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Delay-dependent H_∞ Performance Analysis for Markovian Jump Systems with Mode-dependent Time Varying Delays and Partially Known Transition Rates

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Abstract: This paper deals with the H_∞ performance analysis problems for a class of Markovian jump systems with partially known transition rates and time-delays which are time varying and depend on system mode. Following the recent study on the class of systems, improved sufficient conditions for H_∞ performance of the underlying systems are derived in form of LMI by constructing a new Lyapunov-Krasovskii functional. Illustrative numerical examples are provided to demonstrate the effectiveness of the proposed approach.

Keywords: H_∞ performance, linear matrix inequality (LMI), Markovian jump systems, time-varying delays.

1. INTRODUCTION

In recent years, much attention has been devoted to the time-delayed jump linear systems with Markovian jumping parameters [1-4]. The system is described by a set of time-delayed linear systems with the transitions between models determined by a Markov chain in a finite mode set [5]. With the maturity of H_∞ control theory, many works have been devoted to H_∞ control of time delayed Markovian jump linear systems. Based on the stochastic version of bounded real lemma, necessary and sufficient conditions for the existence of H_∞ controllers for continuous stochastic systems were presented in terms of coupled nonlinear matrix inequalities in the work [6]. The corresponding results for discrete-time systems can be found in the paper [7]. When both parameter uncertainty and time delay appear in a stochastic model, the H_∞ control problem was solved in the work [8], where state feedback controllers were designed under the assumption that all state variables are available. However, the results obtained in [8] cannot be applicable to the case when some of the actual states are not available. Ref [9] and [10] concerned with robust output feedback stabilization and H_∞ control problems for stochastic systems with parameter uncertainties and time delays when the actual state is not available directly.

It is worth pointing out that recent research effort in the study of delay systems are towards developing less conservative delay-dependent results. It has been shown that the conservatism in the existing delay-dependent results are mainly caused by using model transformation to the original delay system or resorting to bounding techniques for some cross terms. Considering this, it can be found that the results in [4] are less conservative by avoiding the use of these techniques

On the other hand, very recently, Markovian jump systems with mode-dependent time delays, where the time delays are dependent of the system modes have been studied. In [12], some robust stability conditions which are less conservative in some cases were presented in terms of LMIs. The robust H_∞ control results in the discrete context can be found in [11]. Robust H_∞ filtering problems were investigated in [12] and [13], and some other results for this class Markovian jump systems can be found in [1, 14-16. etc]. But to the best of the authors' knowledge, the problem of delay-dependent H_∞ performance for such systems has not been fully investigated, which is still open and remains challenging.

We note that the H_∞ performance criteria used in the corresponding results of [1,5,11-16] are all derived by taking the Lyapunov-Krasovskii functional with the similar form as following:

$$V(t) = x^{\mathrm{T}}(t)P(r(t))x(t) + \int_{t-d(r(t),t)}^{t} x^{\mathrm{T}}(s)Qx(s)ds + \eta\int_{-\hbar}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Qx(s)dsd\theta,$$

$$d_i(t) \le h_i, \eta = \max\{|\mu_{ii}|, i \in S\}, \hbar = \max\{h_i, i \in S\}.$$

When revisiting these results of these references, we find that the results still leaves three points for improvement:

1. These results only made use of the information of upper bound $\hbar$, but not the subsystems' upper bounds of the time varying delays, that may bring us some conservativeness.
2. These results for Markovian jump systems with mode-dependent time varying delays are developed based on the assumption that the derivative of subsys-

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tems' time varying delays had the upper bounds $\mu_i < 1, i \in S$. However, in many practical applications, the upper bounds of μ_i are not always restricted to be 1.

3. So far, almost all the issues on Markovian jump systems with mode-dependent time-varying delays have been investigated assuming the complete knowledge of these transition probabilities. However, the likelihood to obtain the complete knowledge of these transition probabilities is questionable and the cost may be probably high [17]. More recently, some attentions have been already drawn to the class of systems without time delays for discrete-time [18,19]. However, to the authors' best knowledge, the problem of delay-dependent stability and H_∞ performance analysis for this class of continuous-time systems with mode-dependent time-varying delays hasn't been studied, say, what is the exact impact of the unknown transition rates to the system with mode-dependent time-varying delays?

Taking these three points into account, we will develop an improved criterion to guarantee the exponential mean-square stability with γ-disturbance attenuation for the Markovian systems with mode-dependent time delays by constructing a different Lyapunov-Krasovskii functional. And then, basing on this new criterion, a corollary will be induced. Last, numerical examples will be provided to demonstrate the effectiveness of the proposed approach, and give the results of comparison. However, it needs to be pointed out that owing to the restrictions on the authors' knowledge and the technique difficulties caused by the mode-dependent time delays, we haven't solve the stability problem of such systems with completely unknown transition rates in this paper, and it needs us to study in the future work.

2. PROBLEM FORMULATION AND PRELIMINARIES

Notation: In this paper, E[•] stands for the mathematical expectation. $\|\bullet\|$ denotes the Euclidean norm for vector or the spectral norm of matrix. $M > 0$ is used to denote a symmetric positive-definite matrix. When $r(t) = i \in S = \{1, \cdots N\}$, we mark $A_i = A(r(t))$.

