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Semiglobal Robust Backstepping Output Tracking for Strict-feedback Form Systems with Nonlinear Uncertainty

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Abstract: Output tracking controller design problem is dealt with for a class of nonlinear systems in strict-feedback form in the presence of time-varying nonlinear uncertainties and unmodeled dynamics with multi-operation points. A new method based on signal compensation is proposed to design a robust controller, which consists of a nominal controller and a robust compensator. It is shown that semiglobal robust tracking property can be achieved. A new feature of our results is that the controller is a linear and time-invariant one and "explosion of complexity" problem is avoided.

Keywords: Backstepping, linear controller, nonlinear system, robust, tracking.

1. INTRODUCTION

The problem of output tracking control of nonlinear systems is a widely encountered problem. A lot of approaches have been proposed under various restrictions on the controlled plants such as matching condition and growth condition [1,2], etc. One of the breakthroughs in nonlinear control theory is the introduction of backstepping algorithms for feedback linearizable systems [3-6]. The relative-degree constraint, matching condition and growth condition were removed by this algorithm. The technique was comprehensively developed as a design tool by [7-11]. In fact, for certain strict-feedback form systems, backstepping could achieve the goals of stabilization and tracking. In [12-14], parametric strict-feedback form systems were addressed. In [15-20], strict-feedback form systems with external disturbance and unknown nonlinearities were considered. Triangular systems whose linearization models were not stablizable were treated in [21,22].

In this paper, we consider robust output tracking control problem for a class of nonlinear strict-feedback form systems in the presence of nonlinear uncertainties and unmodeled dynamics with multi-operation points. The uncertainties are with both higher-order and lower-order growing states, and the unmodeled dynamics is BIBS stable. A new method based on signal compensation [23] is proposed. The key point of this method is to utilize a compensator to produce a signal to reduce the influence of plant uncertainties on the closed-loop control properties. The method involves two key steps: first, a nominal controller is designed to get desired output tracking for the nominal disturbance-free model; then, a robust compensator is added to restrain the effect of uncertainties and external disturbance. It will be shown that semiglobal stabilization of the closed-loop system can be ensured and semiglobal output tracking to a reference output can be achieved.

The basic idea of signal compensation method was first introduced in our early work [24] and [25] to deal with robust output tracking problem for linear time-invariant (LTI) systems with parameter perturbations. Similar approaches, called disturbance observer(DOB) approaches, were proposed in the literature. The DOB methods are concerned with disturbance estimation and have been widely used in many applications. In [26,27] linear minimum phase systems were investigated. In [28,29] a class of strict-feedback form systems with nonlinear uncertainties satisfying matching condition were treated. Specially, in [28] the nonlinear uncertainties are required to be differentiable. In [30-32] a class of SISO plants in normal form with nonlinear uncertainties satisfying matching condition were considered. In the current paper we consider a more general case where the uncertainties are not required to satisfy matching condition or to be smooth. The designed controller is a linear time-invariant one and "explosion of complexity" problem [33,34,17] can be avoid.

This paper is organized as follows. In Section 2, the plant description is presented. The assumptions are made on the uncertainties and reference output. In Section 3, controller design method is shown. Section 4 gives the statement and the proof of the main results. An example is shown in Section 5. Conclusions are stated in Section 6.

Notations: For any $H \in R^{n\times m}$, define $|H| = \left[|h_{ij}|\right]$. For any $H \in R^{n\times m}$, $x \in R^n$ and $y \in R^m$, one has $|x^T Hy| \le |x^T||H||y|$.

For vector $x = [x_1 \quad x_2 \quad \cdots \quad x_n]^T \in R^n$ and matrix

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$y = [y_{ij}] \in R^{n\times m}$, define

$$x_{[i]} = [x_1 \quad x_2 \quad \cdots \quad x_i]^T \in R^{i\times 1}, \quad 1 \le i \le n$$

$$y_{i[j]} = [y_{i1} \quad y_{i2} \quad \cdots \quad y_{ij}] \in R^{1\times j}, \quad 1 \le i \le n, 1 \le j \le m$$

$$\left|x_{[i]}^l\right| = \left[|x_1|^l \quad |x_2|^l \quad \cdots \quad |x_i|^l\right]^T \in R^{i\times 1}$$
$$1 \le i \le n, l \in R, l \ge 1$$

$$\left\|x_{[i]}^l\right\| = \sqrt{\left|x_{[i]}^l\right|^T \left|x_{[i]}^l\right|}, \quad 1 \le i \le n, \quad l \in R, \quad l \ge 1$$

2. PROBLEM DESCRIPTION

Consider a SISO nonlinear plant with M operation points, which has the following description at the ***m***-th operation point

$$\Sigma_x^m \begin{cases} \dot{x}_1(t) = x_2(t) + \phi_{1m}(x, z, d_m, t) \\ \dot{x}_2(t) = x_3(t) + \phi_{2m}(x, z, d_m, t) \\ \quad \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + \phi_{(n-1)m}(x, z, d_m, t) \\ \dot{x}_n(t) = u(t) + \phi_{nm}(x, z, d_m, t), \end{cases} \tag{1}$$

$$\begin{aligned} \Sigma_z^m \quad \dot{z}(t) &= \phi_{zm}(x, z, d_m, t) \\ y_p(t) &= x_1(t) \\ m &= 1, 2, \cdots, M, \end{aligned} \tag{2}$$

where $x_i(t)(i = 1, 2, \cdots, n)$ and $z(t)$ are the states, $y_p(t)$ is the output, $d_m(t)(m = 1, 2, \cdots, M)$ are bounded external disturbance vectors, $\phi_{im}(x, z, d_m, t)(i = 1, 2, \cdots, n; m = 1, 2, \cdots, M)$ are regarded as nonlinear time-varying uncertainties and $\phi_{zm}(x, z, d_m, t)(m = 1, 2, \cdots, M)$ are vector fields describing dynamic uncertainties, which depend on operation point.

It is required to design a linear controller which produces a control input $u(t)$ to drive the output $y_p(t)$ of the plant to track a reference output, denoted by $y_d(t)$.

The plant (1) and (2) and the reference output are assumed to satisfy the following assumption.

