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Improved $L_2 - L_\infty$ Filtering for Stochastic Time-delay Systems

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Abstract: The $L_2 - L_\infty$ filtering problem for continuous-time polytopic uncertain stochastic time-delay systems is investigated. The main purpose is to design a full-order filter guaranteeing a prescribed $L_2 - L_\infty$ attenuation level for the filtering error system. A simple alternative proof is given for an enhanced LMI (linear matrix inequality) representation of $L_2 - L_\infty$ performance. Based on the criterion which keeps Lyapunov matrices out of the products of system dynamic matrices, a sufficient condition for the existence of a robust estimator is formulated in terms of LMIs. The corresponding filter design is cast into a convex optimization problem. A numerical example is employed to demonstrate the feasibility and advantage of the proposed design.

Keywords: $L_2 - L_\infty$ performance, LMI, stochastic systems, time-delay.

1. INTRODUCTION

Over the past decades, considerable attention has been devoted to state estimating problem, and many significant results have been reported in literature. When a priori information on the external noises is not precisely known, the celebrated Kalman filtering scheme is no longer applicable. In such cases, we can resort to H_∞ filtering [1-4] and energy-to-peak ($L_2 - L_\infty$) filtering [5-8]. The $L_2 - L_\infty$ performance was first proposed in [9]. As is mentioned in [5] and [7], the objective of the $L_2 - L_\infty$ filter design is to minimize the peak value of the estimation error for all possible bounded energy disturbance. Time-delays are often encountered in various engineering systems such as long transmission line, chemical processes and nuclear reactors, and so on. The characteristics of dynamic systems can be significantly affected by the presence of uncertainties and time-delays, even to the extent of instability or poor performance [10-12]. Reference [7] proposed the $L_2 - L_\infty$ filter design approaches for polytopic uncertain systems with multiple time-varying state delays. Delay-independent and dependent sufficient conditions for the design of $L_2 - L_\infty$ filter are presented, respectively.

The analysis and design of estimators for systems with stochastic uncertainties have recently received much attention [13,14]. Reference [14] presents the reduced-order H_∞ estimator for stochastic systems with exactly known matrices, and the full-order H_∞ filtering problem has also been solved for systems with both stochastic uncertainties and polytopic uncertain parameters in [15]. To the best of the authors' knowledge, however, up to now, few papers have been made to extend these results to the case of time-delayed stochastic systems, whether with or without parameter uncertainties. Very recently, [6] considered the $L_2 - L_\infty$ filter design problem for uncertain stochastic time-delay systems. Sufficient conditions are formulated in terms of LMIs, which contain the products of system matrices and Lyapunov matrices.

In this paper, we investigate the robust $L_2 - L_\infty$ filtering problem for continuous-time polytopic uncertain stochastic system with time-varying delay based on [6] and so-called "Small Scalar Method" [10,16]. The paper is organized as follows. Section 2 states the class of stochastic time-delay systems for which the filter will be designed. Section 3 gives a new $L_2 - L_\infty$ performance criterion, which exhibits a kind of decoupling between Lyapunov matrices and system dynamic matrices by introducing slack matrices. Section 4 presents a sufficient condition for the existence of the robust $L_2 - L_\infty$ estimator in terms of LMIs based on the preliminary formulation of Section 3. A simulation example is used to illustrate the procedure and performance of the proposed approach in Section 5, which is followed by conclusions in Section 6.

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2. PROBLEM DESCRIPTION

Consider the following stochastic system with time-varying delay:

$$\begin{cases} dx(t) = \left[Ax(t) + A_d x(t-d(t)) + Bw(t)\right]dt \\ \qquad + \left[Mx(t) + M_d x(t-d(t))\right]dv(t) \\ dy(t) = \left[Cx(t) + C_d x(t-d(t)) + Dw(t)\right]dt \\ \qquad + \left[Nx(t) + N_d x(t-d(t))\right]dv(t) \\ z(t) = Lx(t) \\ x(t) = \phi(t), t \in \left[-\bar{d},\, 0\right], \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $y(t) \in \mathbf{R}^m$ is the measurement output, and $z(t) \in \mathbf{R}^p$ is the signal to be estimated, $w(t) \in \mathbf{R}^l$ is the disturbance input which belongs to $L_2[0,\infty)$, and $v(t)$ is a one-dimensional Brownian motion satisfying $\mathbf{E}\{dv(t)\} = 0$, $\mathbf{E}\{dv(t)^2\} = dt$. In addition, $d(t)$ is a time-varying delay satisfying $0 < d(t) \le \bar{d} < \infty$, $\dot{d}(t) \le \tau < 1$, where $\bar{d}$ and τ are real constant scalars, $\phi(t)$ is a real-valued initial vector function that is continuous on the interval $[-\bar{d}, 0]$.

Assumption 1: The state-space data is assumed to be subject to uncertainties in the form of a polytopic model

$$\begin{bmatrix} A & C & M & N \\ A_d & C_d & M_d & N_d \\ B & D & L & 0 \end{bmatrix} \in \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_i & C_i & M_i & N_i \\ A_{di} & C_{di} & M_{di} & N_{di} \\ B_i & D_i & L_i & 0 \end{bmatrix}, \tag{2}$$

where $\alpha \in \Gamma$ and

$$\Gamma := \left\{ (\alpha_1, \alpha_2, \cdots, \alpha_r) : \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \ge 0 \right\}.$$

Consider an estimator or filter described by

$$\begin{cases} dx_F(t) = A_F x_F(t) + B_F dy(t) \\ z_F(t) = C_F x_F(t), \end{cases} \tag{3}$$

where $x_F(t) \in \mathbf{R}^n$ is the filter state vector and A_F, B_F, C_F are appropriately dimensioned filter matrices to be determined.

Augmenting the model (1) to include the states of the filter, we obtain the following filtering error system:

$$\begin{cases} d\xi(t) = \left[\bar{A}\xi(t) + \bar{A}_d \xi(t-d(t)) + \bar{B}w(t)\right]dt \\ \qquad + \left[\bar{M}\xi(t) + \bar{M}_d \xi(t-d(t))\right]dv(t) \\ e(t) = \bar{C}\xi(t) \\ \xi(t) = \left[\phi^T(t) \quad 0\right]^T, \; t \in \left[-\bar{d}, 0\right], \end{cases} \tag{4}$$

where

$$\xi(t) = \left[x^T(t) \quad x_F^T(t)\right]^T, \quad e(t) = z(t) - z_F(t),$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_F C & A_F \end{bmatrix}, \; \bar{A}_d = \begin{bmatrix} A_d & 0 \\ B_F C_d & 0 \end{bmatrix}, \; \bar{B} = \begin{bmatrix} B \\ B_F D \end{bmatrix},$$

$$\bar{M} = \begin{bmatrix} M & 0 \\ B_F N & 0 \end{bmatrix}, \; \bar{M}_d = \begin{bmatrix} M_d & 0 \\ B_F N_d & 0 \end{bmatrix}, \; \bar{C} = \begin{bmatrix} L & -C_F \end{bmatrix}.$$

Before presenting the main objective of this paper, we first introduce the following definitions for the filtering error system (4), which will be essential for our derivation.

Definition 1: The filtering error system (4) with $w(t) = 0$ is said to be mean-square stable if for any $\varepsilon > 0$ there is a $\delta(\varepsilon) > 0$ such that $E\left\{|\xi(t)|^2\right\} < \varepsilon, \quad t > 0$ when $\sup \; E\left\{|\phi(s)|^2\right\} < \delta(\varepsilon) \; (-\bar{d} \le s \le 0)$. In addition, if $\lim_{t\to\infty} E\left\{|\xi(t)|^2\right\} = 0$ for any initial conditions, then the filtering error system (4) is said to be mean-square asymptotically stable.

Definition 2: Given a scalar $\gamma > 0$, the filtering error system (4) is said to be mean-square asymptotically stable with an $L_2 - L_\infty$ disturbance attenuation γ if it is mean-square asymptotically stable and under zero initial conditions, $\|e\|_{E_\infty} < \gamma \|w\|_2$ for all nonzero $w \in L_2[0,\infty)$ where $\|e\|_{E_\infty} = \sup_t \sqrt{E\left\{|e(t)|^2\right\}}$.

