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Second-Order Consensus of Multi-Agent Systems with Unknown but Bounded Disturbance

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Abstract: This paper addresses a consensus problem for second-order agents with unknown but bounded (UBB for short) disturbance which may affect the measure of neighbors' velocities. In this study, the communication topology of the multi-agent system is supposed to be connected. In order to solve this consensus problem, a new velocity estimation called distributed lazy rule is firstly proposed, where each agent can estimate its neighbors' velocities one by one. Then, a group of sufficient conditions for this second-order consensus problem are presented by adopting graph theory and the well-known Barbalat lemma, and the bounded consensus protocol is taken into account due to actuator saturation. Theoretically, the group of agents can reach consensus under the proposed control protocol, which is also validated by some numerical experiments.

Keywords: Barbalat lemma, bounded consensus protocol, distributed lazy rule, multi-agent systems, unknown but bounded disturbance.

1. INTRODUCTION

The distributed control of multi-agent systems gives much insight into the collective behaviors of multiple agents in natural systems, e.g. flocks of birds, schools of fish, herds of animals, colonies of bacteria, etc. Meanwhile, it has also been widely used in many areas such as cooperative control of unmanned air vehicles, formation control, wireless sensor networks, mobile robotic swarms and military applications [1-7]. Generally, the distributed control for a group of agents always aims to seek their consensus state, i.e., reach an agreement on a certain quantity of interest through information exchange among them. The consistent quantity or so-called consensus state, which may depend on the initial state of all agents, can be physical quantities such as altitude, position, temperature, voltage, and so on.

In the literature, Vicsek *et al*. [8] presented a discrete-time model of finite autonomous agents that move in the plane with the same speed but different headings in their pioneering work, where the concept of neighbors of agents was introduced for the first time, and some simulations were provided to demonstrate the nearest neighbor rule. Following this research, Jadbabaie *et al.* [9] provided a theoretical explanation for the behavior observed in [8] by using graph and matrix theories. Besides, Olfati-Saber and Murray [10] addressed a systematical framework to study the consensus problems in networks of dynamic agents with fixed or switching topologies and communication time delays. He and Cao [11] investigated the lth order consensus problem for multi-agent systems and established a linear consensus protocol to solve it. Song *et al.* [12] studied the second-order leader-following consensus problem of nonlinear multi-agent systems with general network topologies, and several effective sufficient conditions were obtained based on graph theory, matrix theory, and LaSalle's invariance principle. In practical applications, agents may update their states with different paces, which entails the consideration of the asynchronous consensus problem. Xiao and Wang [13] studied such asynchronous consensus problem in a continuous-time multi-agent system with discontinuous information transmission by using nonnegative matrix theory and graph theory. More recently, Wang and Cao [14] analyzed the second-order quasi-consensus of leader-following multi-agent systems in four cases, and a unified result was given.

In a multi-agent system, the convergence rate used to measure how quickly the consensus state can be reached has been studied via various methods [15-17]. In all these methods, the communication topology plays a key role in the convergence of consensus processes. Cao *et al.* presented new graph-theoretic results appropriate for the analysis of a variety of consensus problems cast in dynamically changing environments in [15,16], where the concepts of rooted graph, strongly rooted graph, and neighbor-shared graph were introduced and the worst convergence rates of these graphs were derived. In [17],

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Nedic *et al*. considered a constrained consensus problem and addressed a distributed projected consensus algorithm with its convergence rate well established. Moreover, since it may be required that the agreement or consensus is reached in finite time in some practical situations, finite-time consensus becomes a hot topic these years [18-22]. The two recent surveys [23,24] are recommended for a relatively complete coverage of the literature on consensus.

Despite the extensive studies on consensus, only limited approaches considered the influence of disturbances on measuring position, velocity, and other physical quantities. However, in reality, disturbances are ubiquitously presented in multi-agent systems due to various uncertainties such as model mismatches, channel noises, measurement errors, etc. In fact, the consensus for a directed network of agents with disturbance is quite difficult to be achieved explicitly, because it breaks the condition that the row sum of Laplacian matrix must be 0. In [25], Lin *et al*. investigated consensus problems for directed networks of agents with external disturbances and model uncertainty on fixed and switching topologies. Meanwhile, in [26], Xiong and Daniel proposed an impulsive control scheme to solve the consensus problems for directed networks of agents with nonlinear perturbations where a sufficient condition was proposed to guarantee the consensus of all agents. On the other hand, in most literature regarding multi-agent systems, it has been implicitly assumed that the neighbors' states are fully accessible, which however is not always the case in reality. In fact, some information is unmeasurable due to technology limitations or environment disturbance. Therefore, the state estimation for the multi-agent system has become vitally important. In [27], Bauso *et al*. considered stationary consensus protocol for first-order agents in the presence of UBB disturbance, and proposed a rule of state estimation called lazy rule. The main contribution of their work was the introduction and solution of the ε-consensus problem, where the states converge in a target set of radius ε asymptotically or in finite time.

With this background, since many individual systems, especially mechanical systems, are of second-order dynamics [12,28,29], we investigate the consensus problem for second-order agents with UBB disturbance on measuring neighbors' velocities. This consensus problem is resolved by utilizing a distributed lazy rule. Note that here only the bound of the disturbance is needed to establish the theoretical framework and there is no additional requirement of its other statistical properties.

This paper is organized as follows. In Section 2, some basic definitions in graph theory and preliminary results are presented. In Section 3, a new framework to study the consensus problem of second-order agent systems under UBB disturbance is established and the distributed lazy rule is formulated. In Section 4, the consensus problem for second-order agent dynamics with UBB disturbance is theoretically solved. Then, some illustrative examples are provided in Section 5. Finally, the paper is concluded in Section 6.

2. PRELIMINARIES

In this section, some basic definitions in graph theory and preliminary results are briefly introduced for subsequent use.