Assumption A:

A1) There are known positive constant vectors $\xi_{j[i],im} = [\xi_{j1,im} \quad \xi_{j2,im} \quad \cdots \quad \xi_{ji,im}]$ and positive valued functions ς_{im} such that

$$\begin{aligned} |\phi_{im}(x, z, d_m, t)| \le& \sum_{j=1}^{k_i} \xi_{j[i],im} \left|x_{[i]}^{l_{ij}}(t)\right| \\ &+ \varsigma_{im}\left(\|d_m(t)\|, \|z(t_0)\|\right), \end{aligned} \tag{3}$$

where t_0 is the initial time. $l_{ij}(i = 1, 2, \cdots, n; j = 1, 2, \cdots, k_i)$ are known constants great than or equal to 1.

A2) The subsystem Σ_z^m given by (2) is bounded-input ($x(t)$ and $d_m(t)$) bounded-state ($z(t)$) (BIBS) stable.

A3) $y_d(t) \in C^1$ and there exist known and positive constants η_1 and η_2 such that

$$|y_d(t)| \le \eta_1, \quad |\dot{y}_d(t)| \le \eta_2, \quad \forall t \ge t_0.$$

Remark 1: By adding this positive valued function ς_{im}, we can cover nonlinear systems with both higher-order and lower-order growing states, as it is not required $\varsigma_{im}(0,0) = 0$. When $l_{ij} = 1 (i = 1, 2, \cdots, n; j = 1, 2, \cdots, k_i)$, the uncertainties are linear norm-bounded.

Example 1: As a special case of Assumption A1, let

$$\begin{aligned} \phi_{2m}(x, z, d_m, t) =& x_1^2 + 3x_1^5 + 2x_1^{\frac{1}{5}} + 4x_2^{3\frac{1}{3}} + x_2^7 \\ &+ 6|x_2|^{\frac{1}{2}} + \sin x_3 + \frac{1}{|x_4| + 1} + d_m, \end{aligned}$$

where x_1^2, $3x_1^5$, $4x_2^{3\frac{1}{3}}$, x_2^7 are the higher-order terms of states, and $2x_1^{\frac{1}{5}}$, $6|x_2|^{\frac{1}{2}}$ the lower-order terms of states. Due to the states x_3 and x_4, the uncertainty $\phi_{2m}(x, z, d_m, t)$ does not satisfy the lower triangular form, but the bound of the uncertainty $\phi_{2m}(x, z, d_m, t)$ still has lower triangular form. The nonlinear uncertainty satisfies the following inequality

$$\begin{aligned} |\phi_{2m}(x, z, d_m, t)| \le& 2|x_1| + \left|x_1^2\right| + 3\left|x_1^5\right| + 6|x_2| + 4\left|x_2^{3\frac{1}{3}}\right| \\ &+ \left|x_2^7\right| + |d_m| + 10, \end{aligned}$$

which has the form as inequality (3).

For simplicity of statement, in the sequel, $\phi_{im}(x, z, d_m, t)$ will be denoted as $\phi_{im}(t)(i = 1, 2, \cdots, n)$.

3. CONTROLLER DESIGN

In this section, a robust output tracking controller is designed by our new method. The main idea behind the method is that at each step, the influence of the uncertainties and external disturbance is regarded as an equivalent disturbance, and a robust compensator is designed to restrain the effect of the equivalent disturbance on the output tracking properties.

The design of the controller begins with the following subsystem

$$\dot{x}_1(t) = x_2(t) + \phi_{1m}(t)$$

with $x_2(t)$ regarded as a virtual control input and $\phi_{1m}(t)$ as a disturbance. For the above subsystem, a virtual controller is constructed as

$$\hat{x}_2(t) = u_1(t) + f_1 w_1(t), \tag{4}$$

where $u_1(t)$ is a nominal control input given by

$$u_1(t) = -\alpha_1 x_1(t) + \alpha_1 y_d(t), \tag{5}$$

$w_1(t)$ is a robust compensating input given by a robust compensator to be designed, and f_1 is a positive constant to be determined. To perform backstep, apply the variable change

$$y_1(t) = y_p(t) - y_d(t) = x_1(t) - y_d(t) \tag{6}$$

$$y_2(t) = x_2(t) - \hat{x}_2(t) = x_2(t) + \alpha_1 y_1(t) - f_1 w_1(t) \tag{7}$$

then one has

$$\dot{y}_1(t) = -\alpha_1 y_1(t) + \hat{\phi}_{1m}(t) + f_1 w_1(t), \tag{8}$$

where $\hat{\phi}_{1m}(t) = \phi_{1m}(t) + y_2(t) - \dot{y}_d(t)$.

The robust compensating input is constructed as

$$w_1(t) = -F_1(s)\hat{\phi}_{1m}(t), \tag{9}$$

where $F_1(s)$ is a robust filter of the form

$$F_1(s) = \frac{1}{s + f_1}$$

and s is a differential operator (or the Laplace operator). If f_1 is positive and sufficiently large, one can expect that $f_1 w_1(t)$ would approximate $-\hat{\phi}_{1m}(t)$ and restrain the effect of $\hat{\phi}_{1m}(t)$ to obtain robust property.

Since $\hat{\phi}_{1m}(t)$ can be expressed in the form

$$\hat{\phi}_{1m}(t) = (s + \alpha_1) y_1(t) - f_1 w_1(t)$$

to get the robust compensating input $w_1(t)$, only $y_1(t)$ is needed as shown in the following form

$$w_1(t) = -\left(1 + \frac{\alpha_1}{s}\right) y_1(t). \tag{10}$$

From (7), (1), (8) and (9), one has

$$\dot{y}_2(t) = x_3(t) + \tilde{\phi}_{2m}(t), \tag{11}$$

where

$$\begin{aligned}\tilde{\phi}_{2m}(t) = {} & \phi_{2m}(t) - \alpha_1^2 y_1(t) \\ & + (f_1 + \alpha_1)\left[\hat{\phi}_{1m}(t) + f_1 w_1(t)\right].\end{aligned} \tag{12}$$

As the second step, consider the subsystem (11) with $\tilde{\phi}_{2m}(t)$ as a disturbance and $x_3(t)$ as a virtual control input and continue the design procedure. At the ***i*** th step, consider the subsystem

$$\dot{y}_i(t) = x_{i+1}(t) + \tilde{\phi}_{im}(t)$$

and regard $x_{i+1}(t)$ as a virtual control input with the form

$$\hat{x}_{i+1}(t) = u_i(t) + f_i w_i(t),$$

where $u_i(t)$ is a nominal control input given by

$$u_i(t) = -\alpha_i y_i(t)$$

with α_i a positive constant. Let

$$\begin{aligned}y_{i+1}(t) &= x_{i+1}(t) - \hat{x}_{i+1}(t) \\ &= x_{i+1}(t) + \alpha_i y_i(t) - f_i w_i(t).\end{aligned}$$