Our objective is to develop a robust filter of form (3) such that for all admissible uncertainties and time-delays, the filtering error system (4) is mean-square asymptotically stable with an $L_2 - L_\infty$ disturbance attenuation level γ. Filters guaranteeing such a performance are called $L_2 - L_\infty$ filters or energy-to-peak filters.

Throughout this paper, we make the following assumption.

Assumption 2: System (1) is mean-square asymptotically stable.

Remark 1: The original system to be estimated has to be mean-square asymptotically stable, which is a prerequisite for the filtering error system (4) to be mean-square asymptotically stable.

3. FILTERING ANALYSIS

In this section, an improved LMI criterion for $L_2 - L_\infty$ performance analysis is presented. As a preliminary, we first provide an existing one.

Lemma 1 [6]**:** Consider system (1) and suppose that the filter matrices (A_F, B_F, C_F) in (3) are given. Then the filtering error system (4) is mean-square asymptotically stable with an $L_2 - L_\infty$ disturbance attenuation level bound γ if there exist $0 < P \in \mathbf{R}^{2n\times 2n}$ and $0 < Q \in \mathbf{R}^{2n\times 2n}$ satisfying

$$\Psi = \begin{bmatrix} -P & P\bar{M} & P\bar{M}_d & 0 \\ \bar{M}^T P & P\bar{A}+\bar{A}^T P+Q & P\bar{A}_d & P\bar{B} \\ \bar{M}_d^T P & \bar{A}_d^T P & -(1-\tau)Q & 0 \\ 0 & \bar{B}^T P & 0 & -I \end{bmatrix} < 0, \quad (5)$$

$$\begin{bmatrix} P & \bar{C}^T \\ \bar{C} & \gamma^2 I \end{bmatrix} > 0. \quad (6)$$

By some algorithmic transformation and techniques, we can obtain the following conclusion.

Theorem 1: Consider system (1) and suppose that the filter matrices (A_F, B_F, C_F) in (3) are given. Then the filtering error system (4) is mean-square asymptotically stable with an L_2-L_∞ disturbance attenuation level bound γ if there exist $0<P\in\mathbf{R}^{2n\times 2n}$ and $0<Q\in\mathbf{R}^{2n\times 2n}$ and matrices $F,G,V\in\mathbf{R}^{2n\times 2n}$ satisfying (6) and

$$\begin{bmatrix} P-V-V^T & * & * & * & * \\ \bar{M}^T V & F^T\bar{A}+\bar{A}^T F+Q & * & * & * \\ 0 & P-F+G^T\bar{A} & -G-G^T & * & * \\ \bar{M}_d^T V & \bar{A}_d^T F & \bar{A}_d^T G & -(1-\tau)Q & * \\ 0 & \bar{B}^T F & \bar{B}^T G & 0 & -I \end{bmatrix} < 0. \quad (7)$$

Proof: We need only to prove that (7) is equivalent to (5). If (5) holds, due to the strictness of it, there exists a positive scalar θ satisfying

$$\Psi + \frac{\theta}{2}\begin{bmatrix} 0 \\ \bar{A}^T \\ \bar{A}_d^T \\ \bar{B}^T \end{bmatrix}\begin{bmatrix} 0 & \bar{A} & \bar{A}_d & \bar{B} \end{bmatrix} < 0.$$

By the Schur Complement Lemma, the above relation is equivalent to

$$\begin{bmatrix} -P & * & * & * & * \\ \bar{M}^T P & P\bar{A}+\bar{A}^T P+Q & * & * & * \\ 0 & \theta\bar{A} & -2\theta I & * & * \\ \bar{M}_d^T P & \bar{A}_d^T P & \theta\bar{A}_d^T & -(1-\tau)Q & * \\ 0 & \bar{B}^T P & \theta\bar{B}^T & 0 & -I \end{bmatrix} < 0. \quad (8)$$

