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Stabilization of Uncertain Saturated Discrete-Time Switching Systems

Abdellah Benzaouia, Ouahiba Benmessaouda, and Fernando Tadeo

Abstract: This paper presents sufficient conditions for the stabilization of uncertain switching discrete-time linear systems subject to actuator saturations. These conditions are obtained by using output feedback control laws and are formulated in terms of LMIs. Two different sets of LMIs are presented for the output feedback case when saturations are allowed, that differ in complexity and conservativeness. A numerical example is used to illustrate the proposed techniques.

Keywords: Actuator saturations, LMIs, Lyapunov functions, switching systems, uncertain parameters.

1. INTRODUCTION

Switched systems are a class of hybrid systems encountered in many practical situations which involve switching between several subsystems depending on various factors. Generally, a switching system consists of a family of continuous-time subsystems and a rule that supervises the switching between them. This class of systems has numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters and many other fields. Two main problems are widely studied in the literature according to the classification given in [6]: The first one, which is the one solved in this paper, looks for testable conditions that guarantee the asymptotic stability of a switching system under arbitrary switching rules, while the second is to determine a switching sequence that renders the switched system asymptotically stable (see [8,14] and references therein).

A main problem which is always inherent to all dynamical systems is the presence of actuator saturations. Even for linear systems, this problem has been an active area of research for many years. Besides approaches using anti-windup techniques [15] and model predictive controls [9], two main approaches have been developed in the literature: The first is the so-called positive invariance approach which is based on the design of controllers which work inside a region of linear behavior where saturations do not occur (see [5] and the references therein). This approach has already being applied to a class of hybrid systems involving jumping parameters [7]. It has also been used to design controllers for switching systems with constrained control under complete modelling taking into account reset functions at each switch and different system's dimension [4]. The second approach, however allows saturations to take effect while guaranteeing asymptotic stability (see [3,12] and references therein). The main challenge in these two approaches is to obtain large domains of initial states which ensures asymptotic stability for the system despite the presence of saturations.

The objective of this paper is to extend the results of [2] to uncertain switching system subject to actuator saturations. The uncertainty type considered in this work is the polytopic one. This type of uncertainty was also studied, without saturation, in [11]. Thus, this work deals with controllers tolerating saturations to take effect under polytopic uncertainties.

This paper is organized as follows: Section 2 deals with the problem presentation while the third Section presents some preliminary results. The main results of this paper are given in Section 4 together with an illustrative example.

2. PROBLEM PRESENTATION

Let us consider the uncertain saturated switching discrete-time linear system described by:

$$\begin{aligned} x_{k+1} &= A_\alpha(q_\alpha(k))x_k + B_\alpha(q_\alpha(k))sat(u_k), \\ y_k &= C_\alpha(q_\alpha(k))x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k \in \mathbb{R}^p$ are the state, the input and the output respectively, $sat(.)$ is the standard saturation (assumed here to be normalized, i.e., $|sat(u_k)| = min(1, |u_k|)$), function $\alpha(k): \mathbb{N} \mapsto \mathcal{I}$ is a switching rule taking its values $\alpha(k) = i$ in the finite set $\mathcal{I} = \{1, ..., N\}$ and $q_\alpha(k) \in \Gamma_\alpha \subset \mathbb{R}^{d_\alpha}$ are the bounded uncertainties that affect the system parameters in such a way that

$$A_\alpha(q_\alpha(k)) = A_\alpha + \sum_{h=1}^{d_\alpha} A_{\alpha h} q_{\alpha h}(k), \tag{2}$$

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Abdellah Benzaouia and Ouahiba Benmessaouda are with EACPI, Dep. of Physics, Faculty of Sciences Semlalia, University Cadi Ayyad, B.P. 2390, Marrakech 40 000, Morocco (e-mails: benzaouia@ucam.ac.ma, ouahiba04@gmail.com).