Then

$$\dot{y}_i(t) = -\alpha_i y_i(t) + \hat{\phi}_{im}(t) + f_i w_i(t),$$

where

$$\hat{\phi}_{im}(t) = \tilde{\phi}_{im}(t) + y_{i+1}(t).$$

To restrain the effect of $\hat{\phi}_{im}(t)$ the robust compensating input $w_i(t)$ is constructed as

$$w_i(t) = -F_i(s)\hat{\phi}_{im}(t),$$

$$F_i(s) = \frac{1}{s + f_i},$$

where f_i is a positive constant to be determined. Note that

$$\hat{\phi}_{im}(t) = (s + \alpha_i) y_i(t) - f_i w_i(t)$$

so the robust compensating input $w_i(t)$ can also be given by

$$w_i(t) = -\left(1 + \frac{\alpha_i}{s}\right) y_i(t).$$

Differentiating $y_{i+1}(t)$, one has

$$\dot{y}_{i+1}(t) = x_{i+2}(t) + \tilde{\phi}_{(i+1)m}(t),$$

where

$$\begin{aligned}\tilde{\phi}_{(i+1)m}(t) = {} & \phi_{(i+1)m}(t) - \alpha_i^2 y_i(t) \\ & + (f_i + \alpha_i)\left[\hat{\phi}_{im}(t) + f_i w_i(t)\right].\end{aligned}$$

Finally, one has

$$\dot{y}_n(t) = u(t) + \tilde{\phi}_{nm}(t), \tag{13}$$

where

$$\begin{aligned}y_n(t) &= x_n(t) + \alpha_{n-1} y_{n-1}(t) - f_{n-1} w_{n-1}(t), \\ \tilde{\phi}_{nm}(t) &= \phi_{nm}(t) - \alpha_{n-1}^2 y_{n-1}(t) \\ & \quad + (f_{n-1} + \alpha_{n-1})\left[\hat{\phi}_{(n-1)m}(t) + f_{n-1} w_{n-1}(t)\right].\end{aligned}$$

The control input $u(t)$ is constructed as

$$u(t) = u_n(t) + f_n w_n(t)$$

with the nominal control input $u_n(t)$ given by

$$u_n(t) = -\alpha_n y_n(t) \tag{14}$$

and the robust compensating input $w_n(t)$ by

$$w_n(t) = -F_n(s)\hat{\phi}_{nm}(t),$$

$$F_n(s) = \frac{1}{s + f_n},$$

where $\hat{\phi}_{nm}(t) = \tilde{\phi}_{nm}(t)$, α_n and f_n are positive constants. $w_n(t)$ can be expressed as

$$w_n(t) = -\left(1 + \frac{\alpha_n}{s}\right) y_n(t).$$

From (13) and (14) it follows that

$$\dot{y}_n(t) = -\alpha_n y_n(t) + \hat{\phi}_{nm}(t) + f_n w_n(t).$$

Summarizing the design results, one has

$$\begin{bmatrix} \dot{y}_i(t) \\ \dot{w}_i(t) \end{bmatrix} = \begin{bmatrix} -\alpha_i & f_i \\ 0 & -f_i \end{bmatrix} \begin{bmatrix} y_i(t) \\ w_i(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \hat{\phi}_{im}(t)$$
$$i = 1, 2, \cdots, n,$$

and the whole controller description

$$\begin{aligned} u(t) &= -\alpha_n y_n(t) + f_n w_n(t), \\ y_1(t) &= x_1(t) - y_d(t), \\ y_i(t) &= x_i(t) + \alpha_{i-1} y_{i-1}(t) - f_{i-1} w_{i-1}(t), \\ &\quad i = 2, \cdots, n, \\ w_i(t) &= -\left(1 + \frac{\alpha_i}{s}\right) y_i(t), \quad i = 1, \cdots, n. \end{aligned} \tag{15}$$

One sees that the designed controller is a linear time-invariant one and has the same structure for all operation points. Positive constants $\alpha_i (i = 2, 3, \cdots, n)$ are chosen so that $y_i(t)(i = 2, 3, \cdots, n)$ have desired convergence speed to zero, if $\hat{\phi}_{im}(t) = 0 (i = 2, 3, \cdots, n)$. Positive constants $f_i (i = 1, 2, \cdots, n)$ are needed to be determined to achieve semiglobal robust stability and semiglobal robust output tracking properties.

4. CLOSED-LOOP CONTROL PROPERTIES

Let

$$x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T,$$
$$w(t) = [w_1(t) \quad w_2(t) \quad \cdots \quad w_n(t)]^T.$$

It will be shown that the closed-loop control system designed as in previous section has robust properties stated in the following theorem.

Theorem 1: Under Assumption A, the closed-loop system, composed of the controlled plant (1) and (2) and controller (15), has semiglobal robust control property, that is, for any given constants $\varepsilon > 0$, $r_y \geq 0$ and $r_w \geq 0$, if $\|y(t_0)\| \leq r_y, \|w(t_0)\| \leq r_w$, one can find sufficiently large constants $f_i^* (i = 1, 2, \cdots, n)$ and positive constant $T \geq t_0$, such that if $f_i \geq f_i^* (i = 1, 2, \cdots, n)$ and $f_{i+1} \gg f_i (i = 1, 2, \cdots, n-1)$, then at any operation point, the states $x(t)$, $z(t)$ and $w(t)$ are bounded and, moreover

$$\|y(t)\| \leq \varepsilon, \|w(t)\| \leq \varepsilon, t \geq T.$$

If the initial values $y(t_0)$ and $w(t_0)$ are zero, then

$$\|y(t)\| \leq \varepsilon, \|w(t)\| \leq \varepsilon, t \geq t_0.$$

To prove the main results stated in Theorem 1, the following lemma is need.