In a multi-agent system, each agent can only communicate with several other neighboring agents. Its communication topology is always represented by a weighted undirected graph $G=(V,\zeta,A)$ of order n with a set of nodes $V=\{\pi_1,\pi_2,\cdots\pi_n\}$, a set of edges $\zeta\subseteq V\times V$, and a nonnegative symmetric matrix $A=[a_{ij}]$. Here, the node indexes belong to a finite index set $I=\{1,2,\cdots n\}$ and an edge of G is denoted by an unordered pair of vertices, i.e., $e_{ij}=(\pi_i,\pi_j)$. Then, agent i and j can communicate with each other only when $(\pi_i,\pi_j)\in\zeta$ which is equivalent to the condition $a_{ij}>0$. At this time, we say agents i and j are adjacent. Moreover, it is always assumed that the graph has no self-loop, i.e., $a_{ii}=0$ for all $i\in I$. Besides, A is a weight matrix and a_{ij} is the weight of $e_{ij}=(\pi_i,\pi_j)$. The set of neighbors of node π_i is denoted by $N_i=\{\pi_j\,|\,(\pi_j,\pi_i)\in\zeta\}$, and the Laplacian matrix is given as $L=[l_{ij}]\in R^{n\times n}$ with $l_{ii}=\sum_{j\in N_i}a_{ij}$ and $l_{ij}=-a_{ij}$, $i\neq j$. A path in a graph from π_i to π_j is a sequence of different vertices starting with π_i and ending with π_j such that consecutive vertices are adjacent. An undirected graph is connected if there is a path between any pair of nodes.

In the sequel, the dynamics of each agent is described by a second-order differential system:

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=u_i(t),\end{cases}\quad\forall i\in I \tag{1}$$

with initial conditions $x_i(0)=x_{i0}$, $v_i(0)=v_{i0}$, where $x_i(t)=[x_i^{(1)},\cdots x_i^{(m)}]^T\in R^m$, $v_i(t)=[v_i^{(1)},\cdots v_i^{(m)}]^T\in R^m$, and $u_i(t)\in R^m$ denote the position, the velocity, and the control input or the protocol in consensus problem, respectively. For the second-order system (1), the consensus protocol [28,29] is:

$$\begin{aligned}u_i(t)=\alpha\sum_{j\in N_i}a_{ij}(x_j(t)-x_i(t))\\ +\beta\sum_{j\in N_i}a_{ij}(v_j(t)-v_i(t))-bv_i(t),\end{aligned} \tag{2}$$

where $\alpha>0$ and $\beta>0$ are the coupling strengths, $b>0$ denotes the velocity damping gain and $-bv_i(t)$ denotes the velocity damping term which is assumed to be in proportion to the magnitude of velocity.

Definition 1: For the multi-agent system (1)-(2), second-order consensus is considered to be achieved if, for any initial conditions, we have

$$\begin{cases}\lim_{t\to\infty}\left\|x_i(t)-x_j(t)\right\|=0\\ \lim_{t\to\infty}\left\|v_i(t)-v_j(t)\right\|=0,\end{cases}\quad\forall i,j\in I. \tag{3}$$

It is worthy to note that there are two different types of consensus in second-order agent dynamics, i.e., static consensus and dynamical consensus. For static consensus, it is required that $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\ \lim_{t\to\infty} v_i(t)=0,$ $\forall i,j\in I,$ i.e., each agent will eventually static, while for dynamical consensus, it is required that $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\ \lim_{t\to\infty}\|v_i(t)-v_j(t)\|=0$ and $\lim_{t\to\infty} v_i(t)\neq 0,\ \forall i,j\in I,$ i.e., the agents still move in some space, while only the distance between each pair of agents and their relative velocities from each other tend to zero.

Next, the Barbalat lemma [30] is introduced due to its usefulness in the stability analysis of time-varying dynamic systems.

Lemma 1: Let $\phi: R\to R$ be a uniformly continuous function on $[0,\infty)$, suppose that $\lim_{t\to\infty}\int_0^t \phi(\tau)d\tau$ exists and is finite, then $\phi(t)\to 0$ as $t\to\infty$.

Note that $\phi(t)$ is a uniformly continuous function under the condition that the derivative of $\phi(t)$ is bounded. As an alternative, Lemma 1 has the following modified version.

Lemma 2: If the derivative of $\phi(t)$ is bounded on $[0,\infty)$ and $\lim_{t\to\infty}\int_0^t \phi(\tau)d\tau$ exists and is finite, then $\phi(t)\to 0$ as $t\to\infty$.

3. SYSTEM MODEL

In this section, a new framework to study the consensus problem of second-order agent systems (1)-(2) under UBB disturbance is established and the distributed lazy rule is formulated.

In reality, some variables of the agents in a multi-agent system may not be measured precisely due to various kinds of disturbances, such as time delay, model uncertainty, external disturbances and asynchronism of clocks, which might cause the multi-agent system to diverge or oscillate. Since it is believed that UBB disturbance widely exists in many different fields, such as mobile robotics, vision, multi-inventory, data fusion, and unmanned air vehicles, in this paper, it will be considered and integrated into second-order agent dynamics.

When agent j is a neighbor of agent i, $\tilde{v}_{ij}$ is a disturbed measure of v_j obtained by agent i, where $\tilde{v}_{ij}=[\tilde{v}_{ij}^{(1)},\cdots\tilde{v}_{ij}^{(m)}]$ reflects the influence of d_{ij} on estimating v'_{ij}, as shown in Fig. 1. Apparently, we have $v'_{ij}=v_j+d_{ij}$, where $d_{ij}=[d_{ij}^{(1)},\cdots,d_{ij}^{(m)}]\in R^m$ is a UBB disturbance, i.e., $d_{ij}^{(k)}\in[-\xi,\xi],\ \forall k\in\{1,\cdots m\}$ with a prior known constant $\xi>0.$ Note that all agents have a perfect measure of their own velocities, that is, $d_{ii}=0$ for all $i\in I.$

For a given UBB disturbance with a priori known $\xi>0,$ the component of $\tilde{v}_{ij}$ must belong to the interval

$$v_{ij}^{'(k)}-\xi\le\tilde{v}_{ij}^{(k)}\le v_{ij}^{'(k)}+\xi,\quad \forall k\in\{1,\cdots m\}. \tag{4}$$

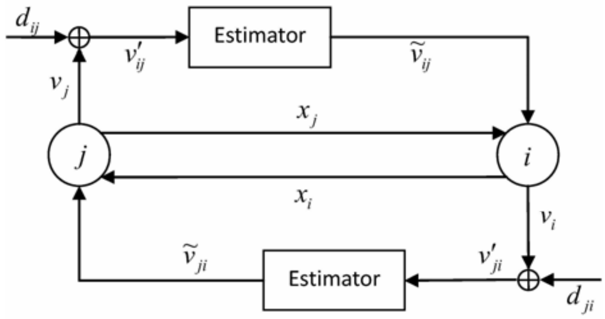


Fig. 1. Velocity estimate between agent i and agent j.

