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Delay-Dependent Robust H_∞ Control for Uncertain Fuzzy Markovian Jump Systems

Yashun Zhang, Shengyuan Xu*, and Jihui Zhang

Abstract: This paper is concerned with the problem of delay-dependent robust H_∞ control for uncertain fuzzy Markovian jump systems with time delays. The purpose is to design a mode-dependent state-feedback fuzzy controller such that the closed-loop system is robustly stochastically stable and satisfies an H_∞ performance level. By introducing slack matrix variables, a delay-dependent sufficient condition for the solvability of the problem is proposed in terms of linear matrix inequalities. An illustrative example is finally given to show the applicability and effectiveness of the proposed method.

Keywords: Delay-dependent H_∞ control, fuzzy systems, Markovian jump systems, uncertain systems.

1. INTRODUCTION

Lots of practical dynamic systems are driven by discrete events such as random component failures or repairs, sudden environmental disturbances and changes in the interconnections of subsystems. Each discrete event changes the structure or parameters of the systems. These complex systems can be described as hybrid system models which consist of two kinds of state variables: continuous state variables and discrete event variables. Markovian jump systems belong to the category of stochastic hybrid systems and the discrete event variables are system modes governed by a discrete-state Markovian process. Markovian jump systems have different system parameters under different system modes. Over the past decades, stability analysis and controller synthesis for Markovian jump linear systems have been extensively studied; see, e.g., [1-3] and the references therein.

For Markovian jump nonlinear systems, however, very few results are available because nonlinear dynamics are extremely difficult to deal with. Recently, the innovative Takagi-Sugeno (T-S) fuzzy-model-based technique becomes quite popular. In a T-S model, a linear system is adopted as the consequent part of each fuzzy rule, which makes a nonlinear system be represented as a weighted sum of some simple linear subsystems. As a result, it provides an efficient approach to taking full advantages of the fruitful modern linear control theory to the nonlinear control. During the past decades, for T-S fuzzy models, many control methods have been studied and many control techniques using the linear matrix inequalities (LMIs) have been investigated in [4-10]. Since fuzzy control has been proved to be a powerful method for the control problem of complex nonlinear systems, the study of fuzzy Markovian jump systems has attracted much attention during the past years. For instance, the stabilization and H_∞ control for fuzzy Markovian jump systems have been studied in [11] and [12], respectively. Recently, the problems of stability analysis and controller design for fuzzy Markovian jump systems have been addressed in [13] by introducing some slack variables to separate Lyapunov matrices from system matrices.

It has been known that the existence of time delays often causes instability or poor performance of a control system. A great number of results on various control issues related to time-delay systems have been presented. For fuzzy systems with time delays, many results have also been reported in [14-18] and the references therein. Recently, much attention has been paid to Markovian jump linear systems with time delays. For example, the problems of delay-independent robust stabilization and H_∞ control were investigated in [19-21]. Delay-depend-ent stabilization conditions were presented in [22-24].

In this paper, we consider the problem of robust H_∞ control for a class of T-S fuzzy Markovian jump systems with time delays and parameter uncertainties. The parameter uncertainties are assumed to be time varying but norm bounded. The aim of this paper is to design a

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Yashun Zhang and Shengyuan Xu are with the School of Automation, Nanjing University of Science and Technology, Nanjing, 210094, P. R. China (e-mails: syxu02@yahoo.com.cn, yashunzhang@gmail.com).

Jihui Zhang is with the School of Automation Engineering of Qingdao University, Qingdao, 266071, P. R. China (e-mail: zhangjihui@qdu.edu.cn).

* Corresponding author.

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mode-dependent fuzzy controller such that the resulting closed-loop system is robustly stochastically stable and satisfies a prescribed H_∞ performance level for all admissible uncertainties. A delay-dependent sufficient condition for the solvability of the problem is given in terms of certain LMIs. Desired state-feedback gains can be obtained by solving these obtained LMIs. Finally, an illustrative example is presented to demonstrate the effectiveness of the design method.

Notation: For real symmetric matrices X and Y, the notation $X \le Y$ and $X < Y$ mean that the matrix $X - Y$ is positive-semidefinite and positive-definite, respectively. I is the identity matrix with appropriate dimensions. The superscript "T" represents the transpose. * is used as an ellipsis for terms that are induced by symmetry. $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. Matrices, if explicitly stated, are assumed to be compatible dimensions for algebra operations.

2. SYSTEM DESCRIPTIONS

The class of uncertain nonlinear time-delay systems with Markovian jump parameters under consideration can be described by the following T-S fuzzy Markovian jump systems with time delays:

Plant Rule i : **IF** $s_1(t)$ is μ_{i1} and $s_2(t)$ is μ_{i2} and … and $s_g(t)$ is μ_{ig} **THEN**

$$\begin{aligned}\dot{x}(t) = &\left[A_i(r(t)) + \Delta A_i(r(t),t)\right]x(t)\\ &+\left[A_{di}(r(t)) + \Delta A_{di}(r(t),t)\right]x(t-\tau)\\ &+\left[B_{1i}(r(t)) + \Delta B_{1i}(r(t),t)\right]u(t)\\ &+B_{2i}(r(t))\omega(t),\end{aligned} \tag{1}$$

$$\begin{aligned}z(t) = &\left[C_i(r(t)) + \Delta C_i(r(t),t)\right]x(t)\\ &+\left[C_{di}(r(t)) + \Delta C_{di}(r(t),t)\right]x(t-\tau)\\ &+\left[D_{1i}(r(t)) + \Delta D_{1i}(r(t),t)\right]u(t)\\ &+D_{2i}(r(t))\omega(t),\end{aligned} \tag{2}$$

$$x(t) = \phi(t), \quad t \in [-\tau, 0], \quad r(0) = r_0, \tag{3}$$

where $i \in S \triangleq \{1,2,...,s\}$, and s is the number of **IF-Then** rules; μ_{ij} is the fuzzy set; $x(t) \in \mathbb{R}^n$ is the system state; $u(t) \in \mathbb{R}^m$ is the control input; $z(t) \in \mathbb{R}^p$ is the control output; $\omega(t) \in \mathbb{R}^q$ is the exogenous disturbance signal in $L_2[0,\infty)$; $s_1(t)$, $s_2(t)$, …, $s_g(t)$ are the premise variables; the scalar $\tau > 0$ is the unknown constant time delay; $A_i(r(t))$, $A_{di}(r(t))$, $B_{1i}(r(t))$, $B_{2i}(r(t))$, $C_{di}(r(t))$, $D_{1i}(r(t))$, and $D_{2i}(r(t))$ are appropriately dimensioned real-valued matrix functions of the Markov process $\{r(t)\}$; in (3), $\phi(t)$ is the continuously differentiable initial function on $[-\tau, 0]$ and r_0 is the initial mode; $\{r(t)\}$ is a continuous-time discrete-state Markov process taking values in a finite set $T=\{1, 2, …, N\}$. The transition probabilities of the process $\{r(t)\}$ are given by

