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A New TSMC Prototype Robust Nonlinear Task Space Control of a 6 DOF Parallel Robotic Manipulator

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Abstract: In this study, a new terminal sliding mode control (TSMC) prototype robust nonlinear task space control approach is developed for 6 degree of freedom (DOF) parallel robotic manipulators in light of TSMC principle integrated with Lyapunov redesign method. Corresponding stability analysis is presented to lay a foundation for analytical understanding in generic theoretical aspects and safe operation for real systems. An illustrative example of a 6 DOF parallel robot is bench tested to validate the effectiveness of the proposed approach.

Keywords: Lyapunov redesign, parallel robot, robot control, terminal sliding mode control.

1. INTRODUCTION

Due to some superior advantages, such as: higher accuracy, higher stiffness, and higher load-carrying capacity over its counterparts, namely, serial manipulators, parallel robotic manipulators have been extensively studied in both academy and industry [1-4]. By virtue of their merits, parallel manipulators can be used as actuators for high precision operation of heavy payload, such as flight simulator, astronomical telescopes and digital control machine-tools [5-8]. It is obvious that the operation precision is one of main objectives pursued by these equipments. As literature [9] has pointed out, after all of the mechanical design problems are solved, the operation precision is mainly determined by control algorithms employed by parallel robots.

In system and control community, parallel robot is a typical multi-input multi-output (MIMO) nonlinear system, which can serve as a test bed for high-performance controller [10]. Owing to complexity of mechanical structure of 6 DOF parallel robots, modeling error can not be avoided. Hence, it is required to cope with system uncertainty caused by modeling error and disturbance for achieving high control precision. To address this problem, some robust control approaches are presented for parallel robots. A robust tracking control is presented in link-space, which can achieve practical stability [1]. Resort to Lyapunov redesign method and the concept of linear sliding mode, literature [10] proposed a robust nonlinear control algorithm for 6 DOF parallel robotic manipulators, it can guarantee tracking error converge to a residual set in finite-time. By using convex integrated design method, a robust control approach is proposed for a planar parallel robot, which can simultaneously satisfy several performance specifications and achieve high control precision [11]. Adaptive robust posture control is studied for a pneumatic muscles driven parallel robot, in which uncertainty bounds can be estimated online [12]. A novel robust learning control is presented for high precision planar parallel manipulator [13].

In real industry, practical stability is more applicable for robot control. However, high control gain is usually needed to achieve higher control precision and faster converging speed in most of robust control algorithms. It is unexpected to use high gains in real industrial robotic manipulator control.

TSMC is a finite-time stability control approach. It has some excellent characteristics, such as, fast convergence, insensitiveness to system uncertainty and external disturbance. This control approach is particularly useful for high precision control without using high gains [14]. By using finite-time reaching law and nonlinear terminal sliding mode (TSM), literature [15] develops a TSMC for serial robots and proves the tracking error can converge to a residual set in the presence of system uncertainty and external disturbance. However, the settling time estimated method is not presented by [15], which is expended to make tracking error converge to the residual set.

Configuration of 6 DOF parallel robots is the most general one in parallel robotic manipulators. Control algorithm designed for them can be extended to other type parallel robot easily. Illuminated by the results of

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literatures [10,15], a new TSMC prototype robust nonlinear control approach is developed for 6 DOF parallel robotic manipulator in this paper. In comparison with literature [10], the proposed approach employs nonlinear TSM and a TSM type feedback control law, literature [10] using linear sliding mode and linear feedback law. Literature [16] claims that TSM has terminal converging ability. Hence, the proposed approach can stabilize tracking error to a smaller residual set with faster converging speed than literature [10]. In comparison with literature [15], Lyapunov redesign method is employed by the proposed approach to analyze system stability and estimate settling time. However literature [15] does not present settling time estimating method under system uncertainty.

In summary, this study emphasizes twofold considerations. The first is for application, the proposed approach may offer an alternative, but more effective algorithm for parallel robot control. The second is, for theory, TSMC is an important and challenging topic in nonlinear theoretical studying. Recently, it has been extensively studied for serial robot control [14,15,17]. Hopefully, for further development, this study can provide a new insight and application incentive in aspect of the theoretical development.

The rest of this paper is organized as follows. In Section 2, the dynamic model of 6 DOF parallel robotic manipulators is described. In Section 3, the new TSMC prototype robust nonlinear control scheme is developed. In Section 4, an illustrative example is presented to validate the performance of the proposed approach. Finally, in Section 5, some concluding remarks are given to summarize this study.

2. DYNAMIC MODEL OF 6 DOF PARALLEL MANIPULATORS

A 6 DOF parallel robotic manipulator is composed of two bodies connected by the six extensible legs. Its task space coordinates of mass center of moving platform can be written as:

$$q = [X \quad Y \quad Z \quad \alpha \quad \beta \quad \gamma]^T, \tag{1}$$

where X, Y, Z is translations, α, β, γ is rotations. In terms of Euler-Lagrangian method, the dynamic model of 6 DOF parallel robotic manipulators can be described as [5,10]:

$$\begin{aligned} &M(q,\sigma)\ddot{q} + C(q,\dot{q},\sigma)\dot{q} + G(q,\sigma) \\ &\quad = J^T(q,\sigma)(f - f_f + f_d), \end{aligned} \tag{2}$$

where $M(\cdot) \in \mathbb{R}^{6\times 6}$ is positive definite symmetry inertia matrix, $C(\cdot)\dot{q} \in \mathbb{R}^6$ is Coriolis and centrifugal force vector, $G(\cdot) \in R^6$ is gravity force vector, $J(\cdot) \in \mathbb{R}^{6\times 6}$ is Jacobian matrix, $f \in \mathbb{R}^6$ is control input, $f_f \in \mathbb{R}^6$ is actuator friction, $f_d \in \mathbb{R}^6$ is unknown external force perturbation and satisfies $\|f_d\| \le d$, $d > 0$ is a positive number, σ is constant or time varying and represents system uncertainties including inertia, modeling error and measuring noise, $\sigma \in \Sigma$, Σ is compact set.