Then, the problem is how to select $\tilde{v}_{ij}$ from the above interval. Different from the lazy rule in [27], we define a distributed lazy rule:

$$\tilde{v}_{ij}^{*(k)}=\arg\left\{\min_{\tilde{v}_{ij}^{(k)}\in[v_{ij}^{'(k)}-\xi,v_{ij}^{'(k)}+\xi]}\left|\tilde{v}_{ij}^{(k)}-v_i^{(k)}\right|\right\},\quad \forall k\in\{1,\cdots m\}, \tag{5}$$

where $\tilde{v}_{ij}^{*}=[\tilde{v}_{ij}^{*(1)},\cdots,\tilde{v}_{ij}^{*(m)}]^T$. According to this rule, each agent estimates its neighbors' velocities as equal to the values v_{ij}^* that induces the minimal of $\left|v-v_i^{(k)}\right|$ on the internal $[v_{ij}^{'(k)}-\xi,v_{ij}^{'(k)}+\xi].$

Remark 1: When $\xi=0,$ i.e., $d_{ij}(t)\equiv 0,\forall t\in[0,\infty),$ it follows from (5) that $v_{ij}^*=v_j$. In other words, for $\xi=0$ (no disturbance), all agents have a perfect measure of their neighbors' velocities. According to the lazy rule in [27], each agent estimates its neighbors' states together, i.e.,

$$\tilde{x}_i^{*(k)}=\arg\left\{\min_{\tilde{x}_{ij}\in\left[x_{ij}^{'(k)}-\xi,x_{ij}^{'(k)}+\xi\right]}\left|\sum_{j\in N_i}\left(\tilde{x}_{ij}-x_i\right)\right|\right\},$$

thus the estimate of each neighbors' state is not unique, and this results in an ε-consensus problem, where the agents converge into a target set of radius ε asymptotically. However, in the distributed lazy rule (5), it is required that each agent estimates its neighbors' velocities one by one, such that the existence and uniqueness of $\tilde{v}_{ij}^*$ is obvious and thus improve the robustness of the estimator, as a result, second-order consensus could be achieved. Moreover, the agent in [27] is first-order dynamical system, and the lazy rule cannot be directly used to establish consensus for second-order agents. Because, in the case of second-order agents, according to the lazy rule, only the velocities of agents may achieve ε-consensus, i.e., $\|v_i(t)-v_j(t)\|\le\varepsilon,\ \forall i,j\in I,$ and the relative positions of agents may tend to infinite.

Given the distributed lazy rule (5), the consensus protocol (2) is transformed as follows:

$$\begin{aligned} u_i(t) &= \alpha \sum_{j\in N_i} a_{ij}(x_j(t)-x_i(t)) \\ &\quad + \beta \sum_{j\in N_i} a_{ij}(\tilde{v}_{ij}^*(t)-v_i(t)) - bv_i(t), \ \ \forall i \in I. \end{aligned} \tag{6}$$

According to (5), there are three cases of the term $\tilde{v}_{ij}^{*(k)} - v_i^{(k)}$:

$$\tilde{v}_{ij}^{*(k)} - v_i^{(k)} = \begin{cases} v_j^{(k)} + d_{ij}^{(k)} - v_i^{(k)} + \xi, \\ \qquad if \ \ v_j^{(k)} + d_{ij}^{(k)} - v_i^{(k)} \le -\xi, \\ 0, \qquad if \ \ \left|v_j^{(k)} + d_{ij}^{(k)} - v_i^{(k)}\right| \le \xi, \\ v_j^{(k)} + d_{ij}^{(k)} - v_i^{(k)} - \xi, \\ \qquad if \ \ v_j^{(k)} + d_{ij}^{(k)} - v_i^{(k)} \ge \xi. \end{cases} \tag{7}$$

In fact, equation (7) is a dead-zone function which has been used in many different fields and applications, especially in the robust adaptive control [31]. Since the UBB disturbance d_{ij} could cause drift in the velocity estimates, the dead-zone function is adopted in this study to analyze second-order consensus, which can overcome this problem and guarantee the velocity convergence. For simplification, denote

$$D_\xi(x) = \begin{cases} x+\xi & if \ \ x \le -\xi, \\ 0 & if \ \ |x| \le \xi, \\ x-\xi & if \ \ x \ge \xi. \end{cases} \tag{8}$$

From (7) and (8), we have

$$\tilde{v}_{ij}^* - v_i = D_\xi(v_j + d_{ij} - v_i), \tag{9}$$

$$\begin{aligned} u_i(t) &= \alpha \sum_{j\in N_i} a_{ij}(x_j(t)-x_i(t)) - bv_i(t) \\ &\quad + \beta \sum_{j\in N_i} a_{ij}(v_{ij}^*(t) - v_i(t)) \\ &= \alpha \sum_{j\in N_i} a_{ij}(x_j(t)-x_i(t)) - bv_i(t) \\ &\quad + \beta \sum_{j\in N_i} a_{ij} D_\xi(v_j + d_{ij} - v_i), \ \ \forall i \in I, \end{aligned} \tag{10}$$

where the function $D_\xi(v_j + d_{ij} - v_i) = [D_\xi(v_j^{(1)} + d_{ij}^{(1)} - v_i^{(1)}), \cdots, D_\xi(v_j^{(m)} + d_{ij}^{(m)} - v_i^{(m)})]^T$. Thus, the second-order multi-agent system with UBB disturbance evolves as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \alpha \sum_{j\in N_i} a_{ij}(x_j(t)-x_i(t)) - bv_i(t) \\ \qquad + \beta \sum_{j\in N_i} a_{ij} D_\xi(v_j + d_{ij} - v_i), \ \ \forall i \in I. \end{cases} \tag{11}$$

Note that d_{ij} is a vector-valued function of t, and (11) is the time-varying dynamic system. In this case, the consensus analysis is more challenging since LaSalle's invariance principle is no longer applicable here. In fact, LaSalle's invariance principle is used to establish the consensus of the time-invariant system. Instead, Barbalat lemma will be adopted for consensus analysis of the proposed algorithm.

