International Journal of Control, Automation, and Systems (2009) 7(4):668-673 http://www.springer.com/12555
DOI 10.1007/s12555-009-0418-y

Robust H-infinity Estimation for Linear Time-Delay Systems: An Improved LMI Approach

Ai-Guo Wu, Jin Dong, and Guang-Ren Duan

Abstract: The problem of robust H_∞ estimation for the polytopic uncertain linear system with state delay is considered. Firstly, by introducing two auxiliary matrices a new LMI representation of H_∞ performance is presented for the linear systems with a single time-varying state delay. The proposed criterion exhibits a kind of separation between the system matrices and the positive definite Lyapunov matrices. So the vertex-dependent Lyapunov functions can be adopted, and thus a less conservative result is expected to be obtained.

Keywords: H-infinity estimation, linear matrix inequalities, linear time-delay systems.

1. INTRODUCTION

The problem of estimation consists of finding an asymptotically stable estimator to estimate the desired signals. The main advantage of H_∞ estimation approach is the fact that it is insensitive to the exact knowledge of the statistics of the noise signals. When the system under consideration is subject to uncertainties, robust H_∞ filtering can provide powerful signal estimation. The aim of robust H_∞ estimation is to design an estimator such that the resulted error system is asymptotically stable and the L_2-induced gain related with the disturbance and the estimation error is less than a prescribed level irrespective of uncertainties [1-3].

On the other hand, it turns out that the noise attenuation level guaranteed by a robust H_∞ estimation design without considering time-delays may collapse if the system actually exhibits non-negligible time-delays. So increasing interest is focused on the H_∞ estimation of the systems with time-delays. In [4], an H_∞ filtering methodology was proposed for precisely known systems with a single time-delayed measurement. In [5] and [6], the problem of robust H_∞ filtering for continuous-time linear systems subject to parameter uncertainty in all the matrices of the system state-space model and multiple time-varying state delays was considered, and an LMI-based approach for the problem was established. In [7], the robust H_∞ filtering problem of the corresponding discrete-time case was investigated. The aforementioned H_∞ estimation design is conservative due to the usage of a common Lyapunov function for all polytopic uncertainties. In order to reduce the conservatism, new H_∞ filtering approaches have been reported recently. In [8] and [9], by introducing a slack matrix new LMI representations for H_∞ performances of discrete time-delay systems are established. Since these criteria exhibit a kind of decoupling between the positive matrices and the system matrices, the parameter-dependent Lyapunov functions are allowed to use. Thus, such estimator design procedures based on these criteria are much less conservative. Moreover, by converting a time-delay system into a descriptor linear system improved H_∞ performance criteria were well established [6,10].

In this paper, we consider the problem of robust H_∞ estimation for continuous-time single state-delayed systems with polytopic uncertainties. Before solving the problem of robust H_∞ estimation, based on an existing result in [11] an improved LMI representation of H_∞ performance, which realizes the separation between the positive definite matrices and the system matrices, is firstly given by introducing two slack matrices. Based on the newly proposed criterion, we provide the procedure of designing robust H_∞ estimators for polytopic uncertain time-delay systems.

2. PROBLEM FORMULATION

Consider the following linear time delay system:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-h(t)) + Bw(t) \\ y(t) = Cx(t) + Dw(t) \\ z(t) = Lx(t) + Tw(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $w(t) \in$

Manuscript received November 26, 2007; revised July 29, 2008; accepted December 29, 2008. Recommended by Editorial Board member Huanshui Zhang under the direction of Editor Young Il Lee. This work was supported by the Program for Changjiang Scholars and Innovative Research Team in University.
Ai-Guo Wu is with Harbin Institute of Technology Shenzhen Graduate School, Shenzhen 518055, P. R. China (e-mails: ag.wu @163.com).
Jin Dong and Guang-Ren Duan are with the Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, P. R. China (e-mails: edison_dj@sina.com, g.r.duan@hit.edu.cn).