Selecting $F=F^T=P$ and $G=G^T=\theta I$, we can obtain

$$\begin{bmatrix} -P & * & * & * & * \\ \bar{M}^T P & F^T\bar{A}+\bar{A}^T F+Q & * & * & * \\ 0 & P-F+G^T\bar{A} & -G-G^T & * & * \\ \bar{M}_d^T P & \bar{A}_d^T F & \bar{A}_d^T G & -(1-\tau)Q & * \\ 0 & \bar{B}^T F & \bar{B}^T G & 0 & -I \end{bmatrix} < 0. \quad (9)$$

In addition, since

$$T = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & \bar{A}^T & 0 & 0 \\ 0 & 0 & \bar{A}_d^T & I & 0 \\ 0 & 0 & \bar{B}^T & 0 & I \end{bmatrix}$$

has full row rank, pre- and post-multiplying the both sides of (9) by T and its transpose, respectively, gives (5). So conditions (9) and (5) are equivalent. In the following, we will show the equivalence between conditions (9) and (7).

By selecting $V=V^T=P$ in condition (9), we can obtain (7). On the other hand, if condition (7) holds, we have $V+V^T-P>0$. Therefore, V is a singular matrix. In addition, by $(V-P)^T P^{-1}(V-P)\geq 0$, we can obtain $V^T P^{-1} V \geq V+V^T-P$. Then we can infer from (7) that

$$\begin{bmatrix} -V^T P^{-1} V & * & * & * & * \\ \bar{M}^T V & F^T\bar{A}+\bar{A}^T F+Q & * & * & * \\ 0 & P-F+G^T\bar{A} & -G-G^T & * & * \\ \bar{M}_d^T V & \bar{A}_d^T F & \bar{A}_d^T G & -(1-\tau)Q & * \\ 0 & \bar{B}^T F & \bar{B}^T G & 0 & -I \end{bmatrix} < 0.$$

Performing congruent transformation to the above inequality by $J=\mathrm{diag}(PV^{-T}, I, I, I, I)$ yields (9). Based on the above deduction, it is known that conditions (7) and (5) are equivalent, which completes this proof.

Remark 2: The above theorem exhibits the separation property between system matrices and Lyapounov matrices with the help of introduced additional matrices.

4. FILTERING SYNTHESIS

In the following, we will focus on the design of L_2-L_∞ filters in the form (3) based on Theorem 1, that is, to determine the filter matrices (A_F, B_F, C_F) which will guarantee the filtering error system (4) to be mean-square asymptotically stable with an L_2-L_∞ disturbance attenuation performance. It may not be directly applicable to L_2-L_∞ filtering design due to the presence of the products of F,G with $\bar{A}$, $\bar{A}_d$, $\bar{B}$, and V with $\bar{M}$, $\bar{M}_d$. To enable the sub-optimal L_2-L_∞ filtering design, the matrices are specialized as

$$F=\Lambda G, \quad V=\Upsilon G,$$

where $\Lambda=\mathrm{diag}(\lambda_1 I_n, \lambda_2 I_n)$, $\Upsilon=\mathrm{diag}(\nu_1 I_n, \nu_2 I_n)$ with $\lambda_1,\lambda_2,\nu_1$ and ν_2 being real scalars. Using the above F,V, (7) can be rewritten as

$$\begin{bmatrix} P-\Upsilon G-G^T\Upsilon & * & * & * & * \\ \bar{M}^T\Upsilon G & G^T\Lambda\bar{A}+\bar{A}^T\Lambda G+Q & * & * & * \\ 0 & P-\Lambda G+G^T\bar{A} & -G-G^T & * & * \\ \bar{M}_d^T\Upsilon G & \bar{A}_d^T\Lambda G & \bar{A}_d^T G & -(1-\tau)Q & * \\ 0 & \bar{B}^T\Lambda G & \bar{B}^T G & 0 & -I \end{bmatrix}<0.$$

The following theorem provides sufficient conditions for the existence of such L_2-L_∞ filters for system (1).

