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Delay-Dependent Robust Stability Criteria for Delay Neural Networks with Linear Fractional Uncertainties

Tao Li, Lei Guo, Lingyao Wu, and Changyin Sun

Abstract: This article investigates the problem of robust stability for neural networks with time-varying delays and parameter uncertainties of linear fractional form. By introducing a new Lyapunov-Krasovskii functional and a tighter inequality, delay-dependent stability criteria are established in term of linear matrix inequalities (LMIs). It is shown that the obtained criteria can provide less conservative results than some existing ones. Numerical examples are given to demonstrate the applicability of the proposed approach.

Keywords: Delay-dependent, linear matrix inequality, neural networks, robust stability.

1. INTRODUCTION

Over the past decades, neural networks have found their important applications in various areas such as pattern recognition, optimization solvers and fixed-point computation. It has been known that time delays are often encountered in neural networks.The existence of time delays is frequently a source of instability for neural networks. Therefore, increasing interest has been focused on stability analysis of neural networks with time delays [1-7,9-22]. Generally speaking, the so-far obtained stability results for delay neural networks can be classified into two types; that is, delay-independent stability [1,3-5,9] and delay-dependent stability [12-16,18]; the former does not include any information on the size of delay while the latter employs such information. It is known that delay-dependent stability conditions are generally less conservative than delay independent ones especially when the size of the delay is small. Thus, much attention has been paid to the delay-dependent type. Recently, a new Lyapunov functional involving many slack variables is constructed in [14]. Based on the Lyapunov functional, a less conservative delay-dependent stability condition for delay neural networks is established for the introduction of many slack variables. However, we notice that it is hard to further reduce the conservatism by using the same types of Lyapunov functional as in [14] and the obtained result in [14] is equivalent to Corollary 1 in [16] by following a similar line to [13], which shows that more variables in conditions do not necessarily make less conservative results. On the other hand, some useful terms are ignored in [16]. Although in [23] consider some useful terms, there is still much useful information is ignored.

In practical implementation of neural networks, the weight coefficients of the neurons depend oncertain resistance and capacitance values, which are called as uncertainties. It is important to ensure that the designed network be stable in thepresence of these uncertainties. The global robuststability of delayed neural networks based on theintervalised network parameters has been studied in [20,21]. A criterion for the robust stability ofdelayed neural networks based on norm-bounded uncertainty has been given in [19,25]. Shortly, anew type uncertainties with linear fractional formis considered in [23,24], which can include thenorm bounded uncertainties asa special case. Butup to our knowledge, the robust stability problemare been touched for delay neural networks with this type uncertainty.

Motivated by the above discussions, in this paper, firstly, a new Lyapunov-Krasovskii functional is constructed, which involves fewer slack variables than those in [14]. Based on the Lyapunov functional, and a tighter inequality, new global asymptotic stability condition is established for delay neural networks. Then the asymptotic stability criterion is extended to uncertain neural networks with linear fractional uncertainties. All obtained results are described in term of LMI, which can be easily tested by using recently developed algorithms solving LMIs. Numerical examples are given to demonstrate the applicability of the proposed approach.

Notation: Throughout this paper, a real symmetric matrix $P > 0$ denotes P being a positive definite. I is used to denote an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $*$.

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2. PROBLEM FORMULATION

Consider the following uncertain neural networks with time-varying delays:

$$\begin{aligned}\dot{z}(t) = &-(C+\Delta C)z(t) + (A+\Delta A)f(z(t)) \\ &+(B+\Delta B)f(z(t-d(t))),\end{aligned} \tag{1}$$

where $z(\cdot)=[z_1(\cdot),\dots,z_n(\cdot)]^T$ is the state vector, $f(z(\cdot)) =[f_1(z_1(\cdot)), f_2(z_2(\cdot)),\dots,f_n(z_n(\cdot))]^T$ are the neuron activation function. A and B are the connection weight matrix and the delayed connection weight matrix. $C = diag\{c_1,...,c_n\}$ is a diagonal matrix with $c_i > 0$. The time delay $d(t)$ is a time-varying continuous function that satisfies

$$0 \le d(t) \le h, \;\; \dot{d}(t) \le \mu, \tag{2}$$

where h and μ are constants. $\Delta C, \Delta A, \Delta B$ are matrices with parametric uncertainties satisfying

$$[\Delta A \quad \Delta B \quad \Delta C] = H\Lambda(t)[N_C \quad N_A \quad N_B], \tag{3}$$

