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Novel Adaptive Particle Filter Using Adjusted Variance and Its Application

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Abstract: Precise estimation of the position of robots, which is essential in mobile robotics, is difficult to achieve. However, particle filter shows great promise in this area. The number of samples used in this study is closely related to the operation time in particle filtering. The main issue in real-time implementation with regard to particle filter is to reduce the operation time, which led to the development of the adaptive particle filter (APF). We propose a new APF which adjusts the variance and then uses the gradient data to generate samples near the high likelihood region. The experiment results show that the new APF performs better, in terms of the total operation time and sample set size, than the standard particle filter and the APF using Kullback-Leibler distance sampling.

Keywords: Kullback-leibler distance, mobile robot, particle filter, ultrasonic beacon.

1. INTRODUCTION

Navigation is the very challenging competences required of a mobile robot. The navigation challenge for a mobile robot involves executing a course of action to reach its goal position. Success in navigation requires success in the four branches of navigation: perception, localization, cognition, and motion control. Localization of a mobile robot involves determining its position and heading with respect to known locations in the environment. This is an important issue for mobile robot research since it is essential to a mobile robot to ensure long-term reliable operations. One of the most reliable solutions to the localization problem is to design and deploy an active beacon system specifically for the target environment. This is the preferred technique used for both industrial and military applications as a way of ensuring the highest possible reliability of localization [1]. The mathematical treatment of navigation is quite difficult because of the uncertainties associated with the sensors used in the process and the variability of the environment. There are various reasons for the occurrence of errors in the accuracy of localization, one of which is that the sensor information is generally unreliable and contains noise. Wheel slippage is also a reason why such errors occur. With the passage of time, these errors accumulate and the location becomes increasingly inaccurate. Thus, researchers have strived to come up with ways to reduce such errors [2,14].

This paper focuses on localization using active ultrasonic beacon systems placed at known positions in the environment [3]. They use ultrasonic pulses to determine the distance between a robot and beacons and estimate the position of a mobile robot. Although these methods experience random large noise, they have the advantage that errors are not accumulated.

The classic optimal solution for linear system models under Gaussian noises is the well known Kalman Filter [4,5]. However, most real-world problems involve elements of nongaussianity and nonlinearity. Consequently, it is usually impossible to derive a solution based on the Kalman Filter. A suboptimal method, the so called Extended Kalman Filter, has been quite popular in dealing with nonlinear stochastic and measurement models. However, it has a drawback in that this approximation method does not take into account all statistical characteristics of the processes, and hence leads to poor results [6].

In nonlinear state space models with nongaussian noise, the Monte Carlo methods show good performance. Monte Carlo methods are very flexible and so do not require any assumptions about the probability distributions of the data.

Particle filtering is most popular among techniques based on the Monte Carlo method. Particle filtering is a modern Bayesian method based on numerical approximation of posterior distributions [7,8]. The drawback of particle filtering is that it usually requires a large number of samples to produce reliable results. In particle filtering, the smaller the size of the sample set, the smaller the computational cost. Therefore, we should keep the sample set size as small as possible in order to improve the efficiency of particle filtering. An improved form of particle filter is the adaptive particle filter (APF), which controls the sample set size and, at the same time, maintains the precision of an estimate [9]. There are two types of APF: likelihood based adaptation, and Kullback-Leibler Distance (KLD) sampling. In both samplings,

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samples are generated randomly, which results in some of them being generated in the low likelihood region [10]. Such samples prevent the estimates from being accurate. Therefore, in order to improve the accuracy of estimates, we intend to consider two factors: variance and gradient.

We propose a new APF which adjusts the variance and then uses the gradient data to generate samples near a high likelihood region. When samples are generated, the proposed APF calculates the gradient, unlike the standard APF. Thus, the operation time increases. However, as the calculated gradient data is used the estimates are more precise. Therefore, less samples need to be generated, which leads to a drop in the total number of samples and eventually reduces the total operation time. In conclusion, both the sample set size and total operation time are reduced. To show the effectiveness of the proposed adaptive particle filter for the localization of a mobile robot, an experiment in ultrasonic beacon systems is presented.

