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REVIEW

Fuzzy Geometric Programming and Its Application

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Abstract Fuzzy geometric programming (GP) is an important optimization type. In this paper, first the origin of fuzzy GP is introduced, some recent research findings of its theory have been summarized. Then a lot of progress achieved in application is introduced in fuzzy GP, including applications in power system, environmental engineering, economic management, etc. The range of its application expects to be further expanded. Besides, as a branch of fuzzy GP, the fuzzy relation GP, rough (extension) and grey one have also been mentioned. Finally, the new research direction of fuzzy GP will be put forward.

Keywords Fuzzy geometric programming · Rough · Non-compatible · Fuzzy relation geometric programming

1. Introduction

GP is an important nonlinear programming type, founded in 1961. Its founders include C. Zener, R.J. Duffin and E.L. Peterson [1]. In general, GP's standard form is

$$\begin{aligned} &\min \quad g_0(x) \\ &\text{s. t.} \;\; g_i(x) \le 1 \ (1 \le i \le p), \\ &\qquad h_j(x) = 1 \ (1 \le j \le q), \\ &\qquad x > 0, \end{aligned} \tag{1}$$

where

$$g_i(x) = \sum_{k=1}^{J_i} g_{ik}(x) = \sum_{k=1}^{J_i} c_{ik} \prod_{l=1}^{m} x_l^{\gamma_{ikl}} (0 \le i \le p) \tag{2}$$

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is posynomial function of variable x,

$$h_j(x) = c_j \prod_{l=1}^{m} x_l^{\gamma_{jl}} (1 \leq j \leq q) \tag{3}$$

is monomial function of variable x, and coefficient $c_{ik} > 0, c_j > 0$, variable $x = (x_1, x_2, \cdots, x_m)^T > 0$, exponent γ_{ikl} $(1 \leq k \leq J_i, 0 \leq i \leq p, 1 \leq l \leq m)$, γ_{jl} $(1 \leq j \leq q, 1 \leq l \leq m)$ is arbitrary real number and "T" denotes the transpose operation.

Without loss of generality, when $g_i(x)$ is posynomial, $h_j(x)$ is monomial, then $G_i(x) = \dfrac{g_i(x)}{h_j(x)}$ is also a posynomial, and when $g_i(x) \leq 1, h_j(x) = 1$, then $G_i(x) = \dfrac{g_i(x)}{h_j(x)} \leq 1$ $(1 \leq i \leq p, 1 \leq j \leq q)$. So an equivalent form of Programming (1) can be written as:

$$\begin{aligned} \min \quad & g_0(x) \\ \text{s. t.} \quad & G_i(x) \leq 1 \ (1 \leq i \leq p), \\ & x > 0. \end{aligned} \tag{4}$$

More than four decades, GP has been applied in communication system, civil engineering, mechanical engineering, structural design and optimization, chemical engineering, optimal control, decision making, network flows, theory of inventory, balance of machinery, analog circuitry, design theory, transportation, fiscal and monetary, management science, electrical engineering, electronic engineering, environmental engineering, nuclear engineering, and technical economical analysis, etc. Up to now, the application scope has been continuously expanded [2].

In the paper, GP introduction is represented in Section 1, and inducing fuzzy GP in Section 2, the advances of fuzzy GP is shown in Section 3, and the new research direction of fuzzy GP in Section 4, and a conclusion is drawn in Section 5.

2. Fuzzy GP

There are a large class of practical problems in classical GP, its constraint conditions and objective functions need softening, including coefficients, exponents, even variables are regarded as fuzzy parameters or fuzzy variables. For example, in economic management, we can often meet with the problem as follows.

If we manufacture a cubical tank, with the bottom and top, for transporting gasoline. The capacity of tank contains 42 gallons or so. The material for the tank costs more than 5 yuan/m^2 (yuan means RMB) and it expenses about 8 yuan to transport a tank of gasoline. What is the cost at least to ship one tank of gasoline? Such a problem can be posed for solution by a GP since a classical GP cannot give a good account of the problem, or obtain a practical solution to it.

Suppose that the length, width and height of tank is x_1, x_2, x_3, respectively, then the problem boils down to solving fuzzy GP

$$\begin{aligned} \min \quad & g_0(x) \\ \text{s.t.} \quad & g_i(x) \lesssim C \ (1 \leq i \leq 3), \\ & x > 0, \end{aligned}$$

where $g_i(x) = \sum\limits_{k=1}^{J_i} c_{ik} \prod\limits_{l=1}^{3} x_l^{\gamma_{ikl}} (0 \leq i \leq 3)$.

