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Exponential Stability of Nonlinear Delay Equation with Constant Decay Rate via Perturbed System Method

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Abstract: This paper studies the exponential stability of nonlinear differential equations with constant decay rate under the assumption that the corresponding crisp equation (without delay, simply, non-delay equation) is exponentially stable. Different from most publications dealing with delay systems by applying Lyapunov-type methods, the perturbed system method is used in this paper. It shall be shown that the considered equations will remain exponentially stable provided the time lag is small enough. Moreover, we formulate and estimate the threshold of delay ensuring exponential stability when a constant decay rate appears explicitly in system model, which is better than the existing results.

Keywords: Nonlinear differential equations, time delay, exponential stability, decay rate, perturbed system method.

1. INTRODUCTION

Stability analysis of time-delay systems is of both practical and theoretical importance since time delays are frequently the main cause of instability and poor performance of a system. Recently, a great number of stability results have been proposed using various methods (see, e.g., [1-8], and the cited references therein). Noting that there exist two notions concerning stability of a delay system, namely, delay dependence and delay independence. Generally, delay-dependent conditions contain a prescribed upper bound for the uncertain delay while the delay-independent ones are also applicable when this bound is arbitrary large.

Perturbed system method, which regards the considered delay system as the perturbed system of the corresponding crisp system (without delay), has been used to analyze stability of delay systems (see, e.g., [2,3] where the delay-independent conditions have been proposed; and [3-7] where the authors presented several delay-dependent results). In the direction of this method, Mao [8] with its modified version [9] investigated the exponential stability of general nonlinear delay systems of the form

$$\dot{x}(t) = f(t, x(t), x(t-\tau)) \tag{1}$$

under the assumption of exponential stability of the corresponding crisp system (without delay)

$$\dot{y}(t) = f(t, y(t), y(t)), \tag{2}$$

and an estimated supper bound of delay ensuring exponential stability was formulated.

The aim of this paper is to formulate the supper bound of delay ensuring exponential stability of the following nonlinear delay system

$$\dot{x}(t) = -ax(t) + Af(x(t)) + Bg(x(t-\tau)). \tag{3}$$

As the special case of system (1), this system appears frequently in science, engineering, physics, biology and economics etc. For instance, cellular neural network model with delay belongs to the class described by (3). In fact, if the right-hand side of (1) has the form $F(x(t)) + G(x(t-\tau))$, then (1) can be rewritten as the form of (3), i.e., $\dot{x}(t) = -ax(t) + [F(x(t)) + ax(t)] + G(x(t-\tau))$. The explicit existence of nonzero decay rate often reduces the conservation of the analytical results; see examples in Section 3 of this paper.

2. MAIN RESULTS

We consider the following delay system

$$\begin{aligned} \dot{x}(t) &= -ax(t) + Af(x(t)) + Bg(x(t-\tau)), \ t \ge t_0, \\ x(t) &= \varphi(t-t_0), \quad t_0 - \tau \le t \le t_0, \end{aligned} \tag{4}$$

where $x = [x_1, x_2, \cdots, x_n]^T$ denotes the state vector, $a>0$ is a constant decay rate, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are real matrices. f and g are continuous vector-value functions over R^n with $f(0) = g(0) = 0$. $\varphi = \{\varphi(s) : -\tau \le s \le 0\} \in C([-\tau, 0]; R^n)$. Throughout this paper, we always assume that the functions f and g satisfy the following assumption.

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(H1) There exists positive constants α and β such that

$$|f(x)-f(y)|\le\alpha|x-y|,\quad |g(x)-g(y)|\le\beta|x-y| \quad (5)$$

hold for any $x, y\in R^n$, where $|z|$ denotes the Euclidean norm of a vector z.

