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Robust Stabilization of T–S Fuzzy Systems: Fuzzy Static Output Feedback under Parametric Uncertainty

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Abstract: This paper addresses robust static output-feedback control problems for a nonlinear system with uncertain fuzzy output in Takagi–Sugeno's (T–S) form. Two cases of the T–S fuzzy system, both continuous- and discrete-time cases are considered. In both cases, sufficient design conditions are derived for asymptotic stabilization in the sense of Lyapunov, in terms of linear matrix inequalities. Results are extended to retain $\mathcal{H}_\infty$ disturbance attenuation performance. An illustrative example on the permanent magnet synchronous motor equipped with uncertain nonlinear output and disturbance is provided to illustrate the effectiveness of the proposed methodology.

Keywords: Linear matrix inequalities, static output-feedback, Takagi–Sugeno (T–S) fuzzy system, uncertain fuzzy output.

1. INTRODUCTION

Most systems have severe nonlinearity and uncertainties. In order to synthesize a controller for an uncertain nonlinear system, extensive research efforts have been made in the last decades, among which a successful approach is the Takagi–Sugeno (T–S) fuzzy model-based control [1]. For this, the commonly adopted controllers are of full state-feedback form. However in real applications, measuring full state may be costly or even impossible due to their economical constraints or practical restrictions. One may avoid the obstacle by exploiting an output feedback with an additional dynamics of order equal to the system. However, the dynamic output-feedback controller increases the dimension of the closed-loop system [2].

A possible remedy is a static output feedback, which leads to a lower-dimensional as well as structurally simpler system than the dynamic output feedback. Nevertheless relatively few research efforts have been devoted to this approach thus a complete solution is not available yet. Necessary and sufficient stability conditions for linear time-invariant (LTI) systems via static output feedback are available, but not numerically tractable [3].

Linear matrix inequalities (LMIs) have recently gained much attention, since the flexibility of LMIs allows one to simultaneously reflect variety of design specifications. Still, necessity and sufficiency of the static output feedback is known to be one ofthe most challenging yet difficult issues. It is due to the fact that the derived stability condition is nonconvex.

A convexification technique is first developed in [4], for the sufficient design condition of the static output-feedback controller targeting at LTI systems in which a linear matrix equality (LME) constraint is additionally introduced. In [5], the scheme is extended to the T–S fuzzy system, which seems the first result in the fuzzy control field. Paper [6] considers the $\mathcal{H}_\infty$ disturbance attenuation under the fuzzy static output feedback. Although the result is elegant, it is not LMI. Therein, the solution is found in an iterative manner. It is noted that the methodologies addressed so far are only applicable for systems in which the output is linearly dependent on the state. In [7], an LMI condition is proposed for the concerned problem under fuzzy output by applying the technique in [4]. They considered the parametric uncertainties as well as the $\mathcal{H}_\infty$ disturbance attenuation performance. However, the uncertainty in the measured output for feedback is not taken into account.

In this paper, robust static output-feedback controller design techniques are presented for both continuous- and discrete-time T–S fuzzy systems in the presence of the parametric uncertainties in the fuzzy, rather than linear, measured output for feedback, in terms of LMIs. The results are extended to H_∞ disturbance attenuation control. An example is included to visualize the theoretical analysis and design.

An ellipsis is adopted for long symmetric matrix expressions, e.g.,

$$K\begin{bmatrix} \mathrm{He}\{S\} & \star \\ M & Q \end{bmatrix}\star := K\begin{bmatrix} S+S^T & M^T \\ M & Q \end{bmatrix}K^T.$$

2. T–S FUZZY SYSTEMS

The i th rule of an uncertain T–S fuzzy system we focus our attention on has the following form:

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$$R^i: \text{IF } z_1 \text{ is about } \Gamma_1^i \text{ and } \cdots \text{ and } z_p \text{ is about } \Gamma_p^i$$
$$\text{THEN} \begin{cases} \dot{x} = A_i x + B_i u \\ y = (C_i + \Delta C_i)x \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state; $u \in \mathbb{R}^m$ is the control input; $y \in \mathbb{R}^p$ is the output; z_j, $j \in \mathcal{I}_P$ are the premise variables injectively mapped from y; Γ_j^i, $(i,j) \in \mathcal{I}_R \times \mathcal{I}_P := \{1,2,\ldots,r\} \times \{1,2,\ldots,p\}$ is the fuzzy set of z_j in R^i; ΔC_i a real-valued matrix function representing parametric uncertainties. Using the center-average deffuzifier, product inference, and singleton fuzzifier, the global dynamics of (1) is inferred as

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \theta_i (A_i x + B_i u) \\ y = \sum_{i=1}^{r} \theta_i (C_i + \Delta C_i)x, \end{cases} \tag{2}$$

where $\Gamma_j^i : \mathbb{R} \to \mathbb{R}_{[0,1]}$ the membership value of z_j in Γ_j^i and

$$\theta_i = \left(\prod_{j=1}^{n} \Gamma_j^i(z_j) \right) \Bigg/ \left(\sum_{i=1}^{r} \left(\prod_{j=1}^{n} \Gamma_j^i(z_j) \right) \right).$$

Throughout this paper, we assume that only y, rather than x, is available for feedback. Under this circumstance, the following controller is taken

$$u = \sum_{i=1}^{r} \theta_i F_i y \tag{3}$$

to robustly asymptotically stabilize (2) in the presence of parametric uncertainty on output, where F_i is the feedback gain matrix. Such a type of uncertainty is not unreasonable to be assumed that, as usual, ΔC_i is norm-bounded and structured.

Assumption 1: The uncertain matrix is represented as $\Delta C_i = H_i \Delta_i(t) E_i$, where H_i and E_i are known real constant matrices of compatible dimensions, and Δ_i is an unknown matrix function with Lebesgue-measurable elements with $\Delta^T \Delta \preccurlyeq I$.

The closed-loop system is then constructed as follows:

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{h=1}^{r} \theta_i \theta_j \theta_h (A_i + B_i F_j (C_h + \Delta C_h))x. \tag{4}$$

3. MAIN RESULTS

Before proceeding, we recall the following matrix inequality which will be needed throughout the proofs.

Lemma 1 [8]**:** Given constant matrices E, H, and S=S^T of appropriate dimensions, the following holds:

$$S + \mathrm{He}\{H \Delta E\} \prec 0 \Leftrightarrow S + \begin{bmatrix} \varepsilon^{-\frac{1}{2}} E^T & \varepsilon^{\frac{1}{2}} H \end{bmatrix} \star \prec 0$$

if and only if for some $\varepsilon \in \mathbb{R}_{>0}$, where $\Delta^T \Delta \preccurlyeq I$.

Theorem 1: The closed-loop T–S fuzzy system (4) is asymptotically stable, if there exist $P = P^T \succ 0$, $M = M^T$, and N_i, such that

$$\begin{bmatrix} \mathrm{He}\{PA_i + B_i N_j C_h\} & \star & \star \\ E_h & -\varepsilon_{ijh} I & \star \\ (B_i N_j H_h)^T & 0 & -\varepsilon_{ijh}^{-1} I \end{bmatrix} \prec 0, \tag{5}$$

$$PB_i - B_i M = 0, \quad (i,j,h) \in \mathcal{I}_R \times \mathcal{I}_R \times \mathcal{I}_R, \tag{6}$$

where the controller gain is given by $F_i = M^{-1} N_i$.