Lemma 1: At the ***m***-th operation point, for any given positive constant ε_ϕ, one can find sufficiently large positive constants $f_{im}^* (i = 1, 2, \cdots, n)$, such that if $f_i > f_{im}^* (i = 1, 2, \cdots, n)$ and $f_{i+1} \gg f_i (i = 1, 2, \cdots, n-1)$, then

$$\frac{|\hat{\phi}_{1m}(t)|}{\sqrt{f_1}} \leq \varepsilon_\phi \left[\|y_{[2]}^l(t)\| + 1 \right], \tag{16}$$

$$\frac{|\hat{\phi}_{im}(t)|}{\sqrt{f_i}} \leq \varepsilon_\phi \left[\|y_{[i+1]}^l(t)\| + \|w_{[i-1]}^l(t)\| + 1 \right], \quad i = 2, 3, \cdots, n-1, \tag{17}$$

$$\frac{|\hat{\phi}_{nm}(t)|}{\sqrt{f_n}} \leq \varepsilon_\phi \left[\|y_{[n]}^l(t)\| + \|w_{[n-1]}^l(t)\| + 1 \right], \tag{18}$$

where $l = max_{1 \leq j \leq i, 1 \leq i \leq n} l_{ij}$

Proof: See Appendix A.

To prove Theorem 1, consider the ***m***-th operation point where $m = 1, 2, \cdots, M$ and consider the following positive function

$$V = \sum_{i=1}^n V_i,$$

where

$$V_i = [y_i(t) \quad w_i(t)] P \begin{bmatrix} y_i(t) \\ w_i(t) \end{bmatrix}, \ i = 1, 2, \cdots, \ nP = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

The derivative of V along the trajectories of the closed-loop system is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n \dot{V}_i \\ &= -2 \sum_{i=1}^n \left[\alpha_i y_i^2(t) + \alpha_i y_i(t) w_i(t) + f_i w_i^2(t) + w_i(t) \hat{\phi}_{im}(t) \right] \\ &= -\sum_{i=1}^n \left[\alpha_i V_i + \alpha_i y_i^2(t) + 2(f_i - \alpha_i) w_i^2(t) + 2 w_i(t) \hat{\phi}_{im}(t) \right] \\ &\leq -\sum_{i=1}^n \left[\alpha_i V_i + \alpha_i y_i^2(t) + (f_i - 2\alpha_i) w_i^2(t) - \frac{\hat{\phi}_{im}^2(t)}{f_i} \right]. \end{aligned}$$

From Lemma 1, it follows that at any operation point, for any given positive constant ε_ϕ, one can find sufficiently large positive constants $f_i^* = \max\{f_{i1}^*, f_{i2}^*, \cdots, f_{iM}^*\} (i = 1, 2, \cdots, n)$, such that if $f_i \geq f_i^* (i = 1, 2, \cdots, n)$ and $f_{i+1} \gg f_i (i = 1, 2, \cdots, n-1)$, then

$$\begin{aligned} \sum_{i=1}^n \frac{\hat{\phi}_{im}^2(t)}{f_i} &\leq 2\varepsilon_\phi^2 \left[\|y_{[2]}^l(t)\|^2 + 1 \right] \\ &\quad + \sum_{i=2}^{n-1} 3\varepsilon_\phi^2 \left[\|y_{[i+1]}^l(t)\|^2 + \|w_{[i-1]}^l(t)\|^2 + 1 \right] \\ &\quad + 3\varepsilon_\phi^2 \left[\|y_{[n]}^l(t)\|^2 + \|w_{[n-1]}^l(t)\|^2 + 1 \right] \\ &\leq 3\varepsilon_\phi^2 n \left[\|y^l(t)\|^2 + \|w^l(t)\|^2 + 1 \right], \end{aligned} \tag{19}$$

$m = 1, 2, \cdots, M.$

Let

$$\alpha_0 = \min\{\alpha_i, i=1,2,\cdots,n\},$$
$$\alpha_M = \max\{\alpha_i, i=1,2,\cdots,n\}.$$

One has

$$\dot{V} \le -\alpha_0 V - \alpha_0 \|y(t)\|^2 - (f_1 - 2\alpha_M)\|w(t)\|^2 + 3\varepsilon_\phi^2 n\left[\left\|y^l(t)\right\|^2 + \left\|w^l(t)\right\|^2 + 1\right].$$

Consider a set $\Omega(r_a, r_b)$ in R^{2n} defined as

$$\Omega(r_a, r_b) = \left\{\begin{pmatrix} y \\ w \end{pmatrix} \mid r_b \le V \le r_a, y \in R^n, w \in R^n\right\}.$$

For the case where $V \le r_a$, one has

$$\|y(t)\|^2 \le \frac{r_a}{\lambda_{p1}}, \quad \|w(t)\|^2 \le \frac{r_a}{\lambda_{p1}},$$

where $\lambda_{p1} = \lambda_{\min}(P)$, and

$$\frac{\left\|y^l(t)\right\|^2}{\|y(t)\|^2} = \|y(t)\|^{2(l-1)} \le \beta,$$
$$\frac{\left\|w^l(t)\right\|^2}{\|w(t)\|^2} = \|w(t)\|^{2(l-1)} \le \beta,$$

where $\beta = \left(\frac{r_a}{\lambda_{p1}}\right)^{l-1}$. If choose f_1 and ε_ϕ to satisfy

$$f_1 \ge 2\alpha_M + \frac{\alpha_0}{2} \tag{20}$$

and

$$\varepsilon_\phi \le \min\left\{\sqrt{\frac{\alpha_0 r_b}{6n\lambda_{p2}}}, \sqrt{\frac{\alpha_0}{6n\beta}}, \sqrt{\frac{f_1 - 2\alpha_M - \frac{\alpha_0}{2}}{3n\beta}}\right\} \tag{21}$$

respectively, where $\lambda_{p2} = \lambda_{\max}(P)$, then for any $\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} \in \Omega(r_a, r_b)$, one has

$$\begin{aligned}
&\alpha_0\|y(t)\|^2 + (f_1 - 2\alpha_M)\|w(t)\|^2 - 3\varepsilon_\phi^2 n\left[\left\|y^l(t)\right\|^2 + \left\|w^l(t)\right\|^2 + 1\right] \\
&= \frac{\alpha_0}{2}\|y(t)\|^2 + \frac{\alpha_0}{2}\|w(t)\|^2 - 3\varepsilon_\phi^2 n + \frac{\alpha_0}{2}\|y(t)\|^2\left[1 - \frac{6\varepsilon_\phi^2 n}{\alpha_0}\frac{\left\|y^l(t)\right\|^2}{\|y(t)\|^2}\right] \\
&\quad + \left(f_1 - 2\alpha_M - \frac{\alpha_0}{2}\right)\|w(t)\|^2\left[1 - \frac{3\varepsilon_\phi^2 n}{f_1 - 2\alpha_M - \frac{\alpha_0}{2}}\frac{\left\|w^l(t)\right\|^2}{\|w(t)\|^2}\right] \\
&\ge \frac{\alpha_0}{2\lambda_{p2}}V(t) - 3\varepsilon_\phi^2 n + \frac{\alpha_0}{2}\|y(t)\|^2\left(1 - \frac{6\varepsilon_\phi^2 n\beta}{\alpha_0}\right) \\
&\quad + \left(f_1 - 2\alpha_M - \frac{\alpha_0}{2}\right)\|w(t)\|^2\left(1 - \frac{3\varepsilon_\phi^2 n\beta}{f_1 - 2\alpha_M - \frac{\alpha_0}{2}}\right) \\
&\ge 0.
\end{aligned}$$