$L_2^p[0,+\infty)$ is the exogenous disturbance signal, $y(t)\in\mathbb{R}^m$ is the measurement output and $z(t)\in\mathbb{R}^q$ is the signal to be estimated. For this system, we assume that the delay is time-varying and satisfies

$$0\le h(t)<\infty,\ \dot{h}(t)\le\rho<1. \tag{2}$$

In the sequel, we denote $\bar{\rho}:=1-\rho.$ The state-space data is assumed to be subject to uncertainties in the form of a polytopic model:

$$\begin{aligned}&(A_0,\ A_1,\ B,\ C,\ D,\ L,\ T)\in\Omega,\\ &\Omega=\{(A_0(\alpha),A_1(\alpha),B(\alpha),C(\alpha),D(\alpha),L(\alpha),T(\alpha))\\ &\quad=\sum_{i=1}^r\alpha_i(A_{0i},\ A_{1i},\ B_i,\ C_i,\ D_i,\ L_i,\ T_i);\ \ \alpha\in\Gamma\},\end{aligned} \tag{3}$$

where Γ is the unit simplex

$$\Gamma:=\left\{(\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_r):\ \sum_{i=1}^r\alpha_i=1,\ \alpha_i\ge 0\right\}.$$

This kind of convex bounded parameter uncertainty has been widely investigated in control and estimation problem [2]. Note that in many practical cases, very frequently, only a few entries of the matrices in state space model are uncertain.

For the system (1) the following linear estimator of order n is introduced

$$\begin{aligned}\dot{\hat{x}}(t)&=A_f\hat{x}(t)+B_fy(t),\\ \hat{z}(t)&=C_f\hat{x}(t)+D_fy(t).\end{aligned} \tag{4}$$

Let $\xi=\mathrm{col}(x,\hat{x}),\ e=z-\hat{z},$ combining (1) and (4) yields the following estimation error dynamics

$$\begin{aligned}\dot{\xi}(t)&=A_{0cl}\xi(t)+A_{1cl}\xi(t-h)+B_{cl}w(t),\\ e(t)&=C_{cl}\xi(t)+D_{cl}w(t)\end{aligned} \tag{5}$$

with

$$A_{0cl}=\begin{bmatrix}A_0&0\\B_fC&A_f\end{bmatrix},\ A_{1cl}=\begin{bmatrix}A_1&0\\0&0\end{bmatrix},\ B_{cl}=\begin{bmatrix}B\\B_fD\end{bmatrix},$$
$$C_{cl}=\begin{bmatrix}L-D_fC&-C_f\end{bmatrix},\ D_{cl}=T-D_fD.$$

The robust H_∞ estimation problem to be addressed in this paper can be stated as follows.

Robust H_∞ estimation problem: For a given $\gamma>0,$ design a full-order linear estimator in the form of (4) such that the estimation error system (5) is asymptotically stable and satisfies the H_∞ performance bound γ, that is, under zero initial condition for any nonzero $w(t)\in L_2^p[0,+\infty),$ the estimation error satisfies $\|e(t)\|_2\le\gamma\|w(t)\|_2.$

3. PERFORMANCE ANALYSIS

In this section, an improved representation of H_∞ performance for the estimation error system (5) is presented by introducing two free matrices. Before giving this criterion, we need the following lemma, which can be found in [11].

Lemma 1: Given a scalar $\gamma>0,$ the estimation error system (5) is asymptotically stable and satisfies the H_∞ performance bound γ if there exist positive definite matrices P and S such that

$$\begin{bmatrix}A_{0cl}^TP+PA_{0cl}+S&PB_{cl}&PA_{1cl}&C_{cl}^T\\B_{cl}^TP&-\gamma I&0&D_{cl}^T\\A_{1cl}^TP&0&-\bar{\rho}S&0\\C_{cl}&D_{cl}&0&-\gamma I\end{bmatrix}<0. \tag{6}$$

In the following a new H_∞ performance condition is established.