Theorem 2: Consider system (1). An admissible L_2-L_∞ filter of the form (3) exists if there exist positive definite matrices $P_{11}\in\mathbf{R}^{n\times n}$, $P_{22}\in\mathbf{R}^{n\times n}$, $Q_{11}\in\mathbf{R}^{n\times n}$, $Q_{22}\in\mathbf{R}^{n\times n}$, matrices $P_{12}\in\mathbf{R}^{n\times n}$, $Q_{12}\in\mathbf{R}^{n\times n}$, $X\in\mathbf{R}^{n\times n}$, $R\in\mathbf{R}^{n\times n}$, $U\in\mathbf{R}^{n\times n}$, $\bar{A}_F\in\mathbf{R}^{n\times n}$, $\bar{B}_F\in\mathbf{R}^{n\times m}$, $\bar{C}_F\in\mathbf{R}^{m\times n}$ and scalars $\lambda_1,\lambda_2,\nu_1,\nu_2$ satisfying LMIs (10-13).

$$\begin{bmatrix} \Omega_{11} & * & * & * & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * & * & * & * \\ \Omega_{31} & \nu_1 M^T R & \Omega_{33} & * & * & * & * & * & * \\ \Omega_{41} & \nu_1 M^T R & \Omega_{43} & \Omega_{44} & * & * & * & * & * \\ 0 & 0 & \Omega_{53} & \Omega_{54} & -X-X^T & * & * & * & * \\ 0 & 0 & \Omega_{63} & \Omega_{64} & -R^T-X-U^T & -R-R^T & * & * & * \\ \Omega_{71} & \nu_1 M_d^T R & \Omega_{73} & \lambda_1 A_d^T R & A_d^T X+C_d^T\bar{B}_F^T & A_d^T R & -(1-\tau)Q_{11} & * & * \\ \Omega_{81} & \nu_1 M_d^T R & \Omega_{83} & \lambda_1 A_d^T R & A_d^T X+C_d^T\bar{B}_F^T & A_d^T R & -(1-\tau)Q_{12}^T & -(1-\tau)Q_{22} & * \\ 0 & 0 & \Omega_{93} & \lambda_1 B^T R & B^T X+D^T\bar{B}_F^T & B^T R & 0 & 0 & -I \end{bmatrix}<0, \quad (10)$$

$$\begin{bmatrix} P_{11} & P_{12} & L^T \\ P_{12}^T & P_{22} & L^T-C_F^T \\ L & L-C_F^T & \gamma^2 I \end{bmatrix}>0, \quad (11)$$

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}>0, \quad (12)$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}>0, \quad (13)$$

where

$$\Omega_{11}=P_{11}-\nu_1 X-\nu_1 X^T,\quad \Omega_{22}=P_{22}-\nu_1 R-\nu_1 R^T,$$
$$\Omega_{21}=P_{12}^T-\nu_1 R^T-\nu_1 X-\nu_2 U^T,$$
$$\Omega_{31}=\Omega_{41}=\nu_1 M^T X+\nu_2 N^T\bar{B}_F^T,$$
$$\Omega_{33}=\lambda_1(X^T A+A^T X)+\lambda_2(\bar{B}_F C+C^T\bar{B}_F^T)+Q_{11},$$
$$\Omega_{43}=\lambda_1(R^T A+A^T X)+\lambda_2(C^T\bar{B}_F^T+\bar{A}_F^T)+Q_{12}^T,$$
$$\Omega_{44}=\lambda_1(R^T A+A^T R)+Q_{22},$$
$$\Omega_{53}=P_{11}-\lambda_1 X+X^T A+\bar{B}_F C,$$
$$\Omega_{54}=P_{12}-\lambda_1 R+X^T A+\bar{B}_F C+\bar{A}_F,$$
$$\Omega_{63}=P_{12}^T-\lambda_1 X-\lambda_2 U^T+R^T A,$$
$$\Omega_{64}=P_{22}-\lambda_1 R+R^T A,\quad \Omega_{93}=\lambda_1 B^T X+\lambda_2 D^T\bar{B}_F^T,$$
$$\Omega_{71}=\Omega_{81}=\nu_1 M_d^T X+\nu_2 N_d^T\bar{B}_F^T,$$
$$\Omega_{73}=\Omega_{83}=\lambda_1 A_d^T X+\lambda_2 C_d^T\bar{B}_F^T.$$