Consider the following stochastic system with Markovian switching:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-d(r(t),t)) \\ \qquad + D_1(r(t))w(t) \\ z(t) = E(r(t))x(t) + E_d(r(t))x(t-d(r(t),t)) \\ \qquad + D_2(r(t))w(t) \\ x(t) = \varphi(t), \quad t \in [-\hbar, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the states; $w(t) \in R^p$ is the noise signal which is assumed to be an arbitrary signal in $L^2([0,\infty])$; $z(t) \in R^q$ is the controlled output; Initial function $\varphi(t) \in L^2_{\mathbb{F}_0}([-\hbar,0];R^n)$, here $L^2_{\mathbb{F}_0}([-\hbar,0];R^n)$ denotes the family of R^n- valued stochastic process $\xi(s)$, $-\hbar \le s \le 0$ such that $\xi(s)$ is $\mathbb{F}_0-$ measurable and $\int_{-\hbar}^{0} \mathrm{E}\|\xi(s)\|^2 ds < \infty$. In the system (1) $d_i(t)$ denotes the mode-dependent time-varying delays which satisfies: $0 \le d_i(t) \le h_i, \dot{d}_i(t) \le \mu_i$, $\hbar = \max_{i\in S}\{h_i\}$. $r(t)$ is a homogenous stationary Markov chain defined on a complete probability space $\{\Omega, \mathbb{F}, \mathrm{P}\}$ and taking values in a finite set $S = \{1, \cdots N\}$. It's state transition rates matrix $\Xi = (\mu_{ij})_{N\times N}$ has the following form:

$$\mathrm{P}\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} \mu_{ij}\Delta t + o(\Delta t) & j \ne i \\ 1 + \mu_{ii}\Delta t + o(\Delta t) & j = i, \end{cases} \tag{2}$$

where $\mu_{ij} \ge 0, j \ne i, \mu_{ii} = -\sum_{j=1, j\ne i}^{N} \mu_{ij}$. In addition, the transition rates of the Markov chain in this paper are considered to be partially available, namely, some elements in matrix Ξ are time-invariant but unknown. For notation clarity, $\forall i \in S$, we denote $S^i_{kn} \triangleq \{j : \text{if } \mu_{ij} \text{ is known}\}$, $S^i_{uk} \triangleq \{j : \text{if } \mu_{ij} \text{ is unknown}\}$, $\mu^i_{kn} \triangleq \sum_{j\in S^i_{kn}} \mu_{ij}$ throughout the paper. Furthermore, we assume the diagonal elements of Ξ are known.

Definition 1: For a given real number $\gamma > 0$, the Markovian jump system (1) is said to be exponentially mean-square stable with γ-disturbance attenuation, if for any initial mode, it is exponentially mean-square stable with $w(t) = 0$ and under zero-initial conditions for any nonzero $w(t) \in L^2([0,\infty])$ the following inequality holds: $\|z\|_{E_2} < \gamma \|w\|_2$.

3. MAIN RESULTS

The purposes of this section are to derive the H_∞ performance criteria for system (1) when the transition rates are partially known and the time-varying delays are mode-dependent.

Theorem 1: For any given $\gamma > 0$, $h_i > 0$, μ_i, the Markovian jump system (1) is exponentially mean-square stable with γ-disturbance attenuation, if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_i > 0$, $S_i > 0$, $Z > 0$, $Q_1 > 0$, $Q_2 > 0$, L_{ki}, M_{ki}, N_{ki}, $k = 1, \cdots 5$, with appropriate dimensions, for any $i = 1, \cdots N$, such that:

$$\begin{bmatrix} (1+\mu^i_{kn})\Psi_{1i} & \Psi^{kn}_{3i} \\ * & \Psi^{kn}_{2i} \end{bmatrix} < 0, \tag{3}$$

$$\begin{bmatrix} \Psi_{1i} & \Psi^{uk}_{3i} \\ * & \Psi^{uk}_{2i} \end{bmatrix} < 0, \quad \forall j \in S^i_{uk}, \tag{4}$$

$$\begin{bmatrix} (1+\mu_{kn}^i)(-Q_1+\mu_{ii}Q_{1i}) & \sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}Q_{1j} \\ * & -\sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}Q_{1j} \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} -Q_1+\mu_{ii}Q_{1i} & Q_{1j} \\ * & -Q_{1j} \end{bmatrix} < 0, \quad \forall j \in S_{uk}^i, \tag{6}$$

$$\begin{bmatrix} (1+\mu_{kn}^i)(-Q_2+\mu_{ii}Q_{2i}) & \sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}Q_{2j} \\ * & -\sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}Q_{2j} \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} -Q_2+\mu_{ii}Q_{2i} & Q_{2j} \\ * & -Q_{2j} \end{bmatrix} < 0, \quad \forall j \in S_{uk}^i, \tag{8}$$

$$R_i < Z, \tag{9}$$

$$S_i < Z, \tag{10}$$

where

$$\Psi_{1i} = \begin{bmatrix} \Pi_{1i} & \Lambda_{1i} & \Lambda_{2i} & \Lambda_{3i} & \Lambda_{4i} & E_i^{\mathrm{T}} & \sqrt{h_i}L_{1i} & \sqrt{h_i}M_{1i} \\ * & \Pi_{2i} & \Lambda_{5i} & \Lambda_{6i} & \Lambda_{7i} & E_{di}^{\mathrm{T}} & \sqrt{h_i}L_{2i} & \sqrt{h_i}M_{2i} \\ * & * & \Pi_{3i} & \Lambda_{8i} & \Lambda_{9i} & 0 & \sqrt{h_i}L_{3i} & \sqrt{h_i}M_{3i} \\ * & * & * & \Pi_{4i} & \Lambda_{10i} & 0 & \sqrt{h_i}L_{4i} & \sqrt{h_i}M_{4i} \\ * & * & * & * & \Pi_{5i} & D_{2i}^{\mathrm{T}} & \sqrt{h_i}L_{5i} & \sqrt{h_i}M_{5i} \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -R_i & 0 \\ * & * & * & * & * & * & * & -S_i \end{bmatrix},$$