Fernando Tadeo is with the Departamento de Ingenieria de Sistemas y Automatica, Universidad de Valladolid, Paseo Prado de la Magdalena, 47005 Valladolid, Spain (e-mail: fernando@autom.uva.es).

$$B_\alpha(q_\alpha(k)) = B_\alpha + \sum_{h=1}^{d_\alpha} B_{\alpha h} q_{\alpha h}(k), \quad (3)$$

$$C_\alpha(q_\alpha(k)) = C_\alpha + \sum_{h=1}^{d_\alpha} C_{\alpha h} q_{\alpha h}(k), \quad (4)$$

where matrices A_α, B_α, C_α represent the nominal matrices and $q_{\alpha h}(k)$ the hth component of vector $q_\alpha(k)$:

$$q_\alpha(k) = [q_{\alpha 1}(k) q_{\alpha 2}(k) \dots\ q_{\alpha h}(k)\ \dots\ q_{\alpha d_\alpha}(k)]^T.$$

The following assumptions are required:

- Γ_α are compact convex sets.
- Matrices C_α are of full rank.

Let the control be obtained by an output feedback control law:

$$u_k = K_\alpha y_k = F_\alpha x_k, \qquad F_\alpha = K_\alpha C_\alpha. \quad (5)$$

Thus, the closed-loop system is given by:

$$x_{k+1} = A_\alpha(q_\alpha(k))x_k + B_\alpha(q_\alpha(k)) sat(K_\alpha C_\alpha(q_\alpha(k))x_k). \quad (6)$$

Defining the indicator function:

$$\xi(k) := [\xi_1(k), \dots, \xi_N(k)]^T, \quad (7)$$

where $\xi_i(k) = 1$ if the switching system is in mode i and 0 otherwise, yields the following representation for the closed-loop system:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k)\big[A_i(q_i(k))x_k + B_i(q_i(k)) sat(K_i C_i(q_i(k))x_k\big]. \quad (8)$$

Let convex sets Γ_i have μ_i vertices $v_{i\kappa}, \kappa = 1, \dots, \mu_i$ so that for every $q_i \in \Gamma_i$, one can write $q_i = \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} v_{i\kappa}$ with $\sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} = 1, 0 \le \beta_{i\kappa} \le 1$. The consequence of this, is that each matrix $A_i(q_i(k))$, $B_i(q_i(k))$ and $C_i(q_i(k))$ can be expressed as a convex combination of the corresponding vertices of the compact set Γ_i as follows:

$$M(q_i) := M_i + \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} M(v_{i\kappa}) = \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} M_{i\kappa},$$
$$M(v_{i\kappa}) = \sum_{h=1}^{d_i} M_{ih} v_{i\kappa h}, \quad M_{i\kappa} = M_i + M(v_{i\kappa}), \quad (9)$$
$$\sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} = 1, \quad 0 \le \beta_{i\kappa} \le 1,$$

where M_i represents the nominal matrix. Matrix M can be taken differently as A, B or C. Note that the system without uncertainties can be obtained as a particular case of this representation by letting the vertices $v_{i\kappa} = 0, \forall i, \forall \kappa$. Besides, (9) are directly related to the dimension d_i of the convex compact set Γ_i. The saturated uncertain switching system given by (8) can be rewritten as:

$$x_{k+1} = \sum_{i=1}^{N} \sum_{\kappa=1}^{\mu_i} \xi_i(k) \beta_{i\kappa}(k)[A_{i\kappa} x_k + B_{i\kappa} sat(K_i C_{i\kappa} x_k)]. \quad (10)$$

The nominal matrices will be represented by A_i, B_i and C_i. The nominal system in closed-loop is then given by:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k)[A_i x_k + B_i sat(K_i C_i x_k)]. \quad (11)$$

2.1. Preliminary results

Consider the following autonomous switching system:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k) A_i x_k. \quad (12)$$

In order to study the stability of switching systems given by (12) one can follow two ways:

- The existence of a common Lyapunov function to the various subsystems guarantees the asymptotic stability of the switching system. Unfortunately, the search for such function is not always obvious [8].
- The Lyapunov-like functions, and the multiple Lyapunov functions were introduced in [8]. They are considered as a strong tool in the analysis of the stability of hybrid systems and in particular of switching systems, so they will be used in this work.