Proof: Choose a positive definite function $V = x^T P x$ for (4). By virtue of the Lyapunov theorem, (4) is asymptotically stable whenever

$$\begin{aligned} &\frac{\mathrm{d}V}{\mathrm{d}t} < 0 \text{ for all } x \in \mathbb{R}^n \setminus \{0\} \\ &\Leftarrow \mathrm{He}\{P(A_i + B_i F_j (C_h + \Delta C_h))\} \prec 0 \qquad (7) \\ &\Leftrightarrow \mathrm{He}\{PA_i + PB_i F_j C_h + PB_i F_j H_h \Delta_h E_h\} \prec 0, \end{aligned}$$

which, however, lacks the joint convexity in P and F_j. Importing (6) and letting $MF_i = N_i$ recovers the convexity with respect to them and further proceeds to:

$$\begin{aligned} (7) &\Leftrightarrow \mathrm{He}\{PA_i + B_i N_j C_h + B_i N_j H_h \Delta_h E_h\} \prec 0 \\ &\Leftrightarrow \mathrm{He}\{PA_i + B_i N_j C_h\} \\ &\quad + \begin{bmatrix} E_h^T & B_i N_j H_h \end{bmatrix} \begin{bmatrix} \varepsilon_{ijh}^{-1} I & 0 \\ 0 & \varepsilon_{ijh} I \end{bmatrix} \star \prec 0 \\ &\Leftrightarrow (5), \end{aligned}$$

where we have used Lemma 1. This completes the proof of Theorem 1.

Similarly to the continuous-time case, sufficient robust asymptotic stability condition for a discrete-time case is presented. A discrete-time T–S fuzzy system closed by a discrete-time fuzzy static output-feedback controller is written as:

$$\begin{cases} x_{k+1} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{h=1}^{r} \theta_i \theta_j \theta_h (A_i + B_i F_j (C_h + \Delta C_h)) x_k. \\ y_k = \sum_{i=1}^{r} \theta_i (C_i + \Delta C_i) x_k, \end{cases} \tag{8}$$

where the subscript $k \in \mathbb{Z}_{\geqslant 0}$ denotes the sequential ordering.

Theorem 2: The discrete-time closed-loop T–S fuzzy system (8) is asymptotically stable, if there exists $P = P^T \succ 0$, $M = M^T$, and N_i, such that

$$\begin{bmatrix} -P & \star & \star & \star \\ PA_i + B_i N_j C_h & -P & \star & \star \\ E_h & 0 & -\varepsilon_{ijh} I & \star \\ 0 & (B_i N_j H_h)^T & 0 & -\varepsilon_{ijh}^{-1} I \end{bmatrix} \prec 0, \quad (9)$$

$$PB_i - B_i M = 0, \quad (i,j,h) \in \mathcal{I}_R \times \mathcal{I}_R \times \mathcal{I}_R, \quad (10)$$

where $F_i = M^{-1} N_i$.

Proof: We set the Lyapunov function as $V = x_k^T P x_k$. It is obvious that if the first forward difference $\Delta V < 0$, (8) is asymptotically stable in the sense of Lyapunov. Our task now is to derive some sufficient condition for this, in terms of LMIs.

$$\begin{aligned}
&\Delta V < 0 \text{ for all } x_k \in \mathbb{R}^n \setminus \{0\} \\
&\Leftarrow (A_i + B_i F_j (C_h + \Delta C_h))^T P (A_e + B_e F_f (C_g + \Delta C_g)) \\
&\quad - P \prec 0 \text{ for all } (i,j,h,e,f,g) \in \underbrace{I_R \times \cdots \times I_R}_{6} \\
&\Leftarrow (A_i + B_i F_j (C_h + \Delta C_h))^T P \star - P \prec 0 \\
&\Leftrightarrow \begin{bmatrix} -P & \star \\ A_i + B_i F_j (C_h + \Delta C_h) & -P^{-1} \end{bmatrix} \prec 0 \\
&\Leftrightarrow \begin{bmatrix} -P & \star \\ A_i + B_i F_j C_h & -P^{-1} \end{bmatrix} \\
&\quad + \begin{bmatrix} E_h^T & 0 \\ 0 & B_i F_j H_h \end{bmatrix} \begin{bmatrix} \varepsilon_{ijh}^{-1} I & \star \\ 0 & \varepsilon_{ijh} I \end{bmatrix} \star \prec 0 \\
&\Leftrightarrow \begin{bmatrix} -P & \star & \star & \star \\ A_i + B_i F_j C_h & -P^{-1} & \star & \star \\ E_h & 0 & -\varepsilon_{ijh} I & \star \\ 0 & (B_i F_j H_h)^T & 0 & -\varepsilon_{ijh}^{-1} I \end{bmatrix} \prec 0 \\
&\Leftrightarrow (9),
\end{aligned}$$