Dynamic equation (2) has the following properties [5,10]:

Property 1: There are positive real numbers $\underline{m}, \overline{m} > 0$ for $q \in D_{qr}$, $D_{qr} = \{q \| \|q\| \le r, r \in [0,\infty)\}$ and $\sigma \in \Sigma$, Σ is compact, inertia matrix $M(q,\sigma)$ satisfies inequality: $\underline{m}I < M(q,\sigma) < \overline{m}I.$

Property 2: Matrix $\dot{M}(\cdot) - 2C(\cdot)$ is skew-symmetric, that is, for $x \in \mathbb{R}^6$, $x^T(\dot{M}(\cdot) - 2C(\cdot))x = 0.$

In this paper, $\|\cdot\|$ denotes L_2 norm for vector and corresponding induced norm for matrix, respectively.

Suppose there are measuring noises in real control system, then, measuring error can be expressed as:

$$\begin{cases} \delta q = q - q' \\ \delta \dot{q} = \dot{q} - \dot{q}', \end{cases} \tag{3}$$

where $\delta q \in \mathbb{R}^6$, $\delta \dot{q} \in \mathbb{R}^6$ are position and velocity measuring error caused by measuring noise, respectively, $q' \in \mathbb{R}^6$, $\dot{q}' \in \mathbb{R}^6$ are position and velocity measuring values, respectively. According to definition (3), dynamic equation (2) can be rewritten as:

$$\begin{aligned} &M(q,\sigma)\ddot{q}' + C(q,\dot{q},\sigma)\dot{q}' + G(q,\sigma) \\ &\quad = J(q,\sigma)(f - f_f + f_d) + h_1, \end{aligned} \tag{4}$$

where $h_1(\cdot) = -M(\cdot)\delta\ddot{q} - C(\cdot)\delta\dot{q}.$

In terms of dynamics characteristic of parallel robot, the following assumptions can be made:

Assumption 1: Task space coordinate q and its time derivative $\dot{q}$ are measurable.

Assumption 2: Each matrix in dynamic equation (2) can be written as nominal part plus uncertain part:

$$\begin{aligned} &M(q,\sigma) = M_0(q',0) + \delta M(q,\sigma), \\ &C(q,\dot{q},\sigma) = C_0(q',\dot{q}',0) + \delta C(q,\dot{q},\sigma), \\ &G(q,\sigma) = G_0(q',0) + \delta G(q,\sigma), \\ &f_f(q,\dot{q},\sigma) = \hat{f}_f(q',\dot{q}',\sigma) + \delta f_f(q,\dot{q},\sigma), \\ &J(q,\sigma) = J_e^T(q',0) + \delta J(q,\sigma). \end{aligned}$$

Assumption 3: Jacobian matrix J is nonsingular.

Remark 1: Because the singularity of J can be avoided in mechanical design and trajectory planning, assumption 3 is reasonable.

Suppose q^d, $\dot{q}^d$ are desired position and velocity, then, position error and velocity error are defined as:

$$\begin{cases} e' = q' - q^d \\ \dot{e}' = \dot{q}' - q^d, \end{cases} \tag{5}$$

where e', $\dot{e}'$ are position and velocity errors, respectively.

The following notions are introduced for simplicity of expression [18]:

$$y^{\gamma}=[y_1^{\gamma},\cdots,y_n^{\gamma}]^T,$$
$$|y|^{\gamma}=[|y_1|^{\gamma},\cdots|y_n|^{\gamma}]^T,$$
$$sig(y)^{\gamma}=[|y_1|^{\gamma}\,sign(y_1),\cdots,|y_n|^{\gamma}\,sign(y_n)]^T,$$

where $y\in\mathbb{R}^n$, $\gamma\in\mathbb{R}$.

Command vector is defined as:

$$\begin{cases} r'=\dot{q}^d-\Lambda sig(e')^{\alpha} \\ \dot{r}'=\ddot{q}^d-\alpha\Lambda diag(|e|^{\alpha-1})\dot{e}, \end{cases} \tag{6}$$

where $\Lambda=diag\{\lambda_i\}$ is positive definite diagonal matrix, $\alpha=\frac{p_1}{p_2}$, $0<p_1<p_2$ are positive odd numbers, $i=1,\cdots,6$.

Due to $\alpha-1<0$, singularity may occur as $e_i=0$ and $\dot{e}_i\neq 0$, that is, $\lim_{e_i\to 0}|e_i|^{\alpha-1}\dot{e}_i\to\infty$, $i=1,\cdots,6$. To avoid singularity, the following definition $e_r=[e_{r1},\cdots,e_{r6}]^T$ is defined as:

$$e'_{ri}=\begin{cases} \alpha|e'_i|^{\alpha-1}\dot{e}'_i & if\ \ |e'_i|\geq\xi \ \ and\ \ \dot{e}'_i\neq 0 \\ \alpha|\xi|^{\alpha-1}\dot{e}'_i & if\ \ |e'_i|<\xi \ \ and\ \ \dot{e}'_i\neq 0 \\ 0 & \dot{e}'_i=0, \end{cases} \tag{7}$$

where $\xi>0$ is a small positive real number, $i=1,\cdots,6$.