A corresponding Lyapunov function for the system is then given by:

$$V(k,x) = x_k^T \Big(\sum_{i=1}^{N} \xi_i(k) P_i\Big) x_k. \quad (13)$$

For P_i a positive definite matrix, we define the following ellipsoid of $\mathbb{R}^n$:

$$\varepsilon(P_i, \rho) := \{x \in \mathbb{R}^n : x^\top P_i x \le \rho\}. \quad (14)$$

Define the following set:

$$\Omega = \bigcup_{i=1}^{N} \varepsilon(P_i, 1). \quad (15)$$

For the feedback gain matrix F_i, denote the the j th row of F_i as F_{ij} and define the region $\mathcal{L}(F_i)$ where the feedback control $u = sat(F_i x)$ is linear:

$$\mathcal{L}(F_i) := \{x \in \mathbb{R}^n : |F_{ij} x| \le 1, \ j \in [1,m]\}. \quad (16)$$

Define D_{is} as a $m \times m$ diagonal matrix with elements either 1 or 0 and $D_{is}^- = \mathbb{I}_m - D_{is}$. Matrices D_{is} and D_{is}^- were introduced in [13] to represent the saturation function as a linear one, as recalled by the following lemma.

Lemma 1 [13]**:** For all $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^m$ such that $|v_l| < 1, l \in [1,m]$

$$sat(u) \in co\{D_{is} u + D_{is}^- v\}, \ s \in [1,\eta], \ \eta = 2^m, i \in \mathcal{I}, \quad (17)$$

where co denotes the convex hull.

As a consequence of (17), there exist $\delta_{i1} \geq 0, \dots, \delta_{i\eta} \geq 0$ with $\sum_{s=1}^{\eta} \delta_{is} = 1$ such that

$$sat(u) = \sum_{s=1}^{\eta} \delta_{is}[D_{is}u + D_{is}^{-}v], \; i \in \mathcal{I}. \tag{18}$$

We now recall a useful stability result for switching systems with input saturations presented by [1] and [2] for the nominal system (11).

Theorem 1 [1]**:** If there exist symmetric positive definite matrices $P_1, \dots, P_N \in \mathbb{R}^{n\times n}$ and matrices $H_1, \dots, H_N \in \mathbb{R}^{m\times n}$ such that

$$\begin{bmatrix} P_i & [A_i + B_i(D_{is}F_i + D_{is}^{-}H_i)]^T P_j \\ * & P_j \end{bmatrix} > 0, \tag{19}$$

$\forall (i,j) \in \mathcal{I}\times\mathcal{I}, \; \forall s \in [1,\eta]$,

and

$$\varepsilon(P_i,1) \subset \mathcal{L}(H_i), \quad \forall i \in \mathcal{I}, \tag{20}$$

where $\eta = 2^m$, then, the closed-loop saturated switching system (11) is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for all switching sequences $\alpha(k)$.

Matrices D_{is} and D_{is}^{-} are defined by Lemma1 for each $i \in \mathcal{I}$. It is worth noting that the proof of this result can be obtained as a particular case of the proof of Theorem 2 by letting $\nu_{i\kappa} = 0, \forall i \in \mathcal{I}, \forall \kappa$.

3. MAIN RESULTS

This section presents sufficient conditions of asymptotic stability of the saturated uncertain switching system given by (10). The synthesis of the controller follows two different approaches, the first one deals firstly with the nominal system and then uses a test to check the asymptotic stability in presence of uncertainties while the second considers the global representation of the uncertain system (10).