$$\Psi_{3i}^{kn} = \begin{bmatrix} \sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}P_j & 0 & 0 \\ 0 & \sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}h_jQ_{1i} & 0 \\ 0 & 0 & \sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}h_jQ_{2i} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{3i}^{uk} = \begin{bmatrix} P_j & 0 & 0 \\ 0 & h_jQ_{1i} & 0 \\ 0 & 0 & h_jQ_{2i} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{2i}^{uk} = \begin{bmatrix} -P_j & 0 & 0 \\ * & -h_jQ_{1i} & 0 \\ * & * & -h_jQ_{2i} \end{bmatrix},$$

$$\Psi_{2i}^{kn} = \begin{bmatrix} -\sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}P_j & 0 & 0 \\ * & -\sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}h_jQ_{1i} & 0 \\ * & * & -\sum_{j\neq i, j\in S_{kn}^i} \mu_{ij}h_jQ_{2i} \end{bmatrix}$$

with

$$\Pi_{1i} = P_iA_i + (P_iA_i)^{\mathrm{T}} + Q_{1i} + Q_{2i} + \hbar Q_1 + \hbar Q_2 + L_{1i}^{\mathrm{T}} + L_{1i} - N_{1i}A_i - (N_{1i}A_i)^{\mathrm{T}} + \mu_{ii}P_i,$$

$$\Pi_{2i} = -(1-\mu_i)Q_{1i} + M_{2i} + M_{2i}^{\mathrm{T}} - L_{2i} - L_{2i}^{\mathrm{T}} - N_{2i}A_{di} - (N_{2i}A_{di})^{\mathrm{T}},$$

$$\Pi_{3i} = -Q_{2i} + \mu_{ii}h_iQ_{2i} - M_{3i} - M_{3i}^{\mathrm{T}},$$

$$\Pi = -\gamma^2 I - N_{5i}D_{1i} - (N_{5i}D_{1i})^{\mathrm{T}},$$

$$\Lambda_{1i} = P_iA_{di} + L_{2i}^{\mathrm{T}} - A_i^{\mathrm{T}}N_{2i}^{\mathrm{T}} + M_{1i} - L_{1i} - N_{1i}A_{di},$$

$$\Lambda_{2i} = L_{3i}^{\mathrm{T}} - A_i^{\mathrm{T}}N_{3i}^{\mathrm{T}} - M_{1i}, \quad \Lambda_{3i} = L_{4i}^{\mathrm{T}} - A_i^{\mathrm{T}}N_{4i}^{\mathrm{T}} + N_{1i},$$

$$\Lambda_{4i} = P_iD_{1i} + L_{5i}^{\mathrm{T}} - A_i^{\mathrm{T}}N_{5i}^{\mathrm{T}} - N_{1i}D_{1i},$$

$$\Lambda_{5i} = M_{3i}^{\mathrm{T}} - L_{3i}^{\mathrm{T}} - A_{di}^{\mathrm{T}}N_{3i}^{\mathrm{T}} - M_{2i},$$

$$\Lambda_{6i} = M_{4i}^{\mathrm{T}} - L_{4i}^{\mathrm{T}} - A_{di}^{\mathrm{T}}N_{4i}^{\mathrm{T}} + N_{2i},$$

$$\Lambda_{7i} = -L_{5i}^{\mathrm{T}} + M_{5i}^{\mathrm{T}} - A_{di}^{\mathrm{T}}N_{5i}^{\mathrm{T}} - N_{2i}D_{1i},$$

$$\Lambda_{8i} = -M_{4i}^{\mathrm{T}} + N_{3i}, \quad \Lambda_{9i} = -M_{5i}^{\mathrm{T}} - N_{3i}D_{1i},$$

$$\Lambda_{10i} = N_{5i}^{\mathrm{T}} - N_{5i}D_{1i}.$$

Proof: When $w(t)$=0, first, in order to cast our model involved into the framework of the Markov processes, we define a new process $x_t(s) = x(t+s)$, $s \in [-2\hbar, 0]$. We choose a Lyapunov–Krasovskii functional:

$$V(x_t, t, r(t)) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where

$$V_1(t) = x^{\mathrm{T}}(t)P(r(t))x(t),$$

$$V_2(t) = \int_{t-d(r(t),t)}^{t} x^{\mathrm{T}}(s)Q_1(r(t))x(s)ds + \int_{t-h(r(t))}^{t} x^{\mathrm{T}}(s)Q_2(r(t))x(s)ds,$$

$$V_3(t) = \int_{-\hbar}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)dsd\theta,$$

$$V_4(t) = \int_{-\hbar}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Q_1x(s)dsd\theta + \int_{-\hbar}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Q_2x(s)dsd\theta,$$

where P_i, Q_{1i}, Q_{2i}, Q_1, Q_2, Z, $i = 1, 2, \cdots N$, are positive definite matrices with appropriate dimensions and:

$$\sum_{j=1}^{N} \mu_{ij}Q_{kj} \le Q_k, \quad k = 1, 2. \tag{11}$$

Let L be the weak infinitesimal generator of the random process $\{x_t, t\geq 0\}$, Then, for each $r(t)=i, i\in S$, it can be shown that:

$$L\{\int_{t-d(r(t),t)}^{t} x^{\mathrm T}(s)Q_1(r(t))x(s)ds\}$$
$$=\lim_{\Delta\to 0^+}\frac{1}{\Delta}E\{\int_{t-d(r(t+\Delta),t+\Delta)+\Delta}^{t+\Delta} x^{\mathrm T}(s)Q_1(r(t+\Delta))x(s)ds\mid r(t)=i-\int_{t-d_i(t)}^{t}x^{\mathrm T}(s)Q_{1i}x(s)ds\}$$
$$=\lim_{\Delta\to 0^+}\frac{1}{\Delta}E\{\int_{t-d_i(t)}^{t} x^{\mathrm T}(s)Q_1(r(t+\Delta))x(s)ds\mid r(t)=i+\int_{t-d(r(t+\Delta),t+\Delta)+\Delta}^{t-d_i(t)}x^{\mathrm T}(s)Q_1(r(t+\Delta))x(s)ds\mid r(t)=i+\int_{t}^{t+\Delta}x^{\mathrm T}(s)Q_1(r(t+\Delta))x(s)ds\mid r(t)=i-\int_{t-d_i(t)}^{t}x^{\mathrm T}(s)Q_{1i}x(s)ds\}$$
$$=\lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\int_{t-d_i(t)}^{t}x^{\mathrm T}(s)[\sum_{j=1}^{N}\mu_{ij}\Delta Q_{1j}+o(\Delta)]x(s)ds+\int_{t-[\sum_{j=1}^{N}\mu_{ij}\Delta d_j(t+\Delta)+d_i(t+\Delta)+o(\Delta)]+\Delta}^{t-d_i(t)}x^{\mathrm T}(s)[\sum_{j=1}^{N}\mu_{ij}\Delta Q_{1j}+o(\Delta)]x(s)ds+\int_{t-[\sum_{j=1}^{N}\mu_{ij}\Delta d_j(t+\Delta)+d_i(t+\Delta)+o(\Delta)]+\Delta}^{t-d_i(t)}x^{\mathrm T}(s)Q_{1i}x(s)ds+\int_{t}^{t+\Delta}x^{\mathrm T}(s)[\sum_{j=1}^{N}\mu_{ij}\Delta Q_{1j}+o(\Delta)]x(s)ds+\int_{t}^{t+\Delta}x^{\mathrm T}(s)Q_{1i}x(s)ds\}$$
$$=\int_{t-d_i(t)}^{t}x^{\mathrm T}(s)(\sum_{j=1}^{N}\mu_{ij}C_{1j})x(s)ds-(1-\dot d_i(t))x^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))+\sum_{j=1}^{N}\mu_{ij}d_j(t)x^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))+x^{\mathrm T}(t)Q_{1i}x(t).$$

Similar to the process above, we can obtain:

$$LV_1(x_t,t,i)=2x^{\mathrm T}(t)P_i(A_ix(t)+A_{di}x(t-d_i(t)))+\sum_{j=1}^{N}\mu_{ij}x^{\mathrm T}(t)P_jx(t),$$
$$LV_2(x_t,t,i)=x^{\mathrm T}(t)Q_{1i}x(t)-(1-\dot d_i(t))x^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))+\sum_{j=1}^{N}\mu_{ij}d_j(t)x^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))+x^{\mathrm T}(t)Q_{2i}x(t)-x^{\mathrm T}(t-h_i)Q_{2i}x(t-h_i)+\int_{t-d_i(t)}^{t}x^{\mathrm T}(s)(\sum_{j=1}^{N}\mu_{ij}Q_{1j})x(s)ds+\sum_{j=1}^{N}\mu_{ij}h_jx^{\mathrm T}(t-h_i)Q_{2i}x(t-h_i)+\int_{t-h_i}^{t}x^{\mathrm T}(s)(\sum_{j=1}^{N}\mu_{ij}Q_{2j})x(s)ds,$$
$$LV_3(x_t,t,i)=\hbar\dot x^{\mathrm T}(t)Z\dot x(t)-\int_{t-\hbar}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)ds,$$
$$LV_4(x_t,t,i)=\hbar x^{\mathrm T}(t)Q_1x(t)+\hbar x^{\mathrm T}(t)Q_2x(t)-\int_{t-\hbar}^{t}x^{\mathrm T}(s)Q_1x(s)ds-\int_{t-\hbar}^{t}x^{\mathrm T}(s)Q_2x(s)ds. \tag{12}$$

By the Newton–Leibniz formula and (1), for any appropriately dimensioned matrices L_i, N_i, M_i, $i=1, \cdots N$, we have:

$$\Upsilon_{1i}=2\eta^{\mathrm T}(t)L_i(x(t)-x(t-d_i(t))-\int_{t-d_i(t)}^{t}\dot x(s)ds)=0, \tag{13}$$
$$\Upsilon_{2i}=2\eta^{\mathrm T}(t)M_i(x(t-d_i(t))-x(t-h_i))-\int_{t-h_i}^{t-d_i(t)}\dot x(s)\,ds)=0, \tag{14}$$
$$\Upsilon_{3i}=2\eta^{\mathrm T}(t)N_i(-A_ix(t)-A_{di}x(t-d_i(t))+\dot x(t))=0, \tag{15}$$

where $\eta^{\mathrm T}(t)=[x^{\mathrm T}(t)\ x^{\mathrm T}(t-d_i(t))\ x^{\mathrm T}(t-h_i)\ \dot x^{\mathrm T}(t)]$. On the other hand, for matrices $Z=Z^{\mathrm T}$, $R_i=R_i^{\mathrm T}$, $T_i=T_i^{\mathrm T}$, $i=1,\cdots N$, which satisfy $R_i<Z$, $T_i<Z$, $i=1, \cdots N$, the following inequalities are true:

$$\Upsilon_{4i}=d_i(t)\eta^{\mathrm T}(t)L_iR_i^{-1}L_i^{\mathrm T}\eta(t)-\int_{t-d_i(t)}^{t}\eta^{\mathrm T}(t)L_iZ^{-1}L_i^{\mathrm T}\eta(t)ds\geq 0, \tag{16}$$
$$\Upsilon_{5i}=(h_i-d_i(t))\eta^{\mathrm T}(t)M_iT_i^{-1}M_i^{\mathrm T}\eta(t)-\int_{t-h_i}^{t-d_i(t)}\eta^{\mathrm T}(t)M_iZ^{-1}M_i^{\mathrm T}\eta(t)\,ds\geq 0. \tag{17}$$