4. CONSENSUS

In this section, some conditions are obtained for solving a consensus problem of second-order agent dynamics (11), based on the following two assumptions:

(A1): the communication topology G is connected;

(A2): $d_{ij}(t) = -d_{ji}(t), \ \ \forall i, j \in I$.

Note that in assumption (A2) the disturbances d_{ij} and d_{ji} are of the same size but opposite direction. As a matter of fact, d_{ij} and d_{ji} have the same size because they exist in the same channel generated by agent i and j. Meanwhile, in the terms $D_\xi(v_j + d_{ij} - v_i)$, $D_\xi(v_i + d_{ji} - v_j)$, $v_j - v_i$ and $v_i - v_j$ are the relative velocities with opposite direction, which infers that d_{ij}, d_{ji} are opposite in direction. The disturbance satisfying assumption (A2) is called anti-symmetric disturbance [32], which has been widely applied in various fields.

The main result of this paper is given in the following theorem:

Theorem 1: Given system (11), assume that both (A1) and (A2) are satisfied, then $\lim_{t\to\infty} \sum_{i=1}^{n} \sum_{j\in N_i} a_{ij} \| x_i - x_j \|^2$ exists, and $\lim_{t\to\infty} v_i(t) = 0, \ \forall i \in I$.

Proof: Consider the following auxiliary function

$$\begin{aligned} V(t) &= \sum_{i=1}^{n} \|v_i\|^2 + \frac{\alpha}{2} \sum_{i=1}^{n} \sum_{j\in N_i} a_{ij} \|x_j - x_i\|^2 \\ &= \sum_{i=1}^{n} v_i^T v_i + \frac{\alpha}{2} \sum_{i=1}^{n} \sum_{j\in N_i} a_{ij} (x_j - x_i)^T (x_j - x_i). \end{aligned} \tag{12}$$

Firstly, in order to study the behavior of $\dot{V}(t)$ for $t \to \infty$, we can write

$$\begin{aligned} \frac{dV}{dt} &= 2\sum_{i=1}^{n} v_i^T \dot{v}_i + \alpha \sum_{i=1}^{n} \sum_{j\in N_i} a_{ij}(x_j - x_i)^T (\dot{x}_j - \dot{x}_i) \\ &= 2\alpha \sum_{i=1}^{n} v_i^T \sum_{j\in N_i} a_{ij}(x_j - x_i) - 2b \sum_{i=1}^{n} v_i^T v_i \\ &\quad + 2\beta \sum_{i=1}^{n} v_i^T \sum_{j\in N_i} a_{ij} D_\xi(v_j + d_{ij} - v_i) \\ &\quad - \alpha \sum_{i=1}^{n} v_i^T \sum_{j\in N_i} a_{ij}(x_j - x_i) \\ &\quad - \alpha \sum_{i=1}^{n} \sum_{j\in N_i} a_{ij} v_j^T (x_i - x_j). \end{aligned} \tag{13}$$

Since the topology G is undirected, we have

$$\sum_{i=1}^{n} \sum_{j\in N_i} a_{ij} v_j^T (x_i - x_j) = \sum_{i=1}^{n} v_i^T \sum_{j\in N_i} a_{ij}(x_j - x_i),$$

and obtain

$$\frac{dV}{dt} = 2\beta \sum_{i=1}^{n} v_i^T \sum_{j \in N_i} a_{ij} D_\xi (v_j + d_{ij} - v_i) - 2b \sum_{i=1}^{n} v_i^T v_i. \quad (14)$$

To show that $\dot{V}(t) \le 0$ for all $t \in [0, \infty)$, we consider the term $\sum_{i=1}^{n} v_i^T \sum_{j \in N_i} a_{ij} D_\xi (v_j + d_{ij} - v_i)$. By the definition of the dead-zone function (8), we can easily get that the dead-zone function is an odd function, i.e.,

$$D_\xi(x) + D_\xi(-x) = 0, \forall x \in R. \quad (15)$$

According to (A2), (15), and the undirected topology G, it follows that

$$\begin{aligned}
&\sum_{i=1}^{n} v_i^T \sum_{j \in N_i} a_{ij} D_\xi (v_j + d_{ij} - v_i) \\
&= -\frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} (v_i - v_j)^T D_\xi (v_i + d_{ji} - v_j) \\
&= -\frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} \sum_{k=1}^{m} \Big[(v_i^{(k)} - v_j^{(k)}) \\
&\quad \times D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) \Big].
\end{aligned} \quad (16)$$

Recall that we have

$$D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) = \begin{cases} v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} + \xi, \\ \qquad \text{if} \quad v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} \le -\xi, \\ 0, \qquad \text{if} \quad \left| v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} \right| \le \xi, \\ v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} - \xi, \\ \qquad \text{if} \quad v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} \ge \xi. \end{cases} \quad (17)$$

In the first case, when $v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} \le -\xi$, it is easy to infer that the two following results hold: $v_i^{(k)} - v_j^{(k)} \le 0$ and $D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) \le 0$, hence, we have $(v_i^{(k)} - v_j^{(k)}) D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) \ge 0$. In the second case, it is trivially satisfied that $(v_i^{(k)} - v_j^{(k)}) D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) = 0$. Finally, in the third case, when $v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)} \ge \xi$, a symmetric argument holds. Hence, it implies that

$$\begin{aligned}
&(v_i^{(k)} - v_j^{(k)}) D_\xi (v_i^{(k)} + d_{ji}^{(k)} - v_j^{(k)}) \ge 0, \\
&\qquad \forall k \in \{1, 2, \cdots, m\}.
\end{aligned} \quad (18)$$

According to (14), (16) and (18), we get

$$\frac{dV}{dt} \le -2b \sum_{i=1}^{n} v_i^T v_i \le 0. \quad (19)$$

Apparently, $V(t)$ is not increasing because $\dot{V}(t) \le 0$. Since $V(t) \ge 0, \forall t \in [0, \infty)$, it yields that $\lim_{t \to \infty} V(t)$ is bounded, and $V(t) \le V(0), \forall t \in [0, \infty)$.