Theorem 2: If there exist symmetric positive definite matrices $P_1, \dots, P_N \in \mathbb{R}^{n\times n}$ and matrices $H_1, \dots, H_N \in \mathbb{R}^{m\times n}$ such that

$$\begin{bmatrix} P_i & [A_{i\kappa} + B_{i\kappa}(D_{is}K_iC_{i\kappa} + D_{is}^{-}H_i)]^T P_j \\ * & P_j \end{bmatrix} > 0, \tag{21}$$

$\forall \kappa = 1, \dots, \mu_i, \; \forall (i,j) \in \mathcal{I}^2, \; \forall s \in [1,\eta]$,

and

$$\varepsilon(P_i,\rho) \subset \mathcal{L}(H_i), \tag{22}$$

then the closed-loop uncertain saturated switching system (10) is asymptotically stable $\forall x_0 \in \Omega := \bigcup_{i=1}^{N} \varepsilon(P_i,\rho)$ and for all switching sequences $\alpha(k)$.

Proof: By using Lemma 1, for all $H_i \in \mathbb{R}^{m\times n}$ with $|H_{ij}x_k| < 1, \quad j \in [1,m]$, where H_{ij} denotes the j th row of matrix H_i, there exist $\delta_{i1} \geq 0, \dots, \delta_{i\eta} \geq 0$ such that $sat(K_iC_{i\kappa}x_k) = \sum_{s=1}^{\eta} \delta_{is}(k)[D_{is}K_iC_{i\kappa} + D_{is}^{-}H_i]x_k$, $\delta_{i1}(k) \geq 0, \;\; \sum_{s=1}^{\eta} \delta_{is}(k) = 1$.

Then the closed-loop system (10) can be rewritten as

$$x_{k+1} = \sum_{s=1}^{\eta}\sum_{i=1}^{N}\sum_{\kappa=1}^{\mu_i} \xi_i(k)\beta_{i\kappa}(k)\delta_{is}(k)Ac_{i\kappa s}x_k, \tag{23}$$

where $Ac_{i\kappa s} := A_{i\kappa} + B_{i\kappa}(D_{is}K_iC_{i\kappa} + D_{is}^{-}H_i)$.

Consider the Lyapunov function candidate $V(x) = x_k^T(\sum_{i=1}^{N} \xi_i(k)P_i)x_k$. Computing its rate of increase along the trajectories of system (10) yields.

$$\Delta V(x_k) = x_{k+1}^T(\sum_{j=1}^{N} \xi_j(k+1)P_j)x_{k+1} - x_k^T(\sum_{i=1}^{N} \xi_i(k)P_i)x_k$$
$$= x_k^T\left\{\Sigma^T(\sum_{j=1}^{N} \xi_j(k+1)P_j)\Sigma - \sum_{i=1}^{N} \xi_i(k)P_i\right\}x_k,$$

where

$$\Sigma = \sum_{s=1}^{\eta}\sum_{i=1}^{N}\sum_{\kappa=1}^{\mu_i} \xi_i(k)\beta_{i\kappa}(k)\delta_{si}(k)Ac_{i\kappa s}.$$

Let condition (21) be satisfied. For each i and j multiply successively by $\xi_i(k)$, $\xi_j(k+1)$, $\beta_{i\kappa}(k)$ and $\delta_{is}(k)$ and sum. As $\sum_{i=1}^{N} \xi_i(k) = \sum_{j=1}^{N} \xi_j(k+1) = \sum_{s=1}^{\eta} \delta_{is}(k) = \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa}(k) = 1$, one gets:

$$\begin{bmatrix} \sum_{i=1}^{N} \xi_i(k)P_i & \Pi \\ * & \sum_{j=1}^{N} \xi_j(k+1)P_j \end{bmatrix} > 0, \tag{24}$$

where

$$\Pi = \Sigma^T(\sum_{j=1}^{N} \xi_j(k+1)P_j).$$

Inequality (24) is equivalent, by Schur complement, to

$$\Sigma^T(\sum_{j=1}^{N} \xi_j(k+1)P_j)\Sigma - \sum_{i=1}^{N} \xi_i(k)P_i < 0.$$