According to the integral algorithms, we get:

$$\int_{t-\hbar}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)ds\geq\int_{t-h_i}^{t-d_i(t)}\dot x^{\mathrm T}(s)Z\dot x(s)ds+\int_{t-d_i(t)}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)ds.$$

Using this and combining (9)-(17), we have:

$$LV(x_t,t,i)\leq LV_1(x_t,t,i)+LV_2(x_t,t,i)+LV_3(x_t,t,i)+LV_4(x_t,t,i)+\Upsilon_{1i}+\Upsilon_{2i}+\Upsilon_{3i}+\Upsilon_{4i}+\Upsilon_{5i}$$
$$\leq 2x^{\mathrm T}(t)P_i(A_ix(t)+A_{di}x(t-d_i(t)))+\sum_{j=1}^{N}\mu_{ij}x^{\mathrm T}(t)P_jx(t)+x^{\mathrm T}(t)Q_{1i}x(t)-(1-\mu_i)x^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))-x^{\mathrm T}(t-h_i)Q_{2i}x(t-h_i)+\sum_{j=1,j\neq i}^{N}\mu_{ij}h_jx^{\mathrm T}(t-d_i(t))Q_{1i}x(t-d_i(t))+x^{\mathrm T}(t)Q_{2i}x(t)+\sum_{j=1}^{N}\mu_{ij}h_jx^{\mathrm T}(t-h_i)Q_{2i}x(t-h_i)+\hbar\dot x^{\mathrm T}(t)Z\dot x(t)$$

$$
\begin{aligned}
&+\hbar x^{\mathrm{T}}(t)Q_1x(t)+\hbar x^{\mathrm{T}}(t)Q_2x(t)+2\eta^{\mathrm{T}}(t)L_i(x(t)\\
&-x(t-d_i(t)))+2\eta^{\mathrm{T}}(t)M_i(x(t-d_i(t))-x(t-h_i))\\
&+h_i\eta^{\mathrm{T}}(t)M_iT_i^{-1}M_i^{\mathrm{T}}\eta(t)+2\eta^{\mathrm{T}}(t)N_i(-A_ix(t)\\
&-A_{di}x(t-d_i(t))+\dot{x}(t))+h_i\eta^{\mathrm{T}}(t)L_iR_i^{-1}L_i^{\mathrm{T}}\eta(t)\\
&-\int_{t-d_i(t)}^{t}[\eta^{\mathrm{T}}(t)L_i+\dot{x}^{\mathrm{T}}(s)Z]Z^{-1}[L_i^{\mathrm{T}}\eta(t)+Z\dot{x}(s)]ds\\
&-\int_{t-h_i}^{t-d_i(t)}[\eta^{\mathrm{T}}(t)M_i+\dot{x}^{\mathrm{T}}(s)Z]Z^{-1}[M_i^{\mathrm{T}}\eta(t)+Z\dot{x}(s)]ds\\
=&\ \eta^{\mathrm{T}}(t)\Sigma_i\eta(t)+2\eta^{\mathrm{T}}(t)L_i(x(t)-x(t-d_i(t)))\\
&+2\eta^{\mathrm{T}}(t)M_i(x(t-d_i(t))-x(t-h_i))\\
&+2\eta^{\mathrm{T}}(t)N_i(-A_ix(t)-A_{di}x(t-d_i(t))+\dot{x}(t))\\
&+\hbar\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)+h_i\eta^{\mathrm{T}}(t)L_iR_i^{-1}L_i^{\mathrm{T}}\eta(t)\\
&+h_i\eta^{\mathrm{T}}(t)M_iT_i^{-1}M_i^{\mathrm{T}}\eta(t)\\
&-\int_{t-d_i(t)}^{t}[\eta^{\mathrm{T}}(t)L_i+\dot{x}^{\mathrm{T}}(s)Z]Z^{-1}[L_i^{\mathrm{T}}\eta(t)+Z\dot{x}(s)]ds\\
&-\int_{t-h_i}^{t-d_i(t)}[\eta^{\mathrm{T}}(t)M_i+\dot{x}^{\mathrm{T}}(s)Z]Z^{-1}[M_i^{\mathrm{T}}\eta(t)+Z\dot{x}(s)]ds,
\end{aligned}
\tag{18}
$$

where

$$
\Sigma_i=\begin{bmatrix}
\Pi_{1i}-L_{1i}^{\mathrm{T}}-L_{1i}+N_{1i}A_i+(N_{1i}A_i)^{\mathrm{T}}+\Omega_{1i} & P_iA_{di} & 0 & 0\\
* & -(1-\mu_i)Q_{1i}+\Omega_{2i} & 0 & 0\\
* & * & \Pi_{3i}+M_{3i}+(M_{3i})^{\mathrm{T}}+\Omega_{3i} & 0\\
* & * & * & \hbar Z
\end{bmatrix}
$$

with

$$
\Omega_{1i}=\sum_{j\neq i,j\in S}\mu_{ij}P_j,\quad \Omega_{2i}=\sum_{j\neq i,j\in S}\mu_{ij}h_jQ_{1i},
$$
$$
\Omega_{3i}=\sum_{j\neq i,j\in S}\mu_{ij}h_jQ_{2i}.
$$