$$\begin{aligned}P_{kl} &= \Pr(r(t+\Delta) = l \mid r(t) = k)\\ &= \begin{cases}\pi_{kl}\Delta + o(\Delta), & k \ne l\\ 1 + \pi_{kk}\Delta + o(\Delta), & k = l,\end{cases}\end{aligned} \tag{4}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, and π_{kl} is the transition probability rate from mode k to mode l satisfying $\pi_{kl} \ge 0$, $k \ne l$ and $\pi_{kk} = -\sum_{l\in T, l\ne k}\pi_{kl}$. For each possible $r(t) = k$, $k \in T$, any matrix as $\Omega(r(t))$ will be denoted by Ω_k. The real-valued unknown matrices representing the time-varying parameter uncertainties are assumed to be of the form

$$\begin{aligned}&\begin{bmatrix}\Delta A_{i,k}(t) & \Delta A_{di,k}(t) & \Delta B_{1i,k}(t)\\ \Delta C_{i,k}(t) & \Delta C_{di,k}(t) & \Delta D_{1i,k}(t)\end{bmatrix}\\ &= \begin{bmatrix}E_{1k}\\ E_{2k}\end{bmatrix} F_k(t)\begin{bmatrix}H_{1i,k} & H_{2i,k} & H_{3i,k}\end{bmatrix},\end{aligned} \tag{5}$$

where E_{1k}, E_{2k}, $H_{1i,k}$, $H_{2i,k}$, and $H_{3i,k}$ are known real constant matrices for any $k \in T$ and $F_k(t)$ is an unknown time-varying Lebesgue measurable matrix function satisfying $F_k^T(t)F_k(t) \le I, \forall k \in T$.

The output of the dynamic fuzzy model in (1)-(3) can be represented by

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{s} h_i(s(t))\Big\{&\left[A_{i,k} + \Delta A_{i,k}(t)\right]x(t)\\ &+\left[A_{di,k} + \Delta A_{di,k}(t)\right]x(t-\tau)\\ &+\left[B_{1i,k} + \Delta B_{1i,k}(t)\right]u(t) + B_{2i,k}\omega(t)\Big\},\end{aligned} \tag{6}$$

$$\begin{aligned}z(t) = \sum_{i=1}^{s} h_i(s(t))\Big\{&\left[C_{i,k} + \Delta C_{i,k}(t)\right]x(t)\\ &+\left[C_{di,k} + \Delta C_{di,k}(t)\right]x(t-\tau)\\ &+\left[D_{1i,k} + \Delta D_{1i,k}(t)\right]u(t) + D_{2i,k}\omega(t)\Big\},\end{aligned} \tag{7}$$

where

$$h_i(s(t)) = \frac{\varpi_i(s(t))}{\sum_{j=1}^{s}\varpi_j(s(t))},$$

$$\varpi_i(s(t)) = \prod_{j=1}^{g}\mu_{ij}(s_j(t)),$$

$$s(t) = \begin{bmatrix}s_1(t) & s_2(t) & \cdots & s_g(t)\end{bmatrix},$$

in which, $\mu_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in μ_{ij}. Then, it can be seen that, for $i \in S$ and all t, $\sum_{i=1}^{s} h_i(s(t)) = 1$, and $h_i(s(t)) \ge 0$.

Next, using the parallel distributed compensation technique, we obtain the following mode-dependent fuzzy controller for the system in (1)-(3):

Controller Rule i **: IF** $s_1(t)$ is μ_{i1} and $s_2(t)$ is μ_{i2} and … $s_g(t)$ is μ_{ig}, **THEN**

$$u(t) = -K_{i,k}x(t), \quad i \in S, k \in T,$$

where $K_{i,k} \in \mathbb{R}^{m\times n}$ are matrices to be determined later. Then the overall state-feedback fuzzy controller is given by

$$u(t) = -\sum_{i=1}^{s} h_i(s(t))K_{i,k}x(t) = -K_k(h)x(t). \tag{8}$$

Any matrix as $\sum_{i=1}^{s} h_i(s(t))\Omega_i$ will be denoted by $\Omega(h)$ to simplify the notation. Then, by the overall fuzzy controller, the closed-loop system is described by

$$\begin{aligned}\dot{x}(t) = &\left[\hat{A}_k(h) - \hat{B}_{1k}(h)K_k(h)\right]x(t)\\ &+ \hat{A}_{dk}(h)x(t-\tau) + B_{2k}(h)\omega(t),\end{aligned} \tag{9}$$

$$\begin{aligned}z(t) = &\left[\hat{C}_k(h) - \hat{D}_{1k}(h)K_k(h)\right]x(t)\\ &+ \hat{C}_{dk}(h)x(t-\tau) + D_{2k}(h)\omega(t),\end{aligned} \tag{10}$$

where

$$\begin{aligned}
\hat{A}_k(h) &= A_k(h) + E_{1k}F_k(t)H_{1k}(h),\\
\hat{A}_{dk}(h) &= A_{dk}(h) + E_{1k}F_k(t)H_{2k}(h),\\
\hat{B}_1(h) &= B_{1k}(h) + E_{1k}F_k(t)H_{3k}(h),\\
\hat{C}_k(h) &= C_k(h) + E_{2k}F_k(t)H_{1k}(h),\\
\hat{C}_{dk}(h) &= C_{dk}(h) + E_{2k}F_k(t)H_{2k}(h),\\
\hat{D}_{1k}(h) &= D_{1k}(h) + E_{2k}F_k(t)H_{3k}(h).
\end{aligned}$$

3. H_∞ PERFORMANCE ANALYSIS

The following theorem provides a condition for H_∞ performance analysis of the open-loop system.

