International Journal of Precision Engineering and Manufacturing (2024) 25:1311–1332
https://doi.org/10.1007/s12541-024-01019-2

Online ISSN 2005-4602
Print ISSN 2234-7593

REVIEW

Geometric Error Measurement of Rotary Axes on Five-Axis Machine Tools: A Review

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Received: 26 December 2023 / Revised: 2 April 2024 / Accepted: 3 April 2024 / Published online: 13 April 2024
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Abstract

For achieving high precision and effectiveness in five-axis computer numerical control (CNC) machine tools, the geometrical accuracy of the rotary axes is a crucial performance criterion. Furthermore, recent advancements in commercial CNCs for machine tools have enabled the numerical compensation for all parameters of geometric errors within rotary axes. As a result, this paper initially delves into the evolution of ISO standards concerning the accuracy testing and error definition in machine tools. Subsequently, the classifications of the rotary axis's geometric errors in five-axis machine tools are described in this paper. Moreover, this paper comprehensively reviews various measurement schemes aimed at identifying the geometric errors of rotary axes. These measurement schemes are categorized based on the measurement instruments or technologies employed. Finally, it is essential to emphasize that this paper offers an overview of diverse measurement theories and technologies pertaining to geometric errors in rotary axes. The primary aim is to contribute to the progression of geometric error measurement and compensation in five-axis machine tools.

Keywords Geometric errors · Rotary axis · Five-axis machine tools · Geometric error measurement

1 Introduction

In recent years, five-axis computer numerical control (CNC) machine tools have gained increasing popularity and are being applied with high efficiency in intelligent manufacturing fields [1, 2]. Compared to traditional three-axis CNC machine tools, five-axis CNC machine tools feature two additional rotary axes [3, 4]. Therefore, five-axis CNC machine tools can provide heightened productivity, improved flexibility, and decreased fixture time [5, 6]. Additionally, ultra-precision five-axis CNC machine tools have been particularly utilized in high-value-added and competitive industries, such as aerospace and medical engineering [7, 8]. It is evident that the machining accuracy of five-axis machine tools plays a pivotal role in modern manufacturing processes involving components with complex geometric structures and sculptured surfaces [9–11].

This paper is an invited paper (Invited Review).

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However, the machining accuracy of five-axis machine tools is influenced by factors such as thermal errors [12–16], structural errors [17–20], motion control errors [21–25], and geometric errors [26–42]. In other words, the various error sources mentioned above contribute to discrepancies between the ideal and actual cutting locations, subsequently impacting the dimensional accuracy of manufactured parts [43–45]. Therefore, the international organization for standardization (ISO) has established a series of standards for machine tools and accuracy testing of these machines. Based on the applicable circumstances, these standards can be categorized into two groups: basic standards and machine-specific standards. Basic standards like the ISO 230 series are designed to be applicable to all types of machine tools universally. On the other hand, machine-specific standards like the ISO 10791 series are applications derived from the basic standards, tailored to a specific type of machine tool. Firstly, in the ISO 230 series, ISO 230-1 specifies methods for testing the accuracy of three-axis machine tools operating either under no-load or quasi-static conditions [46–48]. The assessment of positioning accuracy and repeatability measurement for linear numerically controlled axes is covered in ISO 230-2 and ISO 230-6 [49–51]. ISO 230-3 outlines tests aimed at assessing the impact of thermal effects resulting from linear motion and spindle rotation on the tool center position [52–54]. Circular test is additionally described in ISO 230-4 [55–57]. Furthermore, ISO 230-7 concentrates on standardizing methods for specifying and testing the geometric errors of rotary axes utilized in machine tools[58], a focal point of discussion in this paper [59, 60]. Secondly, ISO 10791-1 to ISO 10791-3 specify test methods for assessing the geometrical conformance to design specifications for various types of machining centers [61–65]. In ISO 10791-4, test methods are prescribed to evaluate the accuracy and repeatability of positioning of linear and rotary axes for machining centers [66–68]. Finally, ISO 10791-6 and ISO 10791-7 specify the standards for the interpolation motion test and the machining accuracy test, respectively, for machining centers [69–72]. As described above, a comprehensive array of error definitions, calibration methods, and accuracy testing protocols for machine tools are established within the ISO standards mentioned.

Periodic error identification and calibration are highly significant tasks essential for characterizing, maintaining, and improving the performance of machine tools. Firstly, formulating the error model and developing measurement methodologies are foundational technological aspects in the process of identifying and calibrating machine tool errors. By quantifying the volumetric errors resulting from thermal, dynamic, motion control, and geometric errors in machine tools, it becomes possible to predict and specify the performance and machining accuracy of the machine tools. Finally, error compensation is a crucial task in ensuring and optimizing the accuracy and performance of machine tools. As a result, it can be seen that measuring, calibrating, and compensating machine tool errors stands as a crucial undertaking throughout the entire lifespan of a machine tool.

In the past decades, several review papers focusing on machine tool error measurement, calibration, and compensation have been published [27]. In 2008 and 2012, Schwenke et al. and Ibaraki et al. conducted reviews on the measurement and compensation of geometric errors in complete machine tools, respectively [8, 73]. In addition, the thermal error measurement and compensation in machine tools were discussed and reviewed by Ramesh et al. and Mayr et al. [74–76]. Lyu et al. conducted a review specifically focusing on the dynamic errors of CNC machine tools in 2020 [77]. Absolutely, the evolution and significance of error measurement and compensation techniques for machine tools indeed carry a lengthy history and crucial importance in enhancing manufacturing precision and efficiency.

Nowadays, in contemporary advanced manufacturing, unlike traditional methods, the machining accuracy of five-axis machine tools is influenced by factors beyond thermal and dynamic errors [78]. Modern advanced manufacturing relies on five-axis machine tools equipped with sub-nanometer position feedback systems and advanced thermal control systems to achieve superior precision and accuracy. As a result, concerning machining accuracy, the proportionate influence of geometric errors in five-axis machine tools has notably increased [79, 80]. Considering the diverse aspects covered in previous review papers regarding errors in entire machine tools, this particular paper primarily concentrates on the measurement of geometric errors specifically related to rotary axes in five-axis machine tools. As a result, this paper serves as a review, updating the state-of-the-art measurement methodologies and compensation strategies for identifying and calibrating geometric errors of rotary axes in five-axis machine tools, building upon the groundwork laid by previous review papers.

This paper provides a review of multiple measurement schemes utilized to identify the geometric errors specifically associated with rotary axes in five-axis machine tools. As a foundation for error measurement, Sect. 2 classifies the geometric errors of the rotary axes in five-axis machine tools according to ISO 230-7. Section 3 provides a review of measurement schemes employed for assessing the geometric errors of rotary axes, categorized based on the measurement instruments utilized. Ultimately, Sect. 4 offers the conclusions drawn from the discussed content within the paper.

2 Classifications of Rotary Axis Geometric Errors

Absolutely, in line with the objective of this paper, which aims to review the state-of-the-art measurement methodologies and compensation strategies for identifying and calibrating geometric errors of rotary axes in five-axis machine tools, this section delivers a succinct review of the definitions and notations concerning the geometric errors of rotary axes outlined in ISO 230-7 and Annex A of ISO 230-1. According to ISO 230-7: 2015, the geometric errors of rotary axes are classified into two categories based on their causes: position and orientation errors, as well as error motions.