Under the definition e_r, command vector r and $\dot{r}$ can be rewritten as:

$$\begin{cases} r'=\dot{q}^d-\Lambda sig(e')^{\alpha} \\ \dot{r}'=\ddot{q}^d-\Lambda e'_r. \end{cases} \tag{8}$$

In light of r, $\dot{r}$ TSM like generalized error can be defined as:

$$\begin{cases} s'=\dot{q}'-r' \\ \dot{s}'=\ddot{q}'-\dot{r}'. \end{cases} \tag{9}$$

Remark 2: Conventional TSMC has singularity problem. Note that the method of this paper to avoid singularity is different from literature [19]. As singularity occurs, control input of literature [19] is set to be zero. By using a small positive number ξ, the proposed approach does not require to set control input to be zero.

Suppose the control input includes two parts, that is, f_{eq} and $J_e^{-T}v$, in terms of command vector r, $\dot{r}$ and TSM like generalized error s', $\dot{s}'$ dynamic equation (4) can be expressed as:

$$\begin{aligned} &M(\cdot)\dot{s}'+C(\cdot)s' \\ &=-M_0(\cdot)\dot{r}'-C_0(\cdot)r'-G_0(\cdot)+J_e^Tf-J_e^T\hat{f}_f+\varphi, \end{aligned} \tag{10}$$

where

$$\varphi=h_1+h_2+h_3+\delta J^TJ_e^{-T}v+Jf_d,$$
$$h_2=-\delta M(\cdot)\dot{r}'-\delta C(\cdot)r'-\delta G(\cdot),$$
$$h_3=\delta J^Tf_{eq}(\cdot)-\delta J^T\hat{f}_f-J^T\delta f_f.$$

The control objective of this paper can be summarized as: Considering dynamic equation (10), develop a TSMC prototype robust nonlinear controller, guarantee system practical stability, namely, system tracking error can converge to a residual set in finite-time.

3. TSMC PROTOTYPE ROBUST NONLINEAR CONTROLLER DESIGN

Without loss generality, several technical assumptions are made to pose the problem in a tractable manner before controller design [10]:

Assumption 4: Measuring errors $\delta\dot{q}$ and δq are bounded.

Assumption 5: $\|\delta J^TJ_e^{-T}\|\leq k<1$, k is a positive number.

Assumption 6: $\|\varphi\|\leq\rho(e',\dot{e}')+k\|v\|$, v is control law to be designed.

Remark 3: If measuring errors and system uncertainty are not bounded, robust controller can not be designed. Under an effective robust controller, system measuring errors and system uncertainty must be bounded otherwise the design of robust controller is failed. Hence, Assumption 4~6 are reasonable.

For 6 DOF parallel robotic manipulators, a TSMC prototype robust nonlinear control law is developed as:

$$\begin{aligned} &f=f_{eq}+J_e^{-T}v, \\ &f_{eq}=J_e^{-T}(M_0(q')\dot{r}'+C_0(q',\dot{q}')r'+G_0(q') \\ &\qquad +J_e^T\hat{f}_f-K_1s'-K_2sig(s')^{\beta}), \\ &v=\begin{cases} -\dfrac{\rho(e',\dot{e}')}{1-k}\dfrac{s'}{\|s'\|} & if\ \ \|s'\|\geq\varepsilon \\ -\dfrac{\rho^2(e',\dot{e}')}{1-k}\dfrac{s'}{\varepsilon} & if\ \ \|s'\|<\varepsilon, \end{cases} \end{aligned} \tag{11}$$

where $K_1=dia\{k_{1i}\}$, $K_2=diag\{k_{2i}\}$, are positive definite diagonal matrices, $0<\beta<1$, $\rho(e',\dot{e}')>0$ is a positive definite scalar function, ε is a small positive number, $i=1,\cdots,6$.

Remark 4: $-K_1s-K_2sig(s)^{\beta}$ is TSM type feedback control law, which can achieve faster converging speed and higher control precision.

Remark 5: Literature [10] presents the following robust nonlinear control law:

$$\begin{aligned} &\tilde{f}=\tilde{f}_{eq}+J_e^{-T}\tilde{v}, \\ &\tilde{f}_{eq}=J_e^{-T}(M_0(q')\dot{\tilde{r}}'+C_0(q',\dot{q}')\tilde{r}'+G_0(q') \\ &\qquad +J_e^T\hat{f}_f-\tilde{K}\tilde{s}'), \end{aligned} \tag{12}$$

$$\tilde{v}=\begin{cases}-\dfrac{\rho(e',\dot{e}')}{1-k}\dfrac{\tilde{s}'}{\|\tilde{s}'\|} & if \quad \|\tilde{s}'\|\geq\varepsilon \\ -\dfrac{\rho^2(e',\dot{e}')}{1-k}\dfrac{(\tilde{s}')}{\varepsilon} & if \quad \|\tilde{s}'\|<\varepsilon,\end{cases}$$

where $\tilde{r}'=\dot{q}^d-\Lambda_2 e'$, $\tilde{s}'=\dot{e}'+\tilde{\Lambda}e'$, $\tilde{\Lambda}=diag\{\tilde{\lambda}_i\}$ and $\tilde{K}=diag\{\tilde{k}_i\}$ are positive definite diagonal matrices, $i=1,\cdots,6$, $-\tilde{K}\tilde{s}'$ is linear feedback control law proposed by literature [10].

Remark 6: Inspired by literature [15], a continuous TSMC (CTSMC) can be designed for task space control of 6 DOF parallel robots, which is given as follows:

$$\begin{aligned} f_{CTSMC}=&J_e^{-T}(M_0(q')\dot{r}'+C_0(q',\dot{q}')r'+G_0(q') \\ &+J_e^T\hat{f}_f-K_{1CTSMC}s'-K_{2CTSMC}sig(s')^\beta),\end{aligned} \tag{13}$$

where $\dot{r}'$, r', s' and β are same as expression (11), $K_{1CTSMC}=diag\{k_{1CTSMCi}\}$, $K_{2CTSMC}=diag\{k_{2CTSMCi}\}$ are positive definite diagonal matrices, $i=1,\cdots,6$. The corresponding stability analysis of control law (13) is similar to [15].

