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Springback Analysis of the Stifened Panel Milling from the Bent Plate

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Abstract

An analytical method considering the redistribution of residual stress in the bent plate is proposed to predict the springback of the stifened panel when milling the panel layer by layer. Two types of stifened panel, namely, a panel with crosswise stifeners and a panel with lengthwise stifeners, were selected as examples and were analyzed during the removal of each layer. Moreover, a fnite-element simulation of the milling process was conducted to make comparisons with the analytical results, which demonstrates similar stress distribution and springback values. The maximum stress variation and springback value appeared when the milling depth reached the initial neutral surface. When the plate thickness decreased, the errors between analytical results and FEM results increased, and the lengthwise-stifened panel was less afected by errors than the crosswise-stifened panel because of a larger moment of inertia. The efects of diferent milling thicknesses per layer, initial plate thickness, and bending radius were also analyzed. Moreover, the milling experiment was performed to make verifcation. The results suggest that the analytical method can predict the springback of the stifened panel efectively. The proposed method can also be applied to other similar forming conditions.

> *m* Hardening index *A* Hardening coefficient

μ Poisson ratio

Resimulation

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1 Introduction

Springback is a critical issue in the aviation manufacturing industry, particularly for the production of stifened panels [\[1](#page-14-0), [2](#page-14-1)]. Generally, in the manufacturing process of these panels, a plate will be milled into a stifened panel and formed into the desired shape by rolling, press bending, peen forming, or creep forming. For panels with large deformation, rolling and press bending are the most efficient processing methods. However, in the bending process, since the stifeners may be damaged and cannot fulfll the high quality requirements [\[3](#page-14-2), [4](#page-14-3)], an alternative manufacturing process for stifened panels has been proposed. In this process, the plate is frst bent or rolled to the desired shape, clamped onto the fxture and milled into the fnal stifened panel using a high-speed milling machine. During the milling process, the residual stress release generates a bending moment and a bending force on the plate, which redistributes the residual stress in the remaining plate. However, this process requires a large clamp force, and the workpiece should be positioned on the fxture. Furthermore, the springback of the panel after unloading also afects the fnal accuracy. Hence, a study of the residual stress relaxation during the milling process and springback of the workpiece is of great value.

Many studies have investigated the relationship between the residual stress and the milling process. Meng [\[5](#page-14-4)] presented a new model to measure the equivalent residual stress and depth of the afected layers generated by milling using the strain change on the opposite side of each removed layer. Krottenthaler [[6\]](#page-14-5) focused on the ion beam milling and digital image correlation to measure the residual stress in thin flms by removing materials to form an H-bar geometry. Sebastiani [[7](#page-14-6)] also evaluated the residual stress and Poisson's ratio using a novel method based on a two-step, fourslot micro-milling process and validated this method through modeling and experiments; however, the methodology was only suitable for small material volumes. Vilček [\[8](#page-14-7)] evaluated the generation of the residual stress in precision milling using a defection-electrochemical etching technique and obtained the residual stress distribution through the milling process.

The milling process also generates a machine-induced residual stress with a high magnitude, which signifcantly affects the surface integrity $[9-11]$ $[9-11]$. The machining parameters, material characteristics and tool geometry should be considered. Sun [\[12](#page-14-10)] performed a series of end-milling experiments and revealed that the compressive residual stress in the feed direction increased with the cutting speed and magnitude. The distribution of residual stress was determined from a highly nonlinear coupling of mechanical and thermal loadings. The maximum machined-surface residual stress (MMSRS) was 400 MPa, and the depth of machined residual stress (DMRS) was less than 50 μm. Arunachalam [[13\]](#page-14-11) investigated the diferences in residual stress caused by the cubic boron nitride and ceramic cutting tools under various cutting conditions such as speed, depth of cut, and tool geometry. The results showed that the MMSRS was 1200 MPa for diferent cutting tool materials. Li [[14\]](#page-14-12) measured the residual stress in diferent zones of thin-walled parts as a function of the depth of cut, where the MMSRS was approximately 200 MPa, and the DMRS was 60–90 μm. Ma [\[15](#page-14-13)] analyzed the relationship between the cutting force and the temperature loading with residual stress and found that the MMSRS was approximately 300 MPa. Although the aforementioned studies suggest that the milling process will generate signifcant residual stress, the depth of cut does not exceed $200 \mu m$, which has minimal effect on the stress redistribution of the entire component. In this work, the additional residual stress caused by the milling process was neglected when the springback and stress distribution were analyzed.

