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REGULAR PAPER

Equivalent Error Based Modelling for Prediction and Analysis of Measuring Accuracy in 3-Axis FXYZ Coordinate Measuring Machines from Position, Repeatability and Reversibility Errors

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Abstract

The measuring accuracy of coordinate measuring machines (CMMs) will be affected by the different geometrical and dynamic errors, including the deviations associated to the axis displacement, the working table and the part to be measured. This work is focused on the analysis of the influence of the position errors, repeatability errors and reversibility errors in 3-axis FXYZ coordinate measuring machines, and it will be developed by a numerical model that is known as EE-based stochastic model. This model implements a new error index that is named equivalent error (EE), which will integrate the totality of machine errors of the CMMs and will allow a global description of all these error sources by means of a unique error parameter. The results obtained by this numerical model have been compared with the application of a traditional method, and it was probed that the EE-based model makes possible an increase of a 13.29% in the linear modelling of the performance of CMMs from the machine errors considered in this work, which implies a relevant improvement for the analysis and description of the effect of the distinct error sources on the achievable measuring accuracy of CMMs. For this reason, the EE-based model will be of special interest for industrial applications such as the quality control to be applied inside the production systems dedicated to manufacture mechanical components of high dimensional accuracy.

Keywords Coordinate measuring machines · Position errors · Repeatability errors · Reversibility errors · Applied physics · Manufacturing engineering

1 Introduction

The analysis and optimization of the performance of coordinate measuring machines (CMMs) with FXYZ structural configuration and 3 linear axes, is of great relevance in order to guarantee the minimization of the uncertainty associated to the measuring process during the dimensional inspection of manufactured products. Among the different factors that contribute to the measuring deviations of CMMs, the geometrical and dynamic errors related to the displacement of the CMM linear axes, the working table of CMM and the part surface must be considered.

There are some technical standards that provide the basis that must be attended with regards to the performance of coordinate measuring machines (CMMs), such as the ASME B89.4.10360.2 [1] and the ISO 10360-2 [2]. These ASME and ISO standards contain the definition of the main error sources that affect the CMMs, as well as certain procedures and methods that are recommended for the calibration or verification of these measuring devices.

The ISO 10360-2 standard [1] describes the definitions and fundamentals of verification tests for CMMs utilized for the measuring of linear dimensions, including the application of laser interferometry or other alternatives in order to check the measuring accuracy of this equipment, while the ASME B89.4.10360.2 standard [2] is specially oriented to the application of ball-bar technique with the objective of registering the level of measuring accuracy that characterize the CMMs.

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There are numerous studies oriented to the analysis and optimization of the dimensional properties of the products generated by the distinct types of manufacturing processes that are utilized in the industry, or to the numerical modelling of these production techniques from the main factors of these processes [3]. In the work edited by Davim [3], there are certain studies about the influence of the processes parameters on the properties of manufactured parts or the numerical modelling of these manufacturing processes for the prediction of part properties. This work provides some guidelines about the experimental methodologies that can be applied for the optimization of these processes and about the computational techniques that can serve to their numerical modelling and simulation.

The coordinate measuring machines are one of the measuring devices that are more frequently employed to check the dimensional properties of the final products. The geometrical and dynamic errors that exist in this inspection devices will also affect the acceptance of the manufactured products, joined to the influence of the different factors that are related to the production techniques. For this reason, the analysis and modelling of the performance of coordinate measuring machines has a great importance for a better understanding about the measuring accuracy of these devices and its influence on the product acceptance.

There are diverse works in which coordinate measuring machines have been applied for dimensional measuring of mechanical products during the optimization of manufacturing process [4, 5]. For example, Wang et al. [4] proposed a method for on-line error measurement and compensation in machining processes to improve the accuracy of product assembly, and they utilized a coordinate measuring system (CMS) to determine the assembly precision. Zhou et al. [5] employed an optical fiber Fabry–Pérot interferometer and machine learning for accurate measuring of the tool profile, runout and tool wear in machining processes and the straightness error of machine tool, and the interferometer measures were compared with the results provided by a CMM.

The influence of distinct machine errors and other factors related to the measuring process has been analyzed in previous works, and some of the most relevant studies about these topics include experimental and theoretical works focused on new numerical models about the measuring accuracy of CMMs [6–17], optimized methods for the error compensation in CMMs [18–20], enhanced techniques for part evaluation by CMMs [21–23], and new calibration procedures for these measuring machines [24–27].

Among the studies oriented to the modelling and analysis of distinct types of machine errors, the works edited by Hocken and Pereira [6] and Sładek [7] specify many of the main error sources that must be attended during the utilization of coordinate measuring machines, and the reduction of the influence of the existing machine errors by error compensation methods.

The work that has been edited by Hocken and Pereira [6] shows the relevance of diverse error sources that can be found in the CMMs, including the position errors, straightness errors, angular errors and squareness errors, as well as other geometrical deviations such as the repeatability errors, which describe the variability originated during the execution of repeated measures in a same working point within the overall working volume of CMM in each linear axis, and the hysteresis errors, which can be also named reversibility errors and represent the variability produced between the measures obtained in a certain point during the displacement in positive and negative directions along each one of the CMM linear axes.

In the study edited by Sładek [7], it is described the basis of some numerical models that can be employed to evaluate the uncertainty associated to the dimensional measurement carried out by CMMs for certain structural configurations and geometrical errors. Among the diverse error sources that were considered in this work, it is included the hysteresis errors (or reversibility errors) that can be found when the dimensional measures are executed in positive and negative direction along a same linear axis, which was conceived as a factor different from other typical deviations such as the position errors that represent the inaccuracies in the probe location during its displacement along the CMM linear axes.

Xijing [8] presented a method for compensation of some of the typical error sources of CMMs with the objective of increasing their measuring performance. The method proposed by these authors includes the analysis of the repeatability errors, as measuring deviations obtained from the repeated measures to be carried out in a same working point, and that differs from the position errors, as the fluctuations registered during the displacement of the probe along the CMM linear axes.

Ramu et al. [9] proposed a theoretical approach focused on the parametric errors originated in multi-sensor and five-axis CMMs, and it was employed to determine the specific strategies that could help for error correction in these measuring devices. Jinwen and Yanling [10] were focused on the measuring deviations in fast probing CMMs, and defined a new model to evaluate the dynamic errors that occur in this inspection equipment.