where H, N_C, N_A and N_B are given matrices. The class of parametric uncertainty $\Lambda(t)$ that satisfies

$$\Lambda(t) = [I - F(t)J]^{-1}F(t) \tag{4}$$

is said to be admissible, where J is also a known matrix satisfying

$$I - JJ^T > 0 \tag{5}$$

and $F(t)$ is uncertain matrix satisfying

$$F^T(t)F(t) \le I. \tag{6}$$

Remark 1: The uncertainty $\Lambda(t)$ satisfying (4)-(6) is referred to as a linear fractional parametric uncertainty. It can be verified that the condition (5) guarantees that $I - F(t)J$ is invertible for all $F(t)$ satisfying (6). Moreover, we note that when $J = 0$, $\Lambda(t)$ reduces to a norm-bounded parametric uncertainty which has been investigated in [18,19,25].

We assume that each neuron activation function $f_i(\cdot), i = 1,2,\dots,n$ satisfy the following condition:

$$f_i^2(z_i) \le k_i z_i f_i(z_i) \le k_i^2 z_i^2, \;\; i = 1,2,\dots,n. \tag{7}$$

3. MAIN RESULTS

In the section, based on a new Lyapunov-Krasovskii functional and LMI technique, we have the following results.

Theorem 1: Given scalars $h \ge 0$ and $0 \le \mu$. Then, for any delay $d(t)$ satisfying the condition (2), the origin of system (1) is robust stable if there exist a positive scalar ε, positive matrices $P_1, Q_i (i = 1,2,3)$, R, positive diagonal matrices $\lambda = diag\{\lambda_1,\lambda_2,\dots,\lambda_n\}$, $D_1 = diag\{d_{11}, d_{12},\dots,d_{1n}\}$, $D_2 = diag\{d_{21}, d_{22},\dots,d_{2n}\}$ and any matrices $P_i (i = 2,\dots,5)$, such that the following LMIs hold:

$$\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & -P_4^T & P_2^T A + KD_1 & P_2^T B & -hP_3^T & -hP_4^T & P_2^T H & -\varepsilon N_C^T \\
* & \Gamma_{22} & 0 & 0 & P_5^T A + \lambda & P_5^T B & 0 & 0 & P_5^T H & 0 \\
* & * & \Gamma_{33} & -P_7^T & 0 & KD_2 & -hP_6^T & -hP_7^T & 0 & 0 \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -2D_1 + Q_2 & 0 & 0 & 0 & 0 & \varepsilon N_A^T \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 & 0 & \varepsilon N_B^T \\
* & * & * & * & * & * & -3hR & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -hR & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon I & -\varepsilon J^T \\
* & * & * & * & * & * & * & * & * & -\varepsilon I
\end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & -P_4^T & P_2^T A + KD_1 & P_2^T B & -hP_3^T & -hP_4^T & P_2^T H & -\varepsilon N_C^T \\
* & \Gamma_{22} & 0 & 0 & P_5^T A + \lambda & P_5^T B & 0 & 0 & P_5^T H & 0 \\
* & * & \Gamma_{33} & -P_7^T & 0 & KD_2 & -hP_6^T & -hP_7^T & 0 & 0 \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -2D_1 + Q_2 & 0 & 0 & 0 & 0 & \varepsilon N_A^T \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 & 0 & \varepsilon N_B^T \\
* & * & * & * & * & * & -hR & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -3hR & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon I & -\varepsilon J^T \\
* & * & * & * & * & * & * & * & * & -\varepsilon I
\end{bmatrix} < 0, \tag{9}$$

where

$$\Gamma_{11} = -P_2^T C - C^T P_2 + P_3 + P_3^T + Q_1 + Q_2,$$
$$\Gamma_{12} = P_1 - P_2^T - C^T P_5,$$
$$\Gamma_{13} = -P_3^T + P_6 + P_4^T,$$
$$\Gamma_{22} = -P_5 - P_5^T + hR,$$
$$\Gamma_{33} = -P_6 - P_6^T + P_7 + P_7^T - (1-\mu)Q_1,$$
$$\Gamma_{66} = -(1-\mu)Q_2 - 2D_2,$$
$$K = diag\{k_1, k_2, \ldots, k_n\}.$$