where we have taken a congruence transformation with $\operatorname{diag}\{I, P, I, I\}$ and used (10) and $MF_i = N_i$.

4. EXTENSION TO $\mathcal{H}_\infty$ CONTROL

In this section we consider the following T–S fuzzy system

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \theta_i (A_i x + B_i u + B_{w_i} w) \\ y = \sum_{i=1}^{r} \theta_i ((C_i + \Delta C_i) x + D_{w_i} w). \end{cases} \quad (11)$$

We still seek to design (3) for (11) to achieve the following $\mathcal{H}_\infty$ disturbance attenuation performance of a given $\gamma \in \mathbb{R}_{>0}$:

$$\int_0^T \| y \|^2 \, \mathrm{d}t \leqslant \int_0^T \| w \|^2 \, \mathrm{d}t \quad (12)$$

for all $w \in \mathcal{L}_2[0,T]$ with $x(0) = 0$.

Theorem 3: The T–S fuzzy system (11) closed by (3) is stable with the $\mathcal{H}_\infty$ disturbance attenuation performance in (12), if there exist $P = P^T \succ 0$, $M = M^T$, and N_i, such that

$$\begin{bmatrix} \mathrm{He}\{PA_i + B_i N_j C_h\} & \star & \star & \star & \star \\ (B_i N_j D_{w_h} + PB_{w_i})^T & -\gamma I & \star & \star & \star \\ C_h & D_{w_h} & -\gamma I & \star & \star \\ E_h & 0 & 0 & -\varepsilon_{ijh} I & \star \\ (B_i N_j H_h)^T & 0 & H_h^T & 0 & -\varepsilon_{ijh}^{-1} I \end{bmatrix} \prec 0, \quad (13)$$

$$PB_i - B_i M = 0, \quad (i,j,h) \in \mathcal{I}_R \times \mathcal{I}_R \times \mathcal{I}_R, \quad (14)$$

where $F_i = M^{-1} N_i$.

Proof: Suppose that there exists a Lyapunov function $V = x^T P x$ satisfying the following Hamilton–Jacobi–Bellman (H–J–B) inequality

$$\frac{\mathrm{d}V}{\mathrm{d}t} + \gamma^{-1} \| y \|^2 - \gamma \| w \|^2 < 0 \quad (15)$$

along (11) for all $(x, w) \in \mathbb{R}^n \setminus \{0\} \times \mathcal{L}_2[0,T]$. For every $T \in \mathbb{R}_{>0}$, integrating over $[0,T]$ (15) gives [9]

$$\begin{aligned}
\gamma^{-1} \int_0^T \| y \|^2 \, \mathrm{d}t - \gamma \int_0^T \| w \|^2 \, \mathrm{d}t &< V(x(0)) - V(x(T)) \\
&\leqslant V(x(0)) \\
&= 0 \\
&\Leftrightarrow (12).
\end{aligned}$$