The corresponding crisp system associated with (4) is of the form

$$\begin{aligned}&\dot y(t)=-ay(t)+Af(y(t))+Bg(y(t)),\quad t\le t_0,\\ &y(t_0)=\varphi(0).\end{aligned} \quad (6)$$

One can see that under the standing hypothesis (H1) (4) (respectively, (6)) has a unique solution denoted by $x(t;t_0,\varphi)$ on $t\ge t_0-\tau$ (respectively, $y(t;t_0,\varphi(0))$ on $t\ge t_0$). For the purpose of this paper, we propose another standing hypothesis:

(H2) Equation (6) is exponentially stable. That is, there exists a pair of constants K and γ such that

$$\left|y\left(t;t_0,\varphi(0)\right)\right|\le K|\varphi(0)|e^{-\gamma(t-t_0)},\quad \text{for any } t\ge t_0.$$

Theorem 1: Suppose that both assumptions (H1) and (H2) hold. Then, (4) is globally exponentially stable provided $\tau<\min\{0.5\delta,\tau^*\}$, where $\tau^*>0$ and $\delta=\gamma^{-1}(\ln K-\ln p)>0$ is the unique positive root to the equation $C_1(\tau^*)-1=0$, in which $p\in(0,1)$ is a free parameter, and

$$\begin{aligned}C_1(\tau)=[&Ke^{-\gamma(\delta-\tau)}+\mu_2\delta+\tau^2\beta\|B\|\mu_1\\ &+2\mu_1a^{-1}e^{-a(\delta-2\tau)}(1-e^{-a\tau})]e^{\mu_2\delta}=1,\end{aligned}$$

where $\mu_1=\beta\|B\|\exp\{(\alpha\|A\|+\beta\|B\|)a^{-1}(1-e^{-2a\delta})\}$, and $\mu_2=\tau\mu_1(a+\alpha\|A\|+\beta\|B\|)$.

Remark 1: Let us define a function $F(\tau)=C_1(\tau)-1$. Because $F(0)=Ke^{-\gamma\delta}-1=p-1<0$ and $F(+\infty)=+\infty$, there exists at least one root to equation $F(\tau)=0$. On the other hand, it is easy to show that $F(\tau)$ is strictly monotonously increasing over $[0,+\infty)$ with respect to τ. Therefore, there exists a unique positive root τ^* to equation $C_1(\tau^*)=1$ and for any $\tau\in[0,\tau^*)$, one sees that $C_1(\tau)<1$.

Proof: The idea of the proof is inspired by the work of Mao's [8]. From (1), it follows that

$$\begin{aligned}e^{at}x(t)=e^{at_0}x(t_0)&+\int_{t_0}^t e^{as}Af(x(s))ds\\ &+\int_{t_0}^t e^{as}Bg(x(s-\tau))ds.\end{aligned}$$

From (3), it follows that

$$\begin{aligned}e^{at}y(t)=e^{at_0}y(t_0)&+\int_{t_0}^t e^{as}Af\left(y(s)\right)ds\\ &+\int_{t_0}^t e^{as}Bg\left(y(s)\right)ds.\end{aligned}$$

Therefore,

$$\begin{aligned}|x(t)-y(t)|&\le\alpha\|A\|\int_{t_0}^t e^{a(s-t)}|x(s)-y(s)|ds\\ &\quad+\beta\|B\|\int_{t_0}^t e^{a(s-t)}|x(s-\tau)-y(s)|ds\\ &\le(\alpha\|A\|+\beta\|B\|)\int_{t_0}^t e^{a(s-t)}|x(s)-y(s)|ds\\ &\quad+\beta\|B\|\int_{t_0}^t e^{a(s-t)}|x(s)-x(s-\tau)|ds.\end{aligned}$$

By means of Gronwall inequality, one sees that

$$\begin{aligned}|x(t)-y(t)|&\le\beta\|B\|a^{-1}\exp\{(\alpha\|A\|+\beta\|B\|)(1-e^{-a(t-t_0)})\}\\ &\times\int_{t_0}^t e^{a(s-t)}|x(s)-x(s-\tau)|ds.\end{aligned}$$

Hence, if $t_0\le t\le t_0+2\delta$,

$$\begin{aligned}|x(t)|&\le|y(t)|\\ &+\beta\|B\|a^{-1}\exp\left\{(\alpha\|A\|+\beta\|B\|)(1-e^{-2a\delta})\right\}\\ &\times\int_{t_0}^t e^{-a(t-s)}|x(s)-x(s-\tau)|ds.\end{aligned} \quad (7)$$