Proof: Choose a new Lyapunov-Krasovskii functional candidate for system (1) as:

$$V(z(t)) = V_1(z(t)) + V_2(z(t)) + V_3(z(t)) + V_4(z(t)),$$

where

$$V_1(z(t)) = \xi_0^T(t) EP \xi_0(t),$$
$$V_2(z(t)) = 2\sum_{i=1}^{n} \lambda_i \int_0^{z_j} f_j(s)ds,$$
$$V_3(z(t)) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}^T(s) R \dot{z}(s) ds d\theta,$$
$$V_4(z(t)) = \int_{t-d(t)}^{t} z^T(s) Q_1 z(s) ds + \int_{t-d(t)}^{t} f^T(z(s)) Q_2 f(z(s)) ds + \int_{t-h}^{t} z^T(s) Q_3 z(s) ds$$

with

$$E = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 & 0 \\ P_2 & P_5 & 0 \\ P_3 & 0 & P_6 \\ P_4 & 0 & P_7 \end{bmatrix},$$
$$\xi_0(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \\ z(t-d(t)) \end{bmatrix},$$

and

$$EP = P^T E^T \geq 0, \ P_1 > 0, \ R > 0, \ Q_i > 0 \ (i = 1,2,3).$$

It is noted that $\xi_0^T(t) EP \xi_0(t)$ is actually $z^T(t) P_1 z(t)$. On the other hand, from the Leibniz-Newton formula and (1), the following equations are true

$$\alpha_1 := -(C+\Delta C)z(t) - \dot{z}(t) + (A+\Delta A) f(x(t)) + (B+\Delta B) f(z(t-d(t))) = 0, \tag{10}$$

$$\alpha_2 := z(t) - z(t-d(t)) - \int_{t-d(t)}^{t} \dot{z}(s) ds = 0, \tag{11}$$

$$\alpha_3 := z(t-d(t)) - z(z-h) - \int_{t-h}^{t-d(t)} \dot{z}(s) ds = 0. \tag{12}$$

Similar to [23], the time derivative of $V_1(z(t))$ along the trajectories of system (1) with (10)-(12) is obtained as

$$\begin{aligned} \dot{V}_1(z(t)) &= 2z^T(t) P_1 \dot{z}(t) \\ &= 2\xi_0^T(t) P^T \begin{bmatrix} \dot{z}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} = 2\xi_0^T(t) P^T \begin{bmatrix} \dot{z}(t) \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \\ &= \xi^T(t)(\Gamma_0 + \Gamma_1)\xi(t), \end{aligned} \tag{13}$$

where

$$\xi(t) = \Big[z^T(t) \quad \dot{z}^T(t) \quad z^T(t-d(t)) \quad z^T(t-h) \quad f^T(z(t)) \quad f^T(z(t-d(t))) \quad (\int_{t-d(t)}^{t} \dot{z}(s)ds)^T \quad (\int_{t-h}^{t-d(t)} \dot{z}(s)ds)^T \Big],$$

$$\Gamma_1 = \begin{bmatrix} \Gamma_{111} & \Gamma_{112} & \Gamma_{113} & -P_4^T & P_2^T A & P_2^T B & -P_3^T & -P_4^T \\ * & \Gamma_{122} & 0 & 0 & P_5^T A & P_5^T B & 0 & 0 \\ * & * & \Gamma_{133} & -P_7^T & 0 & 0 & -P_6^T & -P_7^T \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Gamma_0 = \begin{bmatrix} \Gamma_{01} & -(\Delta C)^T P_5 & 0 & 0 & P_2^T \Delta A & P_2^T \Delta B & 0 & 0 \\ * & -P_5 - P_5^T & 0 & 0 & P_5^T \Delta A & P_5^T \Delta B & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

and

$$\Gamma_{111} = -P_2^T C - C^T P_2 + P_3 + P_3^T,$$
$$\Gamma_{112} = P_1 - P_2^T - C^T P_5,$$
$$\Gamma_{113} = -P_3^T + P_6 + P_4^T,$$
$$\Gamma_{122} = -P_5 - P_5^T,$$
$$\Gamma_{133} = -P_6 - P_6^T + P_7 + P_7^T,$$
$$\Gamma_{01} = -P_2^T \Delta C - (\Delta C)^T P_2.$$