We are in position to formulate (15) in terms of LMIs if

$$\begin{aligned}
&\begin{bmatrix} \mathrm{He}\{P(A_i + B_i F_j (C_h + \Delta C_h))\} & \star \\ (B_i F_j D_{w_h} + B_{w_i})^T P & -\gamma I \end{bmatrix} + \gamma^{-1} \begin{bmatrix} (C_h + \Delta C_h)^T \\ D_{w_h}^T \end{bmatrix} \star \prec 0 \\
&\Leftrightarrow \begin{bmatrix} \mathrm{He}\{P(A_i + B_i F_j C_h)\} & \star & \star \\ (B_i F_j D_{w_h} + B_{w_i})^T P & -\gamma I & \star \\ C_h & D_{w_h} & -\gamma I \end{bmatrix} \\
&\quad + \mathrm{He}\left\{ \begin{bmatrix} PB_i F_j H_h \\ 0 \\ H_h \end{bmatrix} \Delta_h \begin{bmatrix} E_h & 0 & 0 \end{bmatrix} \right\} \prec 0 \\
&\Leftrightarrow \begin{bmatrix} \mathrm{He}\{P(A_i + B_i F_j C_h)\} & \star & * & * & \star \\ (B_i F_j D_{w_h} + B_{w_i})^T P & -\gamma I & * & * & \star \\ C_h & D_{w_h} & -\gamma I & * & \star \\ E_h & 0 & 0 & -\varepsilon_{ijh} I & \star \\ (B_i F_j H_h)^T P & 0 & H_h^T & 0 & -\varepsilon_{ijh}^{-1} I \end{bmatrix} \prec 0 \\
&\Leftrightarrow (13),
\end{aligned}$$

where we have used (14) and denoted $MF_i = N_i$.

Next, we parallel the result such that the following discrete-time closed-loop T–S fuzzy system

$$\begin{cases} x_{k+1} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{h=1}^{r}\theta_i\theta_j\theta_h((A_i + B_iF_j(C_h + \Delta C_h))x_k \\ \qquad + (B_iF_jD_{w_h} + B_{w_i})w_k). \\ y_k = \sum_{i=1}^{r}\theta_i((C_i + \Delta C_i)x_k + D_{w_i}w_k) \end{cases} \tag{16}$$

reveals the following $\mathcal{H}_\infty$ disturbance attenuation performance:

$$\sum_{k=0}^{K}\| y_k \|^2 \leqslant \gamma^2\sum_{k=0}^{K}\| w_k \|^2 \tag{17}$$

for all $K \in \mathbb{Z}_{>0}$ and all $w \in l_2[0, K]$ with $x_0 = 0$.

Theorem 4: The discrete-time closed-loop T–S fuzzy system (16) is stable with the $\mathcal{H}_\infty$ disturbance attenuation performance in (17), if there exist $P = P^T \succ 0$, $M = M^T$, and N_i, such that

$$\begin{bmatrix} -P & \star & \star & \star & \star & \star \\ 0 & -\gamma I & \star & \star & \star & \star \\ PA_i + B_iN_jC_h & B_iN_jD_{w_h} + PB_{w_i} & -P & \star & \star & \star \\ C_h & D_{w_h} & 0 & -\gamma I & \star & \star \\ E_h & 0 & 0 & 0 & -\varepsilon_{ijh}I & \star \\ 0 & 0 & (B_iN_jH_h)^T & H_h^T & 0 & -\varepsilon_{ijh}^{-1}I \end{bmatrix} \prec 0, \tag{18}$$

$$PB_i - B_iM = 0, \quad (i, j, h) \in \mathcal{I}_R \times \mathcal{I}_R \times \mathcal{I}_R, \tag{19}$$

where $F_i = M^{-1}N_i$.