On the other hand, if $t\ge t_0+\tau$, one sees that

$$\begin{aligned}&\int_{t_0+\tau}^t e^{-a(t-s)}|x(s)-x(s-\tau)|ds\\ &\le\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s[(a+\alpha\|A\|)|x(r)|\\ &\quad+\beta\|B\||x(r-\tau)|]dr\\ &=(a+a\|A\|)\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r)|dr\\ &\quad+\beta\|B\|\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r-\tau)|dr.\end{aligned} \quad (8)$$

By changing the order of integration one obtains

Case 1: when $t_0+2\tau\ge t\ge t_0+\tau$,

$$\begin{aligned}&\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r)|dr\\ &\le\tau\left[\int_{t_0}^{t-\tau}|x(r)|dr+\int_{t-\tau}^{t_0+\tau}|x(r)|dr+\int_{t_0+\tau}^t|x(r)|dr\right]\\ &=\tau\int_{t_0}^t|x(r)|dr.\end{aligned}$$

Case 2: when $t\ge t_0+2\tau$,

$$\begin{aligned}&\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r)|dr\\ &\le\int_{t_0+\tau}^t ds\int_{s-\tau}^s|x(r)|dr\le\tau\int_{t_0}^t|x(r)|dr.\end{aligned}$$

Therefore, for any $t\ge t_0+\tau$, we have

$$\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r)|dr\le\tau\int_{t_0}^t|x(r)|dr, \quad (9)$$

and

$$\int_{t_0+\tau}^t e^{-a(t-s)}ds\int_{s-\tau}^s|x(r-\tau)|dr \quad (10)$$

$$\le \tau \int_{t_0}^{t} \left|x(r)\right| dr + \tau^2 \left(\sup_{t_0-\tau\le s\le t_0} \left|x(r)\right|\right).$$

Consequently, substituting (9) and (10) into (8) one obtains that, if $t \ge t_0 + \tau$,

$$\begin{aligned}&\int_{t_0+\tau}^{t} e^{-a(t-s)} \left|x(s) - x(s-\tau)\right| ds \\ &\le \tau\left(a + \alpha\|A\| + \beta\|B\|\right) \int_{t_0}^{t} \left|x(r)\right| dr \\ &\quad + \beta\tau^2 \|B\| \left(\sup_{t_0-\tau\le s\le t_0} \left|x(r)\right|\right).\end{aligned} \tag{11}$$

We now restrict $t_0 - \tau + \delta \le t \le t_0 - \tau + 2\delta$. Substituting (11) into (7) and using hypothesis (H2) one sees that

$$\begin{aligned}\left|x(t)\right| &\le K e^{-\gamma(\delta-\tau)} \left|\varphi(0)\right| \\ &\quad + \beta\|B\| a^{-1} \exp\left\{\left(\alpha\|A\| + \beta\|B\|\right)(1-e^{-2a\delta})\right\} \\ &\quad \times \tau\left(a + \alpha\|A\| + \beta\|B\|\right) \int_{t_0}^{t} \left|x(r)\right| dr \\ &\quad + \tau^2\beta^2 \|B\|^2 a^{-1} \exp\left\{\left(\alpha\|A\| + \beta\|B\|\right)(1-e^{-2a\delta})\right\} \\ &\quad \times \left(\sup_{t_0-\tau\le r\le t_0} \left|x(r)\right|\right) \\ &\quad + \beta\|B\| a^{-1} \exp\left\{\left(\alpha\|A\| + \beta\|B\|\right)(1-e^{-2a\delta})\right\} \\ &\quad \times \int_{t_0}^{t_0+\tau} e^{-a(t-s)} \left|x(s) - x(s-\tau)\right| ds.\end{aligned} \tag{12}$$