The analytical method reveals the physical principle of the residual stress variation during the machining process, but it is also the most difficult method to analyze the residual stress because many assumptions are required to simplify complex issues. Liang [[16\]](#page-14-14) established a predictive model for residual stress using process conditions as the inputs, such as the cutting forces and cutting temperatures. Similarly, Huang [\[17](#page-14-15)] proposed an analytical approach to obtain the residual stress at diferent depths of a machined surface.

The finite-element method (FEM) offers an effective approach to investigate the residual stress distribution because it can provide much detailed information without complicated experiments. As an example, Salahshoor [[18\]](#page-14-16) used the FEM to predict the process-induced residual stress

in high-speed dry milling and validated the method via experiments. Yang [[19\]](#page-14-17) combined the FEM and a statistical model to analyze the residual stress in peripheral milling to improve its efficiency. Hua $[20]$ $[20]$ implemented the hardnessbased fow stress model encapsulated in an FE program to study the residual stress distribution in a machined surface and the efects of cutting conditions and tool geometry. With further development of the FE software, numerical simulations may offer more reliable results.

In this paper, an analytical method is proposed to predict the springback of a stifened panel, which was milled from a cylindrical bent thick plate. The proposed method considers the redistribution of residual stress in the remaining plate after milling. Two types of stifened panels were selected as objects of study, which had crosswise and lengthwise stifeners, respectively. An FEM analysis was conducted to make comparisons with the analytical results. Moreover, the efect of diferent milling thicknesses per layer, initial plate thickness, and bending radius were analyzed. The milling experiment was also performed to make verifcation. The proposed method can also be applied to other similar forming conditions.

2 Residual Stress Analyses During Layer‑by‑Layer Milling

2.1 Residual Stress After Bending

 (a)

Initial

Neutral

Surface

The coordinate system was established as shown in Fig. [1,](#page-2-0) where the original point was set on the middle surface of the plate. A uniform distribution of the residual stress of each layer along the length of the plate was assumed. The thickness of the plate was 2*t*, and the neutral surface was assumed to be always in the center of the plate.

By Hooke's law and Hollomon's law, when the plate was cylindrical bent, the stress in *x*-axis along thickness was:

 \overline{V}

 \mathbf{r}

Fig. 1 Schematic figure of the plate after bending

$$
\sigma_x = \begin{cases} E\varepsilon, & \varepsilon \le \sigma_s/E\\ \sigma_s + A\left(\varepsilon - \frac{\sigma_s}{E}\right)^m, & \varepsilon \ge \sigma_s/E \end{cases}
$$
 (1)

where σ_s is the initial yield stress, *E* is the elastic modulus, ϵ is the strain in *x*-direction, *m* and *A* refer to the hardening index and hardening coefficient, respectively.

According to Mises yield condition, the principal stresses are expressed as:

$$
\sigma_s^* = \sigma_x = \frac{\sigma_s}{\sqrt{1 - \mu + \mu^2}}\tag{2}
$$

And the bending moment to the neutral surface can be expressed as follows [\[21](#page-14-19)]:

$$
M_{bend} = 2 \int_0^{y_s} \sigma_x by dy + 2 \int_{y_s}^t \left[\sigma_s^* + A \left(\varepsilon - \frac{\sigma_s^*}{E} \right)^m \right] by dy
$$

$$
= \sigma_s^* b t^2 - \frac{\sigma_s^{*3} b}{3E^2 \kappa^2} + A \left(\varepsilon - \frac{\sigma_s^*}{E} \right)^m b (t^2 - y_s^2)
$$
 (3)

where *b* is the width of the plate; κ is the curvature of the plate, and y_s is where is the distance between initial yield surface and the neutral surface. When the surface of the plate starts to yield, the bending moment can be obtained:

$$
M_e = 2E\kappa \int_0^t by^2 dy = \frac{2}{3}\sigma_s^* bt^2
$$
 (4)