Inspired by Zadeh's fuzzy sets theory [50], fuzzy GP emerges from the combination of fuzzy sets theory with GP, where models are built in the fuzzy posynomial and the reverse GP.

In 1987, B. Y. Cao proposed the fuzzy GP theory in International Fuzzy Systems Association (IFSA) Congress for the first time [3], since then such problem can be effective to solve. Here fuzzy GP's standard form is shown as Definition 1.

Definition 1 *We call*

$$(\tilde{P}) \qquad \begin{aligned} &\widetilde{\min}\ \ g_0(x) \\ &s.t.\ \ g_i(x) \lesssim 1\ (1 \leq i \leq p) \\ &\qquad x > 0 \end{aligned} \tag{5}$$

the fuzzy posynomial GP, where $x = (x_1, x_2, \cdots, x_m)^T$ *is an* $m-$*dimensional variable vector, and all* $g_i(x) = \sum\limits_{k=1}^{J_i} v_{ik}(x) = \sum\limits_{k=1}^{J_i} c_{ik} \prod\limits_{l=1}^{m} x_l^{\gamma_{ikl}} (0 \leq i \leq p)$ *are fuzzy posynomials of* x, $c_{ik} > 0$ *is a constant,* γ_{ikl} *is an arbitrary real number,* $\widetilde{\min}\ g_0(x) \longleftarrow g_0(x) \lesssim z_0$, *that is, the objective function* $g_0(x)$ *might have to be written as a minimizing goal in order to consider* z_0 *as an upper bound,* z_0 *is an expectation value of objective function* $g_0(x)$, *"*$\lesssim$*" denotes the fuzzified version of "* $\leq$ *" with the linguistic interpretation being "essentially smaller than or equal". The membership functions of fuzzy objective* $g_0(x)$ *and fuzzy constraints* $g_i(x)$ *are like [5], respectively.*

The dual programming of $(\tilde{P})$ is [4, 24]

$$(\tilde{D}) \qquad \begin{aligned} &\widetilde{\max}\ \ \tilde{d}(w) = \left(\frac{\tilde{a}_{00}}{w_{00}}\right)^{w_{00}} \prod_{i=0}^{p} \prod_{k=1}^{J_i} \left(\frac{\tilde{c}_{ik}}{\tilde{a}_i w_{ik}}\right)^{w_{ik}} \prod_{i=1}^{p} w_{i0}^{w_{i0}} \\ &\text{s.t.}\ \ w_{00} = 1, \\ &\qquad \Gamma^{\mathrm{T}} w = 0, \\ &\qquad w \geqslant 0, \end{aligned}$$

where

$$\Gamma = \begin{pmatrix} \gamma_{011} \cdots & \gamma_{01l} \cdots & \gamma_{01m} \\ \vdots & \vdots & \vdots \\ \gamma_{0J_01} \cdots & \gamma_{0J_0l} \cdots & \gamma_{0J_0m} \\ \cdots & \cdots & \cdots \\ \gamma_{p11} \cdots & \gamma_{p1l} \cdots & \gamma_{p1m} \\ \cdots & \cdots & \cdots \\ \gamma_{pJ_p1} \cdots & \gamma_{pJ_pl} \cdots & \gamma_{pJ_pm} \end{pmatrix}$$

denotes structure of exponent to apiece term of variable x_l corresponding to an objective function $g_0(x)$ and each constraint function $g_i(x) (1 \leq i \leq p)$, Γ is called exponent matrix. It contains $J = (J_0 + J_1 + \cdots + J_p)$ row and m column, J is the sum of apiece term in $g_i(x) (0 \leq i \leq p)$, and $w = (w_{01}, \cdots, w_{0J_0}, \cdots, w_{p1}, \cdots, w_{pJ_p})^T$ is a

$J-$dimensional variable vector, $w_{i0} = \sum_{k=1}^{J_0} w_{ik} = w_{i1} + w_{i1} + \cdots + w_{iJ_0} (0 \leq i \leq p)$ is the sum of each dual variables corresponding to an objective function $g_0(x)(i = 0)$ or constraint function $g_i(x)(i \neq 0)$,

$$\tilde{a}_{ik} = \begin{cases} \tilde{a}_{00}, & (k = 0, i = 0) \\ \dfrac{\tilde{c}_{ik}}{\tilde{a}_i}, & (1 \leq k \leq J_i, 0 \leq i \leq p) \end{cases}$$

are fuzzy numbers.