Therefore, for any given constants $\varepsilon > 0$, $r_y \ge 0$ and $r_w \ge 0$, if choose

$$\begin{aligned} r_b &= \lambda_{p1}\varepsilon^2 \\ r_a &\ge \max\{r_b, \lambda_{p2}(r_y^2 + r_w^2)\} \end{aligned} \tag{22}$$

then $V(t_0) \le r_a$, and one can find sufficiently large positive constant f_1 satisfying inequality (20) and sufficiently small positive constant ε_ϕ satisfying inequality (21) such that

$$\dot{V}(t) \le -\alpha_0 V(t), \quad \forall \begin{pmatrix} y(t) \\ w(t) \end{pmatrix} \in \Omega(r_a, r_b), \tag{23}$$

which implies that $y(t)$ and $w(t)$ are bounded, converge exponentially to the following domain and stay in it

$$\left\{\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} \mid \|y(t)\| \le \varepsilon, \|w(t)\| \le \varepsilon\right\}.$$

From above analysis it follows that at any operation point, for any given constants $\varepsilon > 0$, $r_y \ge 0$ and $r_w \ge 0$, if $\|y(t_0)\| \le r_y, \|w(t_0)\| \le r_w$, one can find sufficiently large constants $f_i^* = \max\{f_{i1}^*, f_{i2}^*, \cdots, f_{iM}^*\} (i = 1,2,\cdots,n)$, such that if $f_i \ge f_i^* (i = 1,2,\cdots,n)$ and $f_{i+1} \gg f_i (i = 1, 2,\cdots,n-1)$, then $y(t)$ and $w(t)$, hence $x(t)$ and $z(t)$, are bounded, and $\|y(t)\| \le \varepsilon, \|w(t)\| \le \varepsilon, t \ge T$, where $T = t_0 + \frac{1}{\alpha_0}$ in $\frac{r_a}{r_b}$. If the initial values $y(t_0)$ and $w(t_0)$ are zero, then $\|y(t)\| \le \varepsilon, \|w(t)\| \le \varepsilon, t \ge t_0$.

Remark 2: For the case where the uncertainties are linear norm bounded, one can choose $r_a = \infty$, so global robust output tracking can be achieved.

Remark 3: The values of $f_i (i = 1,2,\cdots,n)$ need to be determined to satisfy (16), (17), (18), (20) and (21) for r_a and r_b chosen as in (22).

5. SIMULATION RESULTS

Consider a series DC motor system which suffers from the considerable nonlinearities including the square of current, the product of current and speed, changeable load torque and parameters uncertainties. The motor can be modeled as [35]

$$J\dot{\omega}(t) = Mi^2(t) - D\omega(t) - T_{Lm}(t) + q_{1m}(\omega(t), d_m(t), t),$$
$$L\dot{i}(t) = -Ri(t) - Mi(t)\omega(t) + u(t) + q_{2m}(i(t), d_m(t), t),$$

where $u(t)$ is the input voltage, $i(t)$ the armature current, $\omega(t)$ the rotational speed of the motor, $T_{Lm}(t)$ the load torque, J the moment of inertia associated with both motor and the load, M the motor constant, D the viscous friction coefficient, L the total armature and field current inductance, R the total armature and field circuit resistance. $q_{1m}(\omega(t), d_m(t), t)$ and $q_{2m}(\omega(t), i(t), d_m(t), t)$ are nonlinear uncertainties, which include the modeling error. $d_m(t)$ is bounded external disturbance, which depends on operation condition. Assume that the system has two operation points, i.e., $m=1,2$. For a 10kw, 1500rpm series DC connected motor [36]

$$J = 0.5\mathrm{Kg\cdot m^2}, \quad M = 0.027\mathrm{N\cdot m/Wb\cdot A},$$
$$D = 0.0004\mathrm{N\cdot m/rad/s}, \quad L = 0.05\mathrm{H}, \quad R = 1\Omega,$$
$$q_{1m}(t) = \Delta D_m\omega(t) + d_{1m}(t),$$
$$q_{2m}(t) = \Delta R_m i(t) + \Delta M_m i(t)\omega(t) + d_{2m}(t)$$

under operation point one, we have

$$\Delta D_1 = 10\%D, \quad \Delta R_1 = 2\%R, \quad \Delta M_1 = 10\%M,$$
$$T_{L1} = 55 + 5\sin(2\pi t),$$
$$d_{11}(t) = 5\sin\left(\frac{\pi}{10}t\right), \quad d_{21}(t) = 5\sin\left(\frac{\pi}{2}t\right)$$

under operation point two, we have

$$\Delta D_2 = -10\%D, \quad \Delta R_2 = -2\%R, \quad \Delta M_2 = -10\%M,$$
$$T_{L2} = 55 + 5\sin\left(\frac{\pi}{5}t\right),$$
$$d_{12}(t) = 5\sin\left(\frac{2\pi}{5}t\right), \quad d_{22}(t) = 5\sin\left(\frac{\pi}{10}t\right).$$

The initial values are given by $\omega(0) = i(0) = 0.1$. The system can be transformed into a strict-feedback form system by means of the following choice of states

$$x(t) = [x_1(t) \quad x_2(t)]^{\mathrm{T}} = \begin{bmatrix} \omega(t) \\ \frac{M}{J}i^2(t) \end{bmatrix}.$$

In this case, the system appears as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) + \phi_m(t),$$
$$y_p(t) = x_1(t) = \omega(t),$$

where $v(t) = \frac{2Mi(t)}{LJ}[u(t) - Ri(t) - Mi(t)\omega(t)]$, and

$$\phi_m(t) = \begin{bmatrix} -D\omega(t) - T_L(t) + q_{1m}(t) \\ \frac{2M}{LJ}i(t)q_{2m}(t) \end{bmatrix}.$$