Remark 7: The main differences between the proposed approach and literature [10] are the generalized error and feedback control law. The proposed generalized error s' and feed back control law $-K_1 s$ $-K_2 sig(s)^\beta$ are illuminated by TSM, which have terminal converging ability. The generalized error $\tilde{s}'$ and feedback control law $-\tilde{K}\tilde{s}'$ are derived from linear sliding mode. It will be seen both in proofs and simulations that the proposed approach has faster converging speed and smaller residual set than those of literature [10]. The main reason is attributed to the proposed control algorithm. Due to the power rule is employed in TSM like generalized error and feedback control law, the proposed approach can achieve higher control precision with faster converging speed [16,20,21]. Though the form of these two control are similar, stability analysis and settling time estimation are more difficult of the propose approach due to nonlinearity of s' and $-K_1 s-K_2 sig(s)^\beta$.

Remark 8: The main difference between the proposed approach and literature [15] is the method to cope with system uncertainty and the stability analysis. The control laws, that is, $-K_1 s'-K_2 sig(s')^\beta$ and v, are developed to overcome system uncertainty by the proposed approach, which is derived from Lyapunov redesign method. However, control law (13) uses feedback control law $-K_{1CTSMC}s'-K_{2CTSMC}sig(s')^\beta$ to overcome system uncertainty, the details can be found in [15]. Hence, the proposed approach is more flexible than that of literature [15] to cope with system uncertainty. Note that the settling time estimating method is proposed in light of Lyapunov redesign method while it is not given in literature [15].

Remark 9: If $\alpha=1$, $\beta=1$, $\tilde{\Lambda}=\Lambda$, $\tilde{K}=K_1+K_2$, the proposed control law (11) will be equal to control law (12). Hence, control law (12) is one of special case of control law (11). Therefore the proposed approach extends literature [10]'s results.

For stability analysis, the following lemma is given [15]:

Lemma 1: If $b_1,b_2,\cdots,b_n>0$ and $0<l<2$, the following inequality is held:

$$(b_1^2+b_2^2+\cdots+b_n^2)^l\leq(b_1^l+b_2^l+\cdots+b_n^l)^2.$$

Theorem 1: Under Assumption 1~6, using Property 1~2, consider dynamic equation (10), subjected to TSMC prototype robust nonlinear control law (11), system tracking error will be practical stability in the neighborhood $D_r=\{s'\in R^6\,\big|\,\|s'\|\leq r, r\in[0,\infty]\}$, namely, position tracking error e' and $\dot{e}'$ will converge to a small residual Ω set in finite-time $t\geq t_1$:

$$t_1=t_0+\frac{2}{\varepsilon}\left(\overline{m}\|s'(t_0)\|^2-\underline{m}\frac{\varepsilon}{2k_{\min}(1+N)}\right),$$

$$\Omega_{s'}=\left\{s'\,\middle|\,\|s'\|\leq\Delta,\Delta=\sqrt{\frac{\overline{m}\varepsilon}{2\underline{m}k_{\min}(1+N)}}\right\},$$

$$\Omega=\left\{e',\dot{e}'\,\middle|\,\left|e_i'\right|\leq\left(\frac{\Delta}{\lambda_i}\right)^{\frac{1}{\alpha}},\left|\dot{e}_i'\right|\leq 2\Delta, i=1,\cdots,6\right\}.$$

Proof: Consider Lyapunov function:

$$V=\frac{1}{2}s'^T Ms'. \tag{14}$$

Using Property 1, the following inequality is held:

$$a_1(\|s'\|)\leq V\leq a_2(\|s'\|), \tag{15}$$

where $a_1(\|s\|)=\frac{1}{2}\underline{m}\|s'\|^2$, $a_2(\|s\|)=\frac{1}{2}\overline{m}\|s'\|^2$. It is obvious that $a_1(\|s\|)$ and $a_2(\|s\|)$ are class $\mathcal{K}$ functions. Differentiate V with respect to time along closed loop function (10), it yields:

$$\begin{aligned}\dot{V}=&s'^T(-M_0\dot{r}'-C_0 r'-G_0+J_e^T f_{eq}+v-J_e^T\hat{f}+\varphi) \\ &-s'^T Cs'+\frac{1}{2}s'^T\dot{M}s'.\end{aligned} \tag{16}$$

By using Property 2, one can get:

$$\begin{aligned}\dot{V}=&s'^T(-M_0\dot{r}'-C_0 r'-G_0+J_e^T f_{eq}+v \\ &-J_e^T\hat{f}+\varphi).\end{aligned} \tag{17}$$

Substitute control law (11) into (17), it can give:

$$\dot{V}=-\sum_{i=1}^{6}k_{1i}s_i'^2-\sum_{i=1}^{6}k_{2i}\left|s_i'\right|^{\beta+1}+s'^T(v+\varphi). \tag{18}$$

When $\|s'\|>\varepsilon$, substitute v of control law (11) into

(18), it yields:

$$\dot{V} \le -\sum_{i=1}^{6} k_{1i} s_i'^2 - \sum_{i=1}^{6} k_{2i} \left| s_i' \right|^{\beta+1}. \tag{19}$$

Suppose $k_{\min\_1}$ and $k_{\min\_2}$ are the minimal eigenvalues of matrices K_1 and K_2, respectively. Define $k_{\min} = \min\{k_{\min\_1}, k_{\min\_2}\}$, then the following inequality must be held:

$$\dot{V} \le -k_{\min} \left(\sum_{i=1}^{6} s_i'^2 + \sum_{i=1}^{6} \left| s' \right|_i^{\beta+1} \right). \tag{20}$$