For any sequence $\{t_k\}_0^\infty$ with $t_{k+1} \ge t_k$ for all k, $t_0 = 0$ and $\lim_{k \to \infty} t_k = \infty$, we can write

$$\begin{aligned}
\lim_{t \to \infty} V(t) - V(0) &= \int_0^\infty \dot{V}(\tau) d\tau \\
&= \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \dot{V}(\tau) d\tau.
\end{aligned} \quad (20)$$

Denote $\Delta(k, r) = \int_{t_k}^{t_{k+r}} \dot{V}(\tau) d\tau$, from Cauchy condition for convergence of a series, it follows that

$$\lim_{k \to \infty} \Delta(k, r) = 0, \text{ for all } r \ge 1. \quad (21)$$

According to (21), we have

$$\int_{t_k}^{t_{k+r}} \dot{V}(\tau) d\tau \le -2b \int_{t_k}^{t_{k+r}} \sum_{i=1}^{n} v_i^T v_i d\tau. \quad (22)$$

Therefore,

$$0 \le \int_{t_k}^{t_{k+r}} \sum_{i=1}^{n} v_i^T v_i d\tau \le -\frac{\Delta(k, r)}{2b}, \quad (23)$$

which implies that

$$\lim_{k \to \infty} \int_{t_k}^{t_{k+r}} \sum_{i=1}^{n} v_i^T v_i d\tau = 0, \text{ for all } r \ge 1. \quad (24)$$

Thus, $\int_0^{+\infty} \sum_{i=1}^{n} v_i^T v_i d\tau$ exists and is finite. Denote $g(t) = \sum_{i=1}^{n} v_i^T v_i$, and $g'(t) = 2 \sum_{i=1}^{n} v_i^T \dot{v}_i$. Since $V(t)$ is bounded in $[0, \infty)$ and $\sum_{i=1}^{n} \|v_i\|^2$, $\sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} \|x_j - x_i\|^2$ are also bounded, we can get that $\|v_i - v_j\|$, $\|x_i - x\|$, $\forall i, j \in I$ are bounded. According to (11), it follows that $\dot{v}_i$, $\forall i, j \in I$ are bounded. From the above analysis, $g'(t)$ is a bounded function on $t \in [0, \infty)$. Consequently, by Lemma 2,

$$\lim_{t \to \infty} g(t) = \lim_{t \to \infty} \sum_{i=1}^{n} v_i^T v_i = 0.$$

By (12), we obtain

$$\begin{aligned}
&\lim_{t \to \infty} \sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} \|x_i - x_j\|^2 \\
&= \frac{2}{\alpha} \left[\lim_{t \to \infty} V(t) - \lim_{t \to \infty} \sum_{i=1}^{n} \|v_i\|^2 \right].
\end{aligned} \quad (25)$$

Hence, $\lim_{t \to \infty} \sum_{i=1}^{n} \sum_{j \in N_i} a_{ij} \|x_i - x_j\|^2$ exists. This completes the proof.

Remark 2: In the robust adaptive control, the sign and the absolute value functions are also very important. However, if we choose the sign or absolute value function, rather than the dead-zone function, on the estimation process, there may be some problems which make it real a challenge to analyze consensus in the multi-agent system introduced here. For example, both the sign and absolute value functions could not hinder the drift in the velocity estimates caused by the UBB disturbance. Meanwhile, for the sign function, since it is discontinuous, then the multi-agent system is also discontinuous on the vector field, in this case, we have to resort to the tools from non-smooth analysis, Filippov solutions for consensus analysis of the proposed algorithm. For the absolute value function, different from the dead-zone function, it is an even function, as a result, the method of Theorem 1 cannot be directly used to establish consensus in this case.

Note that in Theorem 1, the velocities of agents can achieve static consensus asymptotically. Then, another important problem is that under what condition the positions of agents can also achieve consensus asymptotically. The following theorem develops a sufficient condition to establish the position consensus.

Theorem 2: Assume that both (A1) and (A2) are satisfied and there exists a positive number M such that $\left|d_{ij}^{'(k)}\right| \le M, \forall i,j\in I, k\in\{1,\cdots,m\}, t\in[0,\infty)$, then system (11) achieves consensus.

Proof: According to Theorem 1, the velocities of agents can achieve consensus asymptotically. In the following, we show that $\lim_{t\to\infty}\dot{v}_i(t)=0, \forall i\in I$.

Based on system (11), we have

$$\begin{aligned}\dot{v}_i^{(k)}(t) &= \alpha\sum_{j\in N_i} a_{ij}(x_j^{(k)}-x_i^{(k)}) - bv_i^{(k)} \\ &\quad +\beta\sum_{j\in N_i} a_{ij}D_\xi(v_j^{(k)}+d_{ij}^{(k)}-v_i^{(k)}),\ \forall i\in I.\end{aligned} \tag{26}$$

Firstly, we verify that $\dot{v}_i^{(k)}(t)$ is a uniformly continuous function on $[0,\infty)$. In fact,

$$\left[\alpha\sum_{j\in N_i} a_{ij}(x_j^{(k)}-x_i^{(k)})\right]' = \alpha\sum_{j\in N_i} a_{ij}(v_j^{(k)}-v_i^{(k)}). \tag{27}$$

It follows from the boundedness of $V(t)$ that the function $\left[\alpha\sum_{j\in N_i} a_{ij}(x_j^{(k)}-x_i^{(k)})\right]'$ is also a bounded function, i.e., $\exists M_1>0$ such that $\left|\left[\alpha\sum_{j\in N_i} a_{ij}(x_j^{(k)}-x_i^{(k)})\right]'\right| \le M_1,\ \forall t\in$ $[0.\infty)$. Thus, the function $\alpha\sum_{j\in N_i} a_{ij}(x_j^{(k)}-x_i^{(k)})$ is a uniformly continuous function on $[0,\infty)$.

