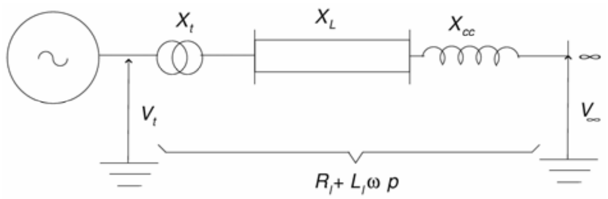

Fig. 2. Single machine infinite bus system.

$$L = \frac{1}{\omega_r}\begin{bmatrix} L_d & kM_f & kM_D & 0 & 0 \\ kM_f & L_f & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix},$$

$$\tau_j = \frac{H}{2w_r},\ L_{sd} = L_d - L_l,\ L_q = L_{sq} + L_l,\ r = r_s + R_l$$

with L_l and R_l are respectively, the inductance and resistance of infinite bus (see Fig. 2).

2.2. Polynomial model approximation

The nonlinear Park's model (3) can be approximated by a polynomial model obtained from a Taylor-series expansions. The description of polynomial systems can be simplified using the Kronecker product and power of vectors and matrices [15,17,26,27] which recognize the advantage of widening the domain of validity of the approximated model of the power system compared to linearized systems.

The exact Park's model (3) characterized by the state vector (2) and controlled by $U = [V_f \quad C_m]^T$ is described by the following polynomial state equation:

$$\dot{X} = F(X,U) = F_1 X + F_2 X^{[2]} + B_0 U + f(\delta), \qquad (4)$$

where

- $F(.)$is a vectorial polynomial function of X;
- $X^{[k]}$ is the kth Kronecker power of the state vector X [15];

$$F_1 = \begin{bmatrix} & & & & & 0 & 0 \\ & & & & & 0 & 0 \\ & & -L^{-1}R & & & 0 & 0 \\ & & & & & 0 & 0 \\ & & & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{D}{3\tau_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad (5)$$

- F_2 is (7×49) matrix such as:

$$F_2(6,4) = \frac{L_d - L_q}{3\tau_j},\quad F_2(6,5) = \frac{kM_Q}{3\tau_j},$$
$$F_2(6,11) = \frac{-kM_f}{3\tau_j},\quad F_2(6,17) = \frac{-kM_D}{3\tau_j}, \qquad (6)$$
$$F_2(1:5, 36:40) = -L^{-1}N,\quad F_2(i,j) = 0$$

for the other values of i and j such as $1 \le i \le 7$ and $1 \le j \le 49$

with $F_2(i_1 : i_2, j_1 : j_2)$ means the sub-matrix of F_2 located between the rows i_1 and i_2 and columns j_1 and j_2. B_0 and $f(\delta)$ are respectively the input matrix and a nonlinear trigonometric vector function of the torque angle δ defined as:

$$B_0 = \begin{bmatrix} -L^{-1} & & 0 \\ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{\tau_j} \\ 0 & 0 \end{bmatrix},$$

$$f(\delta) = \begin{bmatrix} -L^{-1} & & 0 \\ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -\sqrt{3}V_\infty sin(\delta - \alpha) \\ 0 \\ 0 \\ -\sqrt{3}V_\infty sin(\delta - \alpha) \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

Let $P_0(U_0, X_0)$ an operating point of the synchronous machine defined by:

$$\begin{cases} U_0 = [V_{f_0} \quad C_{m_0}]^T \\ X_0 = [i_{d_0} \quad i_{f_0} \quad i_{D_0} \quad i_{q_0} \quad i_{Q_0} \quad \omega_0 \quad \delta_0]^T \\ F(X_0, U_0) = 0 \end{cases} \qquad (7)$$

and consider some variation x around X_0 and u around U_0 which can be expressed in operational form as:

$$X = X_0 + x,\quad U = U_0 + u$$

with $x = [\Delta i_d \ \Delta i_f \ \Delta i_D \ \Delta i_q \ \Delta i_Q \ \Delta\omega \ \Delta\delta]^T$, $u = [\Delta V_f \ \Delta C_m]^T$ and Δ represents some deviation around the operating point.

The equation describing the state variation x can be written as follows:

$$\dot{x} = F_1 x + F_2(X^{[2]} - X_0^{[2]}) + B_0 u + f(\delta) - f(\delta_0). \qquad (8)$$

The development of the nonlinear terms $X^{[2]} - X_0^{[2]}$ and $f(\delta) - f(\delta_0)$ in the first order as:

$$\begin{cases} X^{[2]} - X_0^{[2]} \cong (I_n \otimes X_0 + X_0 \otimes I_n)x \\ f(\delta) - f(\delta_0) \cong \left[\dfrac{\partial f}{\partial \delta}\right]_{\delta_0} \Delta\delta \end{cases} \qquad (9)$$

leads to a linear model of the form:

$$\dot{x} = A_1 x + B_0 u, \tag{10}$$

where

$$A_1 = F_1 + F_2(X_0 \otimes I_7 + I_7 \otimes X_0) + \left[\frac{\partial f}{\partial \delta}\right]_{\delta_0} \xi \tag{11}$$

and:

$$\xi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T. \tag{12}$$

Note that the linearized model (10) is available in a little neighborhood of the operating point. In order to obtain a more accurate model available in a greater domain around the operating point, we propose to consider a third order polynomial approximation of the nonlinear terms in (8):

$$\begin{cases} X^{[2]} - X_0^{[2]} = (I_n \otimes X_0 + X_0 \otimes I_n)x + x^{[2]} \\ f(\delta) - f(\delta_0) \cong \left[\frac{\partial f}{\partial \delta}\right]_{\delta_0} \Delta\delta + \left[\frac{\partial^2 f}{\partial \delta^2}\right]_{\delta_0} (\Delta\delta)^2 \\ \qquad + \left[\frac{\partial^3 f}{\partial \delta^3}\right]_{\delta_0} (\Delta\delta)^3. \end{cases}$$