In light of Lemma 1, one can get the following inequality:

$$\dot{V} \le -k_{\min} \left(\|s'\|^2 + \|s'\|^{\beta+1} \right) \le 0. \tag{21}$$

Choose $a_3\left(\|s'\|\right) = k_{\min}\left(\|s'\|^2 + \|s'\|^{\beta+1}\right)$, it is obvious that $a_3\left(\|s'\|\right)$ is class $\mathcal{K}$ function. Define $N = \|s'\|^{\beta-1}$, then $a_3\left(\|s'\|\right) = k_{\min}\left(1+N\right)\|s'\|^2$. The following inequality is held:

$$\dot{V} \le -a_3\left(\|s'\|\right) \le 0. \tag{22}$$

When $\|s'\| < \varepsilon$, substitute v of control law (11) into (18), it yields:

$$\dot{V} \le -a_3\left(\|s'\|\right) - \frac{\|s'\|^2 \rho^2}{\varepsilon} + \|s'\| \rho. \tag{23}$$

The last terms of inequality (23) satisfy inequality: $-\frac{\|s'\|^2 \rho^2}{\varepsilon} + \|s'\|\rho \le \frac{\varepsilon}{4}$ [22], then inequality can be rewritten as:

$$\dot{V} \le -a_3\left(\|s'\|\right) + \frac{\varepsilon}{4}. \tag{24}$$

Let $\varepsilon < 2a_3\left(a_2^{-1}(a_1(r))\right)$, $\mu = a_3^{-1}\left(\frac{\varepsilon}{2}\right) < a_2^{-1}\left(a_1(r)\right)$, according to Lypunov redesign method [22], there must be:

$$\dot{V} \le -\frac{1}{2} a_3\left(\|s'\|\right), \quad \forall \mu \le \|s'\| < r. \tag{25}$$

Choose $\mu(\varepsilon) < \|s'(t_0)\| < a_2^{-1}\left(a_1(r)\right)$, $s'(t_0)$ is initial value of $s'(t)$ at time t_0, settling time and bounds of $s'(t)$ can be estimated as:

$$t_1 = t_0 + \frac{2}{\varepsilon}\left(\overline{m}\|s'(t_0)\|^2 - \underline{m}\frac{\varepsilon}{2k_{\min}(1+N)} \right), \tag{26}$$

$$\|s'\| \le \sqrt{\frac{\overline{m}\varepsilon}{2\underline{m}k_{\min}(1+N)}} = \Delta. \tag{27}$$

Because $\|s'\| \le \Delta$ means $\left\| s_i' \right\| \le \Delta$, $i = 1, \cdots, 6$, i.e.,

$$\dot{e}_i' + \lambda_{1i} \left| e_i' \right|^{\alpha} sign(e_i') = \phi_i, \ \left|\phi_i\right| \le \Delta. \tag{28}$$

Equation (28) can be rewritten as:

$$\dot{e}_i' + \left(\lambda_{1i} - \frac{\phi_i}{\left| e_i' \right|^{\alpha} sign(e_i')} \right) \left| e_i' \right|^{\alpha} sign(e_i') = 0. \tag{29}$$

Then, when $\lambda_{1i} - \phi_i \Big/ \left| e_i' \right|^{\alpha} sign(e_i') > 0$, equation (29) is still kept in the form of TSM, this also means that position tracking error will converge to the following region:

$$\left| e_i' \right| \le \left(\frac{\Delta}{\lambda_{1i}} \right)^{\frac{1}{\alpha}}, \quad i = 1, \cdots, 6. \tag{30}$$

Furthermore, with TSM dynamics (28), velocity error will converge to the following region:

$$\left| \dot{e}_i' \right| \le 2\Delta, \quad i = 1, \cdots, 6. \tag{31}$$

As $t \ge t_1$, TSM like generalized error s', position tracking error e' and velocity error $\dot{e}'$ will converge to residual sets $\Omega_{s'}$ and Ω, respectively.

Remark 10: If one chooses $\alpha = 1$, $\beta = 1$, literature [10]'s control law (see (12)) can stabilize linear sliding mode like generalized error $\tilde{s}'$, position track-ing error e' and velocity error $\dot{e}'$ to the following residual sets $\tilde{\Omega}_{\tilde{s}'}$ and $\tilde{\Omega}$ in finite-time $\tilde{t}_1$:

$$\tilde{t}_1 = t_0 + \frac{2}{\varepsilon}\left(\overline{m}\|\tilde{s}'(t_0)\|^2 - \underline{m}\frac{\varepsilon}{4k_{\min}} \right), \tag{32}$$

$$\tilde{\Omega}_{\tilde{s}'} = \left\{ \tilde{s}' \middle| \|\tilde{s}'\| \le \tilde{\Delta}, \tilde{\Delta} = \sqrt{\frac{\overline{m}\varepsilon}{4\underline{m}k_{\min}}} \right\}, \tag{33}$$

$$\tilde{\Omega} = \left\{ e', \dot{e}' \middle| \left| e_i' \right| \le \left(\frac{\tilde{\Delta}}{\tilde{\lambda}_i} \right), \left| \dot{e}_i' \right| \le 2\tilde{\Delta}, i = 1, \cdots, 6 \right\}, \tag{34}$$

where $\tilde{k}_{\min}$ is the minimal eigenvalue of matrix $\tilde{K}$.