Theorem 1: Given a positive scalar $\gamma>0,$ the estimation error system is asymptotically stable and satisfies the H_∞ performance bound γ if there exist positive definite matrices P and S and matrices F and G such that

$$\begin{bmatrix}A_{0cl}^TF+F^TA_{0cl}+S&*&*&*&*\\P-F+G^TA_{0cl}&-G-G^T&*&*&*\\B_{cl}^TF&B_{cl}^TG&-\gamma I&*&*\\A_{1cl}^TF&A_{1cl}^TG&0&-\bar{\rho}S&*\\C_{cl}&0&D_{cl}&0&-\gamma I\end{bmatrix}<0. \tag{7}$$

Proof: Condition (6) can be equivalently rewritten as

$$\begin{bmatrix}A_{0cl}^TP+PA_{0cl}+S&PB_{cl}&PA_{1cl}\\B_{cl}^TP&-\gamma I&0\\A_{1cl}^TP&0&-\bar{\rho}S\end{bmatrix}+\gamma^{-1}\begin{bmatrix}C_{cl}^T\\D_{cl}^T\\0\end{bmatrix}\begin{bmatrix}C_{cl}&D_{cl}&0\end{bmatrix}<0.$$

Due to the strictness of the above matrix inequality, there exists a positive scalar $\theta>0$ such that

$$\begin{bmatrix}A_{0cl}^TP+PA_{0cl}+S&PB_{cl}&PA_{1cl}\\B_{cl}^TP&-\gamma I&0\\A_{1cl}^TP&0&-\bar{\rho}S\end{bmatrix}+\gamma^{-1}\begin{bmatrix}C_{cl}^T\\D_{cl}^T\\0\end{bmatrix}\begin{bmatrix}C_{cl}&D_{cl}&0\end{bmatrix}$$

$$+\frac{\theta}{2}\begin{bmatrix} A_{0cl}^T P \\ B_{cl}^T P \\ A_{1cl}^T P \end{bmatrix} P^{-1}\begin{bmatrix} PA_{0cl} & PB_{cl} & PA_{1cl} \end{bmatrix} < 0.$$

This is equivalent to

$$\begin{bmatrix} A_{0cl}^T P + PA_{0cl} + S & PB_{cl} & PA_{1cl} \\ B_{cl}^T P & -\gamma I & 0 \\ A_{1cl}^T P & 0 & -\bar{\rho} S \end{bmatrix} + \begin{bmatrix} \theta A_{0cl}^T P & C_{cl}^T \\ \theta B_{cl}^T P & D_{cl}^T \\ \theta A_{1cl}^T P & 0 \end{bmatrix}$$
$$\begin{bmatrix} 2\theta P & 0 \\ 0 & \gamma I \end{bmatrix}^{-1} \begin{bmatrix} \theta PA_{0cl} & \theta PB_{cl} & \theta PA_{1cl} \\ C_{cl} & D_{cl} & 0 \end{bmatrix} < 0.$$

By Schur Complement, the above inequality is equivalent to

$$\begin{bmatrix} A_{0cl}^T P + PA_{0cl} + S & PB_{cl} & PA_{1cl} & \theta A_{0cl}^T P & C_{cl}^T \\ B_{cl}^T P & -\gamma I & 0 & \theta B_{cl}^T P & D_{cl}^T \\ A_{1cl}^T P & 0 & -\bar{\rho} S & \theta A_{1cl}^T P & 0 \\ \theta PA_{0cl} & \theta PB_{cl} & \theta PA_{1cl} & -2\theta P & 0 \\ C_{cl} & D_{cl} & 0 & 0 & -\gamma I \end{bmatrix} < 0.$$

Applying appropriate congruent transformation to the above matrix inequality, gives

$$\begin{bmatrix} A_{0cl}^T P + PA_{0cl} + S & * & * & * & * \\ P - P + \theta PA_{0cl} & -2\theta P & * & * & * \\ B_{cl}^T P & \theta B_{cl}^T P & -\gamma I & * & * \\ A_{1cl}^T P & \theta A_{1cl}^T P & 0 & -\bar{\rho} S & * \\ C_{cl} & 0 & D_{cl} & 0 & -\gamma I \end{bmatrix} < 0.$$

Selecting $F = F^T = P$ and $G = G^T = \theta P$, we obtain (7).

On the other hand, since the matrix

$$T = \begin{bmatrix} I & A_{0cl}^T & 0 & 0 & 0 \\ 0 & B_{cl}^T & I & 0 & 0 \\ 0 & A_{1cl}^T & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

has full row rank, then pre- and post-multiplying the both sides of (7) by T and T^T, respectively, gives (6). Based on the above two aspects, (6) and (7) are equivalent.