By using the properties of Kronecker product and by adopting the following approximations:

$$\begin{aligned} &sin(\gamma_0 + \Delta\delta) - sin(\gamma_0) \\ &\quad = cos(\gamma_0)(\Delta\delta) - \frac{sin(\gamma_0)}{2}(\Delta\delta)^2 - \frac{cos(\gamma_0)}{6}(\Delta\delta)^3, \\ &cos(\gamma_0 + \Delta\delta) - cos(\gamma_0) \\ &\quad = -sin(\gamma_0)\Delta\delta - \frac{cos(\gamma_0)}{2}(\Delta\delta)^2 + \frac{sin(\gamma_0)}{6}(\Delta\delta)^3, \end{aligned}$$

where $\gamma_0 = \delta_0 - \alpha$, the polynomial model of a synchronous machine connected to an infinite busbar through a transmission line is given by the following state space equation:

$$\dot{x} = A_1 x + A_2 x^{[2]} + A_3 x^{[3]} + Bu \tag{13}$$

$$\begin{cases} A_1 = F_1 + F_2(X_0 \otimes I_7 + I_7 \otimes X_0) + \Upsilon_1 \\ A_2 = F_2 + \Upsilon_2 \\ A_3 = \Upsilon_3 \\ B = B_0 \end{cases} \tag{14}$$

with

$$\Upsilon_1 = \begin{bmatrix} -L^{-1} & & \mathbf{0} \\ & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{3}V_\infty cos(\gamma_0) \\ 0 \\ 0 \\ -\sqrt{3}V_\infty sin(\gamma_0) \\ 0 \\ 0 \\ 0 \end{bmatrix} \xi,$$

$$\Upsilon_2 = \begin{bmatrix} -L^{-1} & & \mathbf{0} \\ & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} V_\infty sin(\gamma_0) \\ 0 \\ 0 \\ -\frac{\sqrt{3}}{2} V_\infty cos(\gamma_0) \\ 0 \\ 0 \\ 0 \end{bmatrix} \xi^{[2]},$$

$$\Upsilon_3 = \begin{bmatrix} -L^{-1} & & \mathbf{0} \\ & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{6} V_\infty cos(\gamma_0) \\ 0 \\ 0 \\ \frac{\sqrt{3}}{6} V_\infty sin(\gamma_0) \\ 0 \\ 0 \\ 0 \end{bmatrix} \xi^{[3]}.$$

Several possibilities for simplifying the Park's model order of the machine are considered in the literature [1,7,8]. Assuming the simplified model of synchronous machine obtained when neglecting the damper transients, the set of equations (13-14) are manipulated to obtain the following reduced 5^{th} order polynomial state-space model:

$$\dot{\tilde{X}} = \tilde{A}_1 \tilde{x} + \tilde{A}_2 \tilde{x}^{[2]} + \tilde{A}_3 \tilde{x}^{[3]} + \tilde{B}u, \tag{15}$$

where

$$\begin{aligned} \tilde{A}_1 &= \Psi.A_1.\Psi^T, \quad \tilde{A}_2 = \Psi.A_2.\Psi^{[2]T} \\ \tilde{A}_3 &= \Psi.A_3.\Psi^{[3]T}, \quad \tilde{B} = \Psi.B_0 \end{aligned} \tag{16}$$

with

$$\tilde{x} = [\Delta i_d \quad \Delta i_f \quad \Delta i_q \quad \Delta\omega \quad \Delta\delta]^T \tag{17}$$

and

$$\Psi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{18}$$

Remark 1: In what follows, the variables notation $\tilde{x}$, $\tilde{B}$ and $\tilde{A}_{i,i=1,2,3}$ are replaced respectively by x, B and $A_{i,i=1,2,3}$, to relieve the writing.

The three mathematical models of the synchronous machine; the original Park's model (3), the linear model (10) and the polynomial model (15) presented above, were translated to a Matlab model in order to simulate the different interesting aspects of the machine. The

Table 1. Parameter values of the synchronous generator (p.u).

$L_d = 1.70$	$L_q = 1.64$	$L_f = 1.65$
$L_D = 1.64$	$L_Q = 1.526$	$r_Q = 0.0540$
$H = 2.37s$	$w_r = 314.1 rd/s$	$M_Q = 1.49\sqrt{\frac{2}{3}}$
$M_f = 1.55\sqrt{\frac{2}{3}}$	$M_R = 1.55$	$M_D = 1.55\sqrt{\frac{2}{3}}$
$r_s = 0.001096$	$r_f = 0.000742$	$r_D = 0.0131$

parameter values of the studied synchronous generator, listed in Table 1 [8], are needed in order to make the mathematical model a representative one of an actual generator.

This machine is connected to an infinite bus (see Fig. 2) characterized by a resistance $R_l = 0.02$ and an inductance $L_l = 0.4$. The working point of the power process is defined by:

$$\begin{aligned} & i_{d_0} = -1.59, \quad i_{f_0} = 1.826, \quad i_{q_0} = 0.701, \\ & \omega_0 = 314.1 rd/s, \quad \delta_0 = 53.735^\circ. \end{aligned} \tag{19}$$

Figs. 3~7 represent the responses of the three proposed models of synchronous machine studied without the controller. It compares the state trajectories of the original Park's model (3), the linear model (10) and the

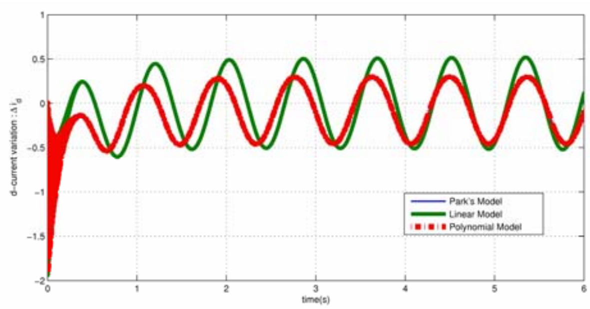

Fig. 3. Evolution of the d-current variation (Δi_d) without controller.