Proof: Suppose that there exists a Lyapunov function $V = x^TPx$ satisfying the following H–J–B inequality

$$\Delta V + \gamma^{-1}\| y_k \|^2 - \gamma\| w_k \|^2 < 0. \tag{20}$$

Summing up (20) from $k = 0$ to K stands for (17). For LMI casting of (20), a similar argument is given as follows:

$$\begin{bmatrix} (A_i + B_iF_j(C_h + \Delta C_h))^T & (C_h + \Delta C_h)^T \\ (B_iF_jD_{w_h} + B_{w_i})^T & D_{w_h}^T \end{bmatrix}\begin{bmatrix} P & \star \\ 0 & \gamma^{-1}I \end{bmatrix}\star - \begin{bmatrix} P & \star \\ 0 & \gamma I \end{bmatrix} \prec 0$$

$$\Leftrightarrow \begin{bmatrix} -P & \star & \star & \star \\ 0 & -\gamma I & \star & \star \\ A_i + B_iF_jC_h & B_iF_jD_{w_h} + B_{w_i} & -P^{-1} & \star \\ C_h & D_{w_h} & 0 & -\gamma I \end{bmatrix} + \mathrm{He}\left\{\begin{bmatrix} 0 \\ 0 \\ B_iF_jH_h \\ H_h \end{bmatrix}\Delta_h\begin{bmatrix} E_h & 0 & 0 & 0 \end{bmatrix}\right\} \prec 0$$

$$\Leftrightarrow \begin{bmatrix} -P & \star & \star & \star & \star & \star \\ 0 & -\gamma I & \star & \star & \star & \star \\ PA_i + PB_iF_jC_h & PB_iF_jD_{w_h} + PB_{w_i} & -P & \star & \star & \star \\ C_h & D_{w_h} & 0 & -\gamma I & \star & \star \\ E_h & 0 & 0 & 0 & -\varepsilon_{ijh}I & \star \\ 0 & 0 & (B_iF_jH_h)^TP & H_h^T & 0 & -\varepsilon_{ijh}^{-1}I \end{bmatrix} \prec 0$$

$\Leftrightarrow$ (18),

where we have taken a congruence transformation with $\mathrm{diag}\{I, I, P, I, I, I\}$ and used (19) and $MF_i = N_i$.

Remark 1: All design conditions can be efficiently solved via semidefinite programming or LMI Control Toolbox by converting the LME into

$$\begin{bmatrix} -\gamma I & \star \\ PB_i - B_iM & -\gamma I \end{bmatrix} \prec 0$$

with a very small $\gamma \in \mathbb{R}_{>0}$.

5. EXAMPLE

To show the effectiveness of the proposed method, the permanent magnetic synchronous motor (PMSM) in [10] is used as a practical test bed. The dynamical behavior of the smooth-air-gap PMSM without the external load torque but with external disturbance w is modeled as

$$\begin{cases} \dot{i}_d = -\frac{R}{L}i_d + n_pi_q\omega + v_d \\ \dot{i}_q = -\frac{R}{L}i_q - n_pi_d\omega - \frac{\psi_r}{L}\omega + v_q \\ \dot{\omega} = \frac{\psi_r}{L}i_q - \frac{\beta}{J}\omega + 2w, \end{cases}$$

where i_d, i_q are the direct and quadrature current components; ω is the motor angular velocity; v_d, v_q stand for the direct and quadrature input voltage components; $R = 0.9\ \Omega$ is the stator winding resistance; $L = 0.01425$ H is the direct and quadrature-axis stator inductors; $n_p = 1$ is the number of pole-pairs; $\psi_r =$ 0.031 Nm/A is the permanent-magnet flux; $\beta = 0.0162$ N/rad/s is the viscous damping coefficient; $J = 4.7 \times 10^{-5}$ Kgm^2 is the polar moment of inertia. We assume only the following uncertain nonlinear outputs are available for feedback:

$$\begin{cases} y_1 = 2(1+\delta)i_d + 4i_q + 4\omega + i_q\omega - \omega^2 + 0.1w \\ y_2 = \omega, \end{cases}$$

where δ, $|\delta| \le 0.1$, denotes the unknown parameter variation.