Since $Z>0$, the last two terms in (18) are all less than 0. To complete the proof, we set:

$$
L_i=\begin{bmatrix}L_{1i}\\ \vdots\\ L_{4i}\end{bmatrix},\quad M_i=\begin{bmatrix}M_{1i}\\ \vdots\\ M_{4i}\end{bmatrix},\quad N_i=\begin{bmatrix}N_{1i}\\ \vdots\\ N_{4i}\end{bmatrix},\quad i\in S=\{1,\cdots N\},
\tag{19}
$$

from (18), (19) and by Schur complement we can see that (9)-(11),(20) holding and $LV(x_t,t,i)<0$ are equivalent, then similar to [5], the exponential mean-square stability can be established.

$$
\Psi_{1i}'+diag\{\Omega_{1i},\Omega_{2i},\Omega_{3i},0,0,0\}<0,
\tag{20}
$$

where

$$
\Psi_{1i}'=\begin{bmatrix}
\Pi_{1i} & \Lambda_{1i} & \Lambda_{2i} & \Lambda_{3i} & \sqrt{h_i}L_{1i} & \sqrt{h_i}M_{1i}\\
* & \Pi_{2i} & \Lambda_{5i} & \Lambda_{6i} & \sqrt{h_i}L_{2i} & \sqrt{h_i}M_{2i}\\
* & * & \Pi_{3i} & \Lambda_{8i} & \sqrt{h_i}L_{3i} & \sqrt{h_i}M_{3i}\\
* & * & * & \Pi_{4i} & \sqrt{h_i}L_{4i} & \sqrt{h_i}M_{4i}\\
* & * & * & * & -R_i & 0\\
* & * & * & * & * & -S_i
\end{bmatrix}.
$$

By Schur complement, note that (11), (20) can be rewritten respectively as:

$$
\begin{bmatrix}
(1+\mu_{kn}^i)(-Q_k+\mu_{ii}Q_{ki}) & \sum_{j\neq i,j\in S_{kn}^i}\mu_{ij}Q_{kj}\\
* & -\sum_{j\neq i,j\in S_{kn}^i}\mu_{ij}Q_{kj}
\end{bmatrix}
+\sum_{j\in S_{uk}^i}\mu_{ij}\begin{bmatrix}-Q_k+\mu_{ii}Q_{ki} & Q_{kj}\\ * & -Q_{kj}\end{bmatrix}<0,\ k=1,2,
\tag{21}
$$

$$
\begin{bmatrix}(1+\mu_{kn}^i)\Psi_{1i}' & \Psi_{3i}^{kn'}\\ * & \Psi_{2i}^{kn}\end{bmatrix}+\sum_{j\in S_{uk}^i}\mu_{ij}\begin{bmatrix}\Psi_{1i} & \Psi_{3i}^{uk'}\\ * & \Psi_{2i}^{uk}\end{bmatrix}<0,
\tag{22}
$$

where

$$
\Psi_{3i}^{kn'}=
\begin{bmatrix}
\sum_{j\neq i,j\in S_{kn}^i}\mu_{ij}P_j & 0 & 0\\
0 & \sum_{j\neq i,j\in S_{kn}^i}\mu_{ij}h_jQ_{1i} & 0\\
0 & 0 & \sum_{j\neq i,j\in S_{kn}^i}\mu_{ij}h_jQ_{2i}\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0
\end{bmatrix},
$$

$$
\Psi_{3i}^{uk}=\begin{bmatrix}
P_j & 0 & 0\\
0 & h_jQ_{1i} & 0\\
0 & 0 & h_jQ_{2i}\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0
\end{bmatrix},
$$

Then, from (2), (21), (22), we obtain when (3)-(10) hold, the system is exponential mean-square stable under partially known transition rates, which is concluded from the obvious fact that no knowledge on $\mu_{ij},\forall j\in S_{uk}^i$ is required in (3)-(10). When $w(t)\neq 0$, and under zero-initial conditions, we set $\eta^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-d_i(t))\ x^{\mathrm{T}}(t-h_i)\ \dot{x}^{\mathrm{T}}(t)\ w^{\mathrm{T}}(t)]$. Then we can complete the proof of Theorem 1 by using the technique in establishing the H_∞ performance in [12]. This is the end of proof.

Remark 1: Theorem 1 develops a H_∞ performance criterion for continuous time Markovian jump systems with mode-dependent time varying delays and partially known transition rates. Comparing with most existing H_∞ performance results of this class of systems, the result of Theorem 1 makes use of the information of the subsystems' upper bounds of the time varying delays, which may bring us less conservativeness. Moreover, the upper bounds of μ_i are not restricted to be 1 in this paper. Therefore, our result is more natural and reasonable to the Markovian jump systems.

Now, the following corollary presents a sufficient condition for the H_∞ performance of system (1) with completely known transition rates.