Position and orientation errors of a rotary axis refer to the location errors associated with the average line of the axis. In other words, they are defined as deviations from the nominal position and orientation of the rotary axis within the machine coordinate system [8, 81]. As a result, they are mainly caused by imperfections in the assembly process [82]. As depicted in Fig. 1, taking the C rotary axis as an example, the position errors in the x and y directions are denoted as E_{XOC} and E_{YOC}, respectively. Additionally, they involve the orientation errors $E_{\mathrm{A(OY)C}}$ and $E_{\mathrm{B(OX)C}}$ around the X and Y axes, respectively, as well as the zero position error, E_{C0C}. Moreover, Table 1 presents a more detailed definition of position and orientation errors for the C rotary axis according to ISO 230-7. In previous literature, various other terms have been used to describe the position and orientation errors of a rotary axis, including terms like position-independent geometric errors (PIGEs) [5, 81, 83–85], link error parameters[86, 87], systematic deviations[88], and location errors[89–95]. As a result, in consideration of its comprehensibility, this paper employs the terms “PIGEs” to characterize the position and orientation errors associated with a rotary axis.

Table 1 Symbol of position and orientation errors (axis shift) of axis average line [58]

Symbol	Description
E_{XOC}	Error of the position of C in X-axis direction
E_{YOC}	Error of the position of C in Y-axis direction
$E_{A(OY)C}$	Error of the orientation of C in A-axis direction; Squareness of C to Y
$E_{B(OX)C}$	Error of the orientation of C in B-axis direction; Squareness of C to X
E_{C0C}	Zero position error of C-axis

On the other hand, error motions of a rotary axis are defined to represent changes in the position and orientation of the axis of rotation as it rotates. Error motions associated with a rotary axis primarily arise from defects in its

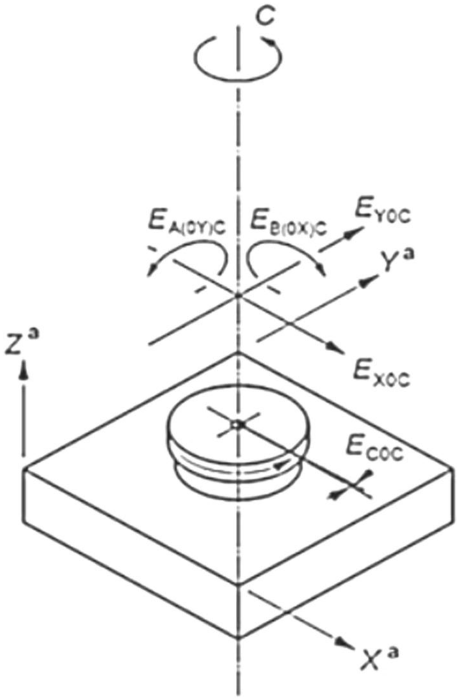

Fig. 1 Position and orientation errors (axis shift) of axis average line according to ISO 230-7 [58]

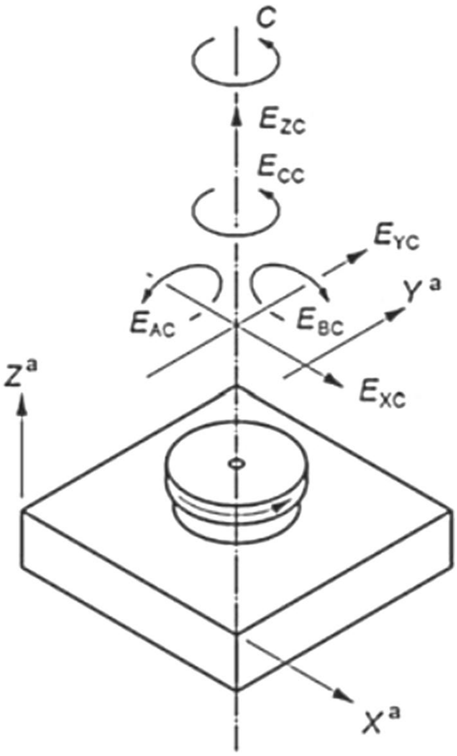

Fig. 2 Error motions of axis of rotation according to ISO 230-7 [58]

Table 2 Symbol of error motions of axis of rotation [58]

Symbol	Description
E_{XC}	Radial error motion of C in X-axis direction
E_{YC}	Radial error motion of C in Y-axis direction
E_{ZC}	Axial error motion of C Reference axis
E_{AC}	Tilt error motion of C around X-axis
E_{BC}	Tilt error motion of C around Y-axis
E_{CC}	Angular positioning error motion of C

segments and components [96]. As shown in Fig. 2, According to ISO230-7, considering the C rotary axis as an example, the six error motions associated with the rotation of C axis consist of the linear deviations including the axial motion error E_{ZC}, the radial motion errors E_{XC} and E_{YC} in the X and Y axis directions, respectively, as well as the angular deviations including the angular positioning error motion E_{CC}, the tilt error motions E_{AC} and E_{BC} around X and Y axis, respectively[97]. For a more comprehensive elucidation in line with ISO 230-7 regarding error motions related to the C rotary axis, refer to Table 2. In prior literature, error motions associated with a rotary axis are also referred to as intra-axis kinematic errors [98], component errors [99], joint kinematic errors[100], and position-dependent geometric errors (PDGEs) [83, 96, 97, 101–103]. Consequently, this paper utilizes the terminology "PDGEs" to characterize the error motions related to a rotary axis.

3 Geometric Errors Measurement for Rotary Axes

3.1 Probes

Despite the widespread recognition of the commercial measurement device for its commendable precision and efficiency, its substantial cost and intricate operational requirements pose considerable challenges. In contrast, touch-trigger probes emerge as a sophisticated solution adept at circumventing these challenges. These probe measurement methods offer automated high precision measurements, adaptable to a wide array of workpiece geometries such as precision spheres or square components, as shown in Figs. 3 and 4. Notably, they are effectively used to measure the geometric errors in machine tools. Furthermore, the measurement methods for the location errors of rotary axes, using a touch-trigger probe and a sphere as proposed by Chen et al., are depicted as follows [89].

As reported in [89], A robust on-machine measurement methodology and calibration algorithm were developed for identifying location errors in the rotary axes of a five-axis CNC machine tool. Initially, the touch-trigger probe was mounted on the spindle, while the precise sphere was mounted on the C-axis rotary table. As show in Fig. 5, the touch-trigger probe was driven to measure the precise sphere at each measurement position of both Pattern I and Pattern II. Finally, the eight location errors were identified using mathematical measurement equations and a calibration algorithm established through the application of homogeneous transformation matrix (HTM) and the least squares method. In the subsequent sections, the studies of measurement techniques are categorized based on the diverse shapes of the workpieces being evaluated. Furthermore, the practical applications and notable advantages of probes in effectively addressing the precision measurement challenges encountered by the industry are also explored.

In our previous works, we also presented a set of methodologies to measure the PIGEs and PDGEs of rotary axes

Fig. 3 Touch-trigger probe with a precision sphere [89]

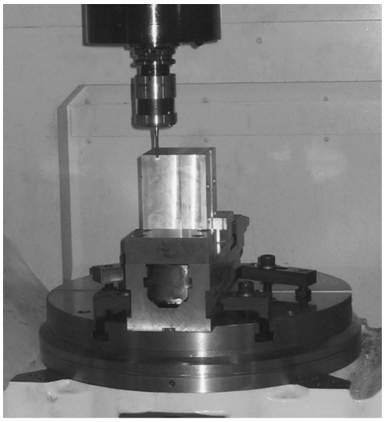

Fig. 4 Square workpiece for calibrating geometric errors of rotary axes [90]