Remark 1: The above theorem proposes a new LMI criteria for H_∞ performance of time-delay systems by introducing two slack matrices. Since such a criteria exhibits a kind of decoupling of Lyapunov matrices and the system matrices, it is allowed to use the vertex-dependent Lyapunov functions.

Remark 2: In the proof of the above theorem, the adopted method to obtain the newly established criterion is beyond the currently prevailing descriptor systems approaches. Due to the introduction of a small positive scalar, the method is called "Small Scalar Method", which has been used in authors' earlier paper [13].

4. ROBUST ESTIMATOR DESIGN

By virtue of the property of polytopic uncertainties, the following conclusion is readily obtained from Theorem 1.

Lemma 2: Consider the system (1) subject to the uncertainty (3). Then the estimation error system (5) is asymptotically stable with H_∞ performance bound γ if there exist positive definite matrices P_i, S_i, $i = 1, 2, \cdots, r$ and matrices F and G such that

$$\begin{bmatrix} A_{0cli}^T F + F^T A_{0cli} + S_i & * & * & * & * \\ P_i - F + G^T A_{0cli} & -G - G^T & * & * & * \\ B_{cli}^T F & B_{cli}^T G & -\gamma I & * & * \\ A_{1cli}^T F & A_{1cli}^T G & 0 & -\bar{\rho} S_i & * \\ C_{cli} & 0 & D_{cli} & 0 & -\gamma I \end{bmatrix} < 0, \quad (8)$$

where

$$A_{0cli} = \begin{bmatrix} A_{0i} & 0 \\ B_f C_i & A_f \end{bmatrix}, \; A_{1cli} = \begin{bmatrix} A_{1i} & 0 \\ 0 & 0 \end{bmatrix}, \; B_{cli} = \begin{bmatrix} B_i \\ B_f D_i \end{bmatrix}$$
$$C_{cli} = \begin{bmatrix} L_i - D_f C_i & -C_f \end{bmatrix}, D_{cli} = T_i - D_f D_i.$$

The above theorem is very efficient to evaluate the H_∞ norm bound for the error system (5) when an estimator (4) is given. However, it may not be directly applicable to the robust H_∞ estimator design problem due to the presence of the product of F with A_{cli} and G with A_{cli}. To enable the sub-optimal robust H_∞ estimator design, motivated by the idea in [12] the matrix F is specialized as

$$F = \Lambda G, \quad (9)$$

where $\Lambda = \text{diag}\ (\lambda_1 I_n, \lambda_2 I_n)$ with λ_1 and λ_2 being real scalars. Using the above F, (8) can be rewritten as

$$\begin{bmatrix} A_{0cli}^T \Lambda G + G^T \Lambda A_{0cli} + S_i & * & * & * & * \\ P_i - \Lambda G + G^T A_{0cli} & -G - G^T & * & * & * \\ B_{cli}^T \Lambda G & B_{cli}^T G & -\gamma I & * & * \\ A_{1cli}^T \Lambda G & A_{1cli}^T G & 0 & -\bar{\rho} S_i & * \\ C_{cli} & 0 & D_{cli} & 0 & -\gamma I \end{bmatrix} < 0. \quad (10)$$

The following result gives a sufficient condition for the existence of a robust H_∞ estimator in the form of (4) for the polytopic uncertain system (1).

Theorem 2: For given scalars λ_1, λ_2, consider the system (1) subject to the uncertainty (3) and let $\gamma > 0$ be a given constant. Then the estimation error system (5) is asymptotically stable with H_∞ performance bound γ if there exist positive definite matrices $P_{11i} \in \mathbb{R}^{n\times n}$, $P_{22i} \in \mathbb{R}^{n\times n}$, $S_{11i} \in \mathbb{R}^{n\times n}$ and $S_{22i} \in \mathbb{R}^{n\times n}$ and matrices $P_{12i} \in \mathbb{R}^{n\times n}$, $S_{12i} \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{n\times n}$, $\overline{A}_f \in \mathbb{R}^{n\times n}$, $\overline{B}_f \in \mathbb{R}^{n\times m}$, $\overline{C}_f \in \mathbb{R}^{m\times n}$ delete it such that for $i = 1, 2, \cdots, r$,