Corollary 1: Consider the Markovian jump system (1) with mode-dependent time-varying delays and completely known transition probabilities. For given scalars $\gamma > 0,\ h_i > 0,\ \mu_i,$ The corresponding system is exponentially mean-square stable with γ-disturbance attenuation, if there exist matrices $P_i > 0,\ Q_{1i} > 0,\ Q_{2i} > 0,\ Q_1 > 0,\ Q_2 > 0,\ R_i > 0,\ S_i > 0,\ Z > 0,\ L_{ki},\ M_{ki},\ N_{ki},$ $k = 1,\cdots 5,$ with appropriate dimensions, for any $i = 1,\cdots N,$ such that:

$$\begin{bmatrix} \Theta_{1i} & \mathrm{H}_{11} & \mathrm{H}_{12} & \mathrm{H}_{13} & \mathrm{H}_{14} & E_i^{\mathrm{T}} & \sqrt{h_i}L_{1i} & \sqrt{h_i}M_{1i} \\ * & \Theta_{2i} & \mathrm{H}_{21} & \mathrm{H}_{22} & \mathrm{H}_{23} & E_{di}^{\mathrm{T}} & \sqrt{h_i}L_{2i} & \sqrt{h_i}M_{2i} \\ * & * & \Theta_{3i} & \mathrm{H}_{31} & \mathrm{H}_{32} & 0 & \sqrt{h_i}L_{3i} & \sqrt{h_i}M_{3i} \\ * & * & * & \Theta_{4i} & \mathrm{H}_{41} & 0 & \sqrt{h_i}L_{4i} & \sqrt{h_i}M_{4i} \\ * & * & * & * & \Theta_{5i} & D_{2i}^{\mathrm{T}} & \sqrt{h_i}L_{5i} & \sqrt{h_i}M_{5i} \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -R_i & 0 \\ * & * & * & * & * & * & * & -S_i \end{bmatrix} < 0, \quad (23)$$

$$\sum_{j=1}^{N} \mu_{ij} Q_{kj} \le Q_k, \quad k = 1,2, \quad (24)$$

$$R_i < Z, \quad (25)$$

$$S_i < Z, \quad (26)$$

where

$$\Theta_{1i} = P_i A_i + (P_i A_i)^{\mathrm{T}} + Q_{1i} + Q_{2i} + \hbar Q_1 + \hbar Q_2 + L_{1i}^{\mathrm{T}} + L_{1i} + \sum_{j=1}^{N} \mu_{ij} P_j - N_{1i} A_i - (N_{1i} A_i)^{\mathrm{T}},$$

$$\Theta_{2i} = -(1-\mu_i) Q_{1i} + \sum_{j=1, j\ne i}^{N} \mu_{ij} h_j Q_{1i} - L_{2i} - L_{2i}^{\mathrm{T}} + M_{2i} + M_{2i}^{\mathrm{T}} - N_{2i} A_{di} - (N_{2i} A_{di})^{\mathrm{T}},$$

$$\Theta_{3i} = -Q_{2i} + \sum_{j=1}^{N} \mu_{ij} h_j Q_{2i} - M_{3i} - M_{3i}^{\mathrm{T}},$$

$$\Theta_{4i} = \hbar Z + N_{4i} + N_{4i}^{\mathrm{T}},$$

$$\Theta_{5i} = -\gamma^2 I - N_{5i} D_{1i} - (N_{5i} D_{1i})^{\mathrm{T}},$$

$$\mathrm{H}_{11} = P_i A_{di} + L_{2i}^{\mathrm{T}} - A_i^{\mathrm{T}} N_{2i}^{\mathrm{T}} + M_{1i} - L_{1i} - N_{1i} A_{di},$$

$$\mathrm{H}_{12} = L_{3i}^{\mathrm{T}} - A_i^{\mathrm{T}} N_{3i}^{\mathrm{T}} - M_{1i},\ \ \mathrm{H}_{13} = L_{4i}^{\mathrm{T}} - A_i^{\mathrm{T}} N_{4i}^{\mathrm{T}} + N_{1i},$$

$$\mathrm{H}_{14} = P_i D_{1i} + L_{5i}^{\mathrm{T}} - A_i^{\mathrm{T}} N_{5i}^{\mathrm{T}} - N_{1i} D_{1i},$$

$$\mathrm{H}_{21} = M_{3i}^{\mathrm{T}} - L_{3i}^{\mathrm{T}} - A_{di}^{\mathrm{T}} N_{3i}^{\mathrm{T}} - M_{2i},$$

$$\mathrm{H}_{22} = M_{4i}^{\mathrm{T}} - L_{4i}^{\mathrm{T}} - A_{di}^{\mathrm{T}} N_{4i}^{\mathrm{T}} + N_{2i},$$

$$\mathrm{H}_{23} = -L_{5i}^{\mathrm{T}} + M_{5i}^{\mathrm{T}} - A_{di}^{\mathrm{T}} N_{5i}^{\mathrm{T}} - N_{2i} D_{1i},$$

$$\mathrm{H}_{31} = -M_{4i}^{\mathrm{T}} + N_{3i},\ \ \mathrm{H}_{32} = -M_{5i}^{\mathrm{T}} - N_{3i} D_{1i},$$

$$\mathrm{H}_{41} = N_{5i}^{\mathrm{T}} - N_{4i} D_{1i}.$$

Proof: By Theorem 1, the desired result can be obtained. This is the end of proof.

Remark 2: We can rapidly obtain that the conditions (3)-(4), (5)-(6), (7)-(8) will reduce to (23), (24) respectively when the i th row of Ξ are all available.

4. NUMERICAL EXAMPLES

To show the advantages of the delay-dependent criterion from H_∞ performance in Corollary 1, we provide the following example.