The reference output is given by

$$\dot{y}_d(t) = -25y_d(t) + 25r(t), \quad r(t) = 150, \quad t \geq 0.$$

It is required that $\varepsilon = 0.005$. The controller is described by (15). Choose $\alpha_i = 25 (i=1,2)$, and choose $f_1=100$, $f_2=1000$. Simulation results are shown in Figs 1 through 4. We can see that, with distinct motor parameters, load torque and external disturbances, the tracking error can be driven into the desired small neighborhood of the

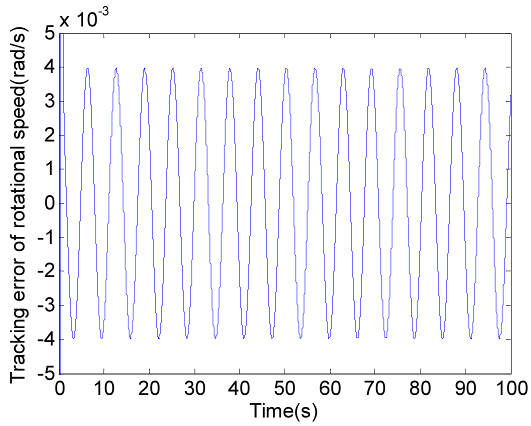


Fig. 1. Plot of tracking error of motor rotational speed at operation point one.

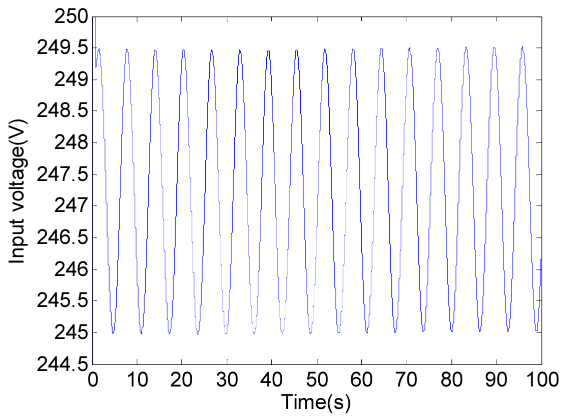


Fig. 2. Plot of robust control input at operation point one.

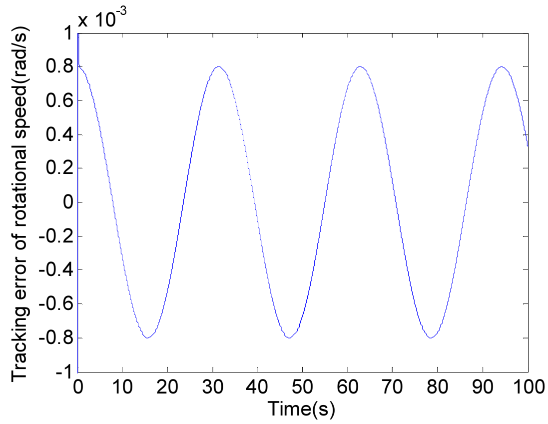


Fig. 3. Plot of tracking error of motor rotational speed at operation point two.

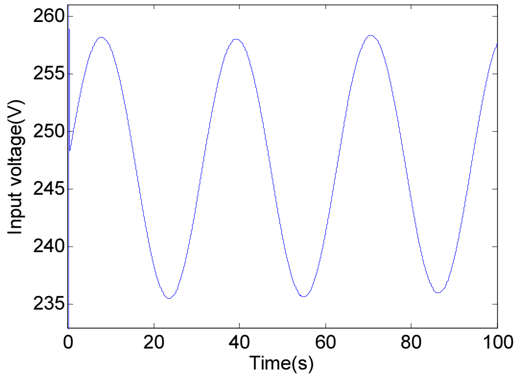
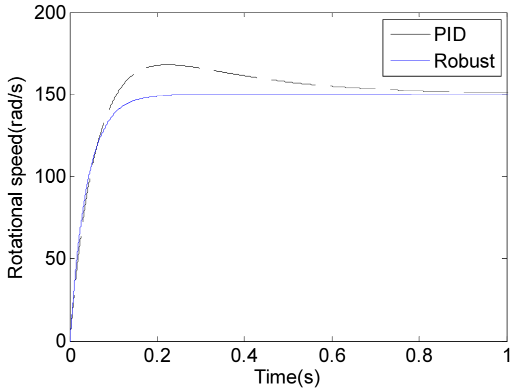
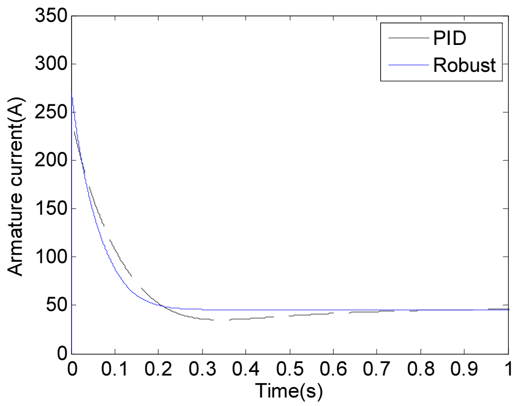


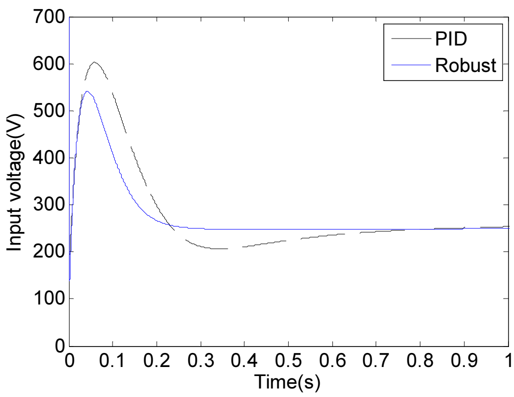
Fig. 4. Plot of robust control input at operation point two.



(a) Rotational speeds.



(b) Armature currents.



(c) Input voltages.

Fig. 5. Plots of transient response for step reference input at operation point one.

origin and the robust control input is bounded. In order to illustrate the effectiveness of the proposed method, classical PID controller [35] is also designed and applied to the motor system. The motor control system is simulated with parameters in condition two.