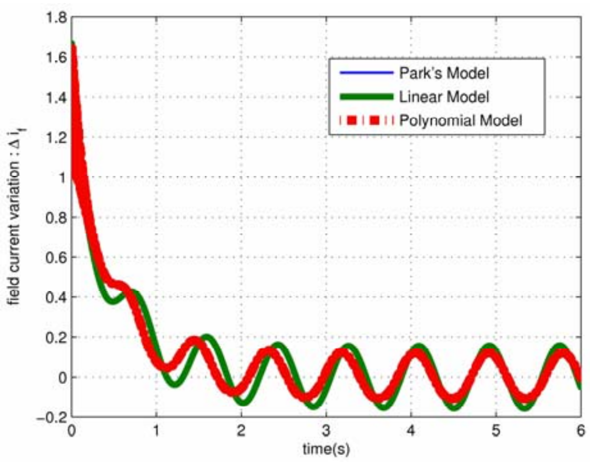

Fig. 4. Evolution of the field current variation (Δi_f) without controller.

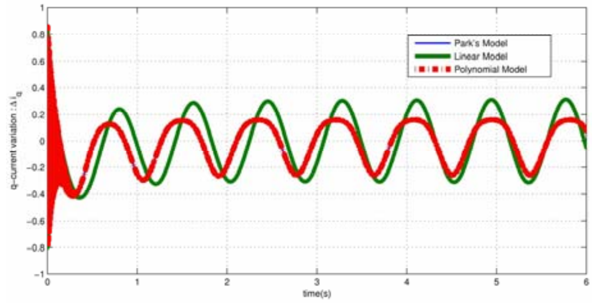

Fig. 5. Evolution of the q-current variation (Δi_q) without controller.

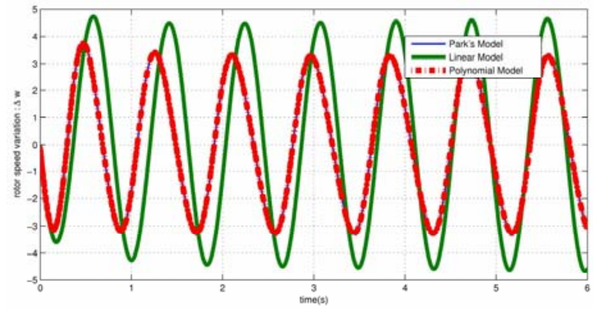

Fig. 6. Evolution of the rotor speed variation ($\Delta\omega$) without controller.

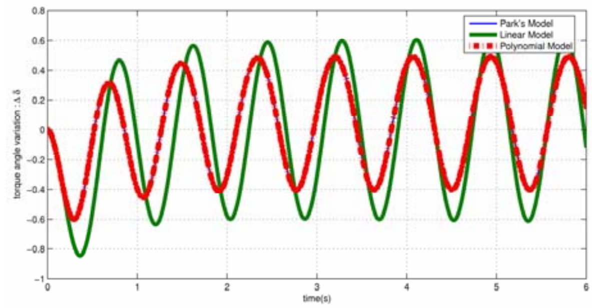

Fig. 7. Evolution of the torque angle variation ($\Delta\delta$) without controller.

polynomial model (15). The study of the digital simulation results shows the superiority of the proposed polynomial model over the linear one. For a disturbance of 100% operating on the inductance current, it appears in the simulations that the polynomial approach is a more objective presentation of the real power process, since, we can't distinguish the real system behavior and the polynomial one. Thus, this polynomial model (15) will be considered in the next section for developing a technique of designing a state feedback control law which stabilizes quadratically the studied system.

3. DESIGN OF NONLINEAR FEEDBACK STABILIZING CONTROLLER

Many research centres continue their efforts towards developing improved power system stabilizers. Since that the electrical power generator is a complex system with highly nonlinear dynamics, many methods of simplification are illustrated in the literature. In fact, control of synchronous machines is usually leaded from linearized models, using a reduction order method or a decoupling of slow and fast dynamics by singular perturbations method, yielding reduced order or composite controllers [1] which were used after for optimal control of synchronous machines. With an ambition to achieve a better performance, we propose a new design for synthesis of a nonlinear polynomial control law using recent results on stabilization of polynomial systems associated to the LMI technique [21].

3.1. Global stabilization condition of controlled synchronous machines

We consider the variation model of synchronous machines around the operating point. We keep unchanged the mechanical torque C_m and we consider only the field voltage V_f to control the machine. The nonlinear model developed as a third order polynomial system is described as the following analytical control-affine state-space equation:

$$\dot{x} = f(x) + Bu, \tag{20}$$

where $f(x)$ is a vectorial polynomial function of the state vector x defined as:

$$f(x) = A_1 x + A_2 x^{[2]} + A_3 x^{[3]} + Bu, \tag{21}$$

where

- A_1, A_2 and A_3 are constant parameter matrices given in (13);
- B is a constant vector;
- $x \in \mathbb{R}^n$ is a variation state vector ($n = 5$);
- $u = \Delta V_f$ is a control input.

The main objective is to find a polynomial feedback control law:

$$u = k(x) = \sum_{i=1}^{3} K_i x^{[i]} \tag{22}$$

with K_1, K_2 and K_3 are constant gain matrices which stabilize asymptotically the equilibrium $X = X_0$ (defined in (19)) of the synchronous machine.