Aggogeri et al. [11] discussed the CMM behaviour during the evaluation of geometrical and dimensional parameters such as the perpendicularity and diameter in distinct surfaces of the mensurand, and defined a numerical model to predict the measurement uncertainty of CMMs from this type of dimensional measures. The proposed model can be adopted to estimate the measuring uncertainty as a function of the main geometry of the part to be inspected, the measuring repetition in distinct days of the week, the different measuring zones on the part surface and the measuring conditions that would be assumed.

The work carried out by Gąska et al. [12] was focused on experimental measure and theoretical modelling of kinematic errors associated to the CMMs in order to increase the measuring accuracy of this metrological equipment. A virtual measuring machine was also implemented, and the proposed model was proved to be useful for deducing the optimum measurement strategy to be assumed. Meng et al. [13] studied the performance of CMMs with six-freedom-degree parallel mechanism, and proposed a new direct-error-compensation method for machines with this configuration. This method serves to evaluate CMM probe deviations such as the errors related to the axis displacement, the machine structure, the control system, the force and thermal state, etc.

Wang et al. [14] proposed a model for identification and compensation of the systematic measurement errors (stage errors) during the dimensional verification of mechanical parts by CMMs, which can be employed to analyze the relationship between the CMM measuring deviations and the stage errors and grid errors. Jia et al. [15] carried out the modelling of thermal errors that are originated by the ambient temperature as one of the factors that affect the measurement accuracy of CMMs. These authors proposed an integrated method for temperature regression that was proved to enhance the overall accuracy that can be obtained by the theoretical modelling of thermal errors.

Moona et al. [16] applied the Monte Carlo method for the modelling of the measurement uncertainty of articulated arm coordinate measuring machines (AACMM). They compared the results provided by this simulation model with the experimental measures obtained through the ISO 10360-12:2016 standard, and an adequate concordance was found between both approaches. Franco and Jodar [17] presented a numerical modelling that consists of using the equivalent error (EE) as a unique parameter that serves to integrate all the different machine errors. This EE-based stochastic model allows an improved theoretical modelling for predicting the measuring accuracy that can be achieved in 3-axis FXYZ coordinate measuring machines, and it was proved to be adequate for evaluating the influence of some of the main machine errors, such as the position and straightness errors associated to the movement of CMM linear axes.

Among the studies related to the definition of new procedures for error compensation in CMMs, the works carried out by Swornowski [18], Mohammadi et al. [19] and Echerfaoui et al. [20] can be remarked. Swornowski [18] presented a new method to optimize the measuring accuracy of CMMs through a matrix of computer aided accuracy (CAA) that provides automatic corrections to be applied, including the utilization of distinct scanning speeds to identify their influence on the measuring accuracy.

Mohammadi et al. [19] developed an autolearning approach that decreases the time needed to prepare the measurement model to be used for dimensional verification of mechanical parts by CMMs. This approach is based on the application of the algorithm Modified Multi-Class Support Vector Machines (iMMC-SVM), which helps to identify the geometrical features of part surface by comparing with the reference part. In the study of Echerfaoui et al. [20], it is proposed a predictive approach that can be applied for compensation of geometrical and dynamic errors of CMMs, with the aim of improving the performance of these inspection devices. This model allows the identification of some of the main factors that affect the CMM dynamic performance, and proves the possibility of reducing more than an 80% in the dynamic errors.

The works of Raghunandan and Venkateswara Rao [21], Zhao et al. [22] and He et al. [23] are oriented to define enhanced techniques that could help to the part evaluation by using CMMs. Raghunandan and Venkateswara Rao [21] discussed about the flatness error in produced parts, and presented a new methodology that could serve to reveal the effect of the sample points and sample size on the process performance, and to determine the optimum sampling conditions to be assumed.

Zhao et al. [22] designed a new method to analyze the circular and cylindrical contours that is based on the combination of statistical process control and a spatial correlation model, and concluded that a technique that implements the spatial lag model (SLM) results more effective for circular and cylindrical profiles. He et al. [23] remarked the sampling strategy to be adopted in the CMMs as a crucial factor for improving the measuring accuracy that can be achieved during the dimensional verification of freeform products. These authors developed an adaptive sampling strategy that is based on the application of a machining error model (MEM) for verification of freeform surfaces, and concluded the validity of the proposed model.

Inside the studies oriented to conceive a better calibration process that could be applied in coordinate measuring machines [24–27], Thompson and Cogdell [24] obtained a new calibration procedure that was specially defined for precision cylindrical CMMs, which allows the minimization of the probe alignment errors by implementing the normal distance between the probe tip and rotation axis. Curran and Phelan [25] presented a new method that could serve as a quick check method for machine calibration, and improves the evaluation of the measuring accuracy of CMMs by using a telescoping ball-bar.

Furutani and Ozaki [26] studied the calibration of 2D planar CMMs, and defined a model that makes possible the registration of the uncertainty of probes and sensors, and the reduction of the measuring uncertainty by assuming the optimum orientation. In the case of the calibration of CMMs with articulated manipulator, the theoretical model of Sultan and Puthiyaveettil [27] facilitates a simplified calibration procedure that is based on stochastic optimization.

It can be identified as traditional methods all the numerical models in which the different machine errors are analyzed by separate and not through an error index that could integrate the totality of these factors. On the contrary, the EE-based stochastic model proposed by these authors will assume the new error index that has been named equivalent error (EE), and will provide a global perspective about the totality of error sources through the evaluation of all of them by a unique error parameter such as the equivalent error.

Swornowski [18], Mohammadi et al. [19], and Echerfaoui et al. [20] have developed optimized methods that could help to achieve the error compensation in coordinate measuring machines, Raghunandan and Venkateswara Rao [21], Zhao et al. [22], and He et al. [23] have described new inspection techniques that could enhance the performance of CMMs for certain applications, and Thompson and Cogdell [24], Curran and Phelan [25], Furutani and Ozaki [26], and Sultan and Puthiyaveettil [27] have deduced some methods that could serve to improve the calibration procedures to be applied in CMMs. Nevertheless, these works do not include the study or modelling of the measuring accuracy of CMMs nor the prediction of the achievable measuring accuracy of these devices. They do not apply a numerical modelling through a traditional method nor through the EE-based model.