Note also that

$$\begin{aligned}\int_{t_0}^{t} |x(r)| dr &= \int_{t_0}^{t_0-\tau+\delta} |x(r)| dr + \int_{t_0-\tau+\delta}^{t} |x(r)| dr \\ &\le \delta\left(\sup_{t_0\le r\le t_0-\tau+\delta} |x(r)|\right) + \int_{t_0-\tau+\delta}^{t} |x(r)| dr,\end{aligned}$$

and

$$\begin{aligned}&\int_{t_0}^{t_0+\tau} e^{-a(t-r)} |x(r) - x(r-\tau)| dr \\ &\le 2a^{-1} e^{-a(\delta-2\tau)} (1-e^{-a\tau}) \left(\sup_{t_0-\tau\le r\le t_0+\tau} |x(r)|\right).\end{aligned}$$

Substituting these into (12), one obtains that, for $t_0 - \tau + \delta \le t \le t_0 - \tau + 2\delta$,

$$\begin{aligned}\left|x(t)\right| &\le \Big[K e^{-\gamma(\delta-\tau)} + \mu_2\delta + \tau^2\beta\|B\|\mu_1 \\ &\quad + 2\mu_1 a^{-1} e^{-a(\delta-2\tau)} (1-e^{-a\tau})\Big] e^{\mu_2\delta} \\ &\quad \left(\sup_{t_0-\tau\le r\le t_0-\tau+\delta} \left|x(r)\right|\right) \\ &\equiv C_1(\tau) \left(\sup_{t_0-\tau\le r\le t_0-\tau+\delta} \left|x(r)\right|\right).\end{aligned}$$

Note that $C_1 < 1$ since $\tau < \tau^*$. Write $C_1 = e^{-\varepsilon\delta}$ with $\varepsilon = -\frac{1}{\delta}\ln C_1$. It then follows from (12) that

$$\sup_{t_0-\tau+\delta\le t\le t_0-\tau+2\delta} \left|x(t; t_0, \varphi)\right| \le e^{-\varepsilon\delta} \left(\sup_{t_0-\tau\le s\le t_0-\tau+\delta} \left|x(s)\right|\right), \tag{13}$$

holds for any $t_0 \ge 0$ and $\varphi \in C([-\tau, 0], R^n)$.

Fixed $t_0 \ge 0$ and $\varphi \in C([-\tau, 0], R^n)$ arbitrarily, and let $k = 1, 2, \cdots$. Denote

$$\begin{aligned}&\hat{x}(t_0 + (k-1)\delta; t_0, \varphi) \\ &\quad = \left\{x(t_0 + (k-1)\delta + s; t_0, \varphi) : -\tau \le s \le 0\right\},\end{aligned}$$

which is regarded as a continuous function.

Note that the solution of (1) has the following flow property: for any $t \ge t_0 + (k-1)\delta$.

$$x(t; t_0, \varphi) = x\left(t; t_0 + (k-1)\delta, \hat{x}\left(t_0 + (k-1)\delta; t_0, \varphi\right)\right).$$

Hence, by (13),

$$\begin{aligned}&\sup_{t_0-\tau+k\delta\le t\le t_0-\tau+(k+1)\delta} \left|x(t; t_0, \varphi)\right| \\ &\quad \le e^{-\varepsilon\delta} \left(\sup_{t_0-\tau+(k-1)\delta\le t\le t_0-\tau+k\delta} \left|x(t; t_0, \varphi)\right|\right).\end{aligned}$$

By mathematical induction,

$$\begin{aligned}&\sup_{t_0-\tau+k\delta\le t\le t_0-\tau+(k+1)\delta} \left|x(t; t_0, \varphi)\right| \\ &\quad \le e^{-\varepsilon k\delta} \left(\sup_{t_0-\tau\le t\le t_0-\tau+\delta} \left|x(t; t_0, \varphi)\right|\right).\end{aligned} \tag{14}$$

On the other hand, it is not difficult to show that there exists a $C_2 > 0$ such that

$$\sup_{t_0-\tau\le t\le t_0-\tau+\delta} \left|x(t; t_0, \varphi)\right| \le C_2 \left(\sup_{-\tau\le s\le 0} \left|\varphi(s)\right|\right). \tag{15}$$

Substituting (15) into (14) yields

$$\sup_{t_0-\tau+k\delta\le t\le t_0-\tau+(k+1)\delta} \left|x(t; t_0, \varphi)\right| \le C_2 e^{-\varepsilon k\delta} \left(\sup_{-\tau\le s\le 0} \left|\varphi(s)\right|\right). \tag{16}$$