Then the function of bending moment and curvature can be expressed as:

$$
\Phi(\kappa) = M \tag{5}
$$

When the plate is unloaded, the springback process is equivalent to the elastic deformation caused by a reverse bending moment *M*′ , whose value is −*M.* Thus the curvature after springback can be derived as:

$$
\kappa_f = \Phi^{-1}(M) - \frac{M}{EI} \tag{6}
$$

where *I* is the moment of inertia of the plate. Based on Eqs. (2) (2) – (6) (6) , the springback ratio can be express as:

$$
\eta = \frac{\kappa_f}{\kappa} = \frac{\Phi^{-1}(M) - M/EI}{\Phi^{-1}(M)} = 1 - \frac{2y_s M}{tM_e}
$$

$$
= \left[1 - \frac{3}{2}\lambda + \frac{1}{2}\lambda^3 - \frac{3A\sigma_s^{*m-1}(1-\lambda)^{m+1}(m+\lambda+1)}{E^m\lambda^{m-1}(m+1)(m+2)}\right]^{-1}
$$
(7)

where λ is equal to y_s/t .

Moreover, the residual stress can be derived as:

$$
\sigma_{res} = \begin{cases} E y \kappa_f, & (-y_s \le y \le y_s) \\ E y (\kappa_f - \kappa) + A (y \kappa - \sigma_s^* / E)^m, & (y_s \le y \le t/2) \\ E y (\kappa_f - \kappa) + A (y \kappa + \sigma_s^* / E)^m, & (-y_s \le y \le -t/2) \end{cases}
$$
(8)

residual stress distribution in the remaining plate satisfes the following conditions:

$$
\begin{cases}\n-F_m = \int_{-t/2}^{-(t/2 - \Delta t)} \sigma_{res} b dz \\
-M_m = \int_{-t/2}^{-(t/2 - \Delta t)} \sigma_{res} y b dz\n\end{cases}
$$
\n(10)

After the frst layer is removed, the stress increment and bending moment increment are $-F_m$ and $-M_m$, respectively. It is supposed that the stress increments on the both sides of the deviated neutral surface are linearly distributed, and the slopes for the upper and lower sides are k_1 and k_2 , respectively. Then the following equations can be derived:

$$
\begin{cases}\nF_m = \int_{\Delta t/2}^{t/2} \left[\sigma_{res} + k_1(y - \Delta t/2) \right] b dy + \int_{-t/2 + \Delta t}^{\Delta t/2} \left[\sigma_{res} + k_2(y - \Delta t/2) \right] b dy \\
M_m = \int_{\Delta t/2}^{t/2} \left[\sigma_{res} + k_1(y - \Delta t/2) \right] (y - \Delta t/2) b dy + \int_{-t/2 + \Delta t}^{\Delta t/2} \left[\sigma_{res} + k_2(y - \Delta t/2) \right] (y - \Delta t/2) b dy\n\end{cases}
$$
\n(11)

2.2 Residual Stress Redistribution During Milling Process

The diference in derivation process between milling from concave and convex surfaces was the coordinate of the neutral surface after the deviation. Here, the case of milling from a concave surface is presented as an example.

When the frst layer was removed, the residual stress was released, which would apply a reverse bending moment and force on the remaining plate. Assuming that the thickness of the removed layer is Δt , the moment and force are expressed as:

$$
\begin{cases}\nF_m = \int_{-t/2}^{-t/2 + \Delta t} \sigma_{res} dy \\
M_m = \int_{-t/2}^{-t/2 + \Delta t} \sigma_{res} y dy\n\end{cases}
$$
\n(9)