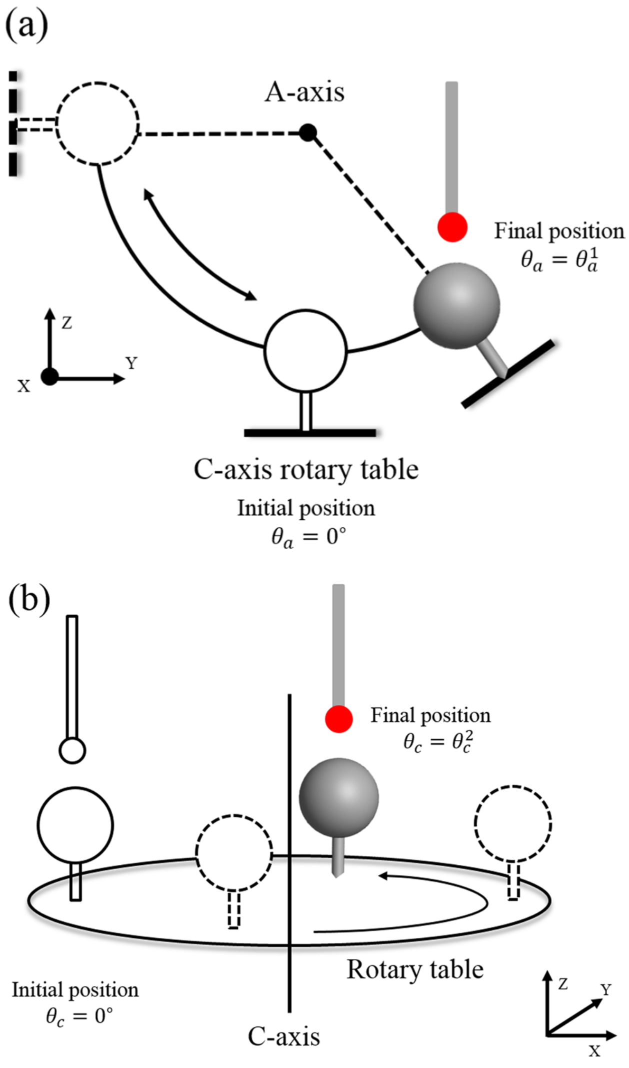

Fig. 5 Probes measurement patterns: **a** pattern I and **b** pattern II [89]

on five-axis machine tools by using a touch-trigger probe and precision spheres [89, 97]. Furthermore, several scholars have extensively investigated similar concepts [91, 95]. They utilized varying quantities of precision spheres and applied diverse mathematical methodologies to identify the errors linked with distinct machine tool configurations. These findings underscore the widespread utilization of precision spheres within advanced manufacturing, serving as essential tools for error measurements and calibrations.

Ibaraki et al. primarily employed cubes or square cylinders as their main workpieces for measuring the geometric errors of rotary axis in five-axis machine tools. According to the proposed study, a cube was strategically positioned on a rotating table [90]. Eight distinct measurement modes were employed to gather data points, and mathematical equations were utilized to decouple the location errors of the two rotary axes. Ibaraki incorporated three test pieces resembling a square column geometry into the experimental setup, and effectively proposed the measurement method for PDGEs of rotary axes [104–106]. Wang et al. and Jiang et al. also conducted analogous studies involving cubes [92, 107].

Previous studies often overlooked the impact of linear axis geometric errors when proposing measurement methods to identify the geometric errors of the rotary axis. As a result, Ibaraki et al. addressed this issue by employing an uncalibrated cylindrical workpiece. They distinguished the geometric errors of the linear axis and rotation axis through four measurement procedures using probes positioned on the workpiece's side [108]. Validation was conducted using the R-test, demonstrating comparable accuracy between the two methods and emphasizing the utility of touch-trigger probes in precision measurement. Subsequently, Ibaraki improved the initial experimental procedure by extending detection to the top surface of the workpiece and introducing an additional rotation axis [109]. This enhancement led to the acquisition of further geometric errors in both linear and rotational axes.

In essence, the touch-trigger probe stands out as an exceptional device due to its remarkable cost-effectiveness and adaptability to diverse workpieces, effectively addressing the limitations of other measurement tools. Its distinctive advantage lies in the capacity to utilize workpieces that don't necessitate high precision for experiments, thereby enhancing convenience in precision measurement applications. This unique feature not only offers a more economically feasible solution for the manufacturing industry but also guarantees heightened accuracy and reliability in the measurement and calibration processes. Consequently, it significantly contributes to augmenting the cost-effectiveness and efficiency of the manufacturing sector.

3.2 Double Ball-Bar

The circular test using a double ball-bar (DBB) has gained widespread acceptance among machine tool builders or users for assessing the geometric accuracy of rotary axes in five-axis machine tools. By mounting the first ball on the tool tip and locating the other ball on the rotary table, the measurement of geometric errors in the rotary axes is conducted. As described in [84], Fig. 6 illustrates four measurement paths, each involving the control of a single rotary axis during the measurement process. During the measurement process, the length of the DBB was recorded, allowing for the calculation of the deviation between the nominal and actual length of the DBB. Ultimately, the eight PIGEs of rotary axes A and C are identified using the proposed paths along with the measurement equations created by HTM. Therefore, this subsection focuses on the measurement of geometric errors in rotary axes by using DBB, particularly in the context of multi-axis machine tools.

Fig. 6 Four DBB measurement paths of PIGEs of rotary axis A and C [84]. **a** Measurement path for offset errors of rotary axis A. **b** Measurement path for squareness errors of rotary axis A. **c** Measurement path for offset errors of rotary axis C. **d** Measurement path for squareness errors of rotary axis C

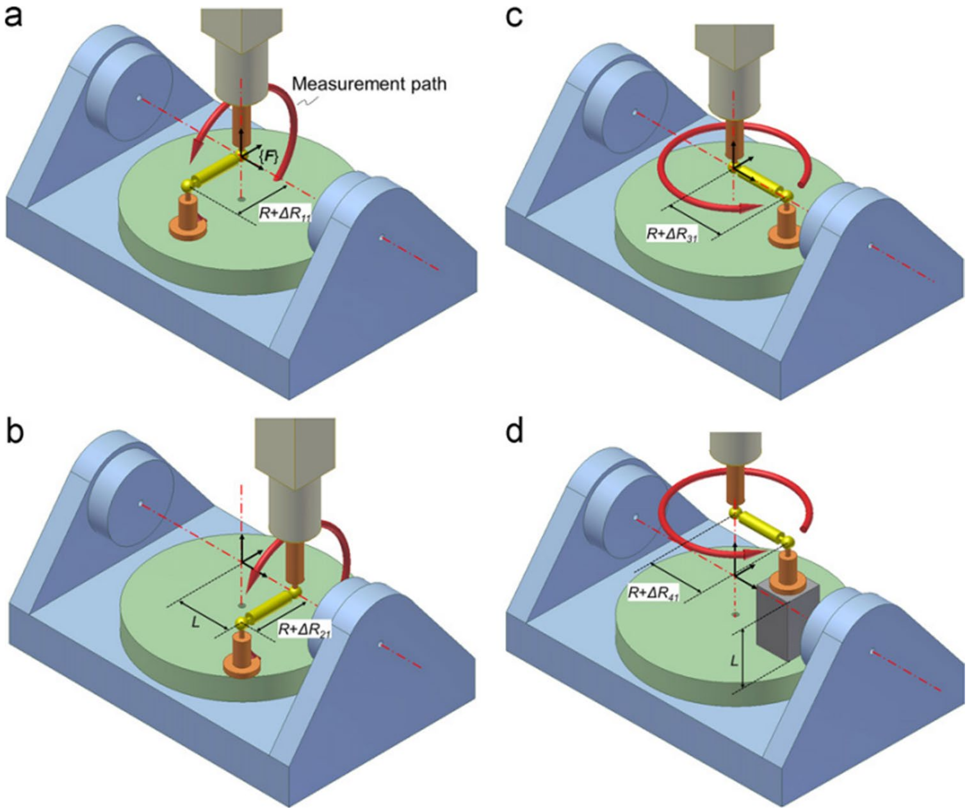

Fig. 7 Ball-bar measurement of five-axis machine tools [115]

Mayer et al. conducted a series of studies in which they modeled motion error sources as polynomial functions for individual joint coordinates [86, 87, 110, 111]. Using these polynomial coefficients, they made predictions regarding tool and workpiece position as well as orientation errors. In their research [110], they eliminated eccentricity from the results obtained through the DBB method and proposed a technique to quantify eccentricity, ensuring comprehensive data processing and avoiding virtual elliptization. Their method outlined in [111] evaluated A-axis motion errors using five tests within a single setup, thereby simplifying the tests and independently addressing issues related to axis motion. Moreover, the systematic approach presented in [86] identified errors on a five-axis machine tool, encompassing joint misalignment, angle deviation, and rotary axis spacing, and applied these findings for machine tool calibration.