The works of Hocken and Pereira [6], Sładek [7], Xijing [8], Ramu et al. [9], Jinwen and Yanling [10], Aggogeri et al. [11], Gąska et al. [12], Meng et al. [13], Wang et al. [14], Jia et al. [15], Moona et al. [16], and Franco and Jodar [17] have been oriented to define new numerical models that could be employed to analyze the measuring accuracy of coordinate measuring machines. The works [6–16] present numerical models that correspond to a traditional method, due to any of them adopt an error index such as the equivalent error (EE) to allow a global perspective about the effect of the totality of machine errors. On the contrary, they are focused on the analysis of the influence of each machine error by separate through the computational algorithms that were implemented in these models.

Among the previous works of the literature that have been mentioned in this section as an example of the studies related to the numerical modelling of the measuring accuracy of CMMs, only the work [17] has applied a numerical model based on an error index that could integrate the totality of geometrical and dynamic errors such as the equivalent error (EE). Nevertheless, this study was limited to some of the distinct machine errors, and not covered other important factors such as the repeatability errors, reversibility errors, etc. that must be also evaluated to discuss and predict the measuring accuracy of CMMs.

For this reason, it is needed to carry out other studies through a numerical model that assumes a new error index such as the equivalent error (EE), in order to make possible to apply this model for evaluating the influence of other geometrical and dynamic errors of coordinate measuring machines, as well as to compare the results of this model with other numerical models that correspond to a traditional method, in which the different machine errors are analyzed by separate and not by an error index that could integrate these factors.

In the present work, the EE-based stochastic model will be applied to evaluate certain machine errors such as the position, repeatability and reversibility errors related to the axis displacement in CMMs, with the objective of trying to identify in detail the influence of these error sources on the CMM performance. During the study of these machine errors by the EE-based model, the straightness and part errors will be also considered in the numerical modelling. Squareness, angular or probe errors and other types of geometrical and dynamic errors were not contained in this work, but they could be also covered in other studies.

The analysis carried out in this work will include the application of both the EE-based model and a traditional approach, and the comparison between both methodologies will serve to prove the benefits of the model based on the equivalent error. The EE-based model will be proved to facilitate the linear modelling between the position, repeatability and reversibility errors and the measuring accuracy of CMMs. The results of this work can be of special interest for the industry for improving the quality control procedures to be applied for the dimensional inspection of mechanical parts inside the production systems.

2 Application of Equivalent Error Based Modelling for the Analysis of Position, Repeatability and Reversibility Errors in 3-Axis CMM

The measuring accuracy that could be achieved during the utilization of coordinate measuring machines (CMMs) is affected by diverse types of error sources of different nature, among which the position errors in each CMM linear axis (or also named as linear displacement errors), the straightness errors in both normal directions to each CMM linear axis, the angular errors for each CMM linear axis (including roll, pitch and yaw errors), and the squareness errors between each pair of CMM linear axes can be considered, as expressed for example in the work edited by Hocken and Pereira [6].

Besides these typical errors of CMMs, there are other numerous error sources that must be also assumed in order to model or study the performance of these machines, such as the probe errors, thermal errors, force deformation errors, part errors, etc., as well as the repeatability errors and reversibility errors related to the measures registered in a same point inside the overall working volume of CMM or in positive and negative directions along a same linear axis respectively, as indicated by Hocken and Pereira [6], Sładek [7], Xijing [8] and other diverse authors.

In this work, the analysis of the achievable measuring accuracy of CMMs will be made by a numerical modelling based on the equivalent error (EE), which can be applied for the dimensional inspection of the mechanical parts that are obtained from the production systems of the companies that belong to the manufacturing sector.

Figure 1 illustrates an schematic representation about the application of both the EE-based model or a traditional method for analyzing and predicting the influence of the position, repeatability and reversibility errors of CMMs with three linear axes and FXYZ structural configuration (3-axis moving bridge coordinate measuring machines) on the expected measuring accuracy of these devices. The influence of other dynamic and geometrical errors such as straightness, angular and part errors are also assumed during the numerical model. The modelling based on the equivalent error (EE) provides a global perspective about the contribution of the totality of error sources to be considered on the CMM measuring deviations.

The following mathematical expression can be applied in order to the study the effect of position, repeatability and reversibility errors associated to the CMM linear axes on the expected measuring deviations to be achieved in the coordinate measuring machines. This equation also contains some terms related to other machine errors such as the straightness, angular and part errors that could serve for the description of these other error sources:

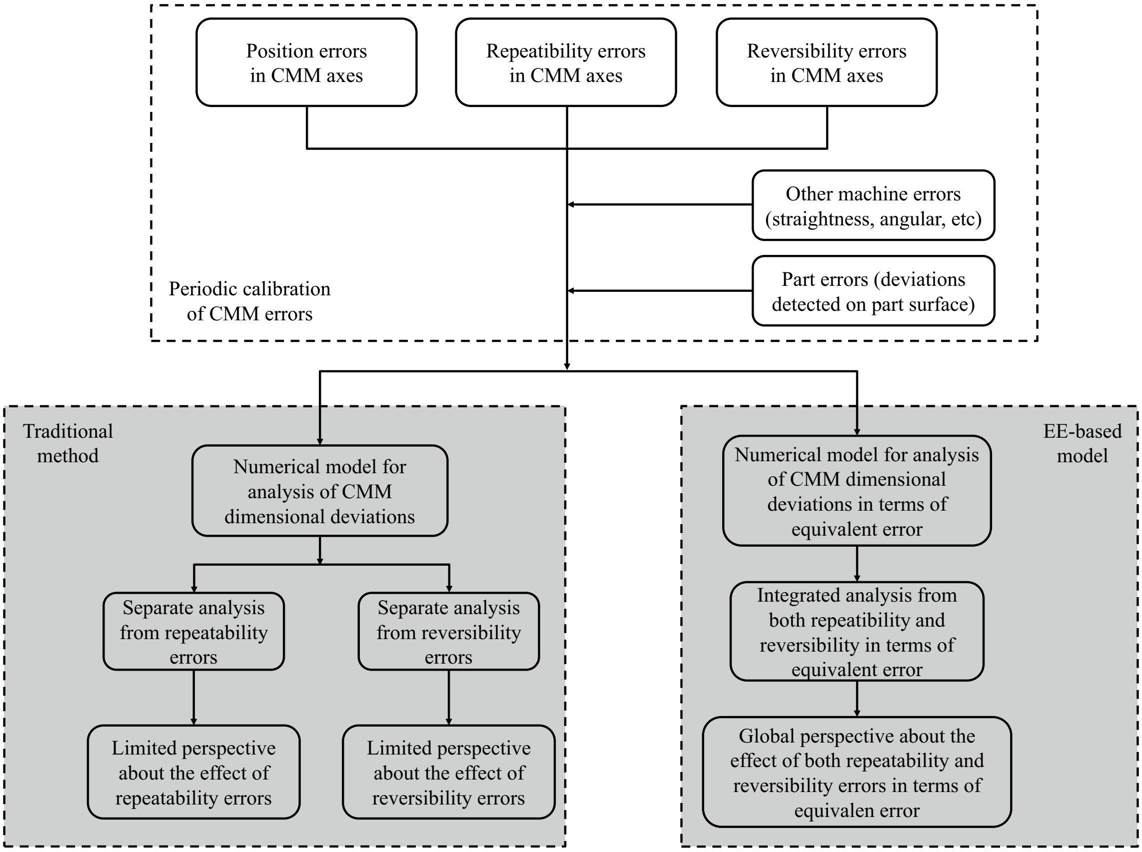


Fig. 1 Schematic representation of numerical analysis of the CMM accuracy from position, repeatability and reversibility errors by the EE-based model and a traditional method

$$\begin{bmatrix} \Delta x_{ijk} \\ \\ \Delta y_{ijk} \\ \\ \Delta z_{ijk} \end{bmatrix} = \begin{bmatrix} \begin{gathered} \left(\varepsilon_{ps,x}\right)_i + \left(\varepsilon_{st,yx}\right)_j + \left(\varepsilon_{st,zx}\right)_k - y\varepsilon_{sq,xy} - z\varepsilon_{sq,zx} - y\left(\varepsilon_{ap,xz}\right)_i + z\left[\left(\varepsilon_{ar,y}\right)_j + \left(\varepsilon_{ay,xy}\right)_i\right] - \\ -y_{p3}\left[\left(\varepsilon_{ar,z}\right)_k + \left(\varepsilon_{ap,xz}\right)_i + \left(\varepsilon_{ay,yz}\right)_j\right] + z_{p3}\left[\left(\varepsilon_{ar,y}\right)_j + \left(\varepsilon_{ay,xy}\right)_i + \left(\varepsilon_{ap,zy}\right)_k\right] + \left(\varepsilon_{rp,x}\right)_i + \left(\varepsilon_{rv,x}\right)_i + \left(\varepsilon_{0,x}\right)_i \end{gathered} \\ \\ \begin{gathered} \left(\varepsilon_{ps,y}\right)_j + \left(\varepsilon_{st,xy}\right)_i + \left(\varepsilon_{st,zy}\right)_k - z\varepsilon_{sq,yz} - z\left[\left(\varepsilon_{ar,x}\right)_i + \left(\varepsilon_{ap,yx}\right)_j\right] + \\ +x_{p3}\left[\left(\varepsilon_{ar,z}\right)_k + \left(\varepsilon_{ap,xz}\right)_i + \left(\varepsilon_{ay,yz}\right)_j\right] - z_{p3}\left[\left(\varepsilon_{ar,x}\right)_i + \left(\varepsilon_{ap,yx}\right)_j + \left(\varepsilon_{ay,zx}\right)_k\right] + \left(\varepsilon_{rp,y}\right)_j + \left(\varepsilon_{rv,y}\right)_j + \left(\varepsilon_{0,y}\right)_j \end{gathered} \\ \\ \begin{gathered} \left(\varepsilon_{ps,z}\right)_k + \left(\varepsilon_{st,xz}\right)_i + \left(\varepsilon_{st,yz}\right)_j + y\left(\varepsilon_{ar,x}\right)_i - x_{p3}\left[\left(\varepsilon_{ar,y}\right)_j + \left(\varepsilon_{ay,xy}\right)_i + \left(\varepsilon_{ap,zy}\right)_k\right] + \\ +y_{p3}\left[\left(\varepsilon_{ar,x}\right)_i + \left(\varepsilon_{ap,yx}\right)_j + \left(\varepsilon_{ay,zx}\right)_k\right] + \left(\varepsilon_{rp,z}\right)_k + \left(\varepsilon_{rv,z}\right)_k + \left(\varepsilon_{0,z}\right)_k \end{gathered} \end{bmatrix} \tag{1}$$

where $(\varepsilon_{ps,x})_i$, $(\varepsilon_{ps,y})_j$ and $(\varepsilon_{ps,z})_k$ denote the position errors detected during the CMM axis displacement with regards to the linear axes x, y and z respectively, $(\varepsilon_{st,xy})_i$, $(\varepsilon_{st,yz})_j$, $(\varepsilon_{st,zx})_k$, etc. are the straightness errors related to the each CMM axis displacement, $(\varepsilon_{rp,x})_i$, $(\varepsilon_{rp,y})_j$ and $(\varepsilon_{rp,z})_k$ are the repeatability errors related to the axis displacement, $(\varepsilon_{rv,x})_i$, $(\varepsilon_{rv,y})_j$ and $(\varepsilon_{rv,z})_k$ are the reversibility errors for each linear axis, $(\varepsilon_{ar,x})_i$, $(\varepsilon_{ap,xz})_i$, $(\varepsilon_{ay,xy})_i$, etc. are the angular errors associated to the linear axes of CMM, and $(\varepsilon_{0,x})_i$, $(\varepsilon_{0,y})_j$ and $(\varepsilon_{0,z})_k$ are the geometrical imperfections on the part surface.

Table 1 shows the equivalent error coefficients to be considered in the EE-based model for evaluating the position, repeatability and reversibility errors of CMMs, which correspond to the coefficient values that optimize the linear regression of these machine errors according to the routines implemented in this numerical model. The initial and optimized values of this table correspond to the case of an application example oriented to the machine errors registered along the longitudinal axis of CMM.

The methodology employed in this study, implies the numerical modelling of the measuring deviations that can be observed in the coordinate measuring machines from the equivalent error (EE) as a novel parameter which integrates all the error sources to be evaluated, from the multi-variable correlation procedure that is implemented in the EE-based model to determine the adequate error coefficients for each type of geometrical and dynamic errors of CMMs.

Table 1 Equivalent error coefficients related to position, repeatability and reversibility errors along the longitudinal axis

Parameters analyzed by ANOVA	Preliminary coefficient	Optimized coefficient
Coefficient value for position error $c_{ps,x}$	1	0.9
Coefficient value for repeatability error $c_{rp,x}$	1	0.2
Coefficient value for reversibility error $c_{rv,x}$	1	0.6

This study is focused on the verification of longitudinal properties of prismatic parts such as the distance between different plane faces, and all the results are referred to the application example that corresponds to a part with an overall length of 50 mm. The numerical analysis will allow the discussion about the effect of different CMMs that could be applied for the dimensional inspection of these manufactured parts, by means of including two distinct distributions of random errors.