Theorem 1: Consider the fuzzy Markovian jump time-delay system in (6), (7) with $u(t) \equiv 0$. Then, given a scalar $\gamma > 0$, for any $\tau \in (0, \bar{\tau}]$ the fuzzy Markovian jump time-delay system in (6), (7) with $u(t) \equiv 0$ is robustly stochastically stable and satisfies $E\left[\int_0^\infty z^T(t)z(t)dt\right] \le \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt$ for any $\omega(t) \in L_2[0,\infty)$ under the condition $x(t) = 0$ for all $t < 0$, if there exist scalars $\alpha_k > 0$, $\beta_k > 0$ and matrices $Q > 0$, $Z > 0$, $P_k > 0$, Y_k, W_k, $U_{ij,k}$, $1 \le i < j \le s$, $k \in T$, such that for all $k \in T$, the following LMIs hold:

$$\Sigma_{ij,k} + \Sigma_{ji,k} < U_{ij,k} + U_{ij,k}^T, \quad 1 \le i < j \le s, \tag{11}$$

$$\begin{bmatrix} \Sigma_{11,k} & * & \cdots & * \\ U_{12,k}^T & \Sigma_{22,k} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ U_{1s,k}^T & U_{2s,k}^T & \cdots & \Sigma_{ss,k} \end{bmatrix} < 0, \tag{12}$$

where

$$\Sigma_{ij,k} = \begin{bmatrix} \bar{\Omega}_{ij,k} & * & * & * \\ L_{1i,k}^T & -\alpha_k I & * & * \\ H_{i,k} & 0 & -\beta_k I & * \\ L_{2i,k} & 0 & 0 & -I + \beta_k E_{2k}E_{2k}^T \end{bmatrix},$$

$$\bar{\Omega}_{ij,k} = \Omega_{ij,k} + \frac{1}{2}\left(\alpha_k H_{i,k}^T H_{j,k} + \alpha_k H_{j,k}^T H_{i,k}\right),$$

$$L_{1i,k} = \left[E_{1k}^T P_k \;\; 0 \;\; 0 \;\; \bar{\tau}E_{1k}^T ZB_{2i,k} + \bar{\tau}E_{2k}^T D_{2i,k} \;\; E_{1k}^T Z\right]^T,$$

$$L_{2i,k} = \left[C_{i,k} \;\; C_{di,k} \;\; 0 \;\; 0 \;\; 0\right],$$

$$H_{i,k} = \left[H_{1i,k} \;\; H_{2i,k} \;\; 0 \;\; 0 \;\; 0\right],$$

$$\Omega_{ij,k} = \begin{bmatrix} \Psi_{1i,k} & * & * & * & * \\ \Psi_{2i,k} & W_k + W_k^T - Q & * & * & * \\ \bar{\tau}Y_k & \bar{\tau}W_k & -\bar{\tau}Z & * & * \\ \Gamma_{1ij,k} & \Gamma_{2ij,k} & 0 & \Gamma_{3ij,k} & * \\ \bar{\tau}ZA_{i,k} & \bar{\tau}ZA_{di,k} & 0 & 0 & -\bar{\tau}Z \end{bmatrix},$$

$$\Gamma_{1ij,k} = B_{2i,k}^T P_k + \bar{\tau}B_{2i,k}^T ZA_{j,k} + D_{2i,k}^T C_{j,k},$$

$$\Gamma_{2ij,k} = \bar{\tau}B_{2i,k}^T ZA_{dj,k} + D_{2i,k}^T C_{dj,k},$$

$$\begin{aligned}\Gamma_{3ij,k} = &\frac{\bar{\tau}}{2}\left(B_{2i,k}^T ZB_{2j,k} + B_{2j,k}^T ZB_{2i,k}\right)\\ &-\gamma^2 I + \frac{1}{2}\left(D_{2i,k}^T D_{2j,k} + D_{2j,k}^T D_{2i,k}\right),\end{aligned}$$

$$\Psi_{1i,k} = P_k A_{i,k} + A_{i,k}^T P_k + \sum_{l=1}^{N}\pi_{kl}P_l + Q - Y_k - Y_k^T,$$

$$\Psi_{2i,k} = A_{di,k}^T P_k + Y_k - W_k^T.$$

Proof: By (11) and (12), we have that, for each $k \in T$,

$$\begin{aligned}
&\sum_{i=1}^{s}\sum_{j=1}^{s} h_i(s(t))h_j(s(t))\Sigma_{ij,k}\\
&= \sum_{i=1}^{s} h_i^2(s(t))\Sigma_{ii,k} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s} h_i(s(t))h_j(s(t))(\Sigma_{ij,k} + \Sigma_{ji,k})\\
&\le \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_s I \end{bmatrix}^T \begin{bmatrix} \Sigma_{11,k} & * & \cdots & * \\ U_{12,k}^T & \Sigma_{22,k} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ U_{1s,k}^T & U_{2s,k}^T & \cdots & \Sigma_{ss,k} \end{bmatrix} \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_s I \end{bmatrix} < 0.
\end{aligned} \tag{13}$$

Therefore, for each $k \in T$,

$$\sum_{i=1}^{s}\sum_{j=1}^{s}h_i(s(t))h_j(s(t))\Sigma_{ij,k} = \begin{bmatrix} \Omega_k(h)+\alpha_k H_k^T(h)H_k(h) & * & * & * \\ L_{1k}^T(h) & -\alpha_k I & * & * \\ H_k(h) & 0 & -\beta_k I & * \\ L_{2k}(h) & 0 & 0 & -I+\beta_k E_{2k}E_{2k}^T \end{bmatrix} < 0, \quad (14)$$

where

$$\Omega_k(h) = \begin{bmatrix} \Psi_{1k}(h) & * & * & * & * \\ A_{dk}^T(h)P_k+Y_k-W_k^T & W_k+W_k^T-Q & * & * & * \\ \bar{\tau}Y_k & \bar{\tau}W_k & -\bar{\tau}Z & * & * \\ \Gamma_{1k}(h) & \Gamma_{2k}(h) & 0 & \Gamma_{3k}(h) & * \\ \bar{\tau}ZA_k(h) & \bar{\tau}ZA_{dk}(h) & 0 & 0 & -\bar{\tau}Z \end{bmatrix},$$

$$\Psi_{1k}(h) = P_kA_k(h)+A_k^T(h)P_k+\sum_{l=1}^{N}\pi_{kl}P_l+Q-Y_k-Y_k^T,$$

$$L_{1k}(h) = \begin{bmatrix} E_{1k}^TP_k & 0 & 0 & \bar{\tau}E_{1k}^TZB_{2k}(h)+\bar{\tau}E_{2k}^TD_{2k}(h) & E_{1k}^TZ \end{bmatrix}^T,$$

$$L_{2k}(h) = \begin{bmatrix} C_k(h) & C_{dk}(h) & 0 & 0 & 0 \end{bmatrix},$$

$$H_k(h) = \begin{bmatrix} H_{1k}(h) & H_{2k}(h) & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{1k}(h) = B_{2k}^T(h)P_k+\bar{\tau}B_{2k}^T(h)ZA_k(h)+D_{2k}^T(h)C_k(h),$$

$$\Gamma_{2k}(h) = \bar{\tau}B_{2k}^T(h)ZA_{dk}(h)+D_{2k}^T(h)C_{dk}(h),$$

$$\Gamma_{3k}(h) = \bar{\tau}B_{2k}^T(h)ZB_{2k}(h)-\gamma^2I+D_{2k}^T(h)D_{2k}(h).$$