Following the research conducted by Lei et al. [112–114], errors within rotary axes were diagnosed using specific circular test paths, with only two rotary axes in motion while the others remained stationary. This methodology effectively pinpointed error sources in rotary axes, specifically detecting servo mismatch within two rotary axes.

Research conducted by Yang et al. employed the use of DBB methods to estimate geometric errors in machine tool rotary axes [84, 85, 102, 115, 116]. Their approach, outlined in Fig. 7 [115], categorized errors into two types: position-dependent and position-independent, and modeled them using nth-degree polynomials to enhance accuracy. In study [84], PIGEs in rotary axes, such as offset and perpendicularity errors, were measured and verified for compensation. Study [85] utilized DBB and single-axis control to measure offset and perpendicularity errors in rotary axes, minimizing setup errors through an innovative fixture and enhancing machine tool positioning accuracy. Additionally, a method to identify and rectify geometric errors on the rotary axis of a five-axis machine tool equipped with a tilted worktable is investigated [55]. This achieved complete separation of PIGESs and PDGEs during measurement by controlling a single rotary axis.

Zhang et al. conducted multiple studies [5, 117, 118], starting with study [117], which employed the DBB technique to evaluate motion errors in the rotary axes of five-axis machine tools. Their study [118] proposed a new measurement method that utilized only one DBB system and an adjustable fixture to simultaneously assess five errors arising from the rotary table, encompassing axial, radial, and tilt errors. Moreover, in study [5], eight position-independent geometric errors on the rotary axis of a five-axis machine tool were identified using three measurement modes. In these modes, the linear axes stayed stationary while only two rotary axes moved to trace circular trajectories. This method completely eliminated the influence of linear axes, thereby enhancing measurement accuracy. Furthermore, Fu et al. presented the TFFIA method, which aimed to enhance accuracy by considering 21 fundamental errors [119]. Using three effective measurement modes, this method captured direction-sensitive errors and required only three workpiece ball positions to comprehensively identify 20 errors across two rotary axes.

Jiang et al. introduced a series systematic approach for identifying and characterizing PIGEs in the rotary axis of five-axis machine tools [120–122]. Among them, their study [121] presented a two-stage DBB method aimed at swiftly diagnosing and identifying PIGEs in the rotary axis of five-axis machine tools. The initial stage allowed for a rapid evaluation of both axes, providing a quick preliminary test. If the errors surpassed the tolerance range, the second stage offered a more accurate but slower inspection, suitable for periodic accuracy checks. Additionally, Chen et al. introduced a method that comprehensively measures and identifies linkage errors and volume errors in the rotary table using the DBB technique [123, 124]. In their study, volume errors were evaluated through circular trajectories, and the circular fitting method was utilized to enhance precision, effectively improving machining quality accuracy.

Lasemi et al. [66] proposed a precise method for identifying geometric errors in five-axis machine tools, particularly focusing on the rotary axis, utilizing the DBB technique [125]. Their model encompasses both PIGEs and PDGEs, employing sensitivity analysis to streamline the model. Zhong et al. conducted a comprehensive assessment of tool position and axis direction errors using nine circular test paths [126]. Through kinematic analysis, Peng et al. developed a mathematical model capable of accurately identifying both angular and displacement errors [103, 127]. Ding et al. employed single-axis drive and optimized parameters to enhance precision [101]. Jiang et al. proposed a novel measurement trajectory that involved simultaneous motion of two rotary axes [128]. Utilizing DBB to sample complex motions, they obtained a true reflection of machine tool errors and suggested a method for estimating PIGEs in the two rotary axes.

Li et al. effectively identified all 12 PDGEs, including commonly overlooked angular positioning errors, by utilizing four DBB installation positions and designing eight measurement modes [129]. Ding et al. investigated the circle-8 test, employing DBB to capture complex motions and extract valuable data, which aids in modeling and reducing errors in five-axis machine tools [130]. The method proposed in [131] proved effective in eliminating installation errors in DBB tests, consequently enhancing measurement accuracy. Yao et al. analyzed eccentricity in trajectories measured by DBB during simultaneous three-axis motion [132]. By compensating for the impact of identified geometric errors on tool center point (TCP) position error, they achieved precise identification and correction of geometric errors.

In summary, this collection of studies presents a wide array of comprehensive methods to evaluate not only geometric errors but also dynamic errors within the rotary axis of five-axis machine tools. Various DBB tests and analysis techniques provide invaluable insights for enhancing accuracy and compensation. These studies encompass various aspects, such as test path design, initiation of test procedures, data processing, and correction mechanisms. They demonstrate adaptability across different models and brands of five-axis CNC machine tools, propelling advancements in precision machining and significantly contributing to the enhancement of efficiency and product quality in manufacturing processes.

3.3 R-test

As mentioned above, Although the DBB has been widely acknowledged as an effective measuring device for identifying geometric errors in the rotational axes of multi-axis machine tools, its application in five-axis machines poses a significant challenge. The DBB operates as a one-dimensional measurement device, capable of measuring only a single direction of displacement at a time. This limitation requires operators to change the measurement device and machine tool settings multiple times to identify all geometric errors accurately, making full automation notably difficult.

To address these challenges, Weikert proposed an alternative device known as the "R-test." This device comprises three (or more) linear displacement sensors along with a precision sphere. The R-test operates by utilizing the linear displacement sensors to measure the three-dimensional displacement of the sphere, thereby enabling the acquisition of three-dimensional error trajectories during the measurement cycle [133]. In general, the geometric errors of the rotary axes are calculated by analyzing these three-dimensional error trajectories. Therefore, Fig. 8 illustrates the prototype setup of the R-test, as demonstrated by Ibaraki et al. Currently, commercial R-test devices are available from IBS Precision Engineering and Fidia. This innovation offers a promising solution to the limitations associated with the

Fig. 8 The prototype setup of the R-test [133]

DBB by facilitating more comprehensive and efficient geometric error measurements in multi-axis machine tools.

Several earlier studies have underscored the efficacy of the R-test in pinpointing geometric inaccuracies in rotary axes by precisely fitting measured data to the kinematics [134–136]. According to the experiments conducted by Florussen et al., the dynamic R-test effectively identified areas that the static R-test failed to detect, showcasing its robust capability in assessing the actual contour performance of five-axis machine tools [137]. Ibaraki et al. employed HTM, Jacobian matrix, and the least squares method to tackle position errors and rotational axis inaccuracies [138]. A similar approach was also adopted in another study [139].

The majority of measurements using R-test involve coordinated movements of both linear and rotary axes. However, numerous prior studies have presumed that the error motion of the linear axis is negligible and can be ignored during the calibration of the rotary axis. Ibaraki et al. demonstrated that the error motions of rotary axes could be calibrated with minimal interference from the error motions of linear axes by strategically positioning the sphere on the nominal axis average line of a rotary table [140, 141]. Moreover, Masashi et al. also utilized the R-test methodology to estimate the motion error of linear axes in machine tools [142].