The transient responses of the rotational speed and the armature current are shown in Fig. 5. It is evident that the transient behavior (settling time and maximum overshoot of speed) of the robust control system is better than that of the conventional PID control system.

6. CONCLUSIONS

For a class of nonlinear systems in strict-feedback form with time-varying nonlinear uncertainties and unmodeled dynamics with multi-operation points, a new method has been proposed to design a robust controller. By this method, a nominal controller is first designed to get exact output tracking property for the nominal plant, and a robust compensator is then added to achieve semiglobal tracking property for the real controlled plant. The robust controller is a linear and time-invariant one, so it can be realized easily.

APPENDIX A

Let

$$l_1 = \max_{1\le j\le k_1} l_{1j},$$

$$\mu_{y1,1m} = \sum_{j=1}^{k_1} \xi_{j1,1m} 3^{l_{1j}-1},$$

$$\mu_{\tau 1,1m} = \sum_{j=1}^{k_1} \xi_{j1,1m} 3^{l_{1j}-1},$$

$$\tilde{\varsigma}_{1m}(t) = \sum_{j=1}^{k_1} \xi_{j1,1m} 3^{l_{1j}} + \varsigma_{1m}\left(\|d_m(t)\|, \|z(t_0)\|\right),$$

$$l_i = \max_{1\le j\le k_i} l_{ij},$$

$$\mu_{y[i],im} = \sum_{j=1}^{k_i} 3^{l_{ij}-1} \begin{bmatrix} \xi_{j1,im} + \alpha_1^{l_{ij}} \xi_{j2,im} \\ \xi_{j2,im} + \alpha_2^{l_{ij}} \xi_{j3,im} \\ \vdots \\ \xi_{j(i-1),im} + \alpha_{i-1}^{l_{ij}} \xi_{ji,im} \\ \xi_{ji,im} \end{bmatrix}^{\mathrm{T}},$$

$$\mu_{w[i-1],im}(f) = \sum_{j=1}^{k_i} 3^{l_{ij}-1} \begin{bmatrix} f_1^{l_{ij}} \xi_{j2,im} \\ f_2^{l_{ij}} \xi_{j3,im} \\ \vdots \\ f_{i-1}^{l_{ij}} \xi_{ji,im} \end{bmatrix}^{\mathrm{T}},$$

$$\mu_{\tau 1,im} = \sum_{j=1}^{k_i} 3^{l_{ij}-1} \xi_{j1,im},$$

$$\tilde{\varsigma}_{im}(t) = \sum_{j=1}^{k_i} 3^{l_{ij}} \xi_{j1,im} + \sum_{j=1}^{k_i} 3^{l_{ij}-1} \sum_{k=2}^{i} \xi_{jk,im}\left(1 + \alpha_{k-1}^{l_{ij}} + f_{k-1}^{l_{ij}}\right).$$

$$+\varsigma_{im}(\|d_m(t)\|,\|z(t_0)\|),$$
$$i=2,3,\cdots,n;\quad m=1,2,\cdots,M.$$

Lemma A: $\phi_{im}(i=1,2,\cdots,n)$ satisfy that

$$|\phi_{1m}(t)|\le\mu_{y1,1m}\left|y_1^{l_1}(t)\right|+\mu_{\tau1,1m}\left|y_d^{l_1}(t)\right|+\tilde{\varsigma}_{1m}(t),$$
$$|\phi_{im}(t)|\le\mu_{y[i],im}\left|y_{[i]}^{l_i}(t)\right|+\mu_{w[i-1],im}(f)\left|w_{[i-1]}^{l_i}(t)\right|$$
$$+\mu_{\tau1,im}\left|y_d^{l_i}(t)\right|+\tilde{\varsigma}_{im}(t),$$
$$i=2,3,\cdots,n.$$

Proof: From the definition of $y_i(i=1,2,\cdots,n)$ it follows that

$$\begin{bmatrix}x_1(t)\\x_2(t)\\\vdots\\x_n(t)\end{bmatrix}=\begin{bmatrix}y_1(t)\\y_2(t)\\\vdots\\y_n(t)\end{bmatrix}-\begin{bmatrix}0\\\alpha_1y_1(t)\\\vdots\\\alpha_{n-1}y_{n-1}(t)\end{bmatrix}+\begin{bmatrix}0\\f_1w_1(t)\\\vdots\\f_{n-1}w_{n-1}(t)\end{bmatrix}+\begin{bmatrix}y_d(t)\\0\\\vdots\\0\end{bmatrix}.$$

So one has

$$\left|x_1^{l_{ij}}(t)\right|\le2^{l_{ij}-1}\left|y_1^{l_i}(t)\right|+2^{l_i-1}\left|y_d^{l_i}(t)\right|$$
$$<3^{l_{ij}-1}\left|y_1^{l_i}(t)\right|+3^{l_{ij}-1}\left|y_d^{l_i}(t)\right|+3^{l_{ij}},$$
$$\left|x_k^{l_{ij}}(t)\right|\le3^{l_{ij}-1}\left(\left|y_k^{l_{ij}}(t)\right|+\alpha_{k-1}^{l_{ij}}\left|y_{k-1}^{l_{ij}}(t)\right|+f_{k-1}^{l_{ij}}\left|w_{k-1}^{l_{ij}}(t)\right|\right)$$
$$\le3^{l_{ij}-1}\left(\left|y_k^{l_i}(t)\right|+\alpha_{k-1}^{l_{ij}}\left|y_{k-1}^{l_i}(t)\right|n+f_{k-1}^{l_{ij}}\left|w_{k-1}^{l_i}(t)\right|\right)$$
$$+3^{l_{ij}-1}\left(1+\alpha_{k-1}^{l_{ij}}+f_{k-1}^{l_{ij}}\right),$$
$$k=2,3,\cdots,i,\quad i=1,2,3,\cdots,n,$$

where we have applied the inequality $a^k\le a^K+1,\forall K\ge k\ge1,a\ge0$. From Assumption A and the inequalities above, one sees that the conclusions of Lemma A hold.