In order to ensure the defined continuity of $\tilde{d}(w)$, we stipulate $(w_{ik})^{w_{ik}}|_{w_{ik}=0} = 1$.

In the last more than 20 years, fuzzy GP has received rapid development in the theory and application [6]. Many scholars have done work in this area, they come from China, India, Iran, China Taiwan, Belgium, Canada, Germany [10], Egypt, Cuba, etc. In 2001, B. Y. Cao published the first monograph of fuzzy GP as applied optimization series (Vol.76), *Fuzzy Geometric Programming*, by Kluwer Academy Publishing (the present Springer) [5], the book gives a detailed exposition to theory and application of fuzzy GP.

The theory development of fuzzy GP is included in references as follows [6,12]:

1) From form
   i) Fuzzy posynomial GP [7].
   ii) Fuzzy reverse GP [8,9].
   iii) Fuzzy multi-objective GP [10,11].
   iv) Fuzzy fractional GP [13].
   v) Extension GP [14].
   vi) Grey GP [48].
   vii) Fuzzy relation GP [15].
   viii) Rough GP [44].
2) From coefficients
   i) GP with interval and fuzzy valued coefficients [16].
   ii) GP with Type $(\cdot, c)$ fuzzy coefficients [17].
   iii) GP with L-R fuzzy coefficients [18, 55].
   iv) GP with T fuzzy coefficients [5].
   v) GP with flat fuzzy coefficients [19, 20].
3) From variable
   i) GP with T-fuzzy variables [21].
   ii) GP with trapezoidal fuzzy variables [5].
4) From algorithm
   i) The primal algorithm [3, 22].
   ii) The dual algorithm [4, 23, 24].
   iii) Lagrange problem of fuzzy GP [25].
   iv) Solving fuzzy GP based on soft computing technique [5].
5) Other special problems

i) The antinomy problems of fuzzy GP [26].
ii) Fuzzy posynomial GP classification and its corresponding class properties [12].
iii) Possibility GP [58].

A large number of applications have been discovered for fuzzy GP in a wide variety of scientific and non-scientific fields for fuzzy GP is more superior to classical GP dealing with question of some fields including:
power system [27];
environmental engineering [28];
postal services [29];
economical analysis [30, 31, 54];
transportation [32];
inventory theory [33, 55, 57];
system designing and optimization [52];
engineering design and optimization [34, 53];
civil engineering [35], etc.

It seems clear that even more remains to be discovered. For these reasons, fuzzy GP arguably has the potential to become a ubiquitous optimization technology the same as fuzzy linear programming, fuzzy objective programming, and fuzzy quadratic programming.

3. The present advances of fuzzy GP

Fuzzy GP has been great development, from the point of view reported in the literature, the study scope of GP mainly concentrated the following three areas:

I. Fuzzy Relation GP [38, 56]
If addition and multiplication operation is replaced by logic synthesis one, the following fuzzy relation GP can be obtained:

Definition 2 *We call*

$$(FRP_1) \qquad \begin{aligned} &\min \bigvee_{l=1}^{m} (c_l \wedge x_l^{\gamma_l}) \\ &s.t.\ \ x \circ A = b \\ &\quad 0 \le x_l \le 1 (1 \le l \le m) \end{aligned} \tag{6}$$

a $(\bigvee, \bigwedge)$ *(max-min) fuzzy relation GP, where* $A = (a_{lj})(0 \le a_{lj} \le 1, 1 \le l \le m, 1 \le j \le q)$ *is an* $(m \times q)$*-dimensional fuzzy matrix,* $x = (x_1, x_2, \cdots, x_m)$ *an* m*-dimensional variable vector,* $c = (c_1, c_2, \cdots, c_m)(c_l > 0)$ *and* $b = (b_1, b_2, \cdots, b_q)(0 \le b_j \le 1)$ *an* q*-dimensional constant vector,* γ_l *an arbitrary real number, and composition operator is* " $\circ$ " $(\vee, \wedge)$, *i.e.,*

$$\bigvee_{l=1}^{m} (x_l \wedge a_{lj}) = b_j (1 \le j \le q).$$

The optimization problem

$$(FRP_2) \qquad \begin{aligned} &\min \bigvee_{l=1}^{m} (c_l \cdot x_l^{\gamma_l}) \\ &s.t.\ x \circ A = b \\ &\quad 0 \le x_l \le 1 (1 \le l \le m) \end{aligned} \tag{7}$$