Thermal deformation also stands as a prominent factor contributing to errors in machine tools. Beyond addressing geometrical errors, the R-test measurement method can also serve to measure the thermal state of a machine tool due to its brief measuring duration. Lee et al. conducted a three-day experiment employing the B-spline interpolation method, unveiling a notable correlation between specific errors and fluctuations in ambient temperature [143]. Hong and Ibaraki also introduced a method to observe heat-induced geometric errors in a rotary axis using the R-test [144, 145].

Furthermore, it is worth acknowledging that the traditional contact-type R-test might be vulnerable to friction effects or dynamics related to the supporting spring in displacement sensors, particularly in dynamic measurement scenarios. To address this concern, Hong and Ibaraki proposed a non-contact R-test employing laser displacement sensors to enhance measurement accuracy and reliability [146, 147]. Additionally, Ibaraki et al. proposed a software which is designed to analyze R-test measurement trajectories, aiming to numerically rectify PIGEs of the rotary axis [148].

In summary, the R-test, functioning as a precision measuring tool for machine tools, has made substantial advancements in addressing the limitations associated with the DBB. It has notably enhanced measurement accuracy and efficiency across multiple facets of evaluation and calibration.

3.4 Tracking Interferometer

The utilization of laser trackers for identifying geometric errors has gained substantial popularity owing to their exceptional capabilities in 3D metrology [149]. The measurement principle relies on the laser tracker's ability to determine the distance to the target, alongside two angles: the azimuth angle and zenith angle, providing positional data within a spherical coordinate system [150]. Prominent brands in use include Leica Geosystems, Faro, and Automated Precision Inc. (API). Laser trackers present various advantages, such as high measurement efficiency, an extensive measurement range, not mandating precise installation, and lacking specific trajectory limitations. In contrast to the DBB and R-test, laser trackers are not limited by a fixed length, allowing them to comprehensively measure the geometric errors of five-axis machine tools more efficiently and with fewer steps [151, 152]. Due to its measurement capabilities and ease of use, the laser tracker is increasingly employed for volumetric error measurements. However, it is crucial to note that the primary contributor to the measurement uncertainty in laser trackers is the angular measurement error. Accompanied by the angular measurement error, the position measurement error deteriorates significantly with increased measurement distances [150]. Consequently, a similar instrument to a laser tracker, known as a laser tracer, has been proposed. By the way, the laser tracer employs the multilateration method to determine the target's position. Compared to conventional laser trackers, employing the laser tracer for position measurement results in reduced measurement uncertainty [151, 153]. As a result, in sectors such as automobile manufacturing, aerospace manufacturing, and the measurement of large machining parts, the use of laser tracers and laser trackers remains indispensable. Furthermore, laser tracking technology is also widely employed in precision measurement and metrology for five-axis machine tools. Therefore, the

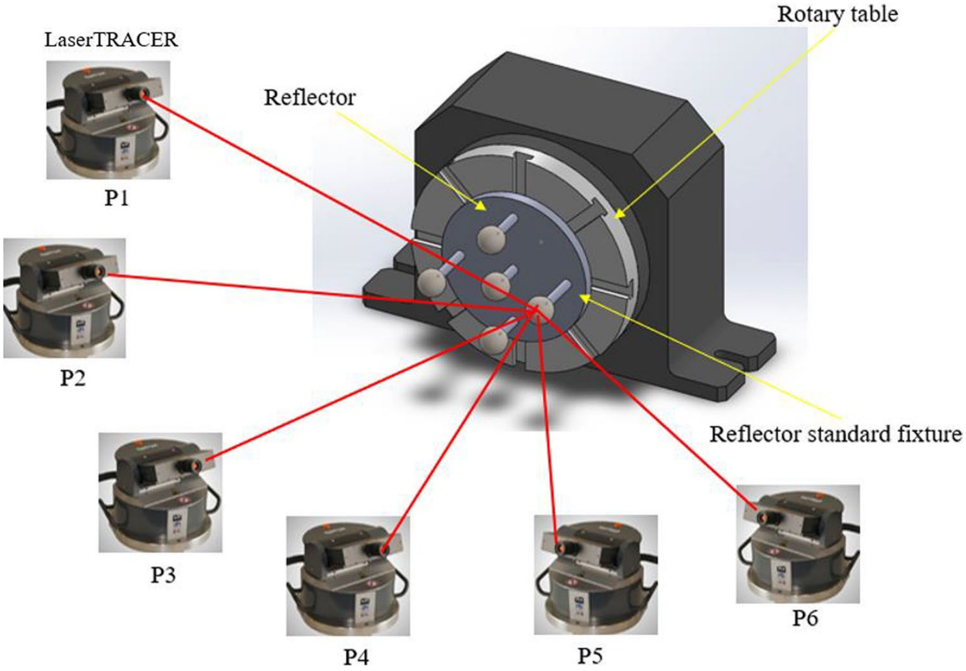

Fig. 9 Schematic diagram illustrating the method for measuring geometric errors using LaserTRACER [154]

following paragraph will depict the measurement methods applied in the field of five-axis machine tools measurement.

As a result, the measurement methods for determining geometric errors in rotary axes using the laserTRACER were outlined in [154]. In the study [154], a novel fixture standard with reflectors was designed and mounted on the pending measurement rotary table. Additionally, the LaserTRACER was installed at different external base stations to identify the location of each reflector at every measurement position, as shown in Fig. 9. Due to the requirement for high measurement accuracy, the principles of multi-station and time-sharing measurements developed by Wang et al. were adopted to obtain and identify the 6-DOF geometric errors of rotary table [155, 156]. In the following paragraph, practical applications for measuring geometric errors of rotary axes using laser tracking technology, as proposed in previous studies, are introduced.

In 2012, Wang et al. introduced a method employing a laser tracker based on the sequential multilateration measurement principle [155]. This method involved measuring the motion trajectory of the linear and rotary axes of a machine tool at different base stations using the laser tracker. In a subsequent study in 2013, Zhang et al. proposed a series of studies for measuring the geometric errors of rotary axes on machine tools [157, 158]. By maintaining continuous measurements of three points and incorporating error model calculations, the tracker was sequentially placed in four positions for measurements, yielding improved repeatability. Yin et al. utilized a laser tracker to identify quasi-static errors of two rotating axes and perpendicularity errors between the centerline of the rotating axis and three linear axes of the gantry-style five-axis machine tool [159]. Furthermore, the laser trackers were also used in conjunction with other instruments. For instance, Acosta et al. utilized linear displacement sensor and self-centring probe to conduct additional measurements focused on radial, pendulum, and axial movement errors [160]. Ellingsdalen et al. presented a compensation strategy for all axes of a five-axis machine tool [161]. This method involved measuring three points on the spindle and worktable at various axis displacements, enabling the calculation of PIGEs. Recently, Yao et al. employed a dual quaternions model for geometric error analysis, employing the Powell algorithm for calculation [152]. This approach allowed the simultaneous determination of PDGEs and PIGEs of rotary axes, simplifying the error decoupling process, as shown in Fig. 10.