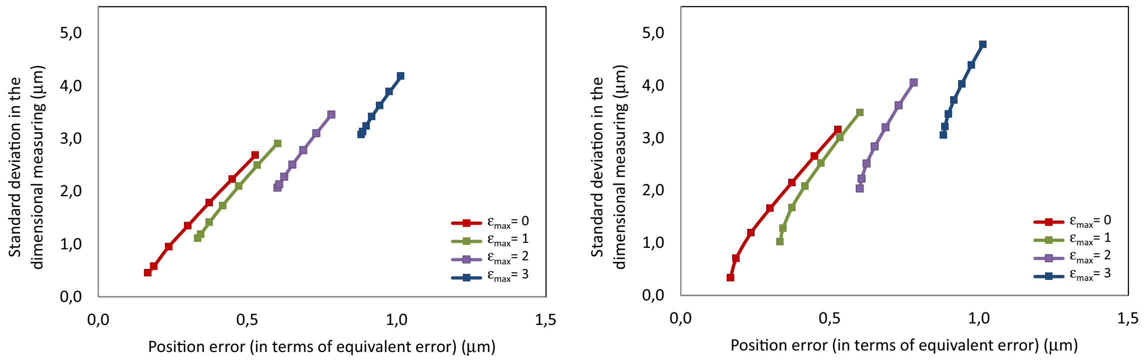
The position, repeatability and reversibility deviations that are produced during the CMM axis displacement will be analyzed, and different levels of straightness and part errors will be also implemented in the numerical modelling. Other geometrical deviations such as the angular errors were not included in the present work, in order to limit the focus of this study to the evaluation of the influence of the error parameters of linear character. The angular errors associated to the CMMs could be added in future works with the purpose of evaluating also their interaction with the rest of error sources.

The utilization of both the EE-based stochastic model and a traditional method will be carried out in the next section of this work, and the proposed model will be proved to contribute to a better understanding about the effect that not only the position errors but also the repeatability and reversibility errors could present on the achievable deviations to be registered in the coordinate measuring machines with three linear axes and FXYZ structural configuration.

3 Results and Discussions

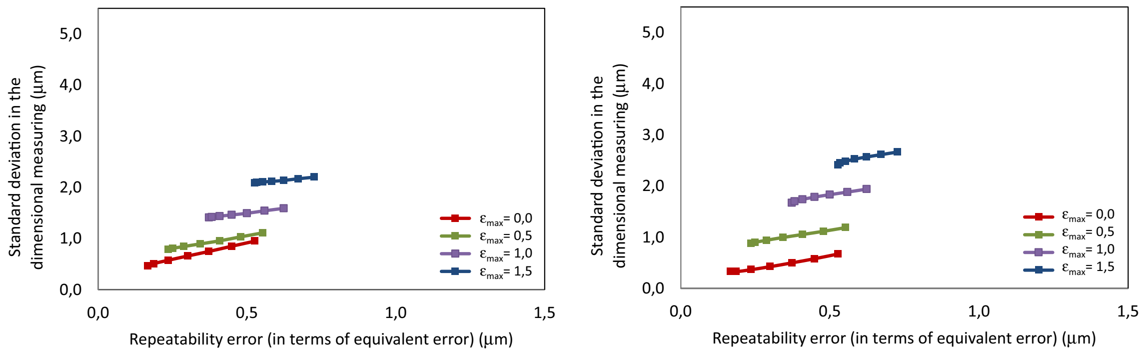
3.1 Evaluation of Measuring Accuracy of CMMs from Position, Repeatability and Reversibility Errors

This section will be oriented to the discussion about the effect of position, repeatability and reversibility errors over the CMM measuring accuracy, when they are studied through the equivalent error. In addition, the advantages of



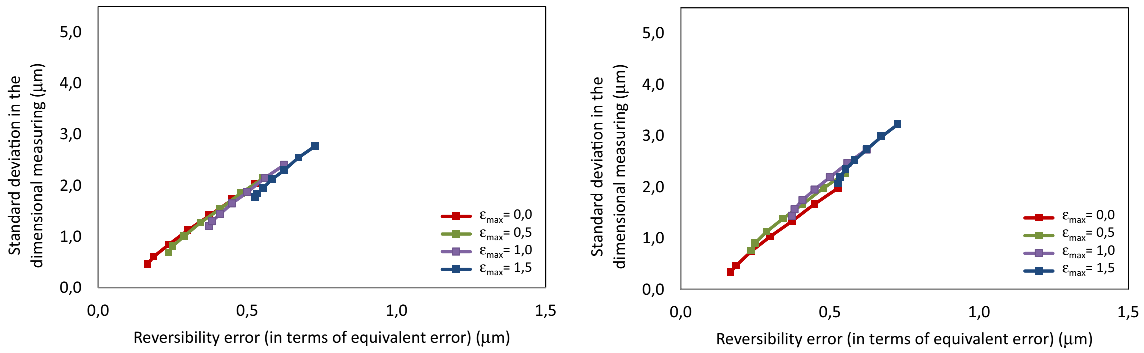
(a) (b)

Fig. 2 Variation in CMM measuring accuracy from position errors through a traditional method (under two different distributions of random errors)



(a) (b)

Fig. 3 Variation in CMM measuring accuracy from repeatability errors through a traditional method (under two different distributions of random errors)

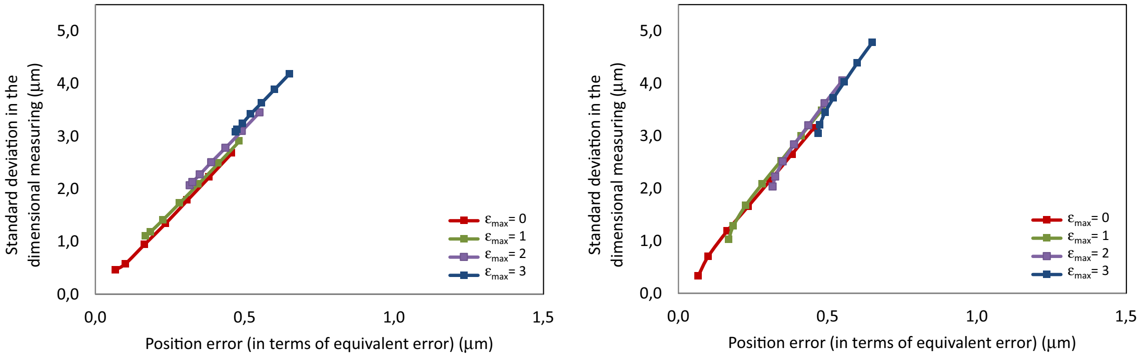


(a) (b)