This paper is organized as follows. Section 2 introduces the general particle filter and APF. For lower computational cost and high accuracy of estimations, the proposed method is presented in Section 3. The experiment setup of the mobile robot with the ultrasonic beacon system is specified in Section 4. The experiment results and the analysis of such results are presented in Section 5. Finally, the conclusion is provided in Section 6.

2. PARTICLE FILTER

2.1. General particle filter

Particle filter represents the required posterior probability density function $p\left(r_{0:t} \mid z_{1:t}\right)$ by a set of random samples with associated weights. *r* denotes a sample, and *z* denotes a measurement. These samples are called 'particles'. The probability assigned to each particle is proportional to the weight [7,9].

In order to explain the details of the algorithm, let $\left\{r_{0:t}^i, w_t^i\right\}_{i=1}^N$ denote a random measure that characterizes the posterior probability density distribution $p\{r_{0:t} \mid z_{1:t}\}$. $\left\{r_{0:t}^i, i=1,\cdots,N\right\}$ is a set of particles with their associated weights, $\left\{w_t^i, i=1,\cdots,N\right\}$. The weights are normalized as $\sum_i w_t^i = 1$. N is the number of samples used in the approximation. The posterior probability density distributions at time *t* can be approximated as

$$p(r_{0:t} \mid z_{1:t}) \approx \sum_{1}^{N} w_t^i \delta(r_{0:t} - r_{0:t}^i), \tag{1}$$

where $\delta(\cdot)$ is the Dirac delta function.

The normalized importance weights w_t^i are chosen using the principle of importance sampling. If samples $r_{0:t}^i$ were drawn from an importance density $q\left(r_{0:t} \mid y_{1:t}\right)$ by the principle of importance sampling, w_t^i is given by

$$w_t^i = \frac{\tilde{w}_t^i}{\sum_{i=1}^{N} \tilde{w}_t^i}, \tag{2}$$

where

$$\tilde{w}_t^i \propto \frac{p(r_{0:t}^i \mid z_{0:t})}{q(r_{0:t}^i \mid z_{0:t})}. \tag{3}$$

According to the Bayes theorem, we can conclude

$$p(r_{0:t} \mid z_{1:t}) \propto p(z_t \mid r_t) p(r_t \mid r_{t-1}) p(r_{0:t-1} \mid z_{1:t-1}). \tag{4}$$

If the importance density is chosen to factorize such that

$$q(r_{0:t} \mid z_{1:t}) \triangleq q(r_t \mid r_{0:t-1}, z_t) q(r_{0:t-1} \mid z_{1:t-1}) \tag{5}$$

then the weight update equation are given by

$$\tilde{w}_t^i = \tilde{w}_{t-1}^i \frac{p(z_t \mid r_t^i) p(r_t^i \mid r_{t-1}^i)}{q(r_t^i \mid r_{0:t-1}^i, z_{1:t})}. \tag{6}$$

The choice of proposal distribution $q(r_t \mid r_{0:t-1}, z_{0:t})$ is one of the most important issues for particle filtering, and has to include the most recent observations. The following equation is chosen to minimize the noise variance:

$$q(r_t \mid r_{0:t-1}, z_{0:t}) = p(r_t \mid r_{0:t-1}, z_{0:t}). \tag{7}$$

For convenience, the most common choice is the prior distribution as follows:

$$q(r_t \mid r_{0:t-1}, z_{0:t}) = p(r_t \mid r_{t-1}). \tag{8}$$

By substituting equations, the weight update equation is simplified as follows:

$$\tilde{w}_t^i \propto \tilde{w}_{t-1}^i \frac{p(z_t \mid r_t^i) p(r_t^i \mid r_{t-1}^i)}{q(r_t^i \mid r_{0:t-1}^i, z_{0:t})} = \tilde{w}_{t-1}^i p(z_t \mid r_t^i). \tag{9}$$

The particle filter described above is called the Sequential Importance Sampling (SIS) [2]. A common problem with the SIS method is that after a few iterations, most particles have negligible weights, called 'degeneracy phenomenon'. The resampling scheme is very important to avoid the degeneracy phenomenon of the particles [9]. Resampling is a method to eliminate particles with small weights and replicate particles with large weights, in which we are interested.