Since the neutral surface is in the middle surface of the plate, it will move upwards by $\frac{\Delta t}{2}$, as shown in Fig. [1](#page-2-0)b. The Then, the formulas can be solved as:

$$
k_1 = k_2 = \frac{-F_m \Delta t}{16I_z} \tag{12}
$$

where I_z is the moment of inertia of the plate after milling. Then, the stress redistribution after the frst layer is milled is expressed as:

$$
\sigma_{res}^1 = \begin{cases}\n\sigma_{res} + k_1(y - \Delta t/2), \Delta t/2 < y < t/2 \\
\sigma_{res} + k_2(y - \Delta t/2), -(t - 2\Delta t)/2 < y < \Delta t/2\n\end{cases} \tag{13}
$$

When the *n*-th layer is milled, the residual stress release is expressed as:

$$
\begin{cases}\nF_m^n = -\int_{-t/2 + (n-1)\Delta t}^{-\left(t/2 - n\Delta t\right)} \sigma_{res}^{n-1} bdz \\
M_m^n = -\int_{-t/2 + (n-1)\Delta t}^{-\left(t/2 - n\Delta t\right)} \sigma_{res}^{n-1} \left[y - \frac{(n-1)\Delta t}{2} \right] bdz\n\end{cases} (14)
$$

where F_{m}^{n} and M_{m}^{n} are the respective stress and bending moment load applied on the remaining plate caused by the removal of the *n*-th layer and satisfy the following equations:

$$
\begin{cases}\nF_m^n = \int_{n\Delta t/2}^{t/2} \left[\sigma_{res}^{n-1} + k_1^n(y - n\Delta t/2) \right] bdy + \int_{-t/2 + n\Delta t}^{n\Delta t/2} \left[\sigma_{res}^{n-1} + k_2^n(y - n\Delta t/2) \right] bdy \\
M_m^n = \int_{n\Delta t/2}^{t/2} \left[\sigma_{res}^{n-1} + k_1^n(y - n\Delta t/2) \right] (y - n\Delta t/2) bdy + \int_{-t/2 + n\Delta t}^{n\Delta t/2} \left[\sigma_{res}^{n-1} + k_2^n(y - n\Delta t/2) \right] (y - n\Delta t/2) bdy\n\end{cases}
$$
\n(15)

where k_1^n and k_2^n are the respective coefficients of linear stress increments after the *n*-th layer is milled, which can be solved as:

$$
\begin{cases}\nk_1^n = \frac{\left[M_m^{n-1} - \Delta t/2(F_m^{n-1} - F_m^n)\right] - 2F_m^{n-1}(t/2 - n\Delta t)}{4I_z} \\
k_2^n = \frac{\left[M_m^{n-1} - \Delta t/2(F_m^{n-1} - F_m^n)\right] + 2F_m^{n-1}(t/2 - n\Delta t)}{4I_z}\n\end{cases} (16)
$$

Then, the stress of the remaining plate is obtained as follows:

$$
\sigma_{res}^{n} = \begin{cases}\n\sigma_{res} + \sum_{i=1}^{n} k_{1}^{n}(y - n\Delta t/2), n\Delta t/2 < y < t/2 \\
\sigma_{res} + \sum_{i=1}^{n} k_{2}^{n}(y - n\Delta t/2), -(t - 2n\Delta t)/2 < y < n\Delta t/2\n\end{cases}
$$
\n(17)

The total stress and bending moment caused by milling process can be expressed as:

$$
\begin{cases}\nM_{fin} = \sum_{i=1}^{n} M_m^i \\
F_{fin} = \sum_{i=1}^{n} F_m^n\n\end{cases}
$$
\n(18)

2.3 Springback Calculation for Diferent Stifened Panels

When the plate is unloaded, the external constraint disappears, and springback occurs. It is assumed that the radius of the neutral surface after springback is *r*′ , which can be derived as follows:

$$
\Delta \kappa = \frac{bM_{fin}}{EI_z} = \frac{1}{r'} - \frac{1}{r + n\Delta t/2} \tag{19}
$$

Here two types of stifened panels are discussed, which have crosswise or lengthwise stifeners (Fig. [2\)](#page-4-0).