Applying this control law to the open-loop system (21), one obtains the following closed-loop system:

$$\begin{aligned} \dot{x} &= h(x) = (f + Bk)x \\ &= \sum_{i=1}^{3} \Gamma_i x^{[i]}, \end{aligned} \tag{23}$$

where

$$\Gamma_i = A_i + BK_i. \tag{24}$$

The designed power system stabilizers of synchronous machine must achieve the following requirements:

(i) Ensures global asymptotic stabilization of the equilibrium, with no need of tuning parameters or trial and error procedures.

(ii) Provides satisfactory performance over a wide range of agressive state perturbations.

Lyapunov's direct method is concerned in this work for

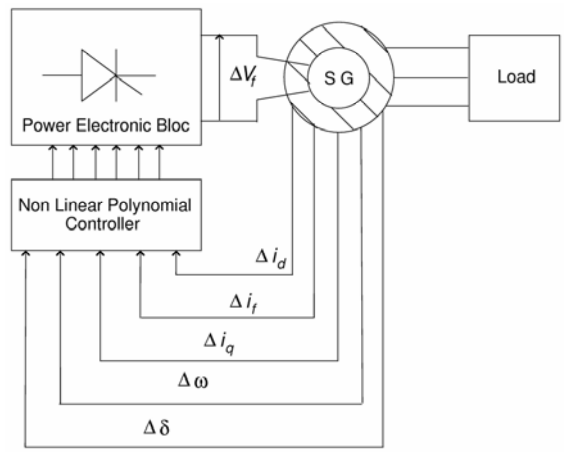

Fig. 8. Closed loop system that representing synchronous generator (SG) and polynomial controller.

assessing the stability analysis and synthesis of the power dynamic system described by a set of nonlinear equations of the form (20-21). For this goal, we consider a quadratic Lyapunov function [28,29]:

$$V(x) = x^T P x, \tag{25}$$

where P is a symmetric positive definite matrix and x is the state vector of the studied system. Then, it comes out that the equilibrium state X_0 of the synchronous generator system (20) is asymptotically stable if the derivative $\dot{V}$ is negative definite along the trajectory $X(t)$ (i.e., when $\dot{V} < 0$). Differentiating $V(x)$ along the trajectory of the system (23-24), we obtain:

$$\begin{aligned} \dot{V}(x) &= \dot{x}^T P x + x^T P \dot{x} \\ &= (\Gamma_1 x + \Gamma_2 x^{[2]} + \Gamma_3 x^{[3]})^T P x \\ &\quad + x^T P (\Gamma_1 x + \Gamma_2 x^{[2]} + \Gamma_3 x^{[3]}) \\ &= \sum_{k=1}^{3} (x^T P \Gamma_k x^{[k]} + x^{[k]^T} \Gamma_k^T P x) \\ &= 2\sum_{k=1}^{3} x^T P \Gamma_k x^{[k]}. \end{aligned}$$

By means of some algebraic manipulations, appealing the vec-function and mat-function defined respectively in Appendix A.1-A.2, one has:

$$\begin{aligned} \dot{V}(x) = 2[&x^T P \Gamma_1 x + x^T P \Gamma_2 x^{[2]} \\ &+ x^{[2]T} U_{n\times n} (P \otimes I_n) M(\Gamma_3) x^{[2]}], \end{aligned}$$

where $U_{n\times n}$ represents the permutation matrix defined in [15,17] and $M(\Gamma_3)$ is expressed as:

$$M(\Gamma_3) = \begin{bmatrix} mat_{(n,n^2)}(\Gamma_3^{1T}) \\ mat_{(n,n^2)}(\Gamma_3^{2T}) \\ mat_{(n,n^2)}(\Gamma_3^{3T}) \\ mat_{(n,n^2)}(\Gamma_3^{4T}) \\ mat_{(n,n^2)}(\Gamma_3^{5T}) \end{bmatrix}$$

with $\Gamma_3^{i,i=1,\ldots,5}$ is the i^{th} row of the matrix Γ_3.
Using some manipulations and exploiting the properties of the Kronecker power, it can be shown that:

$$\begin{aligned}\dot{V}(x) &= 2\mathscr{X}^T \mathscr{P}\mathscr{M}_h\mathscr{X} \\ &= \mathscr{X}^T(\mathscr{P}\mathscr{M}_h + \mathscr{M}_h^T\mathscr{P})\mathscr{X},\end{aligned} \tag{26}$$

where

$$\mathscr{X} = [x^T \quad x^{[2]^T}]^T \tag{27}$$

$$\mathscr{P} = \begin{bmatrix} P & 0_{n\times n^2} \\ 0_{n^2\times n} & P\otimes I_n \end{bmatrix} \tag{28}$$

$$\mathscr{M}_h = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ 0_{n^2\times n} & M(\Gamma_3) \end{bmatrix}. \tag{29}$$

Using the non-redundant Kronecker product power form [15-17], defined in Appendix A.3, the vector $\mathscr{X}$ can be written as:

$$\mathscr{X} = \tau\tilde{\mathscr{X}}, \tag{30}$$

where $\tilde{\mathscr{X}} = [\tilde{x}^T \quad \tilde{x}^{[2]^T}]^T$ and τ is defined by:

$$\tau = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}. \tag{31}$$

Then $\dot{V}(x)$ can be written in the following form:

$$\dot{V}(x) = \tilde{\mathscr{X}}^T\tau^T(\mathscr{P}\mathscr{M}_h + \mathscr{M}_h^T P)\tau\tilde{\mathscr{X}}. \tag{32}$$