Letting λ be the largest eigenvalue among all the above matrices, we obtain that

$$\Delta V(x_k) \leq \lambda x_k^T x_k < 0, \tag{25}$$

which ensures the desired result. Besides, following Theorem 1, (21), (22) also allow for a state belonging to a set $\varepsilon(P_i,1) \subset \mathcal{L}(H_i)$, before the switch, if a switch occurs at time k_s, the switch will drive the state to the desired set $\varepsilon(P_j,1) \subset \mathcal{L}(H_j)$. That means that the set Ω is a set of asymptotic stability of the uncertain saturated switching system.

Remark 1:

- It is worth to note that the result of Theorem 1 can be obtained as a particular case of Theorem 2.
- For the results of Theorem 2 no assumption on the stabilizability of the system is needed as the existence of matrices $P_1,\ldots,P_N$ implies that the system is stabilizable.

This stability result is now used for control synthesis in two ways: the first consists in computing the controllers only with the nominal system and to test their robustness in a second step; while the second consists in computing in a single step the robust controllers.

Theorem 3 [2]**:** If there exist symmetric positive definite matrices X_i, matrices V_i and Z_i such that

$$\begin{bmatrix} X_i & (A_iX_i+B_iD_{is}Y_iC_i+B_iD_{is}^{-}Z_i)^T \\ * & X_j \end{bmatrix} > 0, \tag{26}$$

$$\begin{bmatrix} 1 & Z_{il} \\ * & X_i \end{bmatrix} > 0, \tag{27}$$

$$V_iC_i = C_iX_i, \quad \forall (i,j)\in I^2, \ \forall s\in[1,\eta], \ \forall l\in[1,m] \tag{28}$$

with

$$H_i = Z_iX_i^{-1}, K_i = Y_iV_i^{-1}, P_i = X_i^{-1}, \tag{29}$$

then the closed-loop nominal saturated switching system (11) is asymptotically stable $\forall x_0 \in \Omega,$ and for all switching sequences $\alpha(k)$.

At this step, the stabilizing controllers K_i and H_i of the nominal system are assumed to be known. Then, the following test has to be performed.

Corollary 1: If there exist symmetric positive definite matrices X_i such that

$$\begin{bmatrix} X_i & (A_{i\kappa}X_i+B_{i\kappa}D_{is}K_iC_{i\kappa}X_i+B_{i\kappa}D_{is}^{-}H_iX_i)^T \\ * & X_j \end{bmatrix} > 0, \tag{30}$$

$$\begin{bmatrix} 1 & (H_iX_i)_l \\ * & X_i \end{bmatrix} > 0, \tag{31}$$

$\forall (i,j)\in \mathcal{I}^2, \forall s\in[1,\eta], \forall l\in[1,m], \forall \kappa\in[1,\mu_i],$ with $P_i = X_i^{-1}$, then the closed loop uncertain switching system (10) is asymptotically stable $\forall x_0 \in \bigcup_{i=1}^{N} \varepsilon(P_i,\rho)$ and for all switching sequences $\alpha(k)$.

Proof: The proof is similar to that given in [2].

The second way to deal with robust controller design is to run a global set of LMIs leading, if it is feasible, to the robust controllers directly. However, one can note that this method is computationally more intensive.