From (3), it is noted that

$$\Gamma_0 = S\Lambda(t)N + N^T \Lambda^T(t) S^T, \tag{14}$$

where

$$S=\begin{bmatrix}H^TP_2 & H^TP_5 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T,$$
$$N=\begin{bmatrix}-N_C & 0 & 0 & 0 & N_A & N_B & 0 & 0\end{bmatrix},$$
$$\dot V_2(z(t))=2\sum_{i=1}^{n}\lambda_i f_i(z_i(t))\dot z_i(t)=2f^T(z(t))\lambda\dot z(t), \quad (15)$$

and $\lambda=diag\{\lambda_1,\lambda_2,\ldots,\lambda_n\}$. It follows from Lemma in [8] that we have

$$\begin{aligned}\dot V_3(z(t))&=h\dot z^T(t)R\dot z(t)-\int_{t-h}^{t}\dot z^T(s)R\dot z(s)ds\\&=h\dot z^T(t)R\dot z(t)-\int_{t-h}^{t-d(t)}\dot z^T(s)R\dot z(s)ds\\&\quad-\int_{t-d(t)}^{t}\dot z^T(s)R\dot z(s)ds\\&\le h\dot z^T(t)R\dot z(t)-\min\{\frac{3W_1+W_2}{h},\frac{W_1+3W_2}{h}\},\end{aligned} \quad (16)$$

where

$$W_1=\frac{1}{h-d(t)}(\int_{t-h}^{t-d(t)}\dot z(s)ds)^T R(\int_{t-h}^{t-d(t)}\dot z(s)ds),$$
$$W_2=\frac{1}{d(t)}(\int_{t-d(t)}^{t}\dot z(s)ds)^T R(\int_{t-d(t)}^{t}\dot z(s)ds).$$

Moreover, it can be verifed that

$$\dot V_4(z(t))\le\xi^T(t)\Gamma_2\xi(t) \quad (17)$$

with $\Gamma_2=diag\{(Q_1+Q_3),0,-(1-\mu)Q_1,-Q_3,Q_2,-(1-\mu)Q_2,0,0\}$ it is clear from (7) that

$$k_iz_i(t)f_i(z_i(t))-f_i^2(z_i(t))\ge 0,\ i=1,2,...,n$$
$$k_iz_i(t-d(t))f_i(z_i(t-d(t)))-f_i^2(z_i(t-d(t)))\ge 0,\quad i=1,2,...,n$$

they are equal to

$$z^T(t)KD_1f(z(t))-f^T(z(t))D_1f(z(t))\ge 0$$
$$z^T(t-d(t))KD_2f(z(t-d(t)))-f^T(z(t-d(t)))D_2f(z(t-d(t)))\ge 0$$

From (13), (15)-(17), we obtain

$$\begin{aligned}\dot V(z(t))&=\dot V_1(z(t))+\dot V_2(z(t))+\dot V_3(z(t))+\dot V_4(z(t))\\&\le\xi^T(t)(\Gamma_0+\Gamma_1+\Gamma_3)\xi(t)\\&\quad-\min\{\frac{3W_1+W_2}{h},\frac{W_1+3W_2}{h}\}\\&\quad+2f^T(z(t))\Lambda\dot z(t)+2z^T(t)KD_1f(z(t))\\&\quad+2z^T(t-d(t))KD_2f(z(t-d(t)))\\&\quad-2f^T(z(t))D_1f(z(t))\\&\quad-2f^T(z(t-d(t)))D_2f(z(t-d(t)))\\&=\xi^T(t)(M+\Gamma_0)\xi(t)\\&\quad-\min\{\frac{3W_1+W_2}{h},\frac{W_1+3W_2}{h}\},\end{aligned}$$

$$M=\Gamma_1+\Gamma_3+\Gamma_4,$$
$$\Gamma_4=\begin{bmatrix}0&0&0&0&KD_1&0&0&0\\ *&hR&0&0&\Lambda&0&0&0\\ *&*&0&0&0&KD_2&0&0\\ *&*&*&0&0&0&0&0\\ *&*&*&*&-2D_1&0&0&0\\ *&*&*&*&*&-2D_2&0&0\\ *&*&*&*&*&*&0&0\\ *&*&*&*&*&*&*&0\end{bmatrix}.$$

And Γ_0 is defined in (14). Then it follows from the Lyapunov-Krasovskii stability theorem that if the conditions given in (8), (9) are met, system (1) is guaranteed to be stable.

Remark 2: In the proof of Theorem 1, similar to [23], some useful terms W_1,W_2 are considered. Compared with the result in [23], a tighter inequality is used to W_1,W_2, which will lead to a less conservative stabile criterion.

When restricted to norm-bounded uncertainty case, i.e., $J=0$, the following delay-dependent roust stability criterion is straightforward.