Remark 11: Subjected to CTSMC control law (13), TSM like generalized error s', position tracking error e' and velocity error $\dot{e}'$ will converge to the following residual sets $\Omega_{CTSMCs'}$ and Ω_{CTSMC} in finite-time:

$$\Omega_{CTSMCs'} = \left\{ s' \middle| \|s\| \le \hat{\Delta} = \min(\hat{\Delta}_1, \hat{\Delta}_2) \right\}, \tag{35}$$

$$\hat{\Delta}_1 = \frac{\left\| M_0^{-}(q') \right\| \left(\rho(e', \dot{e}') + k\|v\| \right)}{\underline{k_1}}, \tag{36}$$

$$\hat{\Delta}_2 = \frac{\left\| M_0^{-}(q') \right\| \left(\rho(e', \dot{e}') + k\|v\| \right)}{\underline{k_2}}, \tag{37}$$

where $\underline{k}_1 = \min\{k_{1CTSMCi}\}$, $\underline{k}_2 = \min\{k_{2CTSMCi}\}$, $i=1,\cdots,6$.

$$\Omega_{CTSMC} = \left\{ e', \dot{e}' \left| \left| e_i' \right| \le \left(\frac{\hat{\Delta}}{\lambda_i} \right)^{\frac{1}{\alpha}}, \left| \dot{e}_i' \right| \le 2\hat{\Delta}, i = 1,\cdots,6 \right. \right\}. \quad (38)$$

Note that the estimation method for $\Omega_{CTSMCs'}$ and Ω_{CTSMC} can be found in literature [15], however the settling time estimation method is not presented by [15]. Compare the residual sets $\Omega_{s'}$ and Ω with $\Omega_{CTSMCs'}$ and Ω_{CTSMC}, one can see that $\Omega_{s'}$ and Ω are mainly determined by ε and feedback control gains, however, $\Omega_{CTSMCs'}$ and Ω_{CTSMC} are mainly determined by the bound of system uncertainty and feedback control gains. The control law presented by [15] can only use feedback control gains to obtain small residual set. Due to employing the small positive number ε, the proposed approach can obtain small residual set by using smaller ε besides using feedback control gains. Hence, the proposed approach is more flexible than that of literature [15] in controller design.

Remark 12: Due to $\beta - 1 < 0$, $N \to \infty$ as $s' \to 0$. If controller parameters are selected appropriately, $(1+N) \gg 2$. Then, $\Omega_{s'} \ll \tilde{\Omega}_{\tilde{s}'}$, $\Omega \ll \tilde{\Omega}$. If $\tilde{\Omega} = \Omega$ is expected, controller parameters of literature [10] should satisfy $\tilde{k}_{\min} \gg k_{\min}$. However, high control gain is undesirable in real industrial practice.

Remark 13: If one chooses $\tilde{k}_{\min} = k_{\min}$, $s'(t_0) = \tilde{s}(t_0)$ and make the proposed approach to stabilize TSM like generalized error s' to the same residual set as literature [10], that is $\left\{ s' \left| \|s'\| \le \sqrt{\frac{\bar{m}\varepsilon}{4\underline{m}k_{\min}}} \right. \right\}$, the settling time of the proposed approach is $t_1 = t_0 + \frac{2}{N}\left(\bar{m}\|s'(t_0)\|^2 - \underline{m}\frac{\varepsilon}{4k_{\min}} \right)$. It is obvious that $\frac{2}{N} \ll \frac{2}{\varepsilon}$, then $t_1 \le \tilde{t}_1$. Namely, the proposed approach has faster converging speed than that of literature [10].

The proposed approach employs nonlinear TSM like generalized error and TSM type feedback control, however, literature [10] uses linear sliding mode like generalized error and linear feedback control. Hence, the proposed approach has higher control precision and shorter settling time. The aforementioned analysis has proved these viewpoints. In the next section, these claims will be further validated by an illustrative example.

4. ILLUSTRATIVE EXAMPLE

To validate the proposed approach, SimMechanics 6 DOF parallel robot mode developed by MathWorks was used as a controlled plant. The parameters of 6DOF Stewart Platform were given as: the mass and mass moment of inertia values of upper platform were $m = 1216.9\text{kg}$, $I_X, I_Y(I_Z) = 304.48(608.46)\text{kg}\cdot\text{m}^2$, the mass moment of inertia values of upper and lower part of ith leg were $I_{uX}, I_{uY}(I_{uZ}) = 24.17(0.023)\text{kg}\cdot\text{m}^2$ and $I_{dX}, I_{dY}(I_{dZ}) = 43.02(0.156)\text{kg}\cdot\text{m}^2$, the mass of upper/lower part of ith leg were $(m_u)_i / (m_d)_i = 51.81/92.11\text{kg}$.

Suppose only parameters of upper platform were known, dynamics caused by six legs was considered as system uncertainty in controller design. Six DOF position signals could be measured accurately while velocity measuring signals were polluted by limited band white noise shown by Fig. 1. There were frictions at each actuator joint: $f_{fi} = a_{fi}sign(v_{ei})$, a_f is friction parameter,