Now, for each $k\in T$, define

$$\begin{aligned}\Theta_k = {}& \Omega_k(h)+\left[L_{2k}(h)+E_{2k}F_k(t)H_k(h)\right]^T \\ & \times\left[L_{2k}(h)+E_{2k}F_k(t)H_k(h)\right] \\ & +L_{1k}(h)F_k(t)H_k(h) \\ & +H_k^T(h)F_k^T(t)L_{1k}^T(h).\end{aligned} \quad (15)$$

Then, it follows from (12) that for each $k\in T$,

$$I-\beta_kE_{2k}E_{2k}^T>0. \quad (16)$$

From Lemma 4 in [25] and (16), it can be seen that

$$\begin{aligned}&\left[L_{2k}(h)+E_{2k}F_k(t)H_k(h)\right]^T \\ &\quad\times\left[L_{2k}(h)+E_{2k}F_k(t)H_k(h)\right] \\ &\le L_{2k}^T(h)\left(I-\beta_kE_{2k}E_{2k}^T\right)^{-1}L_{2k}(h) \\ &\quad+\beta_k^{-1}H_k^T(h)H_k(h).\end{aligned} \quad (17)$$

On the other hand, note that, for each $k\in T$,

$$\begin{aligned}0 &\le \alpha_k\left[\alpha_k^{-1}L_{1k}(h)-H_k^T(h)F_k^T(t)\right] \\ &\quad\times\left[\alpha_k^{-1}L_{1k}^T(h)-F_k(t)H_k(h)\right] \\ &\le \alpha_k^{-1}L_{1k}(h)L_{1k}^T(h)+\alpha_kH_k^T(h)H_k(h) \\ &\quad-L_{1k}(h)F_k(t)H_k(h)-H_k^T(h)F_k^T(t)L_{1k}^T(h).\end{aligned} \quad (18)$$

This implies

$$\begin{aligned}&L_{1k}(h)F_k(t)H_k(h)+H_k^T(h)F_k^T(t)L_{1k}^T(h) \\ &\quad\le \alpha_k^{-1}L_{1k}(h)L_{1k}^T(h)+\alpha_kH_k^T(h)H_k(h).\end{aligned} \quad (19)$$

Therefore, from (15), (17), and (19) we obtain

$$\begin{aligned}\Theta_k \le {}& \Omega_k(h)+L_{2k}^T(h)\left(I-\beta_kE_{2k}E_{2k}^T\right)^{-1}L_{2k}(h) \\ &+\beta_k^{-1}H_k^T(h)H_k(h)+\alpha_k^{-1}L_{1k}(h)L_{1k}^T(h) \\ &+\alpha_kH_k^T(h)H_k(h).\end{aligned} \quad (20)$$

Applying the Schur complements to (14), we have

$$\begin{aligned}&\Omega_k(h)+L_{2k}^T(h)\left(I-\beta_kE_{2k}E_{2k}^T\right)^{-1}L_{2k}(h) \\ &\quad+\beta_k^{-1}H_k^T(h)H_k(h)+\alpha_k^{-1}L_{1k}(h)L_{1k}^T(h) \\ &\quad+\alpha_kH_k^T(h)H_k(h)<0.\end{aligned}$$

Hence, for each $k\in T$,

$$\Theta_k<0.$$

Then by applying the Schur complements to this inequality, we can see that there exists a scalar $\sigma>0$ such that, for any τ satisfying $0<\tau\le\bar{\tau}$,

$$\Lambda_k(\tau)<diag(-\sigma I_{n\times n},0,0,0), \qquad k\in T, \quad (21)$$

where

$$\Lambda_k(\tau) = \begin{bmatrix} \hat{\Psi}_{1k}(h)+\tau\hat{A}_k^T(h)Z\hat{A}_k(h)+\hat{C}_k^T(h)\hat{C}_k(h) & * & * & * \\ \hat{\Psi}_{2k}(h)+\tau\hat{A}_{dk}^T(h)Z\hat{A}_k(h)+\hat{C}_{dk}^T(h)\hat{C}_k(h) & \hat{\Psi}_{3k}(h) & * & * \\ \tau Y_k & \tau W_k & -\tau Z & * \\ \hat{\Gamma}_{1k}(h) & \hat{\Gamma}_{2k}(h) & 0 & \hat{\Gamma}_{3k}(h) \end{bmatrix},$$

$$\hat{\Psi}_{1k}(h) = P_k\hat{A}_k(h) + \hat{A}_k^T(h)P_k + \sum_{l=1}^{N}\pi_{kl}P_l + Q - Y_k - Y_k^T,$$
$$\hat{\Psi}_{2k}(h) = \hat{A}_{dk}^T(h)P_k + Y_k - W_k^T,$$
$$\hat{\Psi}_{3k}(h) = W_k + W_k^T - Q + \tau\hat{A}_{dk}^T(h)Z\hat{A}_{dk}(h) + \hat{C}_{dk}^T(h)\hat{C}_{dk}(h),$$
$$\hat{\Gamma}_{1k}(h) = B_{2k}^T(h)P_k + \tau B_{2k}^T(h)Z\hat{A}_k(h) + D_{2k}^T(h)\hat{C}_k(h),$$
$$\hat{\Gamma}_{2k}(h) = \tau B_{2k}^T(h)Z\hat{A}_{dk}(h) + D_{2k}^T(h)\hat{C}_{dk}(h),$$
$$\hat{\Gamma}_{3k}(h) = \tau B_{2k}^T(h)ZB_{2k}(h) - \gamma^2 I + D_{2k}^T(h)D_{2k}(h).$$

Next, denote $x_t = x(t+\theta)$, $-2\tau \le \theta \le 0$, and choose a mode-dependent Lyapunov–Krasovskii functional candidate, for each $k \in T$, as

$$V\left(x_t, r(t) = k\right) = \sum_{i=1}^{3} V_i(x_t, k), \tag{22}$$

where

$$V_1(x_t,k) = x^T(t)P_k x(t),$$
$$V_2(x_t,k) = \int_{-\tau}^{0}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha d\beta,$$
$$V_3(x_t,k) = \int_{t-\tau}^{t} x^T(\alpha)Qx(\alpha)d\alpha.$$