To calculate the moment of inertia of the cross section *I_z*, the middle plane of the plate of the stifened panel was selected as the reference plane. Thus, for the panel in Fig. $2a, I_z$ is calculated as follows:

$$
\begin{cases}\nI_1 = \frac{1}{12}t_1h^3 + (h/2 + t_2/2 - Z_c)^2t_1h \\
I_2 = \frac{1}{12}t_2^3b + Z_c^2t_2b \\
I_z = I_1 + I_2\n\end{cases}
$$
\n(20)

where t_1 is the thickness of the stiffener; t_2 is the thickness of the plate; I_1 is the moment of inertia of the stiffeners; I_2 is the moment of inertia of the plate; *h* is height of the stifener; and Z_c is the distance from the centroid of the cross section of the stifened panel to the reference plane, which is calculated as follows:

$$
Z_c = \frac{t_1 h (h/2 + t_2/2)}{t_1 h + t_2 b} \tag{21}
$$

Fig. 2 Two types of stifened panels

For the panel in Fig. $2b$, I_z is expressed as:

$$
I_z = \frac{t_2^3 b}{12}
$$
 (22)

Before springback, it is assumed that the length of the *y*-th layer after bending is:

$$
\Delta L = (r + y)\alpha \tag{23}
$$

which will change to the following formula after springback:

$$
\Delta L = (r' + y' - \Delta t/2)\alpha \tag{24}
$$

where α and α' are the bending angle of the plate before and after springback, respectively, as presented in Fig. [1](#page-2-0); and *y*′ is equal to *y*. Then, the strain caused by milling after sptingback is derived as:

$$
\varepsilon_e = \frac{\Delta L' - \Delta L}{\Delta L} = \frac{(r' + y' - \Delta t/2)\alpha - (r + y)\alpha}{(r + y)\alpha} \tag{25}
$$

In the derivation, the following condition is also obtained because the strain in the neutral surface is 0:

$$
(r + \Delta t/2)\alpha = r'\alpha'
$$
 (26)

Table 1 Material parameters

Then, Eq. (25) (25) is rewritten as:

$$
\varepsilon_e = \frac{(2y - \Delta t)(2r - 2r' + \Delta t)}{4(r + y)r'}\tag{27}
$$

In addition, the springback will release the stress load and bending moment load. Then, the residual stress in the plate of the stifened panels after springback is expressed as:

$$
\sigma_{res}^s = \sigma_{res}^n + E\varepsilon_e + \frac{(2y - n\Delta t)M_{fin}b}{2I_z} + \frac{F_{fin}}{t - n\Delta t}
$$
(28)

Since no material removal occurs on the stifener, the variation in the residual stress is only caused by the change in strain. Thus, the residual stress in the crosswise stifeners remains unchanged, whereas the residual stress in the lengthwise stifeners can be represented as:

$$
\sigma_x' = \sigma_{res} - E\varepsilon_e \tag{29}
$$

3 Finite‑Element Analyses and Milling Experiments

3.1 Finite Element Model

To validate the analytical model, a fnite-element analysis was conducted to analyze the residual stress distribution and springback. An AL 7B04-T7451 plate was selected as the research object. The material model character belongs to the elastoplastic behavior with isotropic hardening. According

Fig. 3 Finite-element model of the process

her Kant

to the data of uniaxial tensile tests, it is expressed in exponential form. The mechanical properties of the material are listed in Table [1.](#page-5-1)

The simulation process including two procedures is shown in Fig. [3.](#page-5-2) Firstly, the plate was bent into designed radius. The Abaqus/Explicit was used for the procedure and the hardening type was selected as isotropic hardening in the

simulation. The model consists of the multi-point die, the elastic cushion and the plate. The total element number is 72,263. The plate contained 55,000 elements, whose dimensions were 1 mm thick, 2 mm long and 2 mm wide. The element type of the plate and the cushion was C3D8R. To minimize the model, the die was simplifed as a rigid shell of element type R3D4. In bending process, the lower die was fxed and the upper die moved down until the workpiece was in full contact with the lower die. The loading speed was 0.19 m/s, and the friction coefficient was set as 0.15 . The unloading process was conducted with Abaqus/Standard. The bending simulation results were imported as the initial state and all restrains were removed. The central point was selected as the fxed point in order to make it springback spontaneously.