$$\begin{bmatrix} (1,1)_i & * & * & * & * & * & * & * \\ (2,1)_i & (2,2) & * & * & * & * & * & * \\ P_{11i} - \lambda_1 X + X^T A_{0i} + \overline{B}_f C_i & (3,2)_i & -X - X^T & * & * & * & * & * \\ P_{12i}^T - \lambda_1 X - \lambda_2 U^T + R^T A_{0i} & (4,2)_i & (4,3)_i & -R - R^T & * & * & * & * \\ \lambda_1 B_i^T X + \lambda_2 D_i^T \overline{B}_f^T & \lambda_1 B_i^T R & (5,3)_i & B_i^T R & -\gamma I & * & * & * \\ \lambda_1 A_{1i}^T X & \lambda_1 A_{1i}^T R & A_{1i}^T X & A_{1i}^T R & 0 & -\bar{\rho} S_{11i} & * & * \\ 0 & 0 & 0 & 0 & 0 & -\bar{\rho} S_{12i}^T & -\bar{\rho} S_{22i} & * \\ L_i - D_f C_i & (8,2)_i & 0 & 0 & T_i - \overline{D}_f D_i & 0 & 0 & -\gamma I \end{bmatrix} < 0, \quad (11)$$

$$\begin{bmatrix} P_{11i} & P_{12i} \\ P_{12i}^T & P_{22i} \end{bmatrix} > 0,$$

where

$$(1,1)_i = \lambda_1 (X^T A_{0i} + A_{0i}^T X) + \lambda_2 (\overline{B}_f C_i + C_i^T \overline{B}_f^T) + S_{11i},$$
$$(2,1)_i = \lambda_1 (R^T A_{0i} + A_{0i}^T X) + \lambda_2 (C_i^T \overline{B}_f^T + \overline{A}_f^T) + S_{12i}^T,$$
$$(2,2)_i = \lambda_1 (R^T A_{0i} + A_{0i}^T R) + S_{22i},$$
$$(3,2)_i = P_{12i} - \lambda_1 R + X^T A_{0i} + \overline{B}_f C_i + \overline{A}_f,$$
$$(4,2)_i = P_{22i} - \lambda_1 R + R^T A_{0i},$$
$$(4,3)_i = -X - U^T - R^T,$$
$$(5,3)_i = B_i^T X + D_i^T \overline{B}_f^T,$$
$$(8,2)_i = L_i - \overline{D}_f C_i - \overline{C}_f.$$

In addition, an admissible estimator with the form of (4) can be given by

$$A_f = U^{-1} \overline{A}_f, \; B_f = U^{-1} \overline{B}_f, \; C_f = \overline{C}_f, \; D_f = \overline{D}_f. \quad (12)$$

Proof: Since (11) implies

$$\begin{bmatrix} X + X^T & * \\ X + U^T + R^T & R + R^T \end{bmatrix} > 0,$$

where X and R are also nonsingular. Then we can construct the matrix G and G^{-1} as

$$G = \begin{bmatrix} X & X_1 \\ X_2 & X_3 \end{bmatrix}, \; G^{-1} = \begin{bmatrix} R^{-1} & Y_1 \\ Y_2 & Y_3 \end{bmatrix}.$$

Introduce matrices

$$\Psi = \begin{bmatrix} I & \\ & R \end{bmatrix}, \; \Pi_1 = \begin{bmatrix} I & R^{-1} \\ 0 & Y_2 \end{bmatrix}, \; \Pi_2 = \begin{bmatrix} X & I \\ X_2 & 0 \end{bmatrix}$$

then we have $G\Pi_1 = \Pi_2$. Without loss of generality, it is assumed that both Y_2 and X_2 are nonsingular. Therefore, $\Pi_1 \Psi$ is also nonsingular. By some algebraic operations we can obtain