Then, we consider the function

$$\beta\sum_{j\in N_i} a_{ij}D_\xi(v_j^{(k)}+d_{ij}^{(k)}-v_i^{(k)}).$$

By the definition of $D_\xi(x)$, we get

$$\left|D_\xi(x_1)-D_\xi(x_2)\right| \le |x_1-x_2|, \tag{28}$$

Thus,

$$\begin{aligned}&\Big|D_\xi(v_j^{(k)}(t_1)+d_{ij}^{(k)}(t_1)-v_i^{(k)}(t_1)) \\ &\quad -D_\xi(v_j^{(k)}(t_2)+d_{ij}^{(k)}(t_2)-v_i^{(k)}(t_2))\Big| \\ &\le \Big|\left(v_j^{(k)}(t_1)+d_{ij}^{(k)}(t_1)-v_i^{(k)}(t_1)\right) \\ &\quad -\left(v_j^{(k)}(t_2)+d_{ij}^{(k)}(t_2)-v_i^{(k)}(t_2)\right)\Big| \\ &\le \left|v_j^{'(k)}(\eta)+d_{ij}^{'(k)}(\eta)-v_i^{'(k)}(\eta)\right|\cdot|t_1-t_2| \\ &\le \left\{\left|v_j^{'(k)}(\eta)\right|+\left|d_{ij}^{'(k)}(\eta)\right|+\left|v_i^{'(k)}(\eta)\right|\right\}\cdot|t_1-t_2|,\end{aligned} \tag{29}$$

where $\eta\in[t_1,t_2]$. Based on the boundedness of $d_{ij}^{'(k)}(t),\ v_i^{'(k)}(t),\ \forall i,j\in I, k\in\{1,\cdots,m\}$. we have $\exists M_2 > 0$, such that $\left|v_j^{'}(t)\right|+\left|d_{ij}^{'}(t)\right|+\left|v_i^{'}(t)\right| \le M_2, \forall t\in[0,\infty)$. Hence, equation (29) can be further written as

$$\begin{aligned}&\Big|D_\xi(v_j^{(k)}(t_1)+d_{ij}^{(k)}(t_1)-v_i^{(k)}(t_1)) \\ &\quad -D_\xi(v_j^{(k)}(t_2)+d_{ij}^{(k)}(t_2)-v_i^{(k)}(t_2))\Big| \le M_2\,|t_1-t_2|.\end{aligned} \tag{30}$$

From (30), it can be seen that the function

$$\beta\sum_{j\in N_i} a_{ij}D_\xi(v_j^{(k)}+d_{ij}^{(k)}-v_i^{(k)})$$

is a uniformly continuous function on $[0,\infty)$. Based on the above analysis, $\dot{v}_i^{(k)}(t)$ is a uniformly continuous function on $[0,\infty)$. From Theorem 1, we see that $\lim_{t\to\infty}\int_0^t \dot{v}_i^{(k)}(\tau)d\tau$ exists and is finite. According to Lemma 1, $\lim_{t\to\infty}\dot{v}_i^{(k)}(t)=0,\ \forall i\in I,\ k\in\{1,\cdots,m\}$. Hence, $\lim_{t\to\infty}\dot{v}_i(t)=0,\ \forall i\in I$.

Taking limit in (11), we obtain

$$\lim_{t\to\infty}\sum_{j\in N_i} a_{ij}(x_j-x_i)=0,\ \forall i\in I. \tag{31}$$

Therefore, $\lim_{t\to\infty}\left\|x_i-x_j\right\|=0,\ \forall i,j\in I$. This completes the proof.

Note that in Theorem 2, due to the velocity damping term $-bv_i(t)$, the agents achieve static consensus asymptotically. Moreover, the consensus protocol (10) does not explicitly take actuator saturation into account, while, in reality, almost every physical actuator is subject to saturation. So it is important to study the dynamical properties for the multi-agent system with actuator saturation (a bounded consensus protocol). Furthermore, equation (10) can be extended to a bounded consensus protocol, i.e.,

$$\begin{aligned} u_i = \alpha \sum_{j\in N_i} a_{ij} \tanh\left[x_j - x_i\right] - b\tanh\left[v_i\right] \\ + \beta \sum_{j\in N_i} a_{ij} \tanh\left[D_\xi(v_j + d_{ij} - v_i)\right],\ \forall i \in I, \end{aligned} \tag{32}$$

where $\tanh(\cdot)$ is the hyperbolic tangent function and defined component-wise, that is, $\forall[x_1,\cdots,x_m]^T \in R^m$, $\tanh([x_1,\cdots,x_m]^T) = [\tanh(x_1),\cdots,\tanh(x_m)]^T$. Also note that, with (32), one has $\|u\|_\infty \le (\alpha+\beta)\max_{i\in I}\{\deg(i)\} + b$, where $\deg(i)$ is the degree of the node π_i, i.e., $\deg(i) = \sum_{j\in N_i} a_{ij}$. Thus, the second-order multi-agent system with the bounded consensus protocol evolves as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \alpha \sum_{j\in N_i} a_{ij} \tanh\left[x_j - x_i\right] - b\tanh\left[v_i\right] \\ \qquad + \beta \sum_{j\in N_i} a_{ij} \tanh\left[D_\xi(v_j + d_{ij} - v_i)\right],\ \forall i \in I. \end{cases} \tag{33}$$

Corollary 1: Given system (33), assume that both (A1) and (A2) are satisfied, then $\lim_{t\to\infty} v_i(t) = 0,\ \forall i \in I$.

Proof: Consider the following auxiliary function

$$V(t) = \sum_{i=1}^n \|v_i\|^2 + \alpha\sum_{i=1}^n \sum_{j\in N_i} a_{ij} 1_m^T \log\left[\cosh(x_i - x_j)\right], \tag{34}$$

where $\cosh(\cdot)$ is the hyperbolic cosine function and defined component-wise.