Theorem 4: If there exist symmetric positive definite matrices X_i, matrices, Y_i, V_i and Z_i such that

$$\begin{bmatrix} X_i & (A_{i\kappa}X_i+B_{i\kappa}D_{is}Y_iC_{i\kappa}+B_{i\kappa}D_{is}^{-}Z_i)^T \\ * & X_j \end{bmatrix} > 0, \tag{32}$$

$$\begin{bmatrix} 1 & Z_{il} \\ * & X_i \end{bmatrix} > 0, \tag{33}$$

$$V_iC_{i\kappa} = C_{i\kappa}X_i, \quad \forall (i,j)\in I^2, \ \forall s\in[1,\eta], \ \forall l\in[1,m], \ \forall \kappa\in[1,\mu_i] \tag{34}$$

with

$$H_i = Z_iX_i^{-1}, K_i = Y_iV_i^{-1}, P_i = X_i^{-1}, \tag{35}$$

then, the closed-loop uncertain saturated switching system (10) is asymptotically stable $\forall x_0 \in \Omega,$ and for all switching sequences $\alpha(k)$.

Proof: The proof is similar to that given in [2] where the assumption of full rankness of matrices C_i is required (see [10] for more details).

In order to relax the previous LMIs, one can introduce some slack variables as in [10] and [2], as it is now shown:

Theorem 5: If there exist symmetric positive definite matrices X_i, matrices, Y_i, V_i, G_i and Z_i such that

$$\begin{bmatrix} G_i + G_i^T - X_i & \Psi \\ * & X_j \end{bmatrix} > 0, \tag{36}$$

with $\Psi = (A_{i\kappa}G_i + B_{i\kappa}D_{is}Y_iC_{i\kappa} + B_{i\kappa}D_{is}^{-}Z_i)^T$,

$$\begin{bmatrix} 1 & Z_{il} \\ * & G_i+G_i^T-X_i \end{bmatrix} > 0, \tag{37}$$

$$V_iC_{i\kappa} = C_{i\kappa}G_i, \tag{38}$$

$\forall \kappa = 1,\ldots,\mu_i, \ \forall (i,j)\in I^2, \ \forall s\in[1,\eta], \ \forall l\in[1,m]$ with

$$H_i = Z_iG_i^{-1}, K_i = Y_iV_i^{-1}, P_i = X_i^{-1}; \tag{39}$$

then, the closed-loop uncertain saturated switching system (10) is asymptotically stable $\forall x_0 \in \Omega$ and for all switching sequences $\alpha(k)$.

Proof: The proof is similar to that given in [2]. However, the LMI (37) was wrongly given in this reference. In fact, the inclusion condition $\varepsilon(P_i,\rho) \subset \mathcal{L}(H_i), \ \forall i\in\mathcal{I}$ holds if $1 - H_{il}X_iH_{il}^T > 0, \forall l\in[1,m],$ which is equivalent to,

$$1-(H_iG_i)_l(G_i^TX_i^{-1}G_i)^{-1}(H_iG_i)_l^T > 0.$$

That is, by virtue of (39)

$$1-(Z_{il})(G_i^TX_i^{-1}G_i)^{-1}(Z_{il})^T > 0,$$

which is equivalent by Schur complement to,

$$\begin{bmatrix} 1 & Z_{il} \\ * & G_i^TX_i^{-1}G_i \end{bmatrix} > 0. \tag{40}$$

Since $(G_i - X_i)^T X_i^{-1}(G_i - X_i) \ge 0,$ then $G_iX_i^{-1}G_i^T \ge G_i + G_i^T - X_i$. It follows that, the LMI (37) is obtained.

These results can be illustrated with the following example.

Example 1: Consider a SISO saturated switching discrete system with two modes given by the following matrices:

$$A_1(q_1(k)) = \begin{bmatrix} 1 & 1 \\ 0 & 1+q_{11} \end{bmatrix}, \quad B_1(q_1(k)) = \begin{bmatrix} 10 \\ 5 \end{bmatrix},$$
$$C_1(q_1(k)) = [1+q_{12} \quad 1],$$
$$A_2(q_2(k)) = \begin{bmatrix} 0+q_{21} & -1 \\ 0.0001 & 1 \end{bmatrix}, \quad B_2(q_2(k)) = \begin{bmatrix} 0.5 \\ -2+q_{22} \end{bmatrix},$$
$$C_2(q_2(k)) = [2 \quad 3].$$