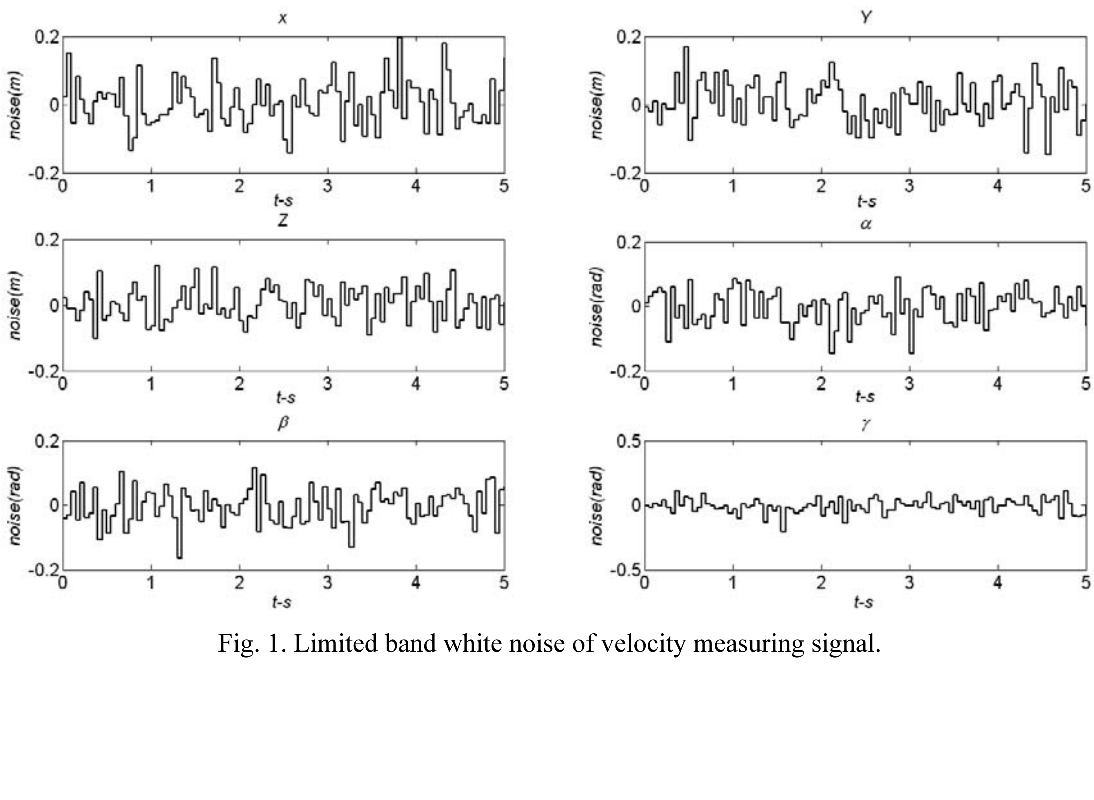

Fig. 1. Limited band white noise of velocity measuring signal.

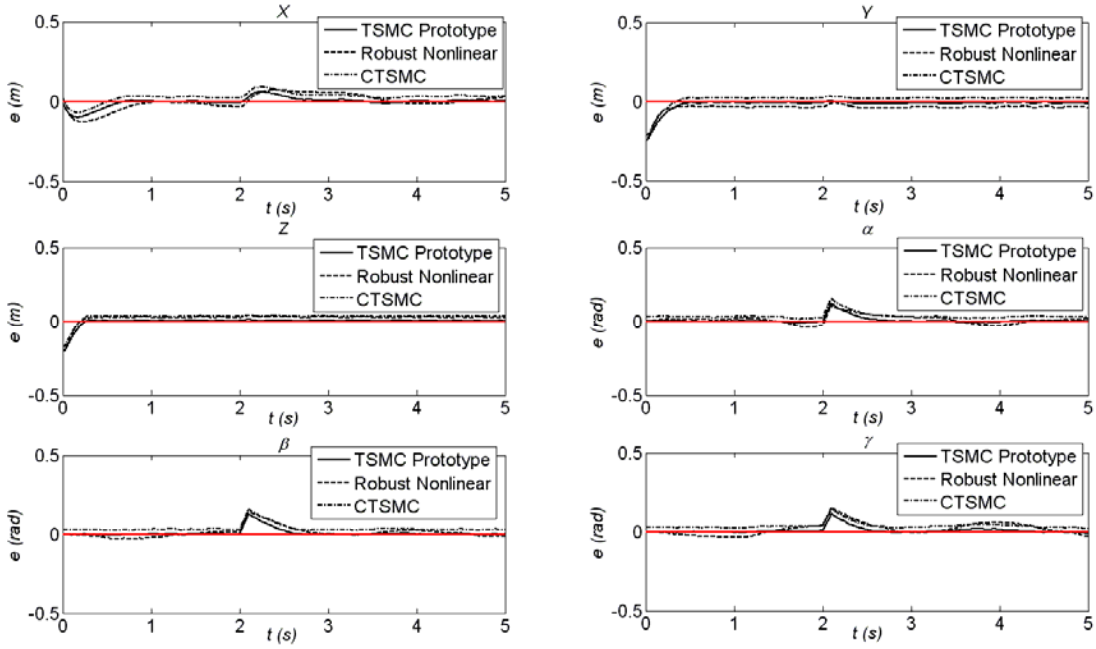

Fig. 2. Position tracking errors of six directions.