Now, for any $t > t_0 - \tau + \delta$, one can find a k such that $t_0 - \tau + k\delta \le t \le t_0 - \tau + (k+1)\delta$ and hence

$$\left|x(t; t_0, \varphi)\right| \le C_2 e^{\varepsilon\delta - \varepsilon(t-t_0)} \left(\sup_{-\tau\le s\le 0} \left|\varphi(s)\right|\right).$$

It is not difficult to show that this holds for any $t_0 \le t \le t_0 - \tau + \delta$ as well. This concludes the proof.

Remark 2: For computational consideration, in order to find the supper bound of delay such that equation (4) is globally exponentially stable provided $\tau < \hat{\tau}$, we suggest the following optimization problem:

$$\text{(P)} \quad \begin{cases} \max \hat{\tau} = \sup\limits_{0<p<1} \left(\min\left\{\dfrac{\delta}{2}, \tau^*\right\}\right) \\ s.t.,\ 1 > p > 0, \quad C_1(\tau^*) = 1, \\ \delta = \gamma^{-1}(\ln K - \ln p) > 0. \end{cases}$$

The above results established for constant-delay case still hold when the time delay is time-varying. More precisely, let $\tau : R^+ \to [0, \overline{\tau}]$ be a Borel measurable function, where $\bar{\tau} > 0$. In this case, equation (4) is rewritten as the form

$$\begin{aligned} &\dot{x}(t) = -ax(t) + Af(x(t)) + Bg(x(t-\tau(t))),\ t \ge t_0, \\ &x(t) = \varphi(t-t_0), \quad t_0 - \bar{\tau} \le t \le t_0, \end{aligned} \tag{17}$$

where $\varphi = \{\varphi(s):\ -\bar{\tau} \le s \le 0\} \in C([-\bar{\tau}, 0]; R^n)$, the matrices A and B, functions f and g are the same as defined in (4). For (17), we have the following result.

Theorem 2: Suppose that both assumptions (H1) and (H2) hold. Then, equation (17) is globally exponentially stable provided $\sup_{t \ge t_0} \tau < \min\{0.5\delta, \tau^*\}$, where δ and τ^* are the same as defined in Theorem 1.

Proof: The proof is similar to that of Theorem 1, and therefore, omitted here.

3. EXAMPLES

In this section, we give two simple examples for illustration.

Example 1: Consider a one-dimensional differential delay equation

$$\begin{aligned} &\dot{x}(t) = -x(t) - 2\sin\left(x(t-\tau)\right), \quad t > t_0, \\ &x(t) = \varphi(t-t_0), \qquad t_0 - \tau \le t \le t_0, \end{aligned} \tag{18}$$

where φ is the same as defined in (4). The corresponding differential equation has the form

$$\begin{aligned} &\dot{y}(t) = -y(t) - 2\sin(y(t)), \quad t > t_0, \\ &y(t_0) = \varphi(0). \end{aligned} \tag{19}$$

It is easy to check that the solution of (19), denoted by $y(t\,; t_0, \varphi(0))$, satisfies

$$\left|y\left(t\,; t_0, \varphi(0)\right)\right| \le |\varphi(0)| e^{-(t-t_0)}, \quad t \ge t_0.$$

Hence, one sees that the standing assumptions (H1) and (H2) are satisfied with $\alpha = 0$, $\beta = 2$, $K = 1$ and $\gamma = 1$.

By solving optimization problem (P) we obtain that when p=0.969 (therefore, $\delta \simeq 0.03149$) and $\tau^* \simeq$ 0.01559 the maximum $\hat{\tau} \simeq 0.01559$. It follows from Theorem 1 that the delay equation (18) remain exponentially stable provided $\tau < 0.01559$. However, if we apply the result in [9], the modified version of [8], the threshold value of the delay ensuring exponential stability will be 0.0093, which is much smaller that our value. Moreover, it is easy to verify that the results in [10-12] are not available for this example.