Fig. 4 Variation in CMM measuring accuracy from reversibility errors through a traditional method (under two different distributions of random errors)

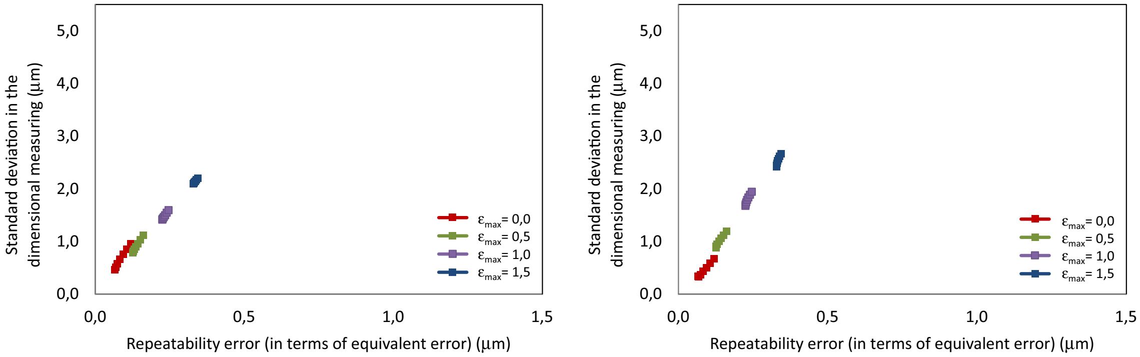
using the equivalent error in order to improve the analysis and prediction of CMM performance as a function of these types of geometrical errors will be evaluated.

The study of the resultant measuring accuracy of CMMs from the position, repeatability and reversibility errors, has been focused on a variation range of these errors from zero to 1.0 μm, which correspond to the possible levels for this type of machine errors. A constant level of part errors equal to 1 μm was assumed in the totality of the numerical predictions, and four different series with distinct values of the rest



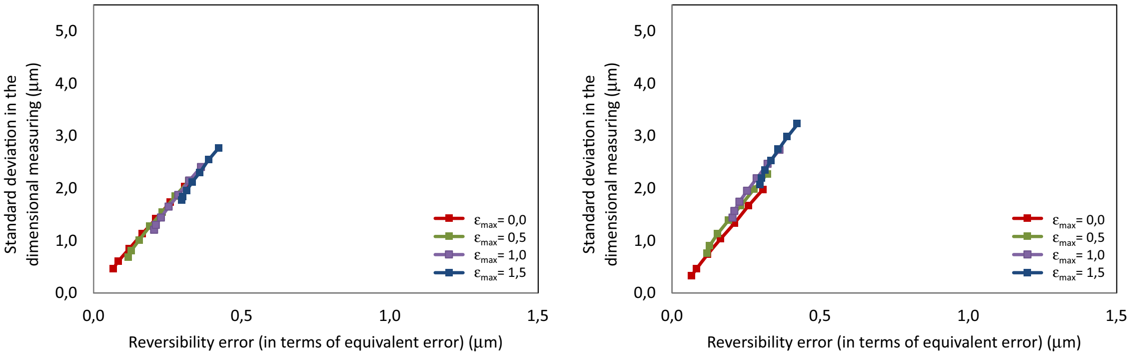
(a) (b)

Fig. 5 Variation in CMM measuring accuracy from position errors through the EE-based model (under two different distributions of random errors)



(a) (b)

Fig. 6 Variation in CMM measuring accuracy from repeatability errors through the EE-based model (under two different distributions of random errors)



(a) (b)

Fig. 7 Variation in CMM measuring accuracy from reversibility errors through the EE-based model (under two different distributions of random errors)

of machine errors were considered in each one of the figures that will be discussed (including straightness errors, as well as position, repeatability and/or reversibility errors).

The possible variation in the CMM performance as a function of the position, repeatability or reversibility errors when these machine errors are studied by the EE-based model or a traditional method, can be observed in Figs. 2, 3 and 4. Secondly, the results obtained when each one of these errors are modelled through the equivalent error are depicted in Figs. 5, 6 and 7. Each one of these figures contains two graphs that provide the results that correspond to the first and second distributions of random errors that have been implemented in the numerical modelling, respectively.

The results about the influence of position errors on the CMM measuring deviations can be seen in Figs. 2 and 5. Figure 2 shows the results obtained from a traditional method, while Fig. 5 is related the EE-based model. Similarly, Figs. 3 and 6 depict the relationship between the repeatability errors associated to the movement of CMM linear axes and the CMM measuring accuracy. Thirdly, the results obtained for the reversibility errors by a traditional method or the EE-based model are illustrated in Figs. 4 and 7 respectively.

As previously indicated in Table 1, the optimized equivalent error coefficients obtained for position, repeatability and reversibility errors are 0.9, 0.2 and 0.6, respectively. These error coefficients were deduced for both distributions of random errors considered in this work, and as a consequence they have been be assumed for the numerical analysis of these machine errors.

When a traditional method is employed, the curves related to position errors show a non-linear behaviour at the beginning, while a certain linearity is revealed at the rest of the points along the curve extension, as can be seen in Fig. 2. Besides, a certain separation in horizontal direction is observed among the curves that correspond to the different series of other error sources, and this distance is increased in the curves with higher values of those machine errors.

As depicted in Fig. 3, in the case of repeatability errors, non-linearity is also encountered at the first points of each curve, followed by a linear tendency in the rest of the curve. An increasing distance is newly detected among the curves that correspond to the different series of results, although in this case the curve separation is produced in vertical direction.

The reversibility errors also evidence a non-linear behaviour at a first stage of the result curves, while a linear tendency is detected during the rest of the curve extension. In the case of these machine errors, the successive curves are not remarkable distanced among them but they are almost superposed (Fig. 4).