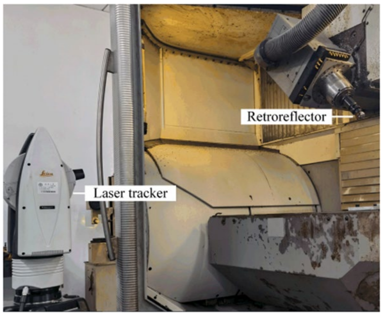

Fig. 10 The experimental setup [152]

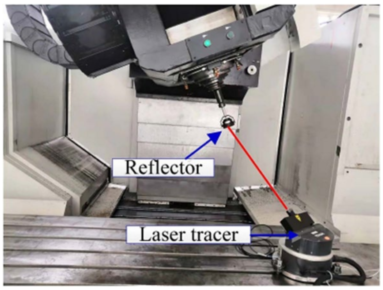

Fig. 11 Experiment setup based on the proposed measurement method [163]

On the other hand, in the laser tracer-based application fields, Zha et al. employed four-station laser tracers to measure and differentiate geometric errors in an NC rotary table [162]. This involved implementing self-calibration algorithms for the laser tracers and employing measurement point-solving algorithms to determine the errors. Deng et al. concurrently incorporated rigid-body motion constraints in multilateration for motion error measurements [151]. Their approach involved a two-step geometric error identification method, enhancing the accuracy of error measurements. Furthermore, Hsu et al. proposed a method to determine the geometric errors of rotary tables using a laser tracer and reflectors mounted on a reflector standard fixture. This technique employed a multi-station and time-sharing measurement approach, employing six distinct reflector positions and six different laser tracer base stations. This method allows measurements without necessitating the operation of the other three external linear axes, thereby reducing setup errors. Importantly, it can be applied to various rotary table configurations. Lastly, Hongdong et al. implemented coordinated movements across all five motion axes to concurrently identify all geometric errors, departing from the conventional method that necessitates separate calibration of linear and rotary axes [163], as shown in Fig. 11. This streamlined approach reduces the measurement process while enhancing accuracy.

3.5 Optical Techniques

Optical measurement falls within the realm of direct measurement. However, due to extensive research and exploration, there has been an increasement in optical methods aimed at concurrently measuring multi-axis motion errors. Consequently, the relevance and importance of optical measurement in this domain have significantly increased. In the field of optical measurements, many studies proposed novel and specific optical measurement systems incorporating laser sources, lenses, detectors, etc., for measuring the geometric errors of rotary axes. As referred in [164], to analyze the geometric errors of rotary axes, a mathematical model of the measurement equations was constructed using a HTM and the skew-ray tracing method. Furthermore, the subsequent paragraph details various methodologies outlined in prior studies for measuring geometric errors associated with rotary axes, employing diverse optical measurement techniques.

Chen et al. introduced and validated a six-degree-of-freedom (six-DOF) optoelectronic motion error measurement system incorporating three laser-diode/position-sensitive detector pairs and a pyramid-polygon-mirror [165]. Skew-ray tracing methods and first-order Taylor series expansion techniques are employed to determine the geometric errors inherent in rotary axes [166]. Subsequently, Murakami et al. devised a straightforward and cost-effective optical measurement system by utilizing rod lenses and ball lenses integrated into the micro-spindles. This system enabled simultaneous measurement of the five degrees-of-freedom error motions in high-speed micro-spindles. Feng et al. proposed three methods for measuring geometric errors of rotary axes employing a combination of prism groups, beam-splitting films, position-sensitive detectors, quadrant detectors, and retroreflectors. Initially, in [167], the deviation of laser light reflected from two mirrors on the rotary axis were measured for enabling simultaneous calculation of five-DOF motion errors. Subsequently, in [80], the laser interferometry principles, laser collimation, and PID closed-loop control were employed achieving precise and simultaneous measurement of all six-DOF motion errors of a rotary axis. Finally, Feng et al. extended the model to linear axes, establishing a direct measurement method for concurrently measuring six-DOF geometric motion errors in both linear and rotary axes, as shown in Figs. 12 and 13 [79]. Liu et al. combined a polygon mirror with a conical lens to simultaneously measure six-DOF for a rotary axis [164, 168]. Zhao et al. employed a high-precision steel ball and a specialized mirror on the rotary axis to measure collimated laser light from three perpendicular directions, providing axial and tilt error measurements for rotary axes [169].

For miniaturized 5-axis machine tools, Park et al. proposed a simple and cost-effective method utilizing only two position-sensitive detectors and a laser diode to identify geometrical errors of a rotary axis through non-contact measurements [170]. Meanwhile, He et al. introduced the dual optical path measurement method (DOPMM) to identify all six volumetric error parameters with straightforward algebraic operations, simplifying machine tool measurements [171].

Furthermore, Yang et al. proposed the position-distance measurement method using multi-wavelength phase-shifting interferometry technology and a high-speed CCD camera to track the spatial posture of a designed microstructure feature during rotary stage movement, as shown in Fig. 14 [172]. Additionally, Yin et al. utilized a monocular camera, presenting a programmable identification method for visually

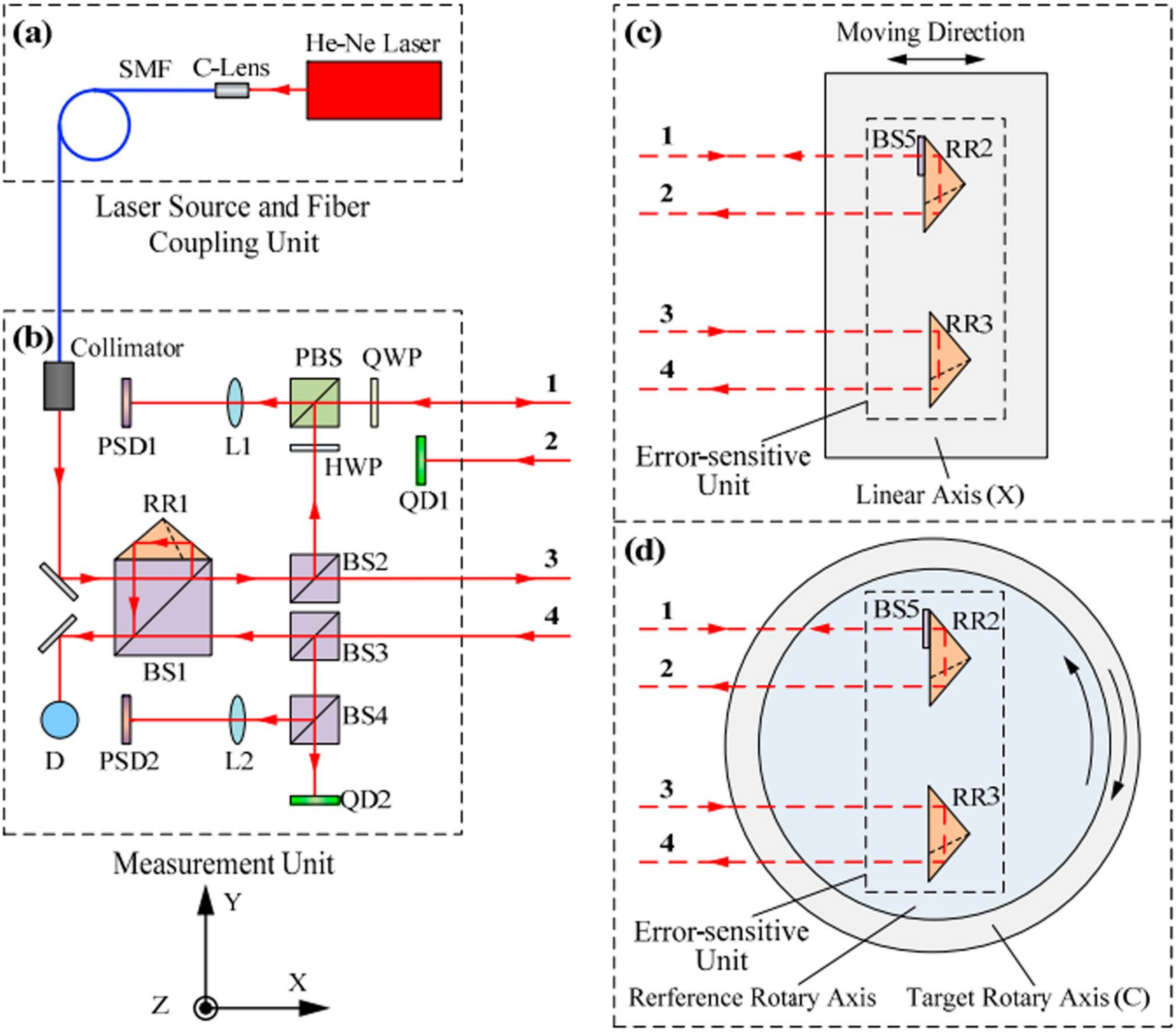

Fig. 12 Schematic diagram for simultaneously measuring 6DOF geometric motion errors of both linear and rotary axes; **a** laser source and fiber coupling unit; **b** measurement unit; **c** error-sensitive unit for measuring a linear axis; **d** error-sensitive unit for measuring a rotary axis [79]