Example 1: Consider a stochastic delay system with Markovian jump parameters in the form of (1) with two modes. The dynamics of the system are described as

$$A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -3.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix},\ A_2 = \begin{bmatrix} -4.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \ D_{12} = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix},\ \ E_1 = \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix},$$

$$E_{d1} = \begin{bmatrix} 0.2 & -0.1 & 0.1 \end{bmatrix},\ \ E_2 = \begin{bmatrix} 0 & 1 & 0.6 \end{bmatrix},$$

$$E_{d2} = \begin{bmatrix} 0.1 & 0.1 & -0.3 \end{bmatrix},\ \ D_{21}=0.1,\ \ D_{22}=0.1.$$

The transition rates matrix is supposed to be:

$$\Xi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}.$$

To compare the stochastic H_∞ performance result in Corollary 1 with the results for systems with mode-dependent time delays in [12-14], we first assume: $h_1 = 0.6,\ h_2 = 0.4,\ \mu_2 = 0.3.$ For given $\mu_1,$ the minimum $\gamma_{\min},$ which satisfies the LMIs in (23)-(26), can be calculated by solving a quasi-convex optimization problem. Table 1 presents the comparison results. Next, we further assume: $h_1 = 0.6,\ \mu_1 = 0.6,\ \mu_2 = 0.3,$ for given $h_2,$ the comparisons of minimum $\gamma_{\min}$ are listed in Table 2. Tables 1 and 2 show that the result in Corollary 1 is much less conservative, and we find that the corresponding methods in [12-14] are unsolvable when $\mu_i \geq 1$, however, by the method of Corollary 1, there's no such restriction.

To illustrate the effectiveness of Corollary 1 when the systems' time varying delays are mode-independent, let us consider the following example:

Example 2: Consider a Markovian jump system in (1) with two modes and the following parameters [4]:

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix},\ A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} 0.0403 \\ 0.6771 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0.5689 \\ -0.2556 \end{bmatrix},$$
$$E_1 = [-0.3775 \quad -0.2959],\ E_2 = [-1.4751 \quad -0.2340],$$
$$E_{d1} = [0 \quad 0],\quad E_{d2} = [0 \quad 0],\quad D_{21} = 0.1184,$$
$$D_{22} = 0.3148,\quad \mu_{11} = -3,\quad \mu_{12} = 3,\quad \mu_{21} = 0.6,$$
$$\mu_{22} = -0.6.$$

We assume that $\mu_1 = \mu_2 = 1.5$. By the method of [4], we obtain the $\hbar$ is 0.2772, whereas by Corollary 1 we obtain the $\hbar$ is 0.3439 when $h_1 = h_2$. It is clear that the obtained result of this paper is significantly better than that in [4].

Now, Example 3 will illustrate how the unknown elements in the transition rates matrix effect on the γ-disturbance attenuation.

Example 3: Consider a stochastic delay system with Markovian jump parameters in the form of (1) with three modes. The dynamics of the system are described as:

$$A_1 = \begin{bmatrix} -0.75 & -0.75 \\ 1.5 & -1.5 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.15 & -0.49 \\ 1.5 & -2.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.3 & -0.15 \\ 1.5 & -1.8 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 0.11 & 0.24 \\ -0.53 & -0.37 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -0.59 & 0.01 \\ -0.07 & -0.61 \end{bmatrix},\ A_{d3} = \begin{bmatrix} 0.52 & 0.24 \\ 0.02 & -0.45 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ D_{12} = \begin{bmatrix} -0.6 \\ 0 \end{bmatrix},\ D_{13} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$
$$E_1 = [0.5 \quad -0.1],\ E_2 = [1 \quad 0.6],\ E_3 = [0 \quad 0.4],$$
$$E_{d1} = [0.2 \ -0.1],\ E_{d2} = [0.1 \ -0.3],\ E_{d3} = [-0.1 \ 0.3],$$
$$D_{21} = 0.1,\ D_{22} = 0.1,\ D_{23} = 0.1,\ \mu_1 = 0.2,\ \mu_2 = 0.3,$$
$$\mu_3 = 0.1,$$

The three cases of the transition rates matrix are considered as:

$$\text{Case 1: } \Xi = \begin{bmatrix} -0.9 & 0.4 & 0.5 \\ 0.1 & -0.9 & 0.8 \\ 0.7 & 0.4 & -1.1 \end{bmatrix},$$

Table 1. For given μ_1, comparisons of minimum allowed $\gamma_{\min}$.

μ_1	0.1	0.6	1.2
$\gamma_{\min}$ by [12-14]	0.87	3.85	Infeasible when $\mu_1 \geq 1$
$\gamma_{\min}$ by Corollary 1	0.56	0.63	0.74

Table 2. For given h_2, comparisons of minimum allowed $\gamma_{\min}$.

h_2	0.1	0.6	1.2
$\gamma_{\min}$ by [12-14]	3.85	3.85	3.85
$\gamma_{\min}$ by Corollary 1	0.68	0.63	0.59

Table 3. The comparisons of $\gamma_{\min}$. under the three cases.

Case 1	Case 2	Case 3
$\gamma_{\min} = 1.30$	$\gamma_{\min} = 1.72$	$\gamma_{\min} = 1.90$

$$\text{Case 2: } \Xi = \begin{bmatrix} -0.9 & ? & ? \\ 0.1 & -0.9 & 0.8 \\ 0.7 & 0.4 & -1.1 \end{bmatrix},$$
$$\text{Case 3: } \Xi = \begin{bmatrix} -0.9 & ? & ? \\ ? & -0.9 & ? \\ 0.7 & 0.4 & -1.1 \end{bmatrix},$$

where "?" represents the inaccessible element. We assume $h_1 = h_2 = h_3 = 0.5$ and under the three cases above, Table 3 lists the minimum of γ which can be computed by the method of Theorem 1 in this paper. Table 3 shows that the $\gamma_{\min}$ increases when the number of unknown elements increases.

5. CONCLUSIONS

In this paper, the H_∞ performance analysis problems for a class of Markovian jump systems with partially known transition rates and mode-dependent time-delays are investigated. By using a different Lyapunov-Krasovskii functional, improved delay-dependent H_∞ performance conditions are obtained in terms of linear matrix inequalities. The results in this paper are more general to deal with this class of systems, and have less conservativeness. Some examples are given to illustrate that the criteria performance is feasible and effective.

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