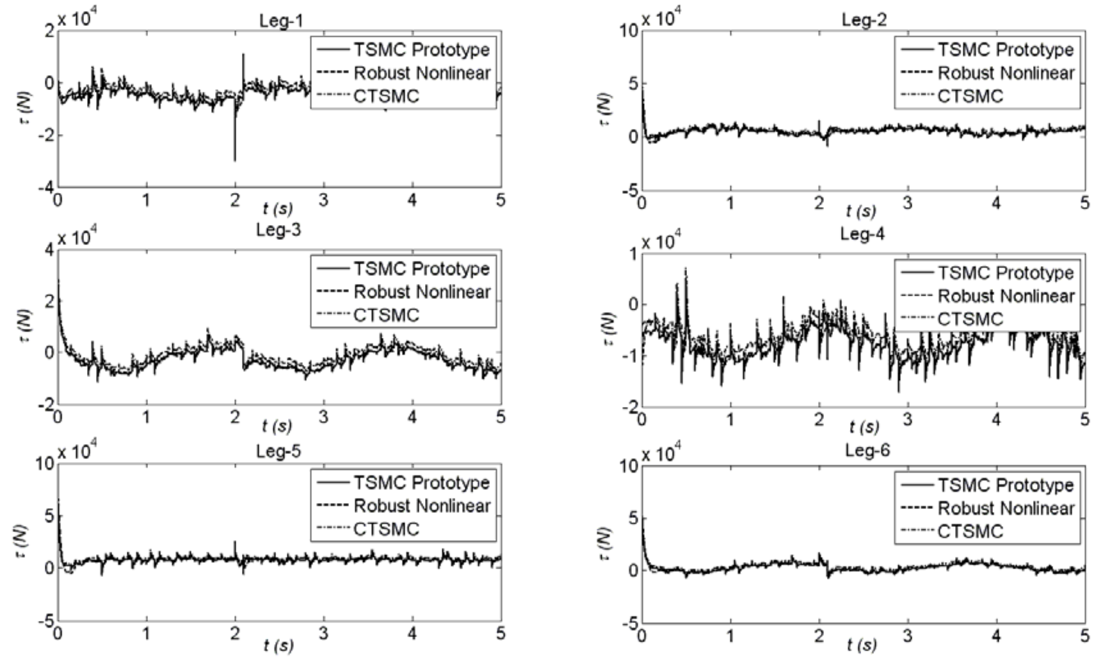

Fig. 3. Control inputs of six actuators.

v_{ei} is velocity. Due to modeling error, suppose only nominal part could be acquired, that is, $\hat{f}_{fi} = \hat{a}_{fi} sign(v_{ei})$, $\hat{f}_{fi}$ and $\hat{a}_{fi}$ were nominal part of f_{fi} and a_f, respectively. In this simulation, a_{fi}=1000 and $\hat{a}_{fi}$=900.

To validate effectiveness of the proposed approach, comparisons between the proposed approaches with the approaches presented by literature [10,15] were presented in this section. Controller parameters of the proposed approach (TSMC Prototype): $\Lambda = diag\{1\}$, $\varepsilon = 0.01$, $\alpha = 0.6$, $\beta = 0.6$, $K_1 = diag\{5000\}$, $K_2 = diag\{5000\}$. Controller parameters of literature [10] (Robust Nonlinear): $\tilde{\Lambda} = diag\{1\}$, $\tilde{K} = diag\{10000\}$, $\varepsilon =$ 0.01. Controller parameters of literature [15]: $\Lambda = diag\{1\}$, $\alpha = 0.6$, $\beta = 0.6$, $K_{CTSMC1} = diag\{5000\}$, K_{CTSMC2}=$diag\{5000\}$.

Remark 14: Compare controller parameters of the proposed approach with literature [10]'s, one can see that except α, β, controller parameters $\Lambda = \tilde{\Lambda}$, $K_1 + K_2 = \tilde{K}$. Compare controller parameters of the proposed approach with literature [15]'s, one can see that except ε, the controller parameters of literature [15] are same

Table 1. ITSE of the three approaches.

ITSE (0~5s)	TSMC Prototype	Robust Nonlinear	CTSMC
X	0.0029	0.0079	0.0051
Y	0.0070	0.0109	0.0063
Z	0.0035	0.0066	0.0043
α	0.0001	0.0016	0.0015
β	0.0001	0.0012	0.0008
γ	0.0002	0.0039	0.0017

as the proposed controller's. It means that these comparisons are fair.

Fig. 2 shows position tracking errors of six directions. Solid line is the proposed approach (TSMC Prototype), dashed line is literature [10]'s approach (Robust nonlinear), dashdotted line is literature [15]'s approach (CTSMC). From this figure, one can see TSMC Prototype can stabilize position tracking error in a smallest residual set with shortest time, which validates effectiveness of the proposed approach. Fig. 3 shows the controller inputs of six actuators, this figure illustrates controller inputs of TSMC Prototype, Robust nonlinear and CTSMC are similar and bounded. Note that high control gains of Robust nonlinear are needed to achieve same control precision as TSMC Prototype does. However, high gain control is unexpected in industrial practice. In theory, if the control inputs are bounded, one can select the actuators such as electromotor according to the bounds of control inputs. In practice, actuators may be selected before the controller designing. In this situation, saturating control approach is required to solve this problem [23,24]. The saturating control approach which considers the actuator dynamics is under the author's research. Any result in this direction will be reported as soon as it is completely developed. To further illustrate the effectiveness of the propose approach, ITSE (0~5) are listed in Table 1. This table also validates the effectiveness of the proposed approach. To valdiate the proposed approach, external force perturbations, that is, $f_{di}(t) = 15000N$, $2\text{s} \le t \le 2.1\text{s}$, $i = 1, \cdots, 6$ were added to the system. From Fig. 2, it can be seen that the tracking performance of Robust nonlinear is affected strongly by the external force perturbation. This comparison also shows that the proposed approach has stronger robustness. From these simulations, one can see that the performances of the proposed approach are better than those of Robust nonlinear and CTSMC. This is attributed to the TSM like generalized error and feedback control law. The detailed explanations are given in Remarks 7 and 8.

5. CONCLUSIONS

This article has studied the issues associated with that of practical stability for task space control of 6 DOF parallel robotic manipulators. In light of TSMC principle and Lyapunov redesign method, a new TSMC prototype control algorithm is developed. Compared with the approaches of literatures [10] and [15], the proposed approach can achieve higher control precision with faster converging speed. Settling time estimation method is presented explicitly. Stability analysis and a numerical example are presented to validate proposed approach. Then, a novel design procedure is developed for parallel robot control, which offers a more effective control approach. It should be mentioned that a series of comprehensive bench tests need to be conducted by simulations and lab demonstrations before applying the approach to control of real parallel robotic manipulators.

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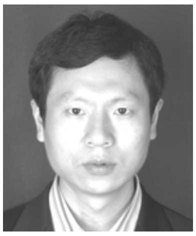

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