In order to cast into a T–S fuzzy system under consideration, it is desired to determine θ_i, A_i, B_i, C_i. Let $\omega \in \mathrm{co}\{\omega_1, \omega_2\}$. Solving $\omega = \theta_1\omega_1 + \theta_2\omega_2$ and $\theta_1 + \theta_2 = 1$ yields $\theta_1 = (-\omega + \omega_2)/(\omega_2 - \omega_1)$ and $\theta_2 = 1 - \theta_1$. Choosing $z_1 = y_2$ and $\Gamma_1^i = \theta_i$, the PMSM is modeled with a two-rule fuzzy system with the following parameters:

$$A_1 = \begin{bmatrix} -\frac{R}{L} & \omega_1 n_p & 0 \\ -\omega_1 n_p & -\frac{R}{L} & -\frac{\psi_r}{L} \\ 0 & \frac{\psi_r}{L} & -\frac{\beta}{J} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -\frac{R}{L} & \omega_2 n_p & 0 \\ -\omega_2 n_p & -\frac{R}{L} & -\frac{\psi_r}{L} \\ 0 & \frac{\psi_r}{L} & -\frac{\beta}{J} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_{w_i} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 & 5 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_i = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

where $x = \mathrm{col}\{i_d, i_q, \omega\}$, $u = \mathrm{col}\{v_d, v_q\}$, and $i \in \mathcal{I}_2$. According to Assumption 1, ΔC_i is decomposed as

$$H_i = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad E_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The disturbance is defined by $w = \cos(100t) \in \mathcal{L}_2[0, T]$. Let $(\omega_1, \omega_2) = (-1, 1)$. It is noted that the approaches in [6] and [7] do not consider uncertainties in the measured output for feedback. Moreover, [5] and [7] conduct only linear output case. Thus the techniques are inapplicable to the controller design problem herein. On the other hand, by applying Theorem 3 and solving the associated LMIs under $\gamma = 0.5$, we obtain the controller gain matrices:

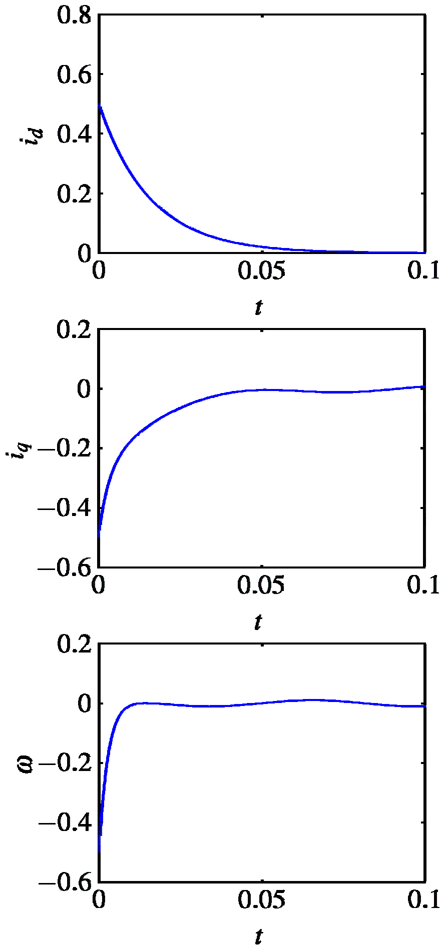

Fig 1: Time responses of the controlled PMSM.

$$F_1 = \begin{bmatrix} 0.0281 & 3.775 \\ 0.0813 & -113.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.0210 & 2.379 \\ 0.0528 & -110.8 \end{bmatrix}.$$

The initial data is set $x(0) = [0.5, -0.5, -0.5]^T$. The parameters δ randomly varies within its allowed interval throughout the simulation process. As the time responses are shown in Fig. 1, the fuzzy static output-feedback controller indeed stabilizes the system and attains the disturbance attenuation effect.

6. CONCLUSIONS

In this paper, we have presented the robust fuzzy static output-feedback controller design methodologies for both continuous- and discrete-time T–S fuzzy system possessing parametric uncertainties in the fuzzy output in the format of LMIs. Simulation result convincingly demonstrated the effectiveness of the developed techniques.

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