It can be shown that:

$$\dot{V}(x) = \tilde{\mathscr{X}}^T \mathscr{Q}_{(\eta\times\eta)}\tilde{\mathscr{X}}, \tag{33}$$

where

$$\mathscr{Q} = \mathscr{Q}(P, K_{i,i=1,2,3}, \mu_{i,i=1,\ldots,\beta}) \tag{34}$$

$$= \tau^T(\mathscr{P}\mathscr{M}_h + \mathscr{M}_h^T\mathscr{P})\tau + \Pi(\mu_{i,i=1,\ldots,\beta}), \tag{35}$$

$$\eta = \sum_{j=1}^{2} n_j = \sum_{j=1}^{2}\binom{n+j-1}{j}. \tag{36}$$

The matrix $\Pi(\mu_{i,i=1,\ldots,\beta})$ is defined as:

$$\Pi(\mu_{i,i=1,\ldots,\beta}) = \sum_{i=1}^{\beta}\mu_i mat_{(\eta,\eta)}(C_i), \tag{37}$$

where the columns $C_{i,i=1,\ldots,\beta}$ and the coefficients μ_i and β are detailed in Appendix A.4.
A sufficient condition of the global asymptotic stability of the equilibrium point $(X = X_0)$ of synchronous machines is that the quadratic form $\dot{V}(x)$ given by (33) is negative definite. This condition can be ensured if there exist $P > 0$, control gain matrices $K_{i,i=1,2,3}$ and parameters $\mu_{i,i=1,\ldots,\beta}$ such that the matrix $\mathscr{Q}(P, K_{i,i=1,2,3}, \mu_{i,i=1,\ldots,\beta})$ is negative definite.
According to (23-24), it can be shown that [30]:

$$\mathscr{M}_h = \mathscr{M}_f + \Theta\mathscr{M}_k, \tag{38}$$

where

$$\mathscr{M}_f = \begin{bmatrix} A_1 & A_2 \\ 0_{n^2\times n} & M(A_3) \end{bmatrix}, \mathscr{M}_k = \begin{bmatrix} K_1 & K_2 \\ 0_{n\times n} & mat_{(n,n^2)}(K_3^T) \end{bmatrix}$$

$$\Theta = \begin{bmatrix} B & 0_{n\times n} \\ 0_{n^2\times 1} & B\otimes I_n \end{bmatrix}.$$

Finally, we obtain the following expression of the symmetric matrix $\mathscr{Q}$:

$$\begin{aligned}\mathscr{Q} &= \tau^T(\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P})\tau + \tau^T(\mathscr{P}\Theta\mathscr{M}_k + \mathscr{M}_k^T\Theta^T\mathscr{P})\tau \\ &\quad + \Pi(\mu_{i,i=1,\ldots,\beta}),\end{aligned} \tag{39}$$

which must be negative definite to guarantee the stability of the controlled system.

3.2. LMI Approach for the controller design

A number of important problems from system and control theory can be numerically solved by reformulating them as convex optimization problems with Linear Matrix Inequalities (LMI) approach [31,32]. This section introduces the LMI-based characterization of a nonlinear control law which can stabilize the considered power system (20-21). According to (39), the controller design problem of the synchronous machine can be formulated as an LMI feasibility problem given as follows:
Find:

- control gain matrices K_1, K_2 and K_3;
- a (5×5) symmetric matrix P;
- real parameters $\mu_{i,i=1,\ldots,\beta}$;

such that:

$$P > 0 \tag{40}$$

$$\begin{aligned}S &= \tau^T(\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P})\tau + \tau^T(\mathscr{P}\Theta\mathscr{M}_k + \mathscr{M}_k^T\Theta^T\mathscr{P})\tau \\ &\quad + \Pi(\mu_{i,i=1,\ldots,\beta}) < 0.\end{aligned} \tag{41}$$

This problem is a NLMI (Non-Linear Matrix Inequalities), since the inequality (41) is bilinear on optimization variables P and $K_{i=1,2,3}$. To overcome this problem, first, we make use of the following inequality [33]:
For any matrices A and B with appropriate dimensions and for any positive scalar $\gamma > 0$, one has:

$$A^T B + B^T A \le \gamma A^T A + \gamma^{-1}B^T B.$$

Then, the inequality (41) yields the following one:

$$\begin{aligned}\mathscr{Q} &\le \tau^T[\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P}]\tau + \Pi(\mu_{i,i=1,\ldots,\beta}) \\ &\quad + \gamma\tau^T\mathscr{P}^T\mathscr{P}\tau + \gamma^{-1}\tau^T\mathscr{M}_k^T\Theta^T\Theta\mathscr{M}_k\tau\end{aligned} \tag{42}$$

with $\gamma > 0$.
Thus, to ensure that the matrix $\mathscr{Q}$ is negative definite, it is sufficient to have:

$$\begin{aligned}&\tau^T[\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P}]\tau + \Pi(\mu_{i,i=1,\ldots,\beta}) \\ &\quad -\tau^T\mathscr{P}^T(-\gamma I)\mathscr{P}\tau - \tau^T\mathscr{M}_k^T\Theta^T(-\gamma^{-1}I)\Theta\mathscr{M}_k\tau < 0.\end{aligned} \tag{43}$$

Using now the Generalized Schur's complement [22], the above inequality is equivalent to:

$$\begin{bmatrix} \tau^T(\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P})\tau + \Pi(\mu_{i,i=1,\dots,\beta}) & (*)^T & (*)^T \\ \mathscr{P}\tau & -\gamma^{-1}I & 0 \\ \Theta\mathscr{M}_k\tau & 0 & -\gamma I \end{bmatrix} < 0. \quad (44)$$

The symmetric terms in a symmetric matrix are denoted by $*$.

When pre-and post-multiplying the inequality (44) by $\Omega = diag(I, I, \gamma^{-1}I)$, we get:

$$\begin{bmatrix} \tau^T(\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P})\tau + \Pi(\mu_{i,i=1,\dots,\beta}) & (*)^T & (*)^T \\ \mathscr{P}\tau & -\gamma^{-1}I & 0 \\ \Theta\mathscr{N}_k\tau & 0 & -\gamma^{-1}I \end{bmatrix} \quad (45)$$

is negative definite, with $\mathscr{N}_k = \gamma^{-1}\mathscr{M}_k$.