$$\Psi^T \Pi_1^T P_i \Pi_1 \Psi = \begin{bmatrix} P_{11i} & P_{12i} \\ P_{12i}^T & P_{22i} \end{bmatrix},$$
$$\Psi^T \Pi_1^T S_i \Pi_1 \Psi = \begin{bmatrix} S_{11i} & S_{12i} \\ S_{12i}^T & S_{22i} \end{bmatrix},$$
$$\Psi^T \Pi_1^T G \Pi_1 \Psi = \begin{bmatrix} X & R \\ X + R^T Y_2^T X_2 & R \end{bmatrix},$$
$$\Psi^T \Pi_1^T G^T A_{0cli} \Pi_1 \Psi =$$
$$\begin{bmatrix} X^T A_{0i} + X_2^T B_f C_i & X^T A_{0i} + X_2^T B_f C_i + X_2^T A_f Y_2 R \\ R^T A_{0i} & R^T A_{0i} \end{bmatrix},$$
$$A_{1cli}^T G \Pi_1 \Psi = \begin{bmatrix} A_{1i}^T X & A_{1i}^T R \\ 0 & 0 \end{bmatrix},$$
$$B_{cli}^T G \Pi_1 \Psi = \begin{bmatrix} B_i^T X + D_i^T B_f^T X_2 & B_i^T R \end{bmatrix},$$
$$C_{cli} \Pi_1 \Psi = \begin{bmatrix} L_i - D_f C_i & L_i - D_f C_i - C_f Y_2 R \end{bmatrix}.$$

Based on the above relations, let $J = \Pi_1 \Psi$, and define

$$U = X_2^T Y_2 R, \; \overline{A}_f = X_2^T A_f Y_2 R, \; \overline{B}_f = X_2^T B_f, \; \overline{C}_f = C_f Y_2 R, \; \overline{D}_f = D_f. \quad (13)$$

Then it can be readily established that (10) reads as

$$\tilde{J}^T \begin{bmatrix} [1,1]_i & * & * & * & * \\ P_i - \Lambda G + G^T A_{0cli} & -G - G^T & * & * & * \\ B_{cli}^T \Lambda G & B_{cli}^T G & -\gamma I & * & * \\ A_{1cli}^T \Lambda G & A_{1cli}^T G & 0 & -\bar{\rho} S_i & * \\ C_{cli} & 0 & D_{cli} & 0 & -\gamma I \end{bmatrix} \tilde{J} < 0$$

with $[1,1]_i = A_{0cli}^T \Lambda G + G^T \Lambda A_{0cli} + S_i$ and $\tilde{J} = \text{diag}(J,$

J, I, I, I). By virtue of the nonsingularity of J, performing congruence transformations to the above inequality by $\tilde{J}^{-1}$ yields (9). Therefore, we can conclude that an estimator can be given from (13). In view of the following relation

$$\bar{C}_f R^{-1} Y_2^{-1} (sI - X_2^{-T} \bar{A}_f R^{-1} Y_2^{-1})^{-1} X_2^{-T} \bar{B}_f + \bar{D}_f$$
$$= \bar{C}_f (sI - U^{-1} \bar{A}_f)^{-1} U^{-1} \bar{B}_f + D_f$$

we can obtain an admissible estimator (12).

Corollary 1: A suboptimal full-order H_∞ estimator in the form of (4) for the system (1) subject to the uncertainty (3) can be found by solving the following optimization problem:

$$\min_{P_{11i},\, P_{22i},\, P_{12i},\, S_{11i},\, S_{22i},\, S_{12i},\, X,\, R,\, U,\, \bar{A}_f,\, \bar{B}_f,\, \bar{C}_f,\, \bar{D}_f,\, \lambda_1,\, \lambda_2} \gamma$$
$$\text{s.t. } (11),\ i = 1, 2, \cdots, r.$$

Observe that for fixed λ_1 and λ_2, (10) are linear with respect to P_{11i}, P_{12i}, X, R, U, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f, \bar{D}_f$ and hence can be solved by LMI Toolbox. The problem is then how to find the optimal values of λ_1 and λ_2. This can be completed by using the Matlab command *fminsearch*.

Note that Theorem 2 has been derived with a specialized matrix F of the form (10) in order to linearize the matrix inequality. This, however, is restrictive. Along the line in [12], in the following we develop an iterative algorithm which can be applied to refine the estimator design using Theorem 2.