In order to study the behavior of $\dot{V}(t)$ for $t \to \infty$, we can write

$$\begin{aligned} \frac{dV}{dt} &= 2\sum_{i=1}^n v_i^T \dot{v}_i + \alpha\sum_{i=1}^n \sum_{j\in N_i} a_{ij}(v_i - v_j)^T \tanh\left[x_i - x_j\right] \\ &= 2\sum_{i=1}^n v_i^T \left\{ \alpha \sum_{j\in N_i} a_{ij} \tanh\left[x_j - x_i\right] - b\tanh\left[v_i\right] \right. \\ &\quad \left. + \beta \sum_{j\in N_i} a_{ij} \tanh\left[D_\xi(v_j + d_{ij} - v_i)\right] \right\} \\ &\quad + \alpha\sum_{i=1}^n \sum_{j\in N_i} a_{ij}(v_i - v_j)^T \tanh\left[x_i - x_j\right] \\ &= 2\beta\sum_{i=1}^n v_i^T \sum_{j\in N_i} a_{ij} \tanh\left[D_\xi(v_j + d_{ij} - v_i)\right] \\ &\quad - 2b\sum_{i=1}^n v_i^T \tanh\left[v_i\right] \\ &\le -2b\sum_{i=1}^n v_i^T \tanh\left[v_i\right] \\ &\le 0. \end{aligned} \tag{35}$$

Under the similar proof in Theorem 1, we obtain $\lim_{t\to\infty} v_i(t) = 0,\ \forall i \in I$. This completes the proof.

The following corollary is similar to Theorem 2, which develops a sufficient condition to establish the position consensus in (33).

Corollary 2: Assume that both (A1) and (A2) are satisfied and there exists a positive number M, such that $\left|d_{ij}^{\prime(k)}\right| \le M,\ \forall i,j \in I,\ k \in \{1,\cdots,m\},\ t \in [0,\infty)$, then system (33) achieves consensus.

5. NUMERICAL SIMULATION

This section presents several numerical examples for the systems (11) and (33) in order to illustrate the theoretical results obtained in the previous sections.

Here, we consider ten agents moving in a 3-dimensional space with the control protocols (11) and (33). The coupling strengths are chosen as $\alpha = 1$, $\beta = 1.5$, and the velocity damping gain is set to $b = 0.2$. Initial positions and velocities of the 10 agents are chosen randomly from the cube [0,20]×[0,20]×[0,20] and [0,4]×[0,4]×[0,4], respectively. Moreover, for each $j \in N_i$, the disturbance $d_{ij}(t) = [d_{ij}^{(1)}(t), d_{ij}^{(2)}(t), d_{ij}^{(3)}(t)]^T$ satisfies $d_{ij}^{(1)}(t) = \cos wt,\ d_{ij}^{(2)}(t) = \sin wt,\ d_{ij}^{(3)}(t) = \frac{t^2}{1+t^2}$, and the parameter w is chosen randomly from [0, 100]. Therefore, with the definition of the dead-zone function D_ξ, we have $\xi = 1$. The interaction topology of the ten agents is presented in Fig. 2.

Here, the communication topology is connected, and the weight matrix is set to be

$$A = \begin{bmatrix} 0 & 0 & 0.9 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0.6 & 0.8 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0.9 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0.8 & 0.7 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0.8 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 \end{bmatrix}.$$

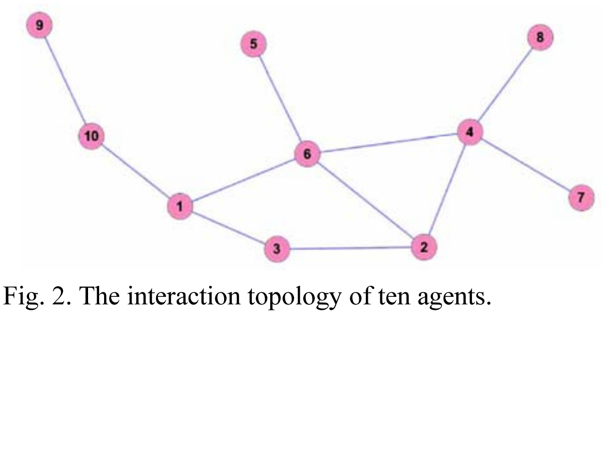
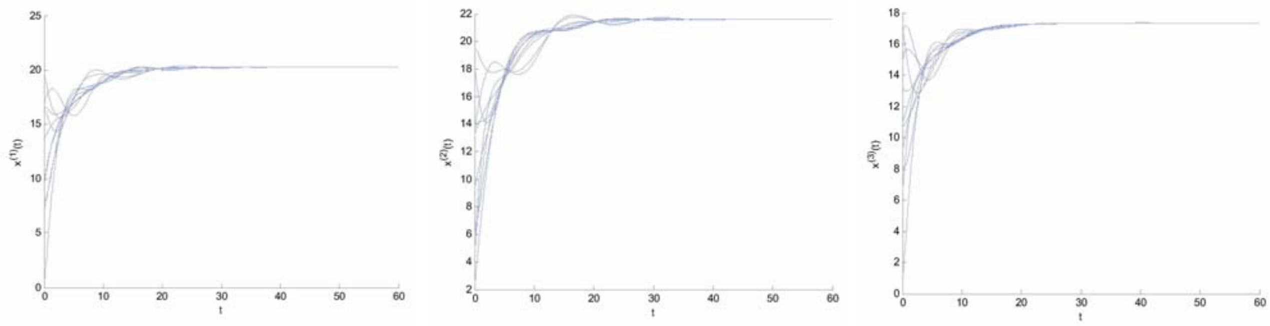
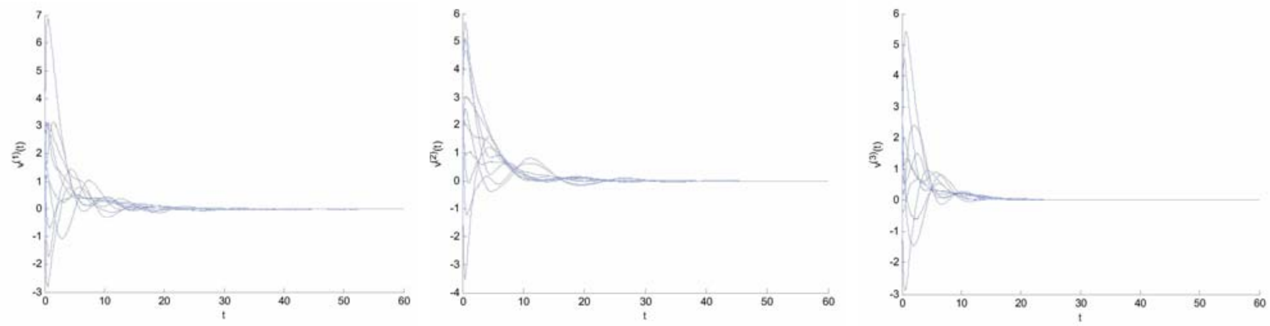


Fig. 2. The interaction topology of ten agents.