In addition, an admissible estimator with the form of (3) can be given by

$$A_F=U^{-1}\bar{A}_F,\quad B_F=U^{-1}\bar{B}_F,\quad C_F=\bar{C}_F. \quad (14)$$

Proof: Since (10) implies

$$\begin{bmatrix} X+X^T & * \\ X+U^T+R^T & R+R^T \end{bmatrix}>0,$$

where X and R are nonsingular. Then we can construct the matrices G and G^{-1} as

$$G=\begin{bmatrix} X & X_1 \\ X_2 & X_3 \end{bmatrix},\quad G^{-1}=\begin{bmatrix} R^{-1} & Y_1 \\ Y_2 & Y_3 \end{bmatrix}.$$

Introduce matrices

$$\Sigma=\begin{bmatrix} I & 0 \\ 0 & R \end{bmatrix},\quad \Pi_1=\begin{bmatrix} I & R^{-1} \\ 0 & Y_2 \end{bmatrix},\quad \Pi_2=\begin{bmatrix} X & I \\ X_2 & 0 \end{bmatrix},$$

then we have $G\Pi_1=\Pi_2$. Without loss of generality, it is assumed that both Y_2 and X_2 are nonsingular. Therefore, $\Pi_1\Sigma$ is also nonsingular. Let

$$J=\Pi_1\Sigma=\begin{bmatrix} I & I \\ 0 & Y_2 R \end{bmatrix}.$$

By some algebraic operations we can obtain

$$J^T PJ=\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix},\quad J^T QJ=\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix},$$

$$J^T G^T\bar{M}J=\begin{bmatrix} X^T M+X_2^T B_F N & X^T M+X_2^T B_F N \\ R^T M & R^T M \end{bmatrix},$$

$$J^T G^T \bar{M}_d J = \begin{bmatrix} X^T M_d + X_2^T B_F N_d & X^T M_d + X_2^T B_F N_d \\ R^T M_d & R^T M_d \end{bmatrix},$$

$$J^T G^T \bar{A} J = \begin{bmatrix} X^T A + X_2^T B_F C & X^T A + X_2^T B_F C + X_2^T A_F Y_2 R \\ R^T A & R^T A \end{bmatrix},$$

$$J^T G J = \begin{bmatrix} X & R \\ X + R^T Y_2^T X_2 & R \end{bmatrix},$$

$$\bar{B}^T G J = \begin{bmatrix} B^T X + D^T B_F^T X_2 & B^T R \end{bmatrix},$$

$$J^T G^T \bar{A}_d J = \begin{bmatrix} X^T A_d + X_2^T B_F C_d & X^T A_d + X_2^T B_F C_d \\ R^T A_d & R^T A_d \end{bmatrix},$$

$$\bar{C} J = \begin{bmatrix} L & L - C_F Y_2 R \end{bmatrix}.$$

Based on the above relations, and define

$$U = X_2^T Y_2 R, \quad \bar{A}_F = X_2^T A_F Y_2 R, \quad \bar{B}_F = X_2^T B_F, \quad \bar{C}_F = C_F Y_2 R. \tag{15}$$

It can be readily established that (10) and (11) read as

$$\begin{bmatrix} J^T(P - \mathrm{Y}G - G^T \mathrm{Y})J & * & * & * & * \\ J^T \bar{M}^T \mathrm{Y} G J & J^T (G^T \Lambda \bar{A} + \bar{A}^T \Lambda G + Q) J & * & * & * \\ 0 & J^T (P - G^T \Lambda + \bar{A}^T G) J & J^T(-G - G^T)J & * & * \\ J^T \bar{M}_d^T \mathrm{Y} G J & J^T \bar{A}_d^T \Lambda G J & J^T \bar{A}_d^T G J & -J^T (1-\tau) Q J & * \\ 0 & \bar{B}^T \Lambda G J & \bar{B}^T G J & 0 & -I \end{bmatrix} < 0 \tag{16}$$

and

$$\begin{bmatrix} J^T P J & J^T \bar{C}^T \\ \bar{C} J & \gamma^2 I \end{bmatrix} > 0. \tag{17}$$

Then by virtue of the nonsingularity of J, performing congruence transformations to (16) by diag$(J^{-T}, J^{-T}, J^{-T}, J^{-T}, I)$ and to (17) by diag(J^{-T}, I) yields (7) and (6), respectively. In addition, the conditions that P and Q are positive definite are equivalent to LMIs (12) and (13).