Example 2: Consider a two-dimensional differential delay equation

$$\begin{aligned} &\dot{x}_1(t) = -2x_1(t) + \sin(x_2(t-\tau)), \\ &\dot{x}_2(t) = -2x_2(t) - 2\sin(x_1(t-\tau)), \quad t > t_0. \end{aligned} \tag{20}$$

The initial value is assumed to be $x(t) = [x_1(t), x_2(t)]^T = \varphi(t-t_0)$ on $t_0 - \tau < t \le t_0$, where $\varphi = \{\varphi(s): -\tau \le s \le 0\} \in C([-\tau, 0]; R^n)$.

The corresponding differential equation has the form

$$\begin{aligned} &\dot{y}_1(t) = -2y_1(t) + \sin(y_2(t)), \\ &\dot{y}_2(t) = -2y_2(t) - 2\sin(y_1(t)) \end{aligned} \tag{21}$$

on $t \ge t_0$ with initial value $y(t_0) = [y_1(t_0),\ y_2(t_0)]^T = \varphi(0)$. It is easy to check that the solution of (21), denoted by $y(t\,; t_0, \varphi(0))$, satisfies $|y(t\,; t_0, \varphi(0))| \le |\varphi(0)| e^{-(t-t_0)},\ t \ge t_0$.

Hence, one sees easily that $K = 1$ and $\gamma = 1$. Note that a=2, $\alpha = 0$, $\beta = 1$, A=0 and $B = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$. Solving Problem (P) yields that p=0.969 (therefore, $\delta \simeq 0.03149$) and $\tau^* \simeq 0.01574$ the maximum $\hat{\tau} \simeq$ 0.01574. Hence, one sees that the delay equation (20) is globally exponentially stable provided $\tau < 0.01574$. However, if we apply the modified result [9] of the reference [8] the threshold value of the delay ensuring exponential stability will be 0.0045574, which is also much smaller that our value.

Example 3: Consider a two-neuron cellular neural network system with delay

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) - 0.5f(x_1(t)) + 0.1f(x_2(t)) \\ \qquad - 0.1f(x_1(t-\tau)) + 0.2f(x_2(t-\tau)), \\ \dot{x}_2(t) = -2x_2(t) + 0.2f(x_1(t)) - 0.1f(x_2(t)) \\ \qquad + 0.2f(x_1(t-\tau)) + 0.1f(x_2(t-\tau)), \end{cases} \tag{22}$$

where $f_i(x) = 0.5(|x+1| - |x-1|),\ i = 1, 2.$

In [12,13], the authors studied the asymptotic stability of the analog of (22) respectively. The upper bounds of delay estimated in [12] and [13] are $\tau^* < 0.17$ and $\tau^* < 0.0279$, respectively.

To estimate the upper bound of delay, let us consider the corresponding crisp system of the form

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) - 0.6f(x_1(t)) + 0.3f(x_2(t)), \\ \dot{x}_2(t) = -2x_2(t) + 0.4f(x_1(t)). \end{cases} \tag{23}$$

We construct the Lyapunov function of the form $V(x) = \|x\|^2$, and estimate its derivative along the solution of (23) as follow

$$\dot{V}(x(t)) \le -2.8219V(x(t)),$$

which implies $V(x(t)) \le V(x(t_0))e^{-2.8219(t-t_0)}$. Therefore, $\|x(t)\| \le \|x(t_0)\| e^{-1.41095(t-t_0)}$. This implies that the exponential convergence rate of (23) is not less than -1.41095. In Theorem 1 of the present paper, letting $\gamma = 1.41095$, we obtain the upper bound of delay is $\tau^* = 0.4886$, which is larger than those in [12,13]. Therefore, our results are less conservative than those given in [12,13]. The numerical simulation of (22) with $\tau = 0.48$ is shown in Fig. 1.

4. CONCLUSIONS

The perturbed-system method has been applied to investigate the preservation of exponential stability of nonlinear delayed equations with constant decay rate. The allowable upper bound of delay has been formulated by an optimization problem that can be solved numerically. Two numerical examples have shown that our results are less conservative than the existing results for case of constant decay rate.

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