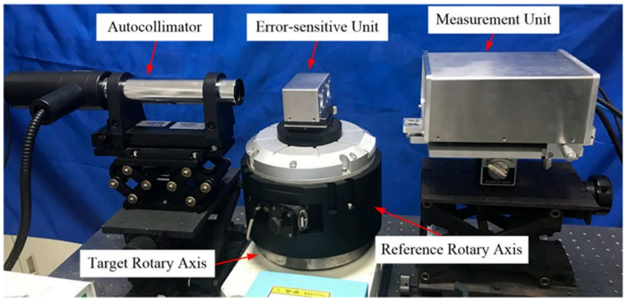

Fig. 13 Measurement experiments of rotary axis [79]

measuring and identifying geometric errors of rotary axes in compact five-axis platforms [173].

3.6 Machining Test

In comparison to traditional non-cutting measurements, machining tests provide a significantly accurate method for elucidating the inherent motion errors in five-axis machine tools during actual machining processes. As a result, several researchers have proposed their own specific shapes of workpieces, such as NAS979, S-shaped,

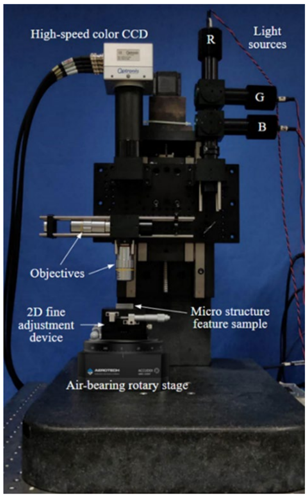

Fig. 14 Experimental platform for simultaneous measurement of 6DOF errors of the precision rotary stage [172]

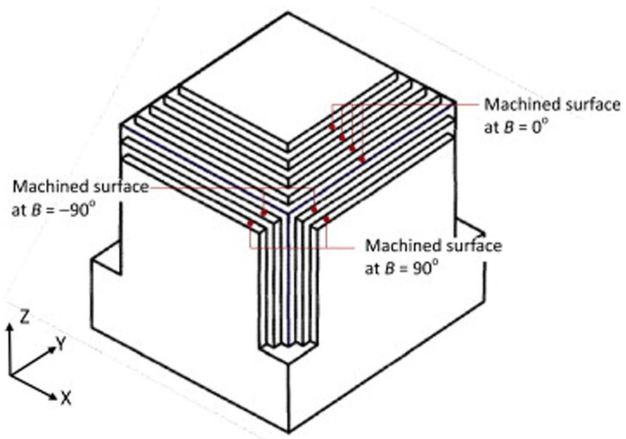

Fig. 15 Finished test piece geometry [172]

and cylindrical-type, for predicting the geometric errors of the rotary axis. For example, Ibaraki et al. employed NAS979 standard workpieces to experimentally validate the prediction and compensation of geometric errors during five-axis machining [174]. Using an error model of the machining center with identified kinematic errors and considering location and geometry of the workpiece, machining geometric error with respect to the nominal geometry of the workpiece is predicted and evaluated.

Subsequently, Hong et al. analyzed top cone truncation for revealing the influence of gravity and geometric errors on radial error motion and tilt error motion [83]. These factors were identified as significant error sources in cone machining tests. Ohta et al. used truncated pyramid workpiece instead of cone shaped workpiece for their study [175]. Their results highlighted how changes in linear axis speeds significantly affected the profile curves of the truncated pyramid.

Unfortunately, the experimental results indicate that the cone shaped workpiece is primarily influenced by the geometric accuracy of the machine tool, demonstrating limited sensitivity towards dynamic accuracy. For addressing this challenge, the development of new testing workpieces was conducted. Ibaraki et al. [176, 177] proposed specialized machining tests for five-axis machine tools aimed at identifying fundamental motion errors by shaping the workpiece to resemble a pyramid, as shown in Fig. 15. Several researchers also adopted a similar approach and employed similar-shaped test pieces to identify various errors of rotary axes [94, 178–181]. Chang et al. utilized Taguchi methods and signal-to-noise ratio (S/N) to calculate effectiveness of direct cutting motion error data [182, 183]. Additionally, Li et al. utilized pyramid-shaped test pieces to assess the quantitative impact of dynamic synchronization errors between rotational and linear axes on machined surfaces [184].

However, Mou et al. noted that the NAS979 tests in the five-axis machine tools might not align with practical industrial demands [185]. To address this issue, Mou et al. proposed the 'S' machining test, demonstrating its feasibility for the industrial applications. Additionally, several researchers compared the machining results of the S-shaped workpieces with the NAS979 workpieces [186–192]. Tao et al. proposed a novel error reduction method and demonstrated its effectiveness through experimental evidence [193]. Furthermore, Osei et al. introduced the S-cone test piece, aiming to delineate the geometric and kinematic characteristics of the machine tools in Fig. 16 [194]. They noted that abrupt alterations in axis speed, acceleration, and jerk significantly influenced its machining process. The widespread adoption of the S-shaped specimen by numerous enterprises highlights its significance as a crucial benchmark for evaluating five-axis machine tool manufacturing technology.

Furthermore, with the proposed S-shaped workpieces, considerable research continued in the realm of uniquely shaped workpieces, aiming to employ straightforward cutting methods for precise error identification [195, 196]. Florussen et al. and Ibaraki et al. tried to identify the maximum geometric errors of the machine tools through machining a cylindrical-type workpieces [197, 198]. Sato et al. proposed hemispherical workpieces as an effective method to