This new inequality (45) is linear on the decision variables P, $K_{i,i=1,\dots,3}$ and $\mu_{i,i=1,\dots,\beta}$, then we can state the following result. The equilibrium ($X = X_0$) of the synchronous machine, described by the polynomial model (20-21), is globally asymptotically stabilizable by the control law (22) if there exist:

- a (5×5)-symmetric positive definite matrix P;
- arbitrary parameters $\mu_{i,i=1,\dots,\beta}$;
- control gain matrices K_1, K_2 and K_3;
- a real $\gamma > 0$;

such that:

$$P > 0 \quad (46)$$

and

$$\begin{bmatrix} \tau^T(\mathscr{P}\mathscr{M}_f + \mathscr{M}_f^T\mathscr{P})\tau + \Pi(\mu_{i,i=1,\dots,\beta}) & (*)^T & (*)^T \\ \mathscr{P}\tau & -\gamma^{-1}I & 0 \\ \Theta\mathscr{N}_k\tau & 0 & -\gamma^{-1}I \end{bmatrix} \quad (47)$$

is negative definite.

The procedure of synthesis of a stabilizing control law for the considered power system (20-21) is as follows:

(i) Solve the LMI feasibility problem i.e., find the matrices $\mathscr{P}$, $\mathscr{N}_k$, the parameters $\mu_{i,i=1,\dots,\beta}$ and a scalar γ such that the inequalities (46), (47) are verified.

(ii) Extract the gain matrices K_1, K_2 and K_3 from the relation $\mathscr{M}_k = \gamma\mathscr{N}_k$.

4. SIMULATION RESULTS

MATLAB software was used in the testing of the LMI stabilization condition of a synchronous machine infinite bus system. The parameters accorded in the simulation program for the considered power system are listed in Table 1. Then, the nonlinear polynomial model of the synchronous machine (20-21) with the constant matrices parameters of the open-loop power system are expressed as follows:

$$A_1 = \begin{bmatrix} -11.3 & 0.1 & -1095.9 & -2.4 & 550.4 \\ 3.9 & -1.6 & 380.2 & 0.8 & -190.9 \\ 1127.2 & 832.0 & -11.3 & 1.8 & 749.6 \\ 0.9 & -24.0 & -89.9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad (48)$$

$A_2(1,18) = -3.49$, $A_2(1,25) = -375.11$, $A_2(2,18) = 1.21$, $A_2(2,25) = 130.13$, $A_2(3,16) = 3.59$, $A_2(3,17) = 2.65$, $A_2(3,25) = 274.98$, $A_2(4,3) = 1.33$, $A_2(4,8) = -34.24$, $A_2(i,j) = 0$ for the others values of i and j such as; $1 \le i \le 5$ and $1 \le j \le 25$,

$A_3(1,125) = -91.73$, $A_3(2,125) = 31.82$,
$A_3(3,125) = -124.94$, $A_3(i,j) = 0$ (49)
for the other values of i and j such as
$1 \le i \le 5$ and $1 \le j \le 125$,

and

$$B = [-186.4 \quad 2116 \quad 0 \quad 0 \quad 0]^T. \quad (50)$$

It's required to stabilize the studied machine with a nonlinear polynomial control law as follows:

$$u = \Delta V_f = k(x) = K_1 x + K_2 x^{[2]} + K_3 x^{[3]}. \quad (51)$$

The dynamic behavior of the state variables of the controlled power system and the performance of the proposed control strategy are illustrated in the next part.

Solving the LMI optimization problem applied to synchronous machine by using LMI Toolbox, we get:

$$P = 10^4 \cdot \begin{bmatrix} 0.5749 & 0.7403 & 0.0314 & -0.0181 & 0.3289 \\ 0.7403 & 1.3021 & -0.0092 & 0.0079 & 0.2490 \\ 0.0314 & -0.0092 & 0.3354 & 0.0090 & -0.1803 \\ -0.0181 & 0.0079 & 0.0090 & 0.3090 & 0.0109 \\ 0.3289 & 0.2490 & -0.1803 & 0.0109 & 1.4799 \end{bmatrix},$$

$\gamma = 2.7066.$

The control gain matrices, extracted from $\mathscr{M}_k$, are given by:

$K_1 = [-0.8153 \quad -0.8153 \quad -0.5435 \quad 4.3744 \quad 3.5591]^T$,

$K_2 = 0_{1\times 25}$.

For i=1,...,125

$K_3(1) = K_3(6) = K_3(27) = K_3(32) = K_3(53) = K_3(58) = 0.4989;$

$K_3(79) = K_3(84) = K_3(94) = K_3(110) = 0.4989;$

$K_3(11) = K_3(37) = K_3(63) = K_3(89) = K_3(115) = 0.3326;$

$K_3(16) = K_3(42) = K_3(68) = K_3(120) = -2.6771;$

$K_3(21) = K_3(47) = K_3(73) = K_3(125) = -2.1782.$

$K_3(1 \le i \le 125) = 0$ for the other values of i.

The system controlled with the obtained polynomial control law was simulated for a perturbation of 100% on the field current. The simulation results (Figs. 9~13), present the state trajectories of the studied power system with the proposed nonlinear polynomial controller (dash-dot line) and without controller (solid line) together in the same figure. The simulation depicted in Fig. 14 describes the evolution of the performed stabilizing polynomial controller. The effect on the damping of the system's oscillation is efficient and the power system is properly stabilized after a very short period of time and, thus, provides a satisfactory performance. The digital simulation study proves the robustness of the proposed controller in the case of more aggressive perturbations.