2.2. Adaptive particle filter

In particle filtering, a large number of samples are required to achieve a certain level of accuracy. Many samples are required for two reasons: a) to accurately approximate posterior density of the state over time, and b) to allow the mobile robot to return to its original position if it loses track of its position. However, the larger the size of the sample set, the larger the computational cost. Therefore, in order to reduce the computational cost, that is, to boost the efficiency of the particle filter, an adaptive particle filter is proposed [10]. This method chooses a small number of samples if the mobile robot tracks its position accurately, whereas it chooses a large number of samples if the position of the mobile robot is not tracked accurately. Fox introduced two kinds of APF: likelihood based adaptation and KLD sampling [10].

1) Likelihood based adaptation: In the likelihood based adaptation, a sum of weight is a measure of uncertainty. Samples are generated until the sum of non-normalized likelihood exceeds a pre-specified threshold. If all sampled particles have low likelihood scores, a sum total does not exceed the threshold. This method has the advantage of introducing no overhead in spite of iterative calculation and the calculation being simple, but it shows poorer performance compared to KLD sampling [10].

2) KLD sampling: Fox showed how to determine the number of samples so that the distance between the sample based Maximum Likelihood Estimate (MLE) and the true posterior does not exceed a pre-specified threshold ε [10]. KLD sampling measures the approximation error by Kullback-Leibler distance between the true distribution and its sampled representation. The KLD is a measure of the difference between two probability distributions p and q:

$$K(p,q) = \sum_x p(x)\log\frac{p(x)}{q(x)}. \tag{10}$$

Through numerical equations, we can identify the relationship between the number of samples and the resulting approximation quality. If we choose the number of samples n_x as

$$n_x = \frac{1}{2\varepsilon} X^2_{k-1,1-\delta} \tag{11}$$

then we can guarantee that with probability $1-\delta$, the KLD between the MLE and the true distribution is less than ε. For details, refer to [10].

In order to determine n_x, we need to compute the values of the chi-square distribution. The best solution is given by the Wilson-Hilferty transformation [13], which yields

$$n_x = \frac{1}{2\varepsilon} X^2_{k-1,1-\delta} \cong \frac{k-1}{2\varepsilon}\left\{1-\frac{2}{9(k-1)}+\sqrt{\frac{2}{9(k-1)}}z_{1-\delta}\right\}^3, \tag{12}$$

where $z_{1-\delta}$ is the upper $1-\delta$ value of the standard normal distribution [10]. This method shows excellent performance in practice.

3. NOVEL ADAPTIVE PARTICLE FILTER

3.1. Adaptive particle filter using adjusted variance

In KLD sampling, samples are generated randomly, most of which are generated near the low likelihood region due to their state being highly uncertain. Although samples are generated, they do not contribute to the improvement of the estimates nor reduce the operation time.

We propose a new APF that can lead to effective sampling by adjusting the variance size. The basic idea is to increase variance inversely proportional to the likelihood and generate samples. Such likelihood is correlated to the KLD. The lower the likelihood, the further away the KLD is. If the KLD is large, it means that the true distribution and the sample are far apart from each other. So the variance size has to be increased by a large quantity in order to generate samples near the true distribution. Using the equation described in (12), the number of required samples can be calculated. The adjusted variance is calculated by using the relationship between the maximum number of samples and the number of required samples. The adjusted variance is given by

$$\sigma_{ad} = \sigma_{lb} + \varepsilon \cdot \frac{n_x}{N_{total}}, \tag{13}$$

where σ_{ad} denotes the adjusted variance, σ_{lb} denotes the low bound variance, ε denotes a maximum value of KLD, n_x denotes the number of required samples, and N_{total} denotes the number of total samples. σ_{lb} is an application specific design parameter, and we set $\sigma_{lb} \ll \varepsilon$ in this paper. If the KLD is small, n_x tends to decrease. For example, n_x / N_{total} becomes very small and σ_{ad} will be similar to σ_{lb}. On the contrary, if the KLD is large, n_x / N_{total} will become closer to 1 and σ_{ad} will be similar to ε.