Notice that

$$A_{0cli} = \begin{bmatrix} A_i & 0 \\ B_f C_i & A_f \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} A_f & B_f \end{bmatrix} \begin{bmatrix} 0 & I \\ C_i & 0 \end{bmatrix}, \tag{14}$$
$$B_{cli} = \begin{bmatrix} B_i \\ B_f D_i \end{bmatrix} = \begin{bmatrix} B_i \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} A_f & B_f \end{bmatrix} \begin{bmatrix} 0 \\ D_i \end{bmatrix}$$

the following iterative procedure can be applied.

Algorithm

Step 1: Given the estimator parameters (A_f, B_f, C_f, D_f), F, G, P_i, and S_i may be found by minimizing γ subject to (8). The initial (A_f, B_f, C_f, D_f) can be the suboptimal estimator designed by (11).

Step 2: With the F, G, P_i and S_i obtained in Step 1, an improved estimator can be obtained by minimizing γ subject to (8) with the consideration of (13).

Step 3: If $|\gamma_{k-1} - \gamma_k| < \mu$, stop the iteration where μ is a prescribed tolerance. Otherwise, increase $k :=$ $k+1$ and go to Step 1.

5. A NUMERICAL EXAMPLE

Consider an uncertain linear system given by (1) with the following parameter [5]:

$$A_0 = \begin{bmatrix} 0 & 3+\theta \\ -4 & -5 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.2+\sigma \end{bmatrix},$$
$$B = \begin{bmatrix} -0.4545 \\ 0.9090 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 100 \end{bmatrix},\ D = 1,\quad L = \begin{bmatrix} 0 & 100 \end{bmatrix},\ T = 0,\ \rho = 0.3,$$

where the uncertain parameters satisfy $|\theta| \le 0.3$, $|\sigma| \le$ 0.1. For this system, a strictly proper estimator is designed by imposing the condition $D_f = 0$. According to the Corollary 1, when (λ_1, λ_2) is chosen to be $(2000, 2000)$, the robust H_∞ estimator is given by

$$A_f = \begin{bmatrix} 0.0032 & 0.0339 \\ 0.0690 & 3.0402 \end{bmatrix},$$
$$B_f = \begin{bmatrix} -0.0002 \\ -0.0148 \end{bmatrix},$$
$$C_f = \begin{bmatrix} 0.0004 & 100.0003 \end{bmatrix}$$

with the H_∞ guaranteed cost value of 0.1384. Using LMI Tool and *fminsearch* command, when the original of (λ_1, λ_2) is chosen to be $(1, 1)$, the robust estimator is given by

$$A_f = \begin{bmatrix} -0.0189 & -0.0029 \\ 0.4146 & 18.1800 \end{bmatrix},$$
$$B_f = \begin{bmatrix} -0.0000 \\ -0.0885 \end{bmatrix},$$
$$C_f = \begin{bmatrix} -0.0006 & 100.0003 \end{bmatrix}$$

with the H_∞ guaranteed cost value of 0.1362, when the parameter (λ_1, λ_2) is $(216.4842, 278.3531)$. By the method proposed based on Lemma 1, the minimum guaranteed H_∞ cost is given by $\gamma^* = 0.1404$. From the comparison it is easily seen that the proposed approach is less conservative.

6. CONCLUSION

An improved LMI representation for the H_∞ performance of the linear system with time-varying state delay is first proposed. The newly proposed criterion exhibits a kind of decoupling between the system matrices and the positive definite matrices, it is thus intended to achieve much less conservativeness since the convex-dependent Lyapunov function can be allowed to use. The new criterion is then applied to design robust H_∞ estimators for polytopic uncertain time-delay systems.

REFERENCES

[1] R. M. Palhares and P. L. D. Peres, "Robust H_∞ filtering design with pole constraints via linear matrix inequality," *Journal of Optimization Theory and Application*, vol. 102, no. 2, pp. 713-723, Feb. 1999.

[2] H. J. Gao and C. H. Wang, "New approaches to robust l2-linf and H_∞ filtering for uncertain discrete-time systems," *Science In China (Series F)*, vol. 46, no. 5, pp. 356-370, Oct. 2003.

[3] Y. S. Lin, J. X. Qian, and S. L. Xu, "A LMI approach to robust H_∞ filtering for discrete-time systems," *Proc. of IEEE Conference on Control Application*, 2003, pp. 230-233.