Denote the filter transfer function from $y(t)$ to $z_F(t)$ by $T_{z_F y} = C_F (sI - A_F)^{-1} B_F + D_F$. By substituting the filter matrices with (14), we have

$$\begin{aligned} T_{z_F y} &= \bar{C}_F R^{-1} Y_2^{-1} (sI - X_2^{-T} \bar{A}_F R^{-1} Y_2^{-1})^{-1} X_2^{-T} \bar{B}_F + \bar{D}_F \\ &= \bar{C}_F (sI - U^{-1} \bar{A}_F)^{-1} U^{-1} \bar{B}_F + \bar{D}_F. \end{aligned} \tag{18}$$

Therefore, we conclude from Theorem 1 that the filter with a state-space (A_F, B_F, C_F) defined in (14) guarantees the filtering error system (4) to be mean-square asymptotically stable with an $L_2 - L_\infty$ disturbance attenuation level bound γ. This completes the proof.

Theorem 2 addresses the $L_2 - L_\infty$ filtering problem for system (1) where the system matrices are all known. However, usually uncertain parameters are presented in the system matrices $(A, A_d, B, M, M_d, C, C_d, D, N, N_d$ $L)$. Now, by virtue of the property of polytopic uncertainties, we present the robust $L_2 - L_\infty$ filtering results for system (1) with polytopic uncertainties in the following theorem without proof which can be obtained along the same lines of reasoning as in the derivation of Theorem 2.

Theorem 3: Consider system (1). An admissible robust $L_2 - L_\infty$ filter of the form (3) exists if for $i = 1, 2, \cdots, r$, there exist positive definite matrices $P_{11i} \in \mathbf{R}^{n\times n}$, $P_{22i} \in \mathbf{R}^{n\times n}$, $Q_{11i} \in \mathbf{R}^{n\times n}$, $Q_{22i} \in \mathbf{R}^{n\times n}$, matrices $P_{12i} \in \mathbf{R}^{n\times n}$, $Q_{12i} \in \mathbf{R}^{n\times n}$, $X \in \mathbf{R}^{n\times n}$, $R \in \mathbf{R}^{n\times n}$, $U \in \mathbf{R}^{n\times n}$, $\bar{A}_F \in \mathbf{R}^{n\times n}$, $\bar{B}_F \in \mathbf{R}^{n\times m}$, $\bar{C}_F \in \mathbf{R}^{m\times n}$ and scalars $\lambda_1, \lambda_2, \nu_1, \nu_2$ satisfying (10-13), where matrices P_{11}, P_{22}, Q_{11}, Q_{22}, P_{12}, Q_{12} take P_{11i}, P_{22i}, Q_{11i}, Q_{22i}, P_{12i}, Q_{12i}, respectively. Then an admissible estimator with the form of (3) can be given by (14).

5. ILLUSTRATIVE EXAMPLE

Example 1: Consider the following stochastic system (1) with the following matrices:

$$A = \begin{bmatrix} 0 & 3 + 0.5\rho \\ -4 & -5 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.2 + 0.3\sigma \end{bmatrix},$$

$$B = \begin{bmatrix} -0.4545 \\ 0.9090 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad D = 1,$$

$$C_d = \begin{bmatrix} 0.5 & 0.3 \end{bmatrix}, \quad L = \begin{bmatrix} 1.5 & 2 \end{bmatrix},$$

$$M = M_d = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad N = N_d = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad \tau = 0.3,$$