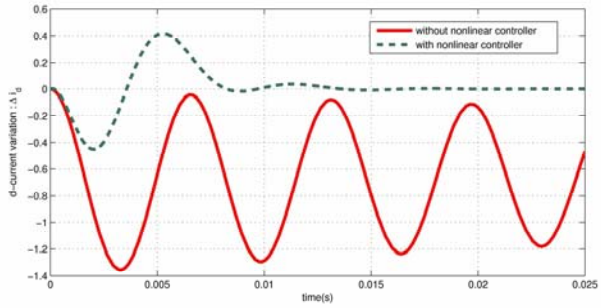

Fig. 9. Simulation curve of the d-current variation (Δi_d) with the proposed feedback controller and without controller for a perturbation of 100% on the field current.

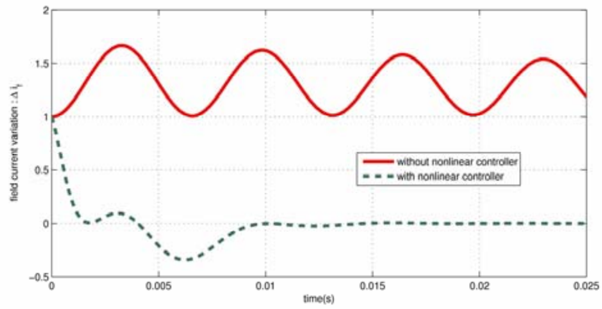

Fig. 10. Simulation curve of the field current variation (Δi_f) with the proposed feedback controller and without controller for a perturbation of 100% on the field current.

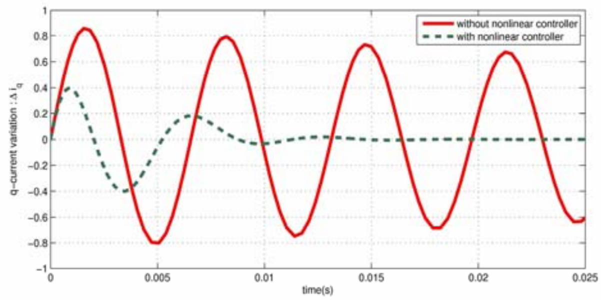

Fig. 11. Simulation curve of the q-current variation (Δi_q) with the proposed feedback controller and without controller for a perturbation of 100% on the field current.

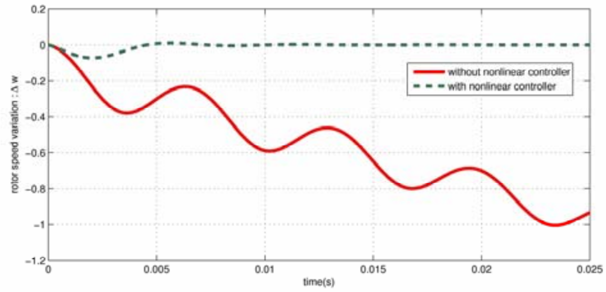

Fig. 12. Simulation curve of the rotor speed variation ($\Delta\omega$) with the proposed feedback controller and without controller for a perturbation of 100% on the field current.

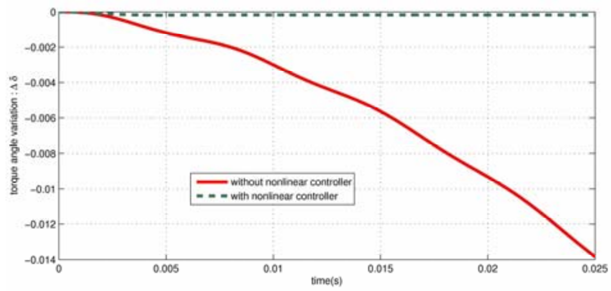

Fig. 13. Simulation curve of the torque angle variation ($\Delta\delta$) with the proposed feedback controller and without controller for a perturbation of 100% on the field current.

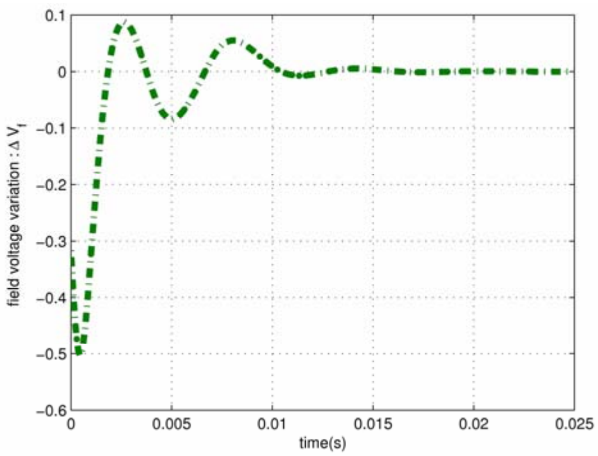

Fig. 14. Simulation of the stabilizing polynomial control (51) of synchronous machine.

5. CONCLUSION

An original technique for the design of nonlinear stabilizing controller for the generator-infinite busbar system have been summarized in this paper. Based on the Park's model of synchronous machine, a polynomial state-space model has been derived, based on the Kronecker product and power state formulation. This polynomial description has the advantage to describe the real dynamic behavior of the machine in a large domain around the working point. Applying Lyapunov's direct method with a quadratic function, sufficient design conditions have been derived as Linear Matrix Inequalities (LMIs).

The simulation results have demonstrate the effectiveness of this control technique to improve the stability and the transient performance of the studied machine under a variety of operating conditions. Moreover, it is clearly seen from the numerical simulations, that the proposed polynomial controller can rapidly damp the system oscillations and greatly enhance the transient stability of the considered power system. On the other hand, the robustness of the controller has been evaluated towards a more aggressive perturbation in the field current. Future work, will extend this approach to robust control with respect to model uncertainties of the synchronous machines.

APPENDIX A

The dimensions of the matrices used in this section are the following: A($p\times q$), $C(q\times f)$, $B(n\times p)$.