The vertices of the domain of uncertainties that affect the first mode are:

$$\nu_{11} = (-0.1, -0.2), \ \nu_{12} = (-0.1, 0.2),$$
$$\nu_{13} = (0.1, -0.2), \ \nu_{14} = (0.1, 0.2).$$

The vertices of the domain of uncertainties that affect the second mode are:

$$\nu_{21} = (-0.2, 0.5), \ \nu_{22} = (-0.2, -0.1),$$
$$\nu_{23} = (0.3, 0.5), \ \nu_{24} = (0.3, -0.1).$$

Using Theorem 3, a stabilizing controller for the nominal system is

$$K_1 = -0.1000, \quad K_2 = 0.1622.$$

To test the robustness, we can use the Corollary 1 which leads to the following results:

$$P_1 = \begin{bmatrix} 0.0208 & -0.0133 \\ -0.0133 & 0.0257 \end{bmatrix}, \ P_2 = \begin{bmatrix} 0.0320 & 0.0023 \\ 0.0023 & 0.0474 \end{bmatrix}.$$

On the other hand, the use of Theorem 4 leads to the following results:

$$K_1 = -0.0902, \ K_2 = 0.1858.$$

Figs. 1, 2, and 3 concern the first method. In Fig. 1, the switching signals $\alpha(k)$ and the evolution of uncertainties used for simulation, are plotted. Fig. 2 shows the obtained level set of stability $\bigcup_{i=1}^{N} \varepsilon(P_i, \rho)$ which is well contained inside the sets of saturations, while Fig. 3 presents some system motions evolving inside the level set starting from different initial states.

Fig. 4 shows the level set of stability $\bigcup_{i=1}^{N} \varepsilon(P_i, \rho)$ using the second method of Theorem 4 which is well

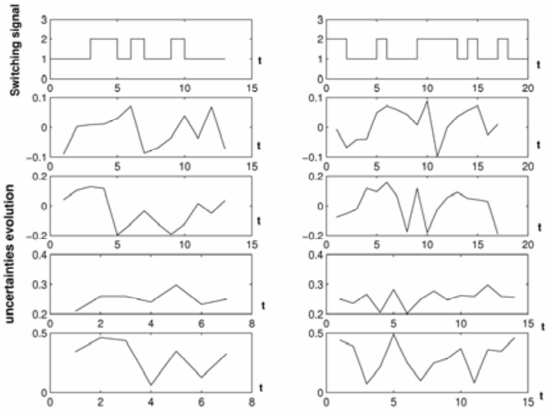

Fig. 1. Switching signals $\alpha(k)$ and uncertainties evolution.

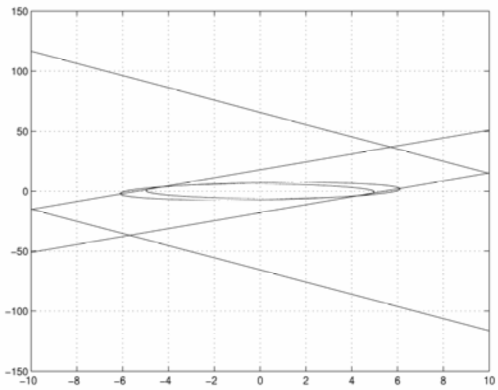

Fig. 2. Inclusion of the ellipsoids inside the polyhedral sets.

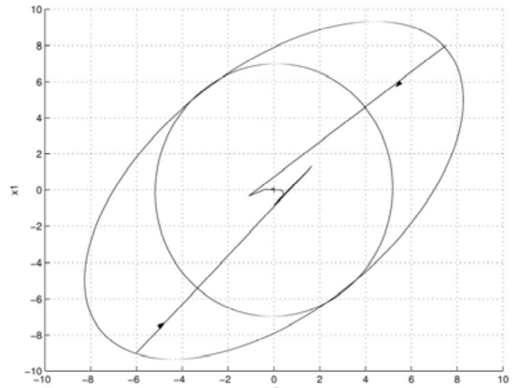

Fig. 3. Motion of the system with controllers obtained with Theorem 2 and Corollary 1.