[4] A. W. Pila, U. Shaked, and C. E. de Souza, "H_∞ filtering for continuous-time linear systems with delay," *IEEE Trans. Automat. Contr*., vol. 44, no. 7, pp. 1412-1417, July 1999.

[5] C. E. de Souza, R. M. Palhares, and P. L. D. Peres, "Robust H_∞ filter design for uncertain linear systems with multiple time-varying state delays," *IEEE Trans. Signal Processing*, vol. 49, no. 3, pp. 569-576, Mar. 2001.

[6] E. Fridman, U. Shaked, and L. Xie, "Robust H_∞ filtering of linear systems with time-varying delay," *IEEE Trans. on Automatic Control*, vol. 48, no. 1, pp. 159-165, Jan. 2003.

[7] C. E. de Souza, R. M. Palhares, and P. L. D. Peres, "Robust H_∞ filtering for uncertain discrete-time state-delayed systems," *IEEE Trans. on Signal Processing*, vol. 49, no. 8, pp. 1696-1703, Aug. 2001.

[8] S. Femmam, "A delay-dependent approach analysis to robust filtering," *Proc. of the Thirty-Sixth Southeastern Symposium on* System Theory, pp. 389-392, 2004.

[9] H. Gao and C. Wang, "A delay-dependent approach to robust H_∞ filtering for uncertain discrete-time state-delayed systems," *IEEE Trans. on Signal Processing*, vol. 52, no. 6, pp. 1631-1640, June 2004.

[10] E. Fridman and U. Shaked, "A new H_∞ filter design for linear time delay systems," *IEEE Trans. on Signal Processing,* vol. 49, no. 11, pp. 2839-2843, Nov. 2001.

[11] E. T. Jeung, J. H. Kim, and H. B. Pari, "H_∞ output controller design for linear systems with time-varying delayed state," *IEEE Trans. on Automatic Control*, vol. 43, no. 7, pp. 971-974, July 1998.

[12] L. H. Xie, L. L. Hu, D. Zhang, and H. S. Zhang, "Robust filtering for uncertain discrete-time: An improved LMI approach," *Proc. of the 42nd IEEE Conference On Decision and Control*, Hawaii, USA, 2003, pp. 906-911.

[13] A. G. Wu, H. F. Dong, and G. R. Duan, "Improved robust Hinf estimation for uncertain continuous-time systems," *Journal of Systems Science and Complexity*, vol. 20, no. 3, pp. 362-369, Sep. 2007.

Ai-Guo Wu was born in Gong'an County, Hubei Province on September 20, 1980. He received his B.Eng. degree in Automation in 2002, M.Eng. degree in Navigation, Guidance and Control in 2004 and Ph.D. degree in Control Science and Engineering in 2008 all from Harbin Institute of Technology. In Oct. 2008, he joined with Harbin Institute of Technology Shenzhen Graduate School as an Assistant Professor. Since Dec. 2007, he has served as a reviewer from American *Mathematical Reviews*. He is the author and co-author of over 40 publications. Now his main research interests include observer design, descriptor linear systems and nonlinear control.

Jin Dong is currently an undergraduate students at the Center for Control Theory and Guidance Technology, Harbin Institute of Technology, China. His research interests include robust control and nonlinear systems.

Guang-Ren Duan received his B.Sc. degree in Applied Mathematics, and both his M.Sc. and Ph.D. degrees in Control Systems Theory. From 1989 to 1991, he was a post-doctoral researcher at Harbin Institute of Technology, where he became a professor of control systems theory in 1991. He visited the University of Hull, UK, and the University of Sheffield, UK from December 1996 to October 1998, and worked at the Queen's University of Belfast, UK from October 1998 to October 2002. Since August 2000, he has been elected Specially Employed Professor at Harbin Institute of Technology sponsored by the Cheung Kong Scholars Program of the Chinese government. He is currently the Director of the Center for Control Theory and Guidance Technology at Harbin Institute of Technology. His main research interests include robust control, descriptor systems, missile autopilot design and magnetic bearing control. Dr. Duan is a Charted Engineer in the UK, a Senior Member of IEEE and a Fellow of IEE.