Corollary 1: Given scalars $h\ge 0$ and $0\le\mu$. Then, for any delay $d(t)$ satisfying $0\le d(t)\le h$ and $\dot d(t)\le\mu$, the origin of system (1) is robust stable if there exist a positive scalar ε, positive matrices $P_1,Q_i(1=1,2,3),R$, positive diagonal matrices $\lambda=diag\{\lambda_1,\lambda_2,\ldots,\lambda_n\}$, $D_1=diag\{d_{11},d_{12},\ldots,d_{1n}\}$, $D_2=diag\{d_{21},d_{22},\ldots,d_{2n}\}$ and any matrices $P_i(i=2,\ldots,5)$ such that the following LMIs hold:

$$\begin{bmatrix}\hat\Gamma_{11}&\Gamma_{12}&\Gamma_{13}&-P_4^T&\Gamma_{15}\\ *&\Gamma_{22}&0&0&P_5^TA+\lambda\\ *&*&\Gamma_{33}&-P_7^T&0\\ *&*&*&-Q_3&0\\ *&*&*&*&\hat\Gamma_{55}\\ *&*&*&*&*\\ *&*&*&*&*\\ *&*&*&*&*\\ *&*&*&*&*\end{bmatrix}$$

$$
\left.\begin{matrix}
P_2^T B-\varepsilon N_C^T N_B & -hP_3^T & -hP_4^T & P_2^T H \\
P_5^T B & 0 & 0 & P_5^T H \\
KD_2 & -hP_6^T & -hP_7^T & 0 \\
0 & 0 & 0 & 0 \\
\varepsilon N_A^T N_B & 0 & 0 & 0 \\
\Gamma_{66} & 0 & 0 & 0 \\
* & -3hR & 0 & 0 \\
* & * & -hR & 0 \\
* & * & * & -\varepsilon I
\end{matrix}\right] < 0, \quad (18)
$$

$$
\begin{bmatrix}
\hat{\Gamma}_{11} & \Gamma_{12} & \Gamma_{13} & -P_4^T & \Gamma_{15} & P_2^T B-\varepsilon N_C^T N_B & -hP_3^T & -hP_4^T & P_2^T H \\
* & \Gamma_{22} & 0 & 0 & P_5^T A+\lambda & P_5^T B & 0 & 0 & P_5^T H \\
* & * & \Gamma_{33} & -P_7^T & 0 & KD_2 & -hP_6^T & -hP_7^T & 0 \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \hat{\Gamma}_{55} & \varepsilon N_A^T N_B & 0 & 0 & 0 \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & -hR & 0 & 0 \\
* & * & * & * & * & * & * & -3hR & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon I
\end{bmatrix} < 0, \quad (19)
$$

where

$$\hat{\Gamma}_{11} = -P_2^T C - C^T P_2 + P_3 + P_3^T + Q_1 + Q_2 + \varepsilon N_C^T N_C,$$

$$\Gamma_{15} = P_2^T A + KD_1 - \varepsilon N_C^T N_A,$$

$$\hat{\Gamma}_{55} = -2D_1 + Q_2 + \varepsilon N_A^T N_A,$$

$$\hat{\Gamma}_{66} = -(1-\mu)Q_2 - 2D_2 + +\varepsilon N_B^T N_B,$$

and $\Gamma_{12}, \Gamma_{13}, \Gamma_{22}, \Gamma_{33}$ are defined in (8), (9).

Remark 2: Theorem 1 and Corollary 1 are delay-dependent with respect to h and μ, unlike [19] which are delay-independent conditions. It is known that delay-dependent stability conditions are generally less conservative than delay-independent ones. Moreover, it is noted that robust stability criteria can be derived by solving LMI, which can be easily tested by using some existing software packages, for example, the Matlab LMI toolbox.

When system (1) without uncertainties, the following result can be obtained by using $\Gamma_1 = 0$ in the proof of Theorem 1.

Corollary 2: Given scalars $h \ge 0$ and $0 \le \mu$. Then, for any delay $d(t)$ satisfying the conditions (2), the origin of system (1) is asymptotically stable if there exist positive matrices $P_1, Q_i (I = 1,2,3), R$, positive diagonal matrices $\lambda = diag\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, $D_1 = diag\{d_{11}, d_{12}, \ldots, d_{1n}\}$, $D_2 = diag\{d_{21}, d_{22}, \ldots, d_{2n}\}$ and any matrices $P_i (i = 2, \ldots, 5)$ such that the following LMIs hold:

$$
\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & -P_4^T & P_2^T A+KD_1 & P_2^T B & -hP_3^T & -hP_4^T \\
* & \Gamma_{22} & 0 & 0 & P_5^T A+\lambda & P_5^T B & 0 & 0 \\
* & * & \Gamma_{33} & -P_7^T & 0 & KD_2 & -hP_6^T & -hP_7^T \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 \\
* & * & * & * & -2D_1+Q_2 & 0 & 0 & 0 \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 \\
* & * & * & * & * & & -3hR & 0 \\
* & * & * & * & * & & & -hR
\end{bmatrix} < 0, \quad (20)
$$

$$
\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & -P_4^T & P_2^T A+KD_1 & P_2^T B & -hP_3^T & -hP_4^T \\
* & \Gamma_{22} & 0 & 0 & P_5^T A+\lambda & P_5^T B & 0 & 0 \\
* & * & \Gamma_{33} & -P_7^T & 0 & KD_2 & -hP_6^T & -hP_7^T \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 \\
* & * & * & * & -2D_1+Q_2 & 0 & 0 & 0 \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 \\
* & * & * & * & * & & -hR & 0 \\
* & * & * & * & * & & & -3hR
\end{bmatrix} < 0. \quad (21)
$$

Remark 3: Corollary 2 involves fewer slack variables than those in [14]. Specifically, in the case when $x(t) \in R^n$, the number of the variables to be determined in (32) is $8.5n^2 + 5.5n,$ while in [14] the number of variables is $14n^2 + 6n$. That is, the variables in [14] are around 1.6 times more than those in Corollary 2. However, our result may provide less conservative results than Theorem 1 in [14] as shown example 2 of Section 4.

Remark 4: Very recently, He, *et. al*. [16] provided thestability condition for delay NNs by introducing slackvariables. It is worth pointing out that Corollary 2 in this paper is equivalent to Theorem 1 in [16] by following Finsler's lemma. That is to say, N_j and $M_j (j = 3,4,5)$ in [16] are useless in reducing the conservatism of the criterion, which shows that more variables in conditions do not necessarily make less conservative results and can be seen from the example 2 in section 4.

4. NUMERICAL EXAMPLES

In this section, we use two examples to show the benefits of our results.

Example 1: Consider the uncertain delayed NNs (1)

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, B = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 \\ -0.1 & -0.1 \end{bmatrix}, N_A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$N_B = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0 \end{bmatrix}, N_C = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$k_1 = 0.4, k_2 = 0.8.$$

For this example, when $J = 0$, it is easy to check that the condition in [19] is not satisfied. It means that they fail to conclude whether this system is robust stable or not. On the other hand, the results in[23] and Theorem 1 in this paper can verify theglobal robust stability in this example. Therefore, Theorem 1 can provide less conservative result than [23] from the comparisons in the following Table 1.

Example 2: Consider the delayed NNs in [12]

$$C = diag\{1.2759;\ 0.6231;\ 0.9230;\ 0.4480\},$$

$$A = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$k_1 = 0.1137, k_2 = 0.1279, k_3 = 0.7994, k_4 = 0.2368.$$

Table 1. Comparisons of maximum allowed delay *h*.

	Method	$\mu = 0.5$	$\mu = 0.9$
J=0	[23]	2.8551	0.8634
	Corollary 1	2.9359	0.9776
J=0.3	[23]	2.9653	0.8629
	Theorem 1	3.0482	0.9750

Table 2. Comparisons of maximum allowed delay

Method	$\mu = 0$	$\mu = 0.9$	Number of variables
[12]	1.4224	--	52
[18]	1.9321	--	70
[14]	--	1.3164	248
[16]	3.2793	1.5847	222
[23]	3.2793	1.5847	158
Corollary2	3.5841	1.8090	158

For this example, it can be checked that Theorem 1in [4] and Theorem 1 in [12] are not satisfied, whichmeans that they fail to conclude whether this systemis asymptotically stable or not. For different μ, TableII gives the comparison results on maximum allowed time delay h via the methods in [12,15,16,18,23], in which "$-$" means that the results are not provided to the corresponding cases.

5. CONCLUSIONS

In this paper, we consider the problem of robust stability for NN with time-varying delays and linear fractional uncertainties. By introducing a tighter inequality when estimating the upper bound of the derivative of Lyapunov functionals, new delay-dependent stability criteria are established in term of LMI. It can be shown that the obtained criteria are less conservative than previously existing results through the numerical examples.

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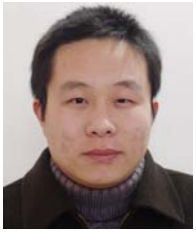

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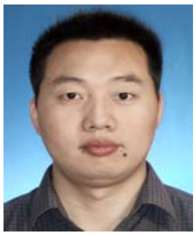

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