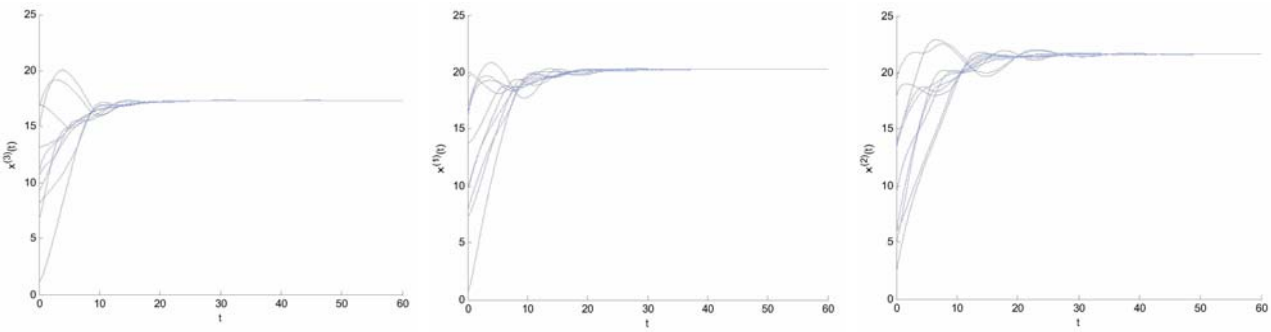
(a) $x^{(1)}$. (b) $x^{(2)}$. (c) $x^{(3)}$.

Fig. 3. Position convergence of ten agents for x axis.



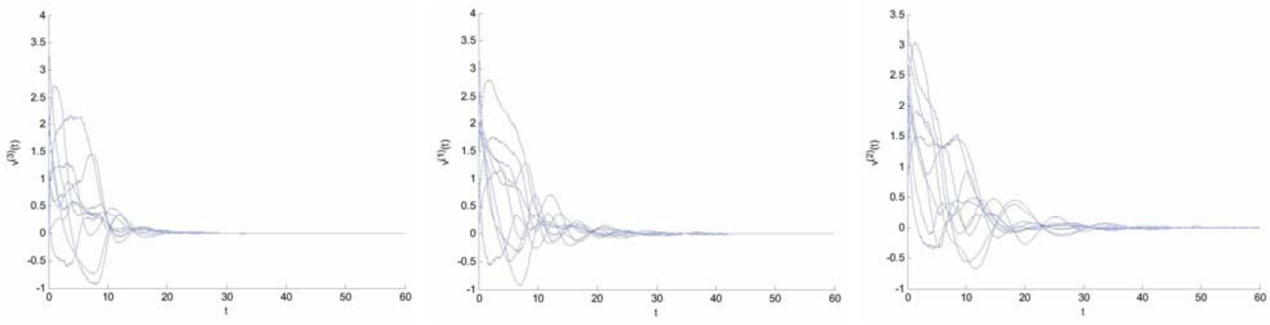
(a) $v^{(1)}$. (b) $v^{(2)}$. (c) $v^{(3)}$.

Fig. 4. Velocity convergence of ten agents for v axis.



(a) $x^{(1)}$. (b) $x^{(2)}$. (c) $x^{(3)}$.

Fig. 5. Position convergence of ten agents for x axis.



(a) $v^{(1)}$. (b) $v^{(2)}$. (c) $v^{(3)}$.

Fig. 6. Velocity convergence of ten agents for v axis.

Case 1: The consensus protocol (11) with UBB disturbance.

As we can see, $\left|d_{ij}^{\prime(k)}(t)\right| \le 100,\ \forall i \in I,\ j \in N_i,\ k \in \{1,2,3\}$. Consequently, it follows from Theorem 2 that second-order consensus in the multi-agent system (12) can be achieved. Fig. 3 depicts the motion trajectories of all agents from t = 0 to 60 s, from which one can see that all agents eventually achieve the same position. Fig. 4 shows the convergence of velocities. Here, the final velocities of the agents are zero, which means that the agents achieve static consensus asymptotically due to the velocity damping term.

Case 2: The consensus protocol (33) with UBB disturbance.

It follows from Corollary 2 that second-order consensus in the multi-agent system (33) can be achieved. Fig. 5 depicts the motion trajectories of all agents from $t = 0$ to 60 s, from which one can see that all agents eventually achieve the same position. Fig. 6 shows the convergence of velocities, where one can see that the final velocities of the agents are also zero.

From the simulation results, we see that distributed lazy rule can make the multi-agent system more robust against external disturbances. Meanwhile, by comparing the results in Case 1 and Case 2, it can be seen that the protocol (11) has better convergence performance than the protocol (33), while the fluctuation of the velocity on the protocol (11) is larger than the protocol (33).

6. CONCLUSION

This paper discussed the consensus problem for second-order agents with UBB disturbance. In this study, it was assumed that there was a UBB disturbance in the neighbors' velocities feedback and only the bound of the UBB disturbance was provided. A new velocity estimation called distributed lazy rule was proposed in order to solve such consensus problem. According to this rule, each agent estimates its neighbors' velocities from a compact set of candidate points. By adopting the graph theory and the Barbalat lemma, sufficient conditions for second-order consensus in the multi-agent system were presented. Furthermore, the bounded consensus protocol was proposed by considering actuator saturation. Finally, the effectiveness of the proposed theoretical results has been demonstrated by several numerical examples.

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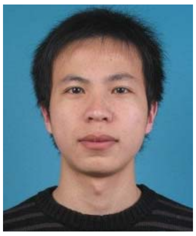

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