is called the $(\bigvee, \cdot)$ *(max-product) fuzzy relation GP, where the sign* " $\circ$ " *is certain types of fuzzy composition operator, i.e.,*

$$\bigvee_{l=1}^{m} (x_l \cdot a_{lj}) = b_j (1 \le j \le q).$$

The objective function of optimization (6) and (7) is a nonconvex function, the feasible region of optimization (6) and (7) is also nonconvex set, so the optimization (6) and (7) is a nonconvex programming problem. It is difficult to find an ideal result by traditional solving nonlinear optimization method, such problems has received some attention of scholars [39, 40, 41, 42, 43, 46, 51, 59].

II. Rough GP [44]

By using theory in a rough set [45], we have proposed rough GP model.

We will abandon an algebra operation sign in GP, and constitute a model of knowledge expression, then the incompatible programming problem becomes a quaternary form

$$M = (U, P, C, Q), \tag{8}$$

where U denotes universe, P denotes substantiality, C denotes character and Q a fix quantify relation between P with regard to C. Moreover this quality value relation is denoted by exponent posynomial $\bar{g}_i(x) = \sum_{k=1}^{J_i} \bar{c}_{ik} \prod_{l=1}^{m} x_\ell^{\gamma_{ikl}}$.

Given the knowledge expression system $M = (U, P, C, Q)$, for every subset $x \subseteq U$ and non-distinct relation $\bar{B}$, the lower and the upper approximation sets of x are defined by

$$B_-(x) = \bigcup \{\bar{g}_i | (\bar{g}_i \in U|\text{ind}(B) \wedge \bar{g}_i \subseteq x)\},$$
$$B^-(x) = \bigcup \{\bar{g}_i | (\bar{g}_i \in U|\text{ind}(B) \wedge \bar{g}_i \cap x \neq \phi)\},$$

where $U|\text{ind}(B) = \{x | (x \subseteq U \wedge \forall x \forall \bar{g}_i \forall b (b(x) = b(\bar{g}_i)))\}$ is a partition of non-distinct relation $\bar{B}$ about U, and it is an equivalence relation.

If (8) is implemented by rough transformation $\mathcal{T}$, then we have

$$\mathcal{T}M = (\mathcal{T}U, \mathcal{T}P, \mathcal{T}C, \mathcal{T}Q),$$

it is called a transformation model knowledge expression of rough GP, and it is a compatible model. If (8) is a compatible model, we can then use a classical method to solving the problem. If (8) is an incompatible model or an opposite one.

The mathematics form in (8) is shown as Definition 3.

Definition 3 *Suppose $\bar{g}_0(x)$ to be a rough objective function defined on $\mathscr{R}$, and $\bar{g}_i(x)(1 \leq i \leq p)$ to be a rough constraint function defined on $\mathscr{R}^p$, then we call*

$$(\bar{P}) \qquad \overline{\min}\ \bar{g}_0(x) \\ s.t.\ \bar{g}_i(x) \lesssim \bar{b}_i (1 \leq i \leq p) \\ x > 0$$

a particular form of rough GP, where

$$\bar{g}_i(x) = \sum_{k=1}^{J_i} \bar{c}_{ik} \prod_{l=1}^{m} x_\ell^{\gamma_{ikl}}$$

is a rough posynomial, $\bar{c}_{ik}, \bar{b}_i$ are the rough numbers, and it is defined by $[(1-\alpha)\mu, (1+\alpha)\mu]$. When $\bar{c}_{ik} > 0$, we call it a rough posynomial GP.

Here, "$\overline{\min}$" denotes minimum value taken for objective function, the rough connection function of $(\bar{P})$ is defined like [44]. We define symbol " $\lesssim$ " as a flexible version of " $\leq$ " at a 'certain degree', or approximately less than or equal to.