3.2. Generation of samples using gradient

Considering the APF, the number of samples is adapted by state uncertainty. When samples are increasing, they are generated randomly and are distributed widely. New samples have low weights because of a large state uncertainty. Therefore, many new samples are generated that do not satisfy a condition, so an increase in wasted operation time is experienced. Therefore, we propose the novel adaptive particle filter for effective sampling by taking advantage of current observation and its gradient information. The basic idea essentially calculates the gradient information from the probability density function of observation and generates new samples toward the high likelihood region, along the gradient-descent direction.

Consider the following nonlinear state space model

$$\begin{aligned} r_t &= f_t(r_{t-1}, w_t), \\ l_t &= h_t(r_t, v_t), \end{aligned} \tag{14}$$

where r_t denotes the state of the system at time t, l_t denotes the output observation, w_t denotes the process noise, and v_t denotes the measurement noise. l_t is measured by using the difference in transmission speed of the signals sent from the beacon. The state equation in (14) characterizes the state transition probability $p(r_t | r_{t-1})$, whereas the measurement equation in (14) describes the probability $p(l_t | r_t)$ which is related to the measurement noise model. The mapping f and h represent the motion and measurement models. If $p(l_t)$ is a Gaussian probability density function of measurement model, the gradient at the state r_t is given by

$$\left.\frac{\partial p(h(r))}{\partial r}\right|_{r=r_t^i} = \frac{\partial}{\partial r}\left[\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{\left(l_t - h(r)\right)^2}{2\sigma^2}\right\}\right]\Bigg|_{r=r_t^i}, \tag{15}$$

where σ is the variance of a Gaussian probability density function.

Depending on whether the gradient is positive or negative, we can identify the relative location of the current sample between the beacon and the observation. Considering the geometric structure of the ultrasonic beacon system, if the gradient is positive, it means that the distance between the current sample and the beacon is shorter than the distance between the observation and the beacon. In other words, the current sample is nearer to the beacon than the observation. Therefore, new samples should be generated farther than the current sample from the beacon according to the adjusted variance in order for the proposed APF to be efficient. On the other hand, if the gradient is negative, the distance between the current sample and the beacon is longer than the distance between the observation and the beacon. In other words, the current sample is farther away from the beacon than the observation. Therefore, new samples should be generated nearer to the beacon according to the adjusted variance. This may be described as follows:

$$r_t^{i+N} = \begin{cases} r_t^i + \sigma_{ad}\cdot\beta, & if\ (\exists i)\ \left.\dfrac{\partial p(h(r))}{\partial r}\right|_{r=r_t^i} > 0 \\ r_t^i - \sigma_{ad}\cdot\beta, & otherwise. \end{cases} \tag{16}$$

where β represents random numbers whose elements are normally distributed with mean 0 and variance 1. The new r_t^{i+N} calculated above is used to update the weight in (9). As a result, new samples may be generated in the high likelihood region by using this method. This ensures localization to be accurate and reduces operation time due to small sample set size.

4. EXPERIMENT SETUP

4.1. Ultrasonic beacon system

The ultrasonic beacon system is composed of a beacon, which emits ultrasonic waves and RF signals, and a listener, which receives the ultrasonic waves and RF signals sent from the beacon. The beacon is installed on the ceiling or walls of a room and the listener is attached to the mobile robot.

The mobile robot uses the ultrasonic waves and RF signals, received by the listener, to identify the distance between itself and the beacon. The difference in the transmission speed of the ultrasonic waves and RF signals is utilized in figuring out such distance. At first, the listener receives the RF signals, which are transmitted faster than the ultrasonic waves, followed by the ultrasonic waves. Then, the mobile robot calculates the distance between itself and the beacon using the time difference of which the signal and wave were received. Finally, the mobile robot utilizes such distance data to estimate its position.