where ρ and σ are uncertain real parameters satisfying

$$|\rho| \le 1, |\sigma| \le 1. \tag{19}$$

Our aim is to design a strictly proper estimator (3) such that the resulting filtering error system (4) is mean-square asymptotically stable with an $L_2 - L_\infty$ noise attenuation level γ. For simplicity, we assume the system matrices are perfectly known, that is, $\rho = \sigma = 0$. From Theorem 2, when $(\lambda_1, \lambda_2, \nu_1, \nu_2)$ are fixed to be $(40, 32, 40, 40)$, the minimum achievable noise attenuation level is given by $\gamma^* = 0.6811$, and the corresponding estimator matrices are given by

$$R=\begin{bmatrix}0.2832 & 0.0990\\0.1053 & 0.1201\end{bmatrix},\quad X=\begin{bmatrix}0.4450 & 0.1631\\0.1475 & 0.1930\end{bmatrix},$$
$$U=\begin{bmatrix}-0.2524 & -0.0868\\-0.1008 & -0.1546\end{bmatrix},\quad \overline{A}_F=\begin{bmatrix}0.6734 & -0.0017\\0.6572 & 0.8371\end{bmatrix},$$
$$\overline{B}_F=\begin{bmatrix}-0.0150\\-0.0356\end{bmatrix},\quad \overline{C}_F=\begin{bmatrix}0.6950 & 1.1455\end{bmatrix}.$$

Then, from (14) the associated matrices for the desired filter (3) are given by

$$A_F=\begin{bmatrix}-1.5542 & 2.4092\\-3.2385 & -6.9862\end{bmatrix},\quad B_F=\begin{bmatrix}-0.0255\\0.2472\end{bmatrix},$$
$$C_F=\begin{bmatrix}0.6950 & 1.1455\end{bmatrix}.$$

When the command Fminsearch is used and the initial value of $(\lambda_1,\lambda_2,\nu_1,\nu_2)$ is chosen to be $(40,32,40,40)$, the minimum guaranteed cost is given by $\gamma^*=0.6577$, when $(\lambda_1,\lambda_2,\nu_1,\nu_2)$ =(14.8729,37.8809,17.0437,49.1361) and the corresponding estimator is given by

$$A_F=\begin{bmatrix}-1.5735 & 2.4729\\-2.8999 & -6.4811\end{bmatrix},\quad B_F=\begin{bmatrix}-0.0292\\0.2501\end{bmatrix},$$
$$C_F=\begin{bmatrix}0.7139 & 1.1796\end{bmatrix}.$$

By the method proposed in [6], the minimum L_2-L_∞ noise attenuation level $\gamma^*=0.7865$.

Now assume the uncertain parameters ρ and σ are as in (19). The uncertain parameters in this example can be modeled as polytopic uncertainty. The parameter uncertainty can be represented by a four-vertex polytope, and the minimum L_2-L_∞ noise attenuation level bound obtained from Theorem 3 is $\gamma^*=0.7460$ when $(\lambda_1,\lambda_2,\nu_1,\nu_2)=(39.3650,29.5234,40.3365,34.3422)$ and the corresponding filter matrices are

$$A_F=\begin{bmatrix}-1.0858 & 2.4672\\-3.4066 & -5.5509\end{bmatrix},\quad B_F=\begin{bmatrix}-0.0625\\0.3291\end{bmatrix},$$
$$C_F=\begin{bmatrix}0.7441 & 1.1107\end{bmatrix}.$$

By the approach proposed in [6], the minimum L_2-L_∞ noise attenuation level g $\gamma^*=0.9236$.

Therefore, for this example, the robust L_2-L_∞ filter design method proposed in this paper produces less conservative result than the results in [6], which is consistent with intuitive analysis and existing conclusions in the literature.

6. CONCLUSIONS

The robust L_2-L_∞ filtering problem of polytopic uncertain stochastic time-delay systems is studied in this paper. A new L_2-L_∞ performance criterion is proposed, which exhibits a kind of decoupling between Lyapunov matrices and system dynamic matrices. Based on this, a sufficient condition for the existence of a robust estimator is provided in terms of LMIs. It is shown that the proposed method is less conservative than some existing ones by introducing some additional matrices. A numerical example is given to demonstrate the feasibility and advantage of the proposed criterion.

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