The weak infinitesimal operator ℓ of the Markov process $\{(x(t), r(t)), t \ge 0\}$ acting on the Lyapunov functional candidate is given by

$$\ell V(x_t, r(t)) = \lim_{\Delta\to 0}\frac{1}{\Delta}\left\{E[V(x_{t+\Delta}, r(t+\Delta)) \mid x_t, r(t)] - V(x_t, r(t))\right\}.$$

Then, we have that, when $t > \tau$,

$$\begin{aligned}
&\ell V_1(x_t,k)\\
&= x^T(t)\left[P_k\hat{A}_k(h) + \hat{A}_k^T(h)P_k + \sum_{l=1}^{N}\pi_{kl}P_l\right]x(t)\\
&\quad + 2x^T(t)P_kB_{2k}(h)\omega(t) + 2x^T(t)P_k\hat{A}_{dk}(h)\\
&\quad \times x(t-\tau) + 2x^T(t-\tau)W_k^T\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha\\
&\quad + 2x^T(t)Y_k^T\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha\\
&\quad - 2x^T(t)Y_k^T[x(t) - x(t-\tau)]\\
&\quad - 2x^T(t-\tau)W_k^T[x(t) - x(t-\tau)],
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\ell V_2(x_t,k)\\
&= \tau[\hat{A}_k(h)x(t) + \hat{A}_{dk}(h)x(t-\tau)]^T Z\\
&\quad \times[\hat{A}_k(h)x(t) + \hat{A}_{dk}(h)x(t-\tau)]\\
&\quad - \int_{t-\tau}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha + 2\tau\omega^T(t)B_{2k}^T(h)Z\\
&\quad \times[A_k(h)x(t) + A_{dk}(h)x(t-\tau)]\\
&\quad + \tau\omega^T(t)B_{2k}^T(h)ZB_{2k}(h)\omega(t),
\end{aligned} \tag{24}$$

$$\ell V_3(x_t,k) = \frac{1}{\tau}\int_{t-\tau}^{t}[x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau)]d\alpha. \tag{25}$$

Then it follows from (22)-(25) that

$$\begin{aligned}
&z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \ell V(x_t, r(t))\\
&= \frac{1}{\tau}\int_{t-\tau}^{t}\varepsilon^T(t,\alpha)\Lambda_k(\tau)\varepsilon(t,\alpha)d\alpha,
\end{aligned} \tag{26}$$

where

$$\varepsilon(t,\alpha) = [x^T(t) \quad x^T(t-\tau) \quad \dot{x}^T(\alpha) \quad \omega(t)]^T.$$

It follows from (21) and (26) that

$$\begin{aligned}
&z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \ell V(x_t, r(t))\\
&\le -\sigma x^T(t)x(t).
\end{aligned} \tag{27}$$

When $\omega(t) = 0$, it can be deduced from (27) that

$$\ell V(x_t, r(t)) \le -\sigma x^T(t)x(t).$$

By this and the result in [3], it is easy to see

$$\lim_{T\to\infty} E\left[\int_0^T x^T(t,\phi,r_0)dt\right] < \infty,$$

where $x(t,\phi,r_0)$ represents the trajectory of the state $x(t)$ at time t. Therefore, the uncertain fuzzy Markovian jump system with time delays is robustly stochastically stable.

Now, by using Dynkin's formula, we have that under the condition $x(t) = 0$ for all $t < 0$,

$$E[V(x_T, r(T))] = E\left[\int_0^T \ell V(x_t, r(t))dt\right]. \tag{28}$$

Define

$$J_T = E\left\{\int_0^T [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)]dt\right\}.$$

Then, from (27), (28) we can deduce

$$\begin{aligned}
J_T &= E\{\int_0^T [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\\
&\quad + \ell V(x_t, r(t))]\} - E[V(x_T, r(T))]\\
&\le \frac{1}{\tau}E[\int_0^T\int_{t-\tau}^{t}\varepsilon^T(t,\alpha)\Lambda_k(\tau)\varepsilon(t,\alpha)d\alpha dt] < 0,
\end{aligned}$$

which implies

$$E\left[\int_0^\infty z^T(t)z(t)dt\right] < \gamma^2\int_0^\infty \omega^T(t)\omega(t)dt.$$

This completes the proof.

Remark 1: Theorem 1 provides a condition guaranteeing an H_∞ performance level of a class of fuzzy Markovian jump systems in terms of LMIs. It should be pointed out that Theorem 1 can be easily extended to the time-varying delay case by using the method in the derivation of Theorem 1.

Remark 2: In the proof of Theorem 1, the weak infinitesimal $\ell V_1(x_t,k)$ remains unaffected when the slack matrix variables Y_k and W_k are introduced. Moreover, the slack matrix variables $U_{ij,k}$ are also introduced in order to obtain the relaxed LMIs. It is worth pointing out that these matrix variables are not required to be symmetric, which is different from [4]. Therefore, a more flexible LMI condition in (11), (12) is obtained and the potential conservatism is thus reduced.

4. ROBUST H_∞ CONTROL

We are now in the position to present the main result on robust H_∞ control for fuzzy Markovian jump systems with time delays.

Theorem 2: Given a scalar $\gamma > 0$. Then, for any $\tau \in (0, \bar{\tau}]$, the fuzzy Markovian jump system with time delay in (6), (7) is robustly stochastically stable and satisfies $E\left[\int_0^\infty z^T(t)z(t)dt\right] \le \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt$ for any nonzero $\omega(t) \in L_2[0,\infty)$ under the condition $x(t)=0$ for all $t<0$ via the fuzzy controller (8), if there exist scalars $\alpha_k > 0$ and matrices $R>0$, $T>0$, $X_k > 0$, $M_{i,k}$, $i \in S$, $k \in T$, and $N_{ij,k}$, $1 \le i < j \le s$, $k \in T$ such that for each $k \in T$ the following LMIs hold:

$$\Xi_{ij,k} + \Xi_{ji,k} < N_{ij,k} + N_{ij,k}^T, \qquad 1 \le i < j \le s, \tag{29}$$

$$\begin{bmatrix} \Xi_{11,k} & * & \cdots & * \\ N_{12,k}^T & \Xi_{22,k} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ N_{1s,k}^T & N_{2s,k}^T & \cdots & \Xi_{ss,k} \end{bmatrix} < 0, \tag{30}$$