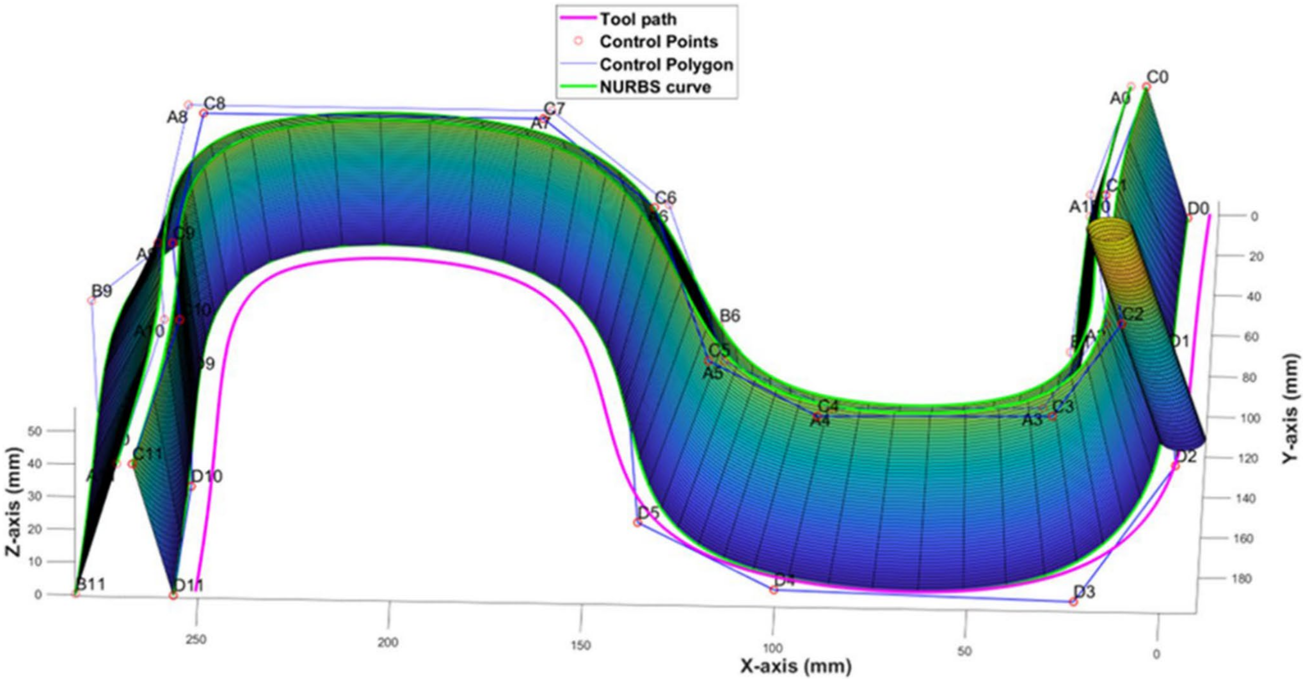

Fig. 16 Development of the two surfaces of the S-cone and the tool movement on the surface [194]

discover the causes of surface defects [199]. Finally, Huang et al. developed a uniquely shaped specimen proficient in identifying PIGEs and PDGEs of the rotary axes [30]. As highlighted in mentioned studies involving extensive experiments and simulations, it is evident that confirming the geometric errors of rotary axes through cutting is a practical and feasible approach.

4 Conclusions

Due to the exceptional processing flexibility and efficiency of five-axis machine tools, they have found widespread use across industries like aviation, shipbuilding, and precision manufacturing. As the rising demand for increased precision in products and components, the identification and compensation of errors in five-axis machine tools become increasingly critical. Existing research indicates that geometric errors of the rotary axes constitute a substantial portion of the overall error sources in five-axis machine tools. Therefore, identifying and compensating for geometric errors stand as pivotal measures to enhance machining accuracy.

In response to this demand, leading manufacturers of machine tool controllers like Heidenhain and Siemens have integrated compensation capabilities for geometric errors in their systems. These functions enable the insertion of the measured geometric errors (including PIGEs and PDGEs) into the controller, facilitating automated adjustments to tool position and orientation. Therefore, researchers worldwide are vigorously engaged in the advancement of measurement theories and technologies. As a result, this paper consolidates and reviews a significant portion of the existing literature concerning measurement techniques for geometric errors in rotary axes. Based on the instrumentation and equipment utilized, these existing literatures can broadly be categorized in five primary types. Their distinct advantages and limitations are described as follows:

4.1 Probes

By probing the designated artifacts, geometric data can be obtained to compute the difference between the ideal and actual probing positions. Through mathematical computations, the correlation between the geometric data and errors can be established, allowing for the derivation the geometric errors of the rotary axis. The ability of this approach to identify diverse geometric errors according to the features of the measured artifacts offers design flexibility, operational simplicity, and cost-effectiveness.

Table 3 Comparison of the six measurement methods

	Measurement times	Measurement types	Operation	Sensing dimensions	Cost
Probes	Time-saving	Contact	Ease	1D	Cost effective
double ball-bar	Time-consuming	Contact	Ease	1D	Cost effective
R-test	Time-saving	Non-contact	Difficulty	3D	High
Tracking Interferometer	Time-consuming	Non-contact	Difficulty	1D	Hgh
Optical techniques	Time-saving	Non-contact	Difficulty	3D	High
Machining test	Time-consuming	Contact	Difficulty	3D	Cost effective

4.2 DBB

The principal measurement approach entails executing circular motion using a calibrated rod in conjunction with two correction balls—one installed on the spindle and another on the worktable. Through calculating the length variations of the rod in specific circular paths, the various geometric errors of rotary axes are identified. Although this method is cost-effective and easier to install compared to the R-TEST methods, it only offers one-dimensional deviation information in a single measurement.

4.3 R-Test

R-test is a widely adopted commercial measurement system designed for assessing geometric errors in rotary axes. Despite the limitations such as the sequential measurement of one-axis errors, complex installation, and relatively higher costs, its three-dimensional sensing capability and remarkable precision have led to substantial research and application advancements.

4.4 Tracking Interferometer

Utilizing a laser tracker or laser tracer to monitor reflector positions enables the measurement of position information and analysis of diverse geometric errors in rotary axes. Despite its advantages, like high precision, non-contact measurement, and extensive measurement range, challenges such as high costs and the requirement for ample space during equipment installation remain important considerations.

4.5 Optical Techniques

These designs involved creating dedicated optical pathways using laser light, reflectors, sensors, and beam splitters to identify the geometric errors of the rotary axes. Moreover, multiple theoretical measurement methodologies for geometric errors in rotary axes have been formulated. Although the optical measurement techniques promise high accuracy and broad applicability, there is currently a lack of commercially available measurement systems specifically designed for identifying geometric errors in the rotary axes of five-axis machine tools.

4.6 Machining Test

Machining the specific workpieces on the five-axis machine tools and subsequently measuring the resultant features reveals the dimension errors of the workpieces. Through analysis the dimension errors of the workpieces, the geometric errors of the rotary axes were obtained. Although this method directly reflects the impact of the errors on machining accuracy, it is really a significant challenge for disciplining the geometric error from the other machine errors. Hence, it remains an area with potential for further development.

Furthermore, Table 3 presents a comparison of the six measurement methods used for identifying the geometric errors of rotary axes in five-axis machine tools. Although the modeling, identification, and compensation methods of errors in machine tools have reached a significant level of maturity over the last few decades, they still demand a high level of engineering and metrology expertise [27, 89, 97]. Additionally, the measurement and compensation of geometric errors in the rotary axes of five-axis machine tools are time-consuming and demand skilled engineers to operate the measuring equipment. Consequently, the requirements mentioned above pose significant barriers to the advancement of intelligent manufacturing. In our view, the development trend in the machine tool error calibration field is towards the creation of a measurement system that offers efficient and automated calibration procedures for each periodic measurement. As a result, the integration of inexpensive sensors and intelligent measurement algorithms embedded into machine tools and their control systems enables precise and intelligent manufacturing. This paper summarizes multiple measurement methodologies and the efforts made towards achieving the future goal of true intelligent manufacturing.

Author Contributions YTC Writing—Original Draft preparation and Writing—Review and Editing. CSL Writing—Review and Editing, Supervision and project administration. HFX Writing—Original Draft and Resources. WCS Writing—Original Draft and Resources. CLC Writing—Original Draft and Resources. QHY Writing—Original Draft and Resources. BKL Writing—Original Draft and Resources. THC Data Curation and Resources. YYH Data Curation and Resources.

Funding The authors gratefully acknowledge the financial support provided to this study by the National Science and Technology Council of Taiwan under grant numbers NSTC 112-2221-E-218-022-, 111-2222-E-218-001- and 112-2218-E-002-045.

Availability of Data and Materials Not applicable.

Code Availability Not applicable.

Declarations

Ethics Approval and Consent to Participate Not applicable.

Consent to Publication Not applicable.

Competing interests The authors declare that they have no competing interests.

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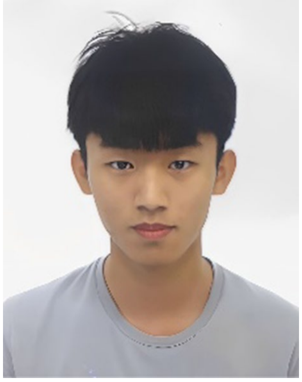

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