4.2. Modeling

(X_1, Y_1) coordinates are absolute coordinates based on the robot's external position. (X_2, Y_2) coordinates are local coordinates in which the heading direction of the robot is X_2-axis. (x, y) is the center of the mobile robot. θ is the heading angle. In the real world, error occurs due to wheel slippage and the rough surface of the ground. Assuming that the mobile robot moves in a two dimensional space, the motion model is given as follows:

$$\begin{aligned} r_{t+1} &= f(r_t, u_t, w_t) \\ \begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \end{bmatrix} &= \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} \cos\theta_t & 0 \\ \sin\theta_t & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} d_t \\ \Delta\theta_t \end{bmatrix} + w_t, \end{aligned} \tag{17}$$

r_t is a measurement and we set $r_t = (x_t, y_t, \theta_t)$, where (x_t, y_t) is a position vector and θ_t is a heading angle at time t. $u_t = (d, \Delta\theta_t)$ represents input signal, and w_t is independent noise. The measurement equation represents sensor uncertainty using probability. The measurement equation is given as follows:

$$\begin{aligned} l_t &= h(r_t, v_t) \\ &= \left[(x_t - x_B)^2 + (y_t - y_B)^2 + (z_t - z_B)^2\right]^{1/2} + v_t \end{aligned} \tag{18}$$

where v_t is mutually independent noise, and (x_B, y_B, z_B) is position of beacons.

4.3. Parameter set up

The operation time of particle filtering is important for localization in real time and is directly linked to the number of samples. To demonstrate the relationship between the number of samples and operation time, we experimented with the localization of mobile robots using a particle filter with fixed samples. We set the maxi-

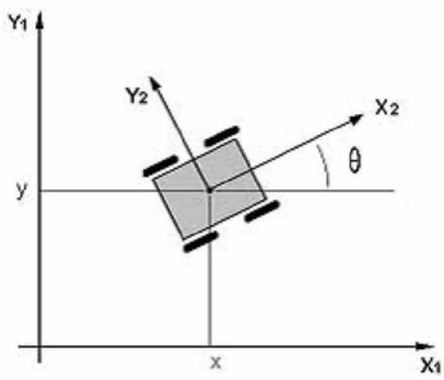

Fig. 1. Coordinates of mobile robot.

mum number of APF samples taking such results into account, and then performed an experiment on localization using the standard APF and the proposed APF. Finally, we compared the experiment results in terms of Root Mean Square Error (RMSE), total samples, and operation time. In the experiment, we set the robot's speed at 30cm/s and angle variation at $6°/s$. σ_{lb} is determined based on the result of the experiment. Each covariance of noise is set up as follows:

$$P_{w_t} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & pi/180 \end{bmatrix}, \quad P_{v_t} = [2].$$

5. EXPERIMENTAL RESULTS

5.1. Localization using particle filter with fixed samples

A particle filter with various samples is used to identify the relationship between efficiency and accuracy. The accuracy of the particle filter is measured by RMSE between actual and estimated state. Fig. 2 shows the result of localization.

The more samples are used in a particle filter, the more the state is accurately estimated. However, the operation time increases in proportion to the number of samples, and the performance of the particle filter is closely related to the number of samples. We found out that the accuracy of a particle filter with more than 20,000 samples was similar to that of a particle filter with 20,000 samples. Given this, it was deemed sufficient to represent the posterior density function using a particle filter with 20,000 samples. Therefore, we set the maximum number of samples at 20,000 for APF.

5.2. Localization using APF

5.2.1 Set up of σ_{lb}

Table 1 shows the localization accuracy and operation time with respect to the changes in σ_{lb}. Setting the σ_{lb} smaller than the initial variance $(\sigma_{lb} = 5)$ tends to result in better performance in terms of the sample set size, RMSE, and operation time. For example, when the σ_{lb} is set at 0.5, the sample set size and operation time show the best performance. Thus, we set the σ_{lb} at 0.5 and conducted the experiment.

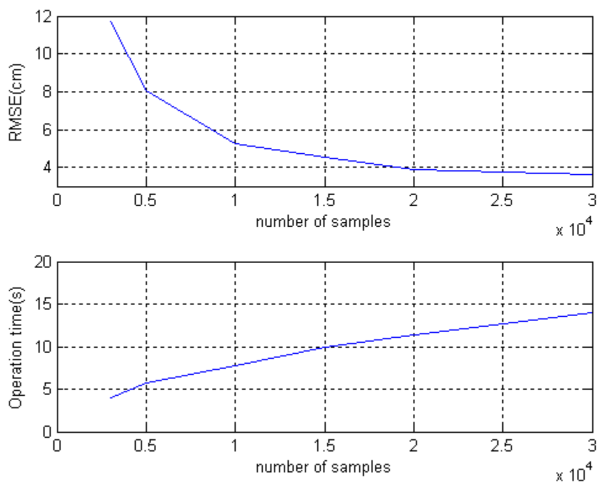

Fig. 2. RMSE and Operation time by number of samples.