As has been discussed with regards to Figs. 2, 3 and 4, the presence of a non-linear tendency has been identified in the results provided by a traditional method. On the contrary, when the EE-based stochastic model is adopted for numerical modelling of CMM measuring accuracy, a linear tendency can be revealed in the totality of the curves that show the results obtained for position, repeatability and reversibility errors, as depicted in Figs. 5, 6 and 7 respectively. A clear linear relationship can be observed

Table 2 Results provided by ANOVA about the equivalent error coefficient with regards to the position errors of the CMM

Parameters analyzed by ANOVA	Mathematical modelling of error sources by separate	Mathematical modelling of error sources by equivalent error
Regression coefficient for coefficient calculation from the first distribution of random errors by ANOVA	0.92143	0.99084
Regression coefficient for coefficient calculation from the second distribution of random errors by ANOVA	0.82942	0.99119
Regression coefficient for coefficient calculation from an average value for both distributions of random errors by ANOVA	0.87543	0.99102
Improvement percentage for the linear regression from an average value that corresponds to both distributions of random errors		13.20%

Table 3 Results provided by ANOVA about the equivalent error coefficient with regards to the repeatability errors of the CMM

Parameters analyzed by ANOVA	Mathematical modelling of error sources by separate	Mathematical modelling of error sources by equivalent error
Regression coefficient for coefficient calculation from the first distribution of random errors by ANOVA	0.71039	0.99334
Regression coefficient for coefficient calculation from the second distribution of random errors by ANOVA	0.65114	0.99293
Regression coefficient for coefficient calculation from an average value for both distributions of random errors by ANOVA	0.68077	0.99314
Improvement percentage for the linear regression from an average value that corresponds to both distributions of random errors		45.86%

Table 4 Results provided by ANOVA about the equivalent error coefficient with regards to the reversibility errors of the CMM

Parameters analyzed by ANOVA	Mathematical modelling of error sources by separate	Mathematical modelling of error sources by equivalent error
Regression coefficient for coefficient calculation from the first distribution of random errors by ANOVA	0.98320	0.98887
Regression coefficient for coefficient calculation from the second distribution of random errors by ANOVA	0.98584	0.98316
Regression coefficient for coefficient calculation from an average value for both distributions of random errors by ANOVA	0.98452	0.98601
Improvement percentage for the linear regression from an average value that corresponds to both distributions of random errors		0.15%

in the result curves that correspond to the analysis of these machine errors when this numerical model is applied.

On the other hand, a different variation rate (or curve slope) has been evidenced between the curves obtained through a traditional approach or the EE-based model for position, repeatability and reversibility errors. This discrepancy between the results of both models is more evident in the case of position and repeatability errors, due to not-aligned curves were deduced when these errors are analyzed by separate during the mathematical modelling. In addition, these three machine errors exhibit a similar curve slope when they are described by the equivalent error.

The analysis of variance (ANOVA) has been utilized for evaluating the linear regression that could be established from the results shown in the previous figures, which were obtained by the EE-based stochastic model or a traditional method. The results facilitated by ANOVA from the influence of position, repeatability and reversibility errors on the expected performance of CMM are represented in Tables 2, 3 and 4, respectively.

These tables show the regression coefficients that can be achieved between the position, repeatability or reversibility errors and the CMM measuring accuracy when these machine errors are modelled by each one of both numerical approaches that have been considered in this work.

According to Table 2, the possibility of evaluating the measuring deviations of CMM by a linear modelling can be increased about 13.20% when the position errors are studied by the EE-based model, which implies that the contribution of position errors over the CMM performance could be more easily discussed from the linear relationship found between both factors.

Table 3 shows that a great enhancement of about 45.86% can be observed in the linear modelling of the CMM measuring accuracy from the existing levels of repeatability errors. Again it will allow a better evaluation about the influence of these machine errors on the expected CMM performance when the EE-based model is employed.

In the case of the reversibility errors, a moderated variation of about a 0.15% has been evidenced in the possibility of linear regression between these geometrical errors and the CMM measuring accuracy by using the equivalent error, as can be seen in Table 4.

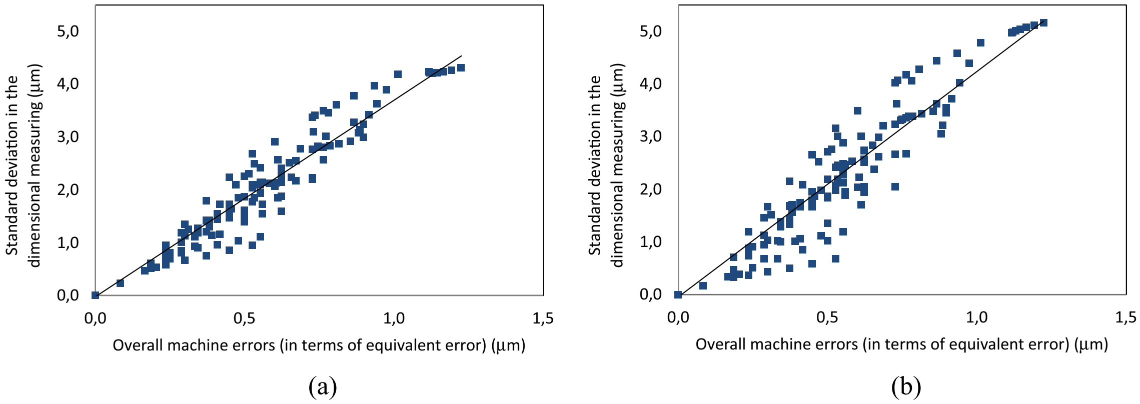


Fig. 8 Variation in CMM measuring accuracy from overall machine errors through a traditional method (under two different distributions of random errors)

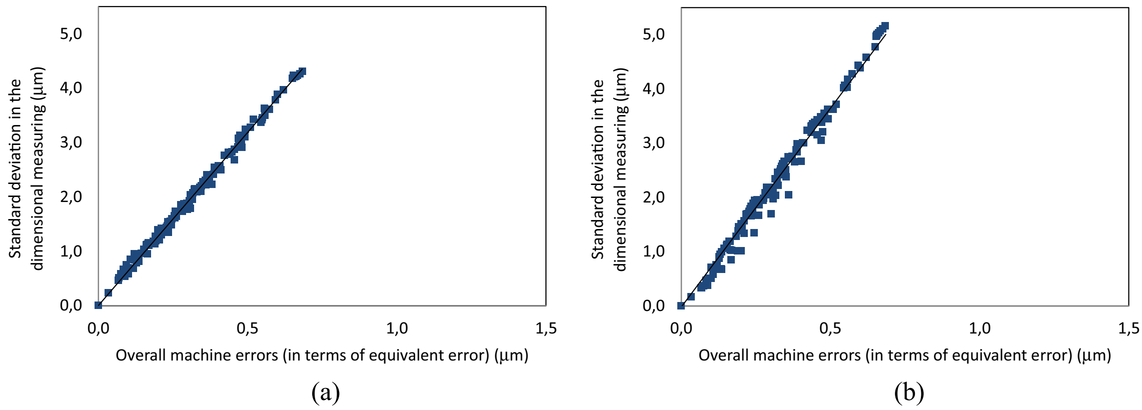


Fig. 9 Variation in CMM measuring accuracy from overall machine errors through the EE-based model (under two different distributions of random errors)