where

$$\Xi_{ij,k} = \begin{bmatrix} \tilde{\Omega}_{ij,k} + \alpha_k \tilde{L}_k \tilde{L}_k^T & * \\ \tilde{H}_{ij,k} & -\alpha_k I \end{bmatrix},$$

$$\tilde{\Omega}_{ij,k} = \begin{bmatrix} \tilde{\Upsilon}_{1ij,k} + \pi_{kk} X_k & * & * & * & * & * & * \\ R A_{di,k}^T & -3R & * & * & * & * & * \\ \bar{\tau} T & -\bar{\tau} T & -\bar{\tau} T & * & * & * & * \\ B_{2i,k}^T & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{\tau} A_{i,k} X_k - \bar{\tau} B_{1i,k} M_{j,k} & \bar{\tau} A_{di,k} R & 0 & \bar{\tau} B_{2i,k} & -\bar{\tau} T & * & * \\ C_{i,k} X_k - D_{1i,k} M_{j,k} & C_{di,k} R & 0 & D_{2i,k} & 0 & -I & * \\ \Gamma_k^T & 0 & 0 & 0 & 0 & 0 & -\Phi_k \end{bmatrix},$$

$$\tilde{\Upsilon}_{1ij,k} = A_{i,k} X_k + X_k A_{i,k}^T - B_{1i,k} M_{j,k} - M_{j,k}^T B_{1i,k}^T - 2X_k,$$

$$\tilde{L}_k = \begin{bmatrix} E_{1k}^T & 0 & 0 & 0 & E_{1k}^T & E_{2k}^T & 0 \end{bmatrix}^T,$$

$$\tilde{H}_{ij,k} = [H_{1i,k} X_k - H_{3i,k} M_{j,k} \quad H_{2i,k} R \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Gamma_k = \begin{bmatrix} \sqrt{\pi_{k1}} X_k & \cdots & \sqrt{\pi_{k,k-1}} X_k & \sqrt{\pi_{k,k+1}} X_k & \cdots & \sqrt{\pi_{kN}} X_k & X_k \end{bmatrix},$$

$$\Phi_k = diag(X_1 \quad \cdots \quad X_{k-1} \quad X_{k+1} \quad \cdots \quad X_N \quad R).$$

In this case, desired state-feedback gains can be chosen as, for each $k \in T$,

$$K_{i,k} = M_{i,k} X_k^{-1}.$$

Proof: Following similar manipulations as in (22)-(26), we can obtain that the uncertain closed-loop system (9), (10) is stochastically stable and satisfies $E\left[\int_0^\infty z^T(t)z(t)dt\right] \le \lambda^2 \int_0^\infty \omega^T(t)\omega(t)dt$ for any nonzero $\omega(t) \in L_2[0,\infty)$ under the condition $x(t)=0$ for all $t<0$, if for each $k \in T$ and all $F_k(t)$ satisfying $F_k^T(t)F_k(t) \le I$ the following matrix inequality holds:

$$\tilde{\Lambda}_K(\tau) = \begin{bmatrix} \tilde{\Psi}_{1K}(h) + \tau \tilde{A}_k^T(h) Z \tilde{A}_k(h) + \tilde{C}_k^T(h)\tilde{C}_k(h) & * & * & * \\ \tilde{\Psi}_{2K}(h) + \tau \hat{A}_{dk}^T(h) Z \tilde{A}_k(h) + \hat{C}_{dk}^T(h)\tilde{C}_k(h) & \tilde{\Psi}_{3k}(h) & * & * \\ \tau Y_k & \tau W_k & -\tau Z & * \\ \tilde{\Gamma}_{1k} & \tilde{\Gamma}_{2k} & 0 & \tilde{\Gamma}_{3k} \end{bmatrix} < 0,$$

where

$$\tilde{A}_k(h) = \hat{A}_k(h) - \hat{B}_{1k}(h) K_k(h),$$

$$\tilde{\Psi}_{1k}(h) = P_k \tilde{A}_k(h) + \tilde{A}_k^T(h) P_k + \sum_{l=1}^N \pi_{kl} P_l + Q - Y_k - Y_k^T,$$

$$\tilde{\Psi}_{2k}(h) = \hat{A}_{dk}^T(h) P_k + Y_k - W_k^T,$$

$$\tilde{\Psi}_{3k}(h) = W_k + W_k^T - Q + \tau \hat{A}_{dk}^T(h) Z \hat{A}_{dk}(h) + \hat{C}_{dk}^T(h)\hat{C}_{dk}(h),$$

$$\tilde{\Gamma}_{1k} = B_{2k}^T(h) P_k + \tau B_{2k}^T(h) Z \tilde{A}_k(h) + D_{2k}^T(h)\tilde{C}_k(h),$$

$$\tilde{\Gamma}_{2k} = \tau B_{2k}^T(h) Z \hat{A}_{dk}(h) + D_{2k}^T(h)\hat{C}_{dk}(h),$$

$$\tilde{\Gamma}_{3k} = \tau B_{2k}^T(h) Z B_{2k}(h) - \gamma^2 I + D_{2k}^T(h) D_{2k}(h),$$

$$\tilde{C}_k(h) = \hat{C}_k(h) - \hat{D}_{1k}(h) K_k(h).$$

By the Schur complements, we obtain that $\tilde{\Lambda}_k(\tau) < 0$, $k \in T$, for any τ satisfying $0 < \tau \le \bar{\tau}$, if the following matrix inequality holds:

$$\begin{bmatrix} \tilde{\Psi}_{1k}(h) & * & * & * & * & * \\ \tilde{\Psi}_{2k}(h) & W_k + W_k^T - Q & * & * & * & * \\ \bar{\tau} Y_k & \bar{\tau} W_k & -\bar{\tau} Z & * & * & * \\ B_{2k}^T(h) P_k & 0 & 0 & -\gamma^2 I & * & * \\ \bar{\tau} Z \tilde{A}_k(h) & \bar{\tau} Z \hat{A}_{dk}(h) & 0 & \bar{\tau} Z B_{2k}(h) & -\bar{\tau} Z & * \\ \tilde{C}_k(h) & \hat{C}_{dk}(h) & 0 & D_{2k}(h) & 0 & -I \end{bmatrix} < 0. \quad (31)$$

Now let

$$X_k = P_k^{-1}, \quad M_{i,k} = K_{i,k} X_k, \quad R = Q^{-1},$$
$$T = Z^{-1}, \quad Y_k = P_k, \quad W_k = -Q.$$