Lemma B: $\hat{\phi}_{im}(t)(i=1,2,\cdots,n)$ satisfy the following inequalities.

$$\left|\hat{\phi}_{1m}(t)\right|\le|y_2(t)|+\hat{\mu}_{y_{1,1m}}\left|y_1^l(t)\right|+\hat{\varsigma}_{1m}(t),$$
$$\left|\hat{\phi}_{im}(t)\right|\le|y_{i+1}(t)|+\hat{\mu}_{y[i],im}(f)\left|y_{[i]}^l(t)\right|$$
$$+\hat{\mu}_{w[i-1],im}(f)\left|w_{[i-1]}^l(t)\right|+\hat{\varsigma}_{im}(f,t),$$
$$i=2,3,\cdots,n-1,$$
$$\left|\hat{\phi}_{nm}(t)\right|\le\hat{\mu}_{y[n],nm}(f)\left|y_{[n]}^l(t)\right|$$
$$+\hat{\mu}_{w[n-1],nm}(f)\left|w_{[n-1]}^l(t)\right|+\hat{\varsigma}_{nm}(f,t),$$

where

$$l=\max_{1\le i\le n}l_i,$$
$$\hat{\mu}_{y_{1,1m}}=\mu_{y_{1,1m}},$$
$$\hat{\mu}_{y[i],im}(f)=(f_{i-1}+\alpha_{i-1})[\hat{\mu}_{y[i-1],(i-1)m}(f)\quad 1]$$
$$+\mu_{y[i],im}+[0_{1\times(i-2)}\quad\alpha_{i-1}^2\quad 0],$$
$$i=2,\cdots,n,$$
$$\hat{\mu}_{w1,2m}(f)=\mu_{w1,2m}+f_1(f_1+\alpha_1),$$
$$\hat{\mu}_{w[i-1],im}(f)=(f_{i-1}+\alpha_{i-1})\left[\hat{\mu}_{w[i-2],(i-1)m}(f)\quad f_{i-1}\right]$$
$$+\mu_{w[i-1],im}(f),$$
$$i=3,4,\cdots,n,$$
$$\hat{\varsigma}_{1m}(t)=\tilde{\varsigma}_{1m}(t)+\mu_{\tau1,1m}\left|y_d^{l_1}(t)\right|+|\dot{y}_d(t)|+\hat{\mu}_{y_{1,1m}},$$
$$\hat{\varsigma}_{im}(f,t)=(f_{i-1}+\alpha_{i-1})\hat{\varsigma}_{(i-1)m}(f,t)+\tilde{\varsigma}_{im}(t)$$
$$+\mu_{\tau1,im}\left|y_d^{l_i}(t)\right|+\sum_{k=1}^{i}\hat{\mu}_{yk,im}(f)+\sum_{k=1}^{i-1}\hat{\mu}_{wk,im}(f),$$
$$i=2,3,\cdots,n.$$

Proof: From the definition of $\hat{\phi}_{im}(t)(i=1,2,\cdots,n)$, one has that

$$\hat{\phi}_{1m}(t)=y_2(t)+\phi_{1m}(t)-\dot{y}_d,$$
$$\hat{\phi}_{im}(t)=y_{i+1}(t)-\alpha_{i-1}^2y_{i-1}(t)+\phi_{im}(t)$$
$$+(f_{i-1}+\alpha_{i-1})\left[f_{i-1}w_{i-1}(t)+\hat{\phi}_{(i-1)m}(t)\right]$$
$$i=2,3,\cdots,n-1,$$
$$\hat{\phi}_{nm}(t)=-\alpha_{n-1}^2y_{n-1}(t)+\phi_{nm}(t)$$
$$+(f_{n-1}+\alpha_{n-1})\left[f_{n-1}w_{n-1}(t)+\hat{\phi}_{(n-1)m}(t)\right].$$

Together with Assumption A and Lemma A, the conclusions of Lemma B hold.

Proof of Lemma 1: From the definition of $\hat{\mu}_{yj,im}(f)$, $\hat{\mu}_{wj,im}(f)$ and $\hat{\varsigma}_{im}(f,t)$, one sees that $\hat{\mu}_{yj,im}(f)$ $(j=1,2,\cdots,i)$, $\hat{\mu}_{wj,im}(f)(j=1,2,\cdots,i-1)$ and $\hat{\varsigma}_{im}(f,t)$ $(i=1,2,\cdots,n)$ involve only $f_j(j=1,2,\cdots,i-1)$ Therefore, for any given positive constant ε_i, one can find sufficiently large positive constants f_{im}^* $(i=1,2,\cdots,n)$, such that if $f_i>f_{im}^*(i=1,2,\cdots,n)$ and $f_{i+1}\gg f_i,(i=1,2,\cdots,n-1)$, then

$$\frac{1}{\sqrt{f_i}}\le\varepsilon_i,$$
$$\frac{\left|\hat{\mu}_{yj,im}(f)\right|}{\sqrt{f_i}}\le\varepsilon_i,\ j=1,2,\cdots,i,$$
$$\frac{\left|\hat{\varsigma}_{im}(f,t)\right|}{\sqrt{f_i}}\le\varepsilon_i,\qquad\text{(A1)}$$
$$i=1,2,\cdots,n,$$
$$\frac{\left|\hat{\mu}_{wj,im}(f)\right|}{\sqrt{f_i}}\le\varepsilon_i,\ j=1,2,\cdots,i-1,$$
$$i=2,\cdots,n.$$

We only show the proof of (17). The proof of (16) and (18) can be performed in a similar way. For $i=2,3,\cdots,n-1$, from Lemma B and (A1), one has

$$\frac{\left|\hat{\phi}_{im}(t)\right|}{\sqrt{f_i}} \le \frac{1}{\sqrt{f_i}} | y_{i+1}(t) | + \frac{\hat{\mu}_{y[i],im}(f)}{\sqrt{f_i}} \left| y_{[i]}^l(t) \right|$$
$$+ \frac{\hat{\mu}_{w[i-1],im}(f)}{\sqrt{f_i}} \left| w_{[i-1]}^l(t) \right| + \frac{1}{\sqrt{f_i}} \hat{\varsigma}_{im}(f,t)$$
$$\le \varepsilon_i \left[\sum_{k=1}^{i+1} |y_k^l(t)| + \sum_{k=1}^{i-1} |w_k^l(t)| + 1 \right]$$
$$\le \varepsilon_i \left[\sqrt{i+1} \left\| y_{[i+1]}^l(t) \right\| + \sqrt{i-1} \left\| w_{[i-1]}^l(t) \right\| + 1 \right].$$

If choose ε_i such that $\varepsilon_i \sqrt{i+1} \le \varepsilon_\phi$, then (17) holds.

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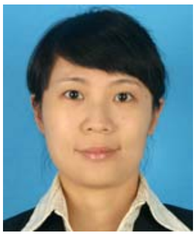

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