From the results of Tables 2, 3 and 4, it can be concluded that a better linear regression can be determined between the CMM measuring accuracy and the position, repeatability and reversibility errors when the EE-based model is employed. It can be useful for the linear modelling of the measuring process to be developed in coordinate measuring machines, and for the prediction of the CMM performance that can be achieved as a function of these machine errors.

The mathematical expression that could serve to deduce the expected measuring accuracy of CMMs with 3 linear axes and FXYZ configuration from the distinct levels of position, repeatability and reversibility errors by the application of the EE-based model will be presented in Sect. 3.2, which is dedicated to the discussion about the mathematical prediction from the totality of machine errors when this numerical model is adopted.

3.2 Evaluation of Measuring Accuracy of CMMs from Overall Machine Errors

The results obtained through the EE-based model or a traditional method, can be seen in the curves depicted in Figs. 8 and 9, respectively. This work implements two different distributions of random errors for the position, repeatability and reversibility errors, with the objective of making possible the discussion about the expected performance of CMMs from the distinct levels of the totality of machine errors.

When a traditional method is employed, a linear tendency cannot be deduced from the results obtained for the different values of position, repeatability and reversibility errors, due to the excessive dispersion associated to the numerical predictions (Fig. 8). Nevertheless, an excellent linear correlation can be evidenced between these geometrical errors and the measuring accuracy of CMM through the EE-based model, with an elevated regression coefficient of $R^2 = 0,991$ (Fig. 9).

Table 5 presents the results obtained by the analysis of variance (ANOVA) for the CMM performance from all the

Table 5 Results provided by ANOVA about the equivalent error coefficient with regards to the overall machine errors of the CMM

Parameters analyzed by ANOVA	Mathematical modelling of error sources by separate	Mathematical modelling of error sources by equivalent error
Regression coefficient for coefficient calculation from the first distribution of random errors by ANOVA	0.90322	0.99489
Regression coefficient for coefficient calculation from the second distribution of random errors by ANOVA	0.84697	0.98792
Regression coefficient for coefficient calculation from an average value for both distributions of random errors by ANOVA	0.87510	0.99140
Improvement percentage for the linear regression from an average value that corresponds to both distributions of random errors		13.29%

geometrical errors studied in this work, by means of both the EE-based model and a traditional method.

As can be seen in Table 5, a remarkable enhancement of a 13.29% can be obtained in the possible utilization of a linear approach for the numerical analysis of CMM performance, when the EE-based stochastic model is employed. This higher possibility to assume a linear modelling allows the validation of the EE-based model to predict the expected measuring deviations in CMMs with three linear axes and FXYZ structural configuration.

As has been evidenced in Fig. 9, a linear modelling can be adopted to predict the relationship between the position, repeatability and reversibility errors and the CMM performance, when the totality of error sources are represented by the equivalent error. The following linear expression can be deduced to explain the effect that these geometrical errors provoke on the CMM measuring accuracy:

$$\begin{bmatrix} \Delta x_{ijk} \\ \Delta y_{ijk} \\ \Delta z_{ijk} \end{bmatrix} = \begin{bmatrix} 6.86\left(\varepsilon_{eq,x}\right)_{ijk} \\ 6.86\left(\varepsilon_{eq,y}\right)_{ijk} \\ 6.86\left(\varepsilon_{eq,z}\right)_{ijk} \end{bmatrix} \tag{2}$$

This equation will serve to predict the measuring deviations to be observed during the inspection of geometrical properties in mechanical parts through 3-axis FXYZ coordinate measuring machines, or on the contrary to define the maximum admissible level of machine errors that would guarantee the desired measuring accuracy in the CMMs. In the case of other levels of machine errors or other structural configurations, a distinct value could be possibly determined for the coefficient contained in this equation.

4 Conclusions

In this work, the contribution of position, repeatability and reversibility errors on the achievable performance of coordinate measuring machines (CMMs) was analyzed by numerical modelling, and it was focused on CMMs with 3 linear axes and configuration type FXYZ. The CMM measuring accuracy was evaluated by the EE-based stochastic model, which considers the equivalent error (EE) to integrate the totality of error sources, and the results obtained by this theoretical model were also discussed in comparation with a traditional method, which evaluates each machine error by separate. A better linear correlation between the measuring accuracy of CMMs and the existing level of geometrical errors was evidenced in general terms by the EE-based model, with an improvement of about 13.20% and 45.86% in the linear modelling of the measuring process in the case of position and repeatability errors respectively, while a little variation was detected in the case of reversibility errors. In addition, an important enhancement on the possibility of using a linear approach of about 13.29% for the overall machine errors has been also evidenced through the EE-based model. According to the results of this work, it has been probed that this numerical model will facilitate the linear correlation between the measuring accuracy of each CMM and its machine errors, with all these errors expressed in terms of equivalent error. As a consequence, by this model it will be easier to deduce the measuring accuracy of each CMM from the level of machine errors that has been identified in this device, or on the contrary to establish the maximum limit that cannot be exceeded in the machine errors in terms of equivalent errors in order to guarantee a certain CMM measuring accuracy. It will be helpful in the industry to predict the fragment of the manufacturing tolerance that would be lost from the influence of the existing machine errors, or also to determine the CMMs that could be employed for dimensional verification of high accuracy manufactured products or even the ranges of the working volume of a specific CMM that could be selected to verify the products with severe specifications about their dimensional properties. Then the EE-based model will be of special interest to improve the quality control procedures to be applied for the dimensional inspection of mechanical parts inside the production systems of companies specialized in the manufacturing sector.

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Declarations

Conflict of interest The authors declare no competing interests.

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