A.1. vec(.) function:

An important vector valued function of matrix denoted *vec*(.) was defined in [15] as follows:

$$A=[A_1 \quad A_2 \quad \dots \quad A_q]\in\mathbb{R}^{p\times q}, \tag{A.1}$$

where

$$\begin{aligned}&\forall i\in\{1,\dots,q\}, A_i\in\mathbb{R}^p,\\ &vec(A)=[A_1 \quad A_2 \quad \dots \quad A_q]^T\in\mathbb{R}^{pq}.\end{aligned} \tag{A.2}$$

We recall the following useful properties of *vec-function*:

$$vec(BAC)=(C^T\otimes B)vec(A), \tag{A.3}$$

$$vec(A^T)=U_{p\times q}vec(A). \tag{A.4}$$

A.2. mat(.) function

A special function $mat_{(n,m)}(.)$ can be defined as follows:

If V is a vector of dimension $p=n.m$ then $M=mat_{(n,m)}(V)$ is the $(n\times m)$ matrix verifying $V=vec(M)$.

A.3. Non-redundant Kronecker product power form

$\tilde{X}^{[i]}\in\mathbb{R}^{n_i}$ where $n_i=\begin{pmatrix}n+i-1\\ i\end{pmatrix}$, is the non-redundant Kronecker power of the state vector X defined as:

$$\begin{aligned}&\tilde{X}^{[1]}=X^{[1]}=X\\ &\forall\ i\geq 2,\\ &\tilde{X}^{[i]}=[x_1^i,x_1^{i-1}x_2,\dots,x_1^{i-1}x_n,\dots,x_1^{i-2}x_n^2,\dots,x_1^{i-3}\,x_2^3,\dots,x_n^i]^T\end{aligned} \tag{A.5}$$

i.e., the components of $\tilde{X}^{[i]}$ are the same as those of $X^{[i]}$ with omission of the repeated terms.

The relation between the redundant and the non-redundant Kronecker power of the state vector X can be expressed by the transition matrix denoted $T_i\in\mathbb{R}^{n^i\times n_i}$ as follows:

$$X^{[i]}=T_i\tilde{X}^{[i]} \quad (\forall i\in N). \tag{A.6}$$

A.4. Definition of the matrix $\Pi(\mu_{i,i=1,\dots,\beta})$

According to the equality (34) and using the *vec*-function (see Appendix A.1), it comes out:

$$vec^T\left(\mathscr{Q}-\tau^T(\mathscr{P}\mathscr{M}_h+\mathscr{M}_h^T\mathscr{P})\tau\right)\tilde{\mathscr{X}}^{[2]}=0. \tag{A.7}$$

But, it can be easily checked that $\tilde{\mathscr{X}}^{[2]}$ can be written as:

$$\tilde{\mathscr{X}}^{[2]}=\mathscr{R}\tilde{\mathscr{X}}_2, \tag{A.8}$$

where

$$\tilde{\mathscr{X}}_2=[\tilde{\mathscr{X}}^{[2T]} \qquad \tilde{\mathscr{X}}^{[3T]} \qquad \tilde{\mathscr{X}}^{[4T]}]^T \tag{A.9}$$

and $\mathscr{R}$ is the matrix defined by:

$$\mathscr{R}=\tau^{+[2]}\cdot\mathscr{U}\cdot\mathscr{H}\cdot\tilde{\tau} \tag{A.10}$$

with

$$\tilde{\tau}=diag(T_{j,j=2,3,4}),$$

$$\mathscr{U}=\begin{bmatrix}U_{n\times(n+n^2)} & 0\\ 0_{n^2\times(n+n^2)} & U_{n^2\times(n+n^2)}\end{bmatrix}, \tag{A.11}$$

$$\mathscr{H}=\begin{bmatrix}I_{n(n+n^2)} & 0\\ 0_{n^2(n+n^2)\times n(n+n^2)} & I_{n^2(n+n^2)}\end{bmatrix}. \tag{A.12}$$

The proof of the expression (A.8) is given in [17].

Therefore, we get the following equation:

$$\mathscr{R}^T vec(\mathscr{S})=0 \tag{A.13}$$

with $\mathscr{S}=\mathscr{Q}-\tau^T(\mathscr{P}\mathscr{M}_h+\mathscr{M}_h^T\mathscr{P})\tau$.

The η^2-vector $vec(\mathscr{S})$ can be expressed as:

$$vec(\mathscr{S})=\left(\mathscr{R}^{+T}\mathscr{R}^T-I_{\eta^2}\right)\mathscr{Y}, \tag{A.14}$$

where $\mathscr{Y}$ is an arbitrary vector of $\mathbb{R}^{\eta^2}$ and η is defined in (36).

The matrix $\mathscr{S}$ is symmetric since $\mathscr{Q}$ is symmetric, then we can write:

$$\mathscr{S}=\frac{1}{2}(\mathscr{S}+\mathscr{S}^T) \tag{A.15}$$

and, by using the second property of *vec*-function (see Appendix A.1), we obtain:

$$vec(\mathscr{S})=\frac{1}{2}(I_{\eta^2}+U_{\eta\times\eta})vec(\mathscr{S})=\sum_{i=1}^{\beta}\mu_i C_i, \tag{A.16}$$

where

- $\beta=rank\left[\left(I_{\eta^2}+U_{\eta\times\eta}\right)\left(\mathscr{R}^{+T}\mathscr{R}^T-I_{\eta^2}\right)\right]$
- $C_{i,i=1,\dots,\beta}$ are β linearly independent columns of

$$\left(I_{\eta^2}+U_{\eta\times\eta}\right)\left(\mathscr{R}^{+T}\mathscr{R}^{T}-I_{\eta^2}\right) \tag{A.17}$$

- $\mu_{i,i=1,\ldots,\beta}$ are arbitrary values.

Finally, the matrix $\mathscr{C}$ can be expressed by the relation (39), where $\Pi(\mu_{i,i=1,\ldots,\beta})=\sum_{i=1}^{\beta}\mu_i mat_{(\eta,\eta)}(C_i)$.

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