5.2.2 Comparison of performance: particle filter, standard APF and proposed APF

Experimental results are shown in Figs. 3 through 7. Figs. 3, 4 and 5 show the useful samples generated, where ∘ represents the true motion of a mobile robot and ∗ represents the useful samples generated using PF, APF

Table 1. Localization accuracy and operation time vs σ_{lb}.

σ_{lb}	Sample set size	RMSE (cm)	Operation time (s)
5	5431	4.846	4.153
3	5254	4.636	3.972
1	5071	4.557	3.811
0.5	4863	4.412	3.706
0	5017	4.434	3.799

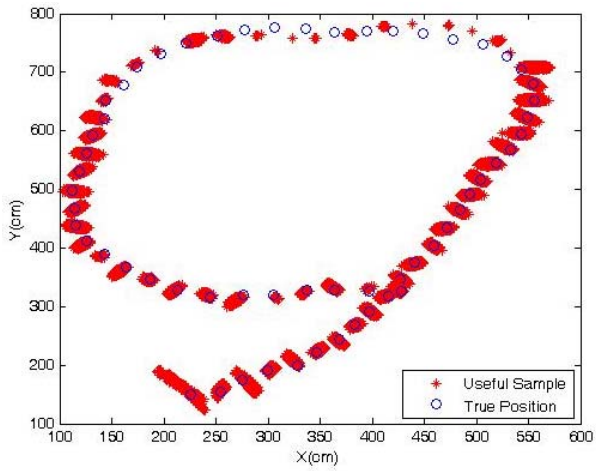

Fig. 3. The useful samples generated using the PF (5000 samples per a step).

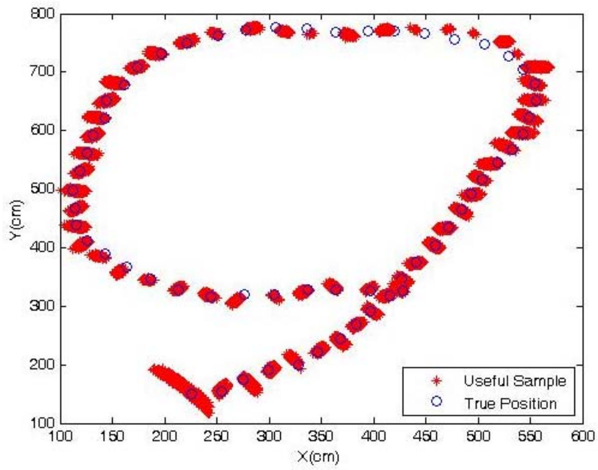

Fig. 4. The useful samples generated using the standard APF.

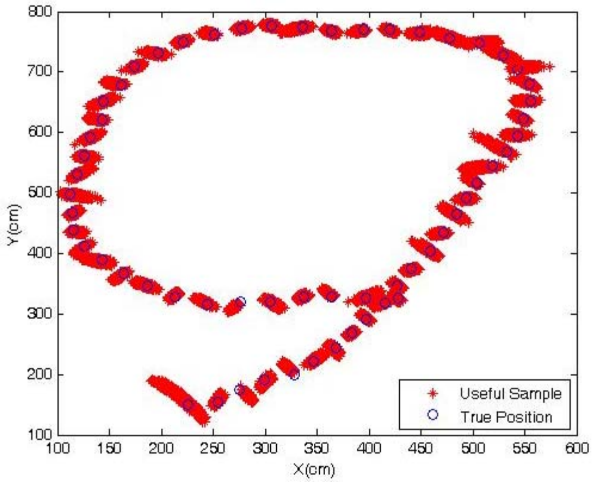

Fig. 5. The useful samples generated using the proposed APF.