By pre- and post-multiplying (31) by $diag\left(X_k, R, T, I, T, I\right)$ and its transpose, respectively, and applying the Schur complements, we obtain that (31) holds if the following matrix inequality holds for each $k \in T$,

$$\tilde{\Omega}_k(h) + \tilde{L}_k F_k(t) \tilde{H}_k(h) + \tilde{H}_k^T(h) F_k^T(t) \tilde{L}_k^T < 0, \quad (32)$$

where

$$\tilde{H}_k(h) = [H_{1k}(h) X_k - H_{3k}(h) M_k(h) \quad H_{2k}(h) R \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\tilde{\Upsilon}_{1k}(h) = A_k(h) X_k + X_k A_k^T(h) - B_{1k}(h) M_k(h) - M_k^T(h) B_{1k}^T(h) - 2X_k,$$

$$\tilde{\Omega}_k(h) = \begin{bmatrix} \tilde{\Upsilon}_{1k}(h) + \pi_{kk} X_k & * & * & * & * & * & * \\ R A_{dk}^T(h) + R + X_k & -3R & * & * & * & * & * \\ \bar{\tau} T & -\bar{\tau} T & -\bar{\tau} T & * & * & * & * \\ B_{2k}^T(h) & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{\tau} A_k(h) X_k - \bar{\tau} B_{1k}(h) M_k(h) & \bar{\tau} A_{dk}(h) R & 0 & \bar{\tau} B_{2k}(h) & -\bar{\tau} T & * & * \\ C_k(h) X_k - D_{1k}(h) M_k(h) & C_{dk}(h) R & 0 & D_{2k}(h) & 0 & -I & * \\ \Gamma_k^T & 0 & 0 & 0 & 0 & 0 & -\Phi_k \end{bmatrix} < 0.$$

By using the similar manipulations as in (18), we obtain that (32) holds for all $F_k(t)$ satisfying $F_k^T(t) F_k(t) \le I$, if the following matrix inequality holds for each $k \in T$ with scalars $\alpha_k > 0$,

$$\tilde{\Omega}_k(h) + \alpha_k \tilde{L}_k \tilde{L}_k^T + \alpha_k^{-1} \tilde{H}_k^T(h) \tilde{H}_k(h) < 0. \quad (33)$$

Then by the Schur complements, we have that (33) holds for each $k \in T$, if the following matrix inequality holds,

$$\sum_{i=1}^{s} \sum_{j=1}^{s} h_i(s(t)) h_j(s(t)) \Xi_{ij,k} < 0. \quad (34)$$

By using the relaxed technique in (13), for each $k \in T$, we have that (34) holds for any τ satisfying $0 < \tau \le \bar{\tau}$, if the LMIs in (29), (30) hold. Therefore, we have $\tilde{\Lambda}_k(\tau) < 0$. This completes the proof.

5. ILLUSTRATIVE EXAMPLE

In this section, we apply the above design method to robust H_∞ control of a computer simulated single link robot arm in [12]. We consider the following model of the single robot arm

$$\dot{x}_1(t) = x_2(t), \quad (35)$$

$$\dot{x}_2(t) = -\frac{M_k g l}{J_k} \sin(x_1(t)) - \frac{D(t)}{J_k} x_2(t) + \frac{1}{J_k} u(t) + 0.1\omega(t), \quad (36)$$

$$z(t) = x_1(t) + 0.2\omega(t), \qquad k = 1,2,3, \quad (37)$$

where $x_1(t)$, $x_2(t)$, $u(t)$, and $z(t)$ are the angle of the arm, the angular velocity, the control input, and the control output, respectively; $\omega(t)$ is the exogenous disturbance input with $\omega(t) \in L_2[0,\infty)$. The mass M_k and the inertia J_k have three modes: $M_1 = J_1 = 1$, $M_2 = J_2 = 5$, $M_3 = J_3 = 10$. The transition rate of the operation modes is given by

$$\Pi = \begin{bmatrix} -0.3 & 0.25 & 0.05 \\ 0.1 & -0.2 & 0.1 \\ 0.03 & 0.07 & -0.1 \end{bmatrix}.$$

The values of the length l, the acceleration of gravity g, and the damping $D(t)$ are given as $l = 0.5$, $g = 9.81$, and $D(t) \in [1.8, 2.2]$. We assume that $x_2(t)$ is perturbed by time delays to illustrate the proposed design method on the Markovian nonlinear time-delay system. The delayed model is given as

$$\dot{x}_1(t) = \mu x_2(t) + (1-\mu) x_2(t-\tau), \quad (38)$$

$$\dot{x}_2(t) = -\frac{M_k g l}{J_k} \sin(x_1(t)) - \frac{\mu D(t)}{J_k} x_2(t) + 0.1\omega(t) - \frac{(1-\mu) D(t)}{J_k} x_2(t-\tau) + \frac{1}{J_k} u(t), \quad (39)$$

$$z(t) = x_1(t) + 0.2\omega(t), \qquad k = 1,2,3, \quad (40)$$

where $\mu \in [0,1]$ is the constant representing the retarded coefficient. In this example, we assume $\mu = 0.7$. Without time delays, the example was studied in [12] in which the proposed design method can not be applied to this time-delay system.

Similar to [26], we set the fuzzy basis functions as

$$h_1(x_1(t)) = \begin{cases} \dfrac{\sin(x_1(t)) - \rho x_1(t)}{x_1(t)(1-\rho)}, & x_1(t) \neq 0 \\ 1, & x_1(t) \neq 0, \end{cases}$$

$$h_2(x_1(t)) = \begin{cases} \dfrac{x_1(t) - \sin(x_1(t))}{x_1(t)(1-\rho)}, & x_1(t) \neq 0 \\ 0, & x_1(t) \neq 0, \end{cases}$$

where $\rho = 10^{-2}/\pi$. Then, we represent the Markovian jump nonlinear time-delay system in (38)-(40) as the following T-S model, for $k = 1,2,3,$