III. Grey GP [48]

Definition 4 *Assuming $c_{i1}, c_{i2}, \ldots, c_{iJ_i}$ are positive, $\gamma_{ik}(\otimes)$ is the arbitrary grey interval, meaning $\gamma_{ik}(\otimes) \in [\underline{\gamma}_{ik}, \overline{\gamma}_{ik}]$ $(k = 1, 2, \ldots, J_i,\ i = 1, 2, \ldots, p)$, then we call $(\otimes)g_i(x) = \sum\limits_{k=1}^{J_i} c_{ik} \prod\limits_{l=1}^{m} x_\ell^{\gamma_{ikl}(\otimes)}$ grey polynomial. If $x = (x_1, x_2, \ldots, x_m)^T$, we call*

$$((\otimes)P) \qquad \min\ (\otimes)g_0(x) \\ s.t.\ (\otimes)g_i(x) \leq 1\ (1 \leq i \leq p) \\ x > 0$$

a grey polynomial GP, where '$(\otimes)$' denotes grey arithmetic operators. When $c_{ik} > 0$, we call it a grey posynomial GP.

4. The New Research Direction of Fuzzy GP

In the real world, the problems, which can be expressed once by an exponent posynomial functions in the relation under fuzzy environment, are concluded as a fuzzy GP. Such research directions as mentioned previously are not limited to those fields. Fuzzy GP will attract us to further research because many aspects remain untouched. In the basic field, we shall consider the following topics.

1) We should continue to conduct research on the fuzzy GP, including the following aspects：

a. Fuzzy reverse GP, including a GP problem with mixed sign-terms, is much more complex than the fuzzy convex (resp. concave) GP and fuzzy posynomial GP, which differ a lot in their properties.

b. Fuzzy fractional GP model and its method remain to be researched.

c. The discussion of the GP with fuzzy coefficients and fuzzy variables is yet to be further perfected as well as extended.

d. It is very interesting to solve antinomy in the realistic world by using fuzzy GP further.

e. Intuitionistic fuzzy GP.

f. Solving fuzzy relation GP.

g. Decision making based on fuzzy GP [36].

h. Solving fuzzy GP through meta-heuristic algorithms, such as genetic algorithm, simulated annealing algorithm, artificial neural network, tabu search, ant algorithms, it is worthy research how to effective apply those algorithms to solving fuzzy GP [37].

i. It will be a creative job to combine fuzzy GP and other fuzzy programming with Data Mining before dealing with their data and classification.

j. Further study of the antinomy of fuzzy GP and its application in engineering and economic management.

2) The research of Fuzzy relation GP has only just begun, we need to focus on the following three aspects:

a. A more appropriate definition of fuzzy relation GP should be given.

b. More efficient algorithms of fuzzy relation GP should be established based on different logical operators.

c. Search for the application of fuzzy relation GP.

3) The research of Non-compatible GP

- We have developed the concepts of extension convex function in extendable-valued sets based on the extendable sets [47] and its convex sets [49], built a matter-element model and a mathematical model in extendable GP, and discussed solution properties in an extendable mathematical model and given an algorithm of the programming.
- Recently, we have proposed rough posynomial GP on foundation of rough sets and rough convex sets， antinomy of the more-for-less paradox is solved with an arithmetic in rough posynomial GP given.
- In the future, we can combine the above two cases to discuss fuzzy non-compatible, and in turning a non-compatible problem into a compatible one.

4) Grey GP

A model of grey polynomial GP has conducted research. In the model of GP, values of parameters cannot be gotten owing to data fluctuation and incompletion. But when the model contains grey numbers, it is hard for common programming method to solve them. By combining the common programming model with the grey system theory, and by using some analysis strategies, a model of θ positioned GP and their quasi-optimum solution or optimum solution are put forward and an algorithm for the problem is developed. We can combine the above-mentioned of GP of fuzzy number and non-compatible GP conduct research, and to further explore their application, especially the applications in the forecasting and decision-making.

5. Conclusion

We have obtained a series of results in fuzzy GP since we began to study it 23 years ago. It looks like a mine containing rich resources which remains to be excavated. Meanwhile, it can be generalized into investigation in many other fields.

In the real world, the problems, which can be expressed once by an exponent posynomial function in the relation under fuzzy environment, are concluded as a fuzzy GP. The fuzzy GP has received the application in electrical power and postal service, etc, that was only part of the practical application. In practice, fuzzy GP bears broad prospects in application in fields such as optimal design, management, electronics, chemical industry, biology and automation control [60].

Such research directions as mentioned above are not limited to those fields. In general, take an application of stochastic factors to fuzzy GP for example, it has captivating prospects.

It is not totally confirmed whether the application of fuzzy GP is successful since this branch is like a newborn baby. We belief that fuzzy GP will become an important branch of fuzzy mathematics after scholars over the world make an effort to study it, and that it will add luster to the development of science in the world and bring benefit to humanity.

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