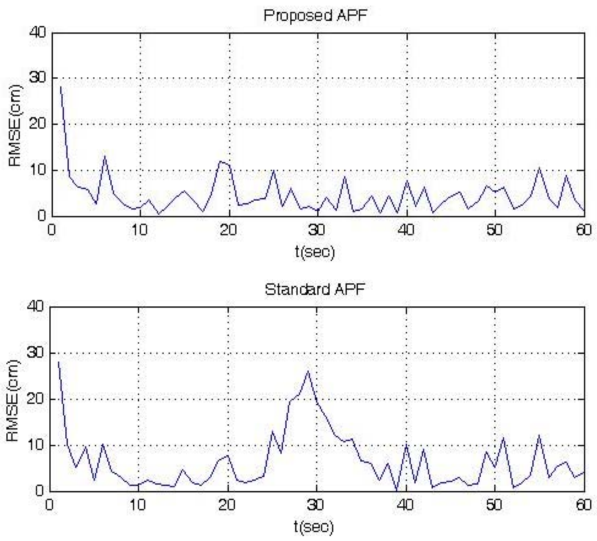

Fig. 6. RMSE of standard APF and proposed APF.

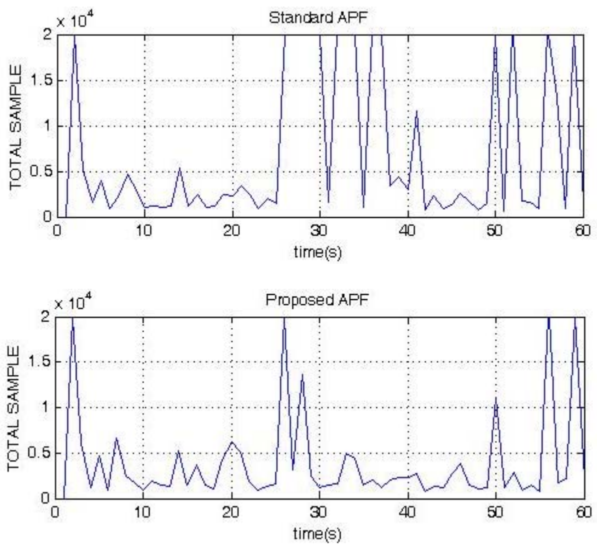

Fig. 7. Total samples using standard APF and proposed APF.

Table 2. Localization accuracy and operation time.

	Total Samples	RMSE (cm)	Operation time (s)
PF (fixed samples)	3,000*60	11.239	9.422
	5,000*60	7.562	13.890
	10,000*60	4.823	25.110
	20,000*60	3.849	47.031
Standard APF	409,238	6.479	8.141
Proposed APF	228,890	4.549	6.422
Improvement	78.8%	42.4%	26.8%

and the proposed method, respectively. The results of the PF with the 5000 fixed samples are shown in Fig. 3. In this case, though many more samples are used in total localization than the proposed methods, its performance is poor in a corner. Moreover, we see that the proposed method is similar to the standard APF when the robot goes straight, but the proposed method is better when the robot turns a corner as shown in Figs. 4 and 5.

Fig. 6 shows the estimate errors on each case. Fig. 7 shows the results when the proposed APF with adjusted variance was compared to the standard APF in terms of the total number of samples. Moreover, Table 2 represents the RMSE of robot states and the elapsed CPU time per unit of PF, APF and the proposed APF method, which indicate the localization accuracy and operation time, respectively.

We can clearly confirm that the total number of samples using the proposed APF is much smaller than that of the standard APF. In addition, we can greatly reduce the operation time. The proposed method is better when the robot turns a corner and Table 2 shows that the improvements of RMSE and operation time are 42.4% and 26.8%, respectively.

6. CONCLUSIONS

In this paper, we proposed a new APF by adjusting the variance and using gradient. This algorithm generates new samples in the high likelihood region as uncertainty increases. Hence the proposed method is more accurate and reduces computational costs. The approach has been implemented and evaluated on the localization of a mobile robot in an ultrasonic beacon system. From the results of the experiment, the proposed APF was found to show better performance in terms of sample set size, RMSE and total operation time.

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