Plant Rule 1: IF $x_1(t)$ is μ_{i1}, **THEN**

$$\begin{aligned} \dot{x}(t) &= \left[A_{i,k} + \Delta A_{i,k}(t)\right]x(t) + \left[A_{di,k} + \Delta A_{di,k}(t)\right]x(t-\tau) \\ &\quad + \left[B_{1i,k} + \Delta B_{1i,k}(t)\right]u(t) + B_{2i}\omega(t), \\ z(t) &= C_i x(t) + D_{2i}\omega(t), \end{aligned}$$

where μ_{11} is about 0 rad, μ_{21} is about π rad or $-\pi$ rad and

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad A_{1,1} = \begin{bmatrix} 0 & \mu \\ -gl & -2\mu \end{bmatrix},$$

$$A_{1,2} = \begin{bmatrix} 0 & \mu \\ -gl & -0.4\mu \end{bmatrix}, \ A_{1,3} = \begin{bmatrix} 0 & \mu \\ -gl & -0.2\mu \end{bmatrix},$$

$$A_{2,1} = \begin{bmatrix} 0 & \mu \\ -\rho gl & -2\mu \end{bmatrix}, \ A_{2,2} = \begin{bmatrix} 0 & \mu \\ -\rho gl & -0.4\mu \end{bmatrix},$$

$$A_{2,1} = \begin{bmatrix} 0 & \mu \\ -\rho gl & -2\mu \end{bmatrix}, \ A_{2,2} = \begin{bmatrix} 0 & \mu \\ -\rho gl & -0.4\mu \end{bmatrix},$$

$$A_{2,3} = \begin{bmatrix} 0 & \mu \\ -\rho gl & -0.2\mu \end{bmatrix},$$

$$A_{d1,1} = A_{d2,1} = \begin{bmatrix} 0 & 1-\mu \\ 0 & -2(1-\mu) \end{bmatrix},$$

$$A_{d1,2} = A_{d2,2} = \begin{bmatrix} 0 & 1-\mu \\ 0 & -0.4(1-\mu) \end{bmatrix},$$

$$A_{d1,3} = A_{d2,3} = \begin{bmatrix} 0 & 1-\mu \\ 0 & -0.2(1-\mu) \end{bmatrix},$$

$$B_{11,1} = B_{12,1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{11,2} = B_{12,2} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$B_{11,3} = B_{12,3} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \ B_{21} = B_{22} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$C_1 = C_2 = [1 \quad 0], \qquad D_{21} = D_{22} = 0.2.$$

The uncertain parameters $\Delta A_{i,k}(t)$, $\Delta A_{di,k}(t)$, and $\Delta B_{1i,k}(t)$ satisfy (5) with

$$E_{1k} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad H_{1i,k} = \begin{bmatrix} 0 & 0 \\ 0 & \mu \end{bmatrix},$$

$$H_{2i,k} = \begin{bmatrix} 0 & 0 \\ 0 & 1-\mu \end{bmatrix}, \ H_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

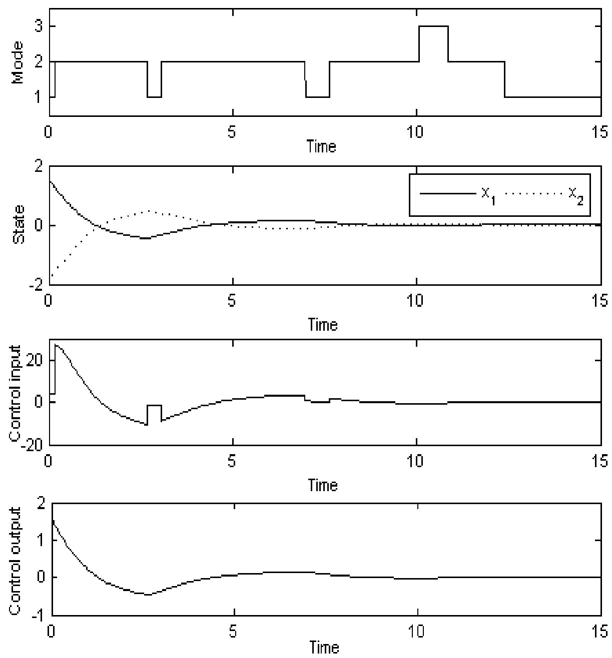

Fig. 1. Operation mode and control results of the closed-loop system.

The purpose of this example is to develop a fuzzy controller such that the resulting closed-loop system is robustly stochastically stable and satisfies an H_∞ performance level γ. Based on Theorem 2, we obtain that, when the prescribed γ is 0.3, the maximum allowable size of the delay τ for the above robust H_∞ control problem is 1.2. Then by using the Matlab LMI Control Toolbox to solve the LMIs in (29), (30) we obtain the parameters of the fuzzy controller as follows:

$$\begin{aligned} K_{1,1} &= [-1.8873 \quad 1.8151], \\ K_{1,2} &= [-9.2868 \quad 15.2084], \\ K_{1,3} &= [-18.0308 \quad 32.2445], \\ K_{2,1} &= [3.0021 \quad 1.8151], \\ K_{2,2} &= [15.1601 \quad 15.2084], \\ K_{2,3} &= [30.8630 \quad 32.2445]. \end{aligned}$$

Now, we set the initial conditions as $r_0 = 1$ and $\phi(t) = [0.5\pi, -2]^T$, $t \in [-1.2, 0]$. We further assume that

$$D(t) = 2 + 0.2\sin(t), \quad \omega(t) = \frac{1}{0.5 + 1.2t}, \quad t \geq 0.$$

We now apply the designed fuzzy controller in the form of (8) to the Markovian nonlinear system in (38)-(40). The simulation is shown in Fig. 1. The result shows that the designed fuzzy controller can effectively stabilize the uncertain Markovian jump nonlinear time-delay system in (38)-(40) with an H_∞ performance level γ.

6. CONCLUSION

The problem of robust H_∞ control for a class of fuzzy Markovian jump systems with time delays and norm-bounded parameter uncertainties has been investigated. A delay-dependent sufficient condition for the solvability of the problem has been obtained in terms of LMIs. An illustrate example has shown the effectiveness of the proposed method.

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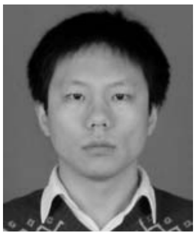

Yashun Zhang received the B.S. and M.S. degrees in Control Science and Control Engineering from Hefei University of Science and Technology in 2003 and 2006. He is currently a Ph.D. student in Control Science and Control Engineering, Nanjing University of Science and Technology. His research interests include fuzzy control, sliding mode control and nonlinear control.

Shengyuan Xu received the Ph.D. degree in Control Science and Control Engineering from Nanjing University of Science and Technology in 1999. His research interests include robust filtering and control, singular systems, time-delay systems and nonlinear systems.

Jihui Zhang is a Professor in the School of Automation Engineering of Qingdao University, China. His main areas of interest are discrete event dynamic systems, production planning and control, and operations research.