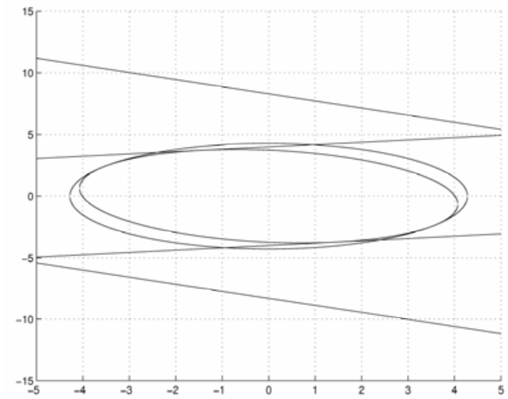

Fig. 4. Inclusion of the ellipsoids inside the polyhedral sets obtained with Theorem 4.

contained inside the sets of saturations.

The use of Theorem 5 leads to the following results:

$$K_1 = -0.0752, \ K_2 = 0.1386;$$

Fig. 5 shows the level set of stability $\bigcup_{i=1}^{N} \varepsilon(P_i, \rho)$ obtained with Theorem 5, which is also well contained inside the sets of saturations.

An illustrative example is studied by using the direct resolution of the proposed LMIs. A comparison of the obtained solutions is also given.

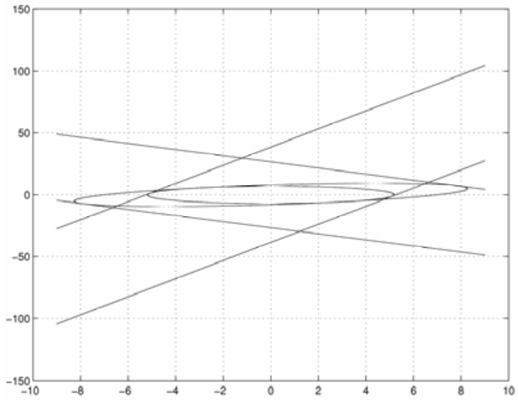

Fig. 5. Inclusion of the ellipsoids inside the polyhedral sets obtained with Theorem 5.

4. CONCLUSION

In this work, two different sufficient conditions of asymptotic stability are obtained for output feedback control of uncertain switching discrete-time linear systems subject to actuator saturations. These conditions allow the synthesis of stabilizing controllers inside a large region of saturation under LMIs formulation.

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Abdellah Benzaouia was born in Attaouia (Marrakech) in 1954. He received the degree of Electrical Engineering at the Mohammedia school (Rabat) in 1979 and the Doctorat (Ph.D.) at the University Cadi Ayyad in 1988. He is a professor at the University of Cadi Ayyad (Marrakech) where he is also head of a team of research on Robust and Constrained Control (EACPI). His research interests are mainly constrained control, robust control, pole assignment, systems with Markovian jumping parameters, hybrid systems and greenhouses. He collaborates with many teams in France, Canada, Spain and Italy.

Ouahiba Benmessaouda was born in Marrakech, Morocco, in 1978. She received the master's degrees with specialisation in regulation of industrial system from faculty of sciences Semlalia, Marrakech, Morocco. She is preparing her PhD on saturated switching systems.

Fernando Tadeo was born in 1969. He received the B.Sc. degree in Physics in 1992, and in Electronic Engineering in 1994, both from the University of Valladolid, Spain. After he received the M.Sc. degree in Control Engineering from the University of Bradford, U.K., he got his Ph.D. degree from the University of Valladolid, Spain, in 1996. Since 1998 he is a lecturer ('Profesor Titular') at the University of Valladolid, Spain. His current research interests include robust control, process control, control of systems with constraints and reinforcement learning, applied in several areas, from neutralization processes to robotic manipulators.