Table 3 Dimensions and the milling parameters of the stiffened panels	
--	--

For example, when the variable is "milling depth", in the corresponding row the variable value of the "milling depth" are "2, 4, 6, 8, 10, 12". The other parameters are constants: the "bending radius" is 600 mm, the "plate thickness" is 20 mm, and the "milling layer thickness" is 2 mm

Fig. 4 Milling experiment. **a** Bending process, **b** bent plate and **c** milling process

The simulation of milling process was conducted with Abaqus/Explicit. In the process, the stress distribution status after springback was frstly imported into the model. Then the bottom of the plate was fxed, and the plate was milled into desired shape by controlling the path of milling tools. The diameter of the milling tools was 10 mm and the rotate speed was 10,000 rpm. The ductile damage model was used to calculate the failure of the material. The relevant parameters were obtained from material properties and were listed in Table [2](#page-6-0). The interaction was surface-to-surface contact. After milling, the springback procedure was conducted again with Abaqus/Standard.

The milling process with two types of stifened panels was simulated to make comparisons with the analytical values. The results at diferent milling depths were analyzed such as 2, 4, 6, 8, 10, and 12 mm, and each removed layer was 2 mm thick. Moreover, the milling process of diferent milling layer thicknesses, bending radii and plate thicknesses was simulated. The relevant dimensions, milling depth and milling layer thickness of each condition are shown in Table [3](#page-6-1).

3.2 Milling Experiment

Twenty-millimeter-thick AL 7B04-T7451 plates were prepared for the experiment and cut into the dimensions of $200 \text{ mm} \times 50 \text{ mm}$. The plates were first bent into different bending radius (200, 400 and 600 mm) with multi-point forming process as shown in Fig. [4a](#page-6-2). The die was regulated into desired shape, and positioned on the mold holder. The plate covered with elastic cushion was put on the lower die. The upper die was moved down until it was in full contact with the workpiece. After holding the pressure for a while, the die was moved away. Bending process was carried out on a hydraulic press whose capacity was 800 t. After the profle was measured, the bent plate was

Fig. 6 Initial residual stress through thickness after bent for R400 mm

clamped and milled into stifened panel with a 3-axis CNC milling machine.

Workpiece measurement was conducted by an NDI Procam 3000 three-dimensional laser scanner as shown in Fig. [5.](#page-7-0) During measurement, the three-dimensional coordinates of the workpiece were generated and uploaded to the Geomagic Qualify 2013 software. Then the profle data was required. Through the comparison with the profle data acquired from the analytical method, the errors of the two methods were obtained.

Fig. 5 The measurement of the workpiece. **a** The three-dimensional laser scanner and **b** the milled stifened panel

Fig. 7 Residual stress distribution of the crosswise-stifened panel at cutting depths of **a**–**f** 2, 4, 6, 8, 10 and 12 mm

4 Result Analyses and Discussion

4.1 The Bending Residual Stress Distribution

Figure [6](#page-7-1) shows the residual stress distribution in *x*-direction along the thickness when the bending radius was R400 mm. The comparisons of FEM value and analytical value were made to validate the formula ([8](#page-3-0)). Since the target shape was symmetry, the central section (A–B) was selected as study region and the average residual stress distribution in plate area (the region will be milled away) and stifener area (the region will be retained as stifener) is obtained respectively. As shown in the fgure, the simulation result of the residual stress in the plate matched well with the analytical result. The maximum error appeared in the elasto plastic interface and reached 74 MPa. The standard deviation of two curves was 41 MPa. However, for the result in the stifener, there are greater diferences in the stress distribution above the neutral layer. The main reason is the width of the stifener is smaller and the data points are fewer, leading to larger error.

4.2 Residual Stress Distribution in Milling Process

The stress results in *x*-direction of the plate with the crosswise-stifened panel are presented in Fig. [7.](#page-8-0) The residual stress distribution curves of two methods have identical variation trends and similar stress values. Compared with Fig. [6,](#page-7-1) when the plate was milled to 2 mm, the stress increased from -8 mm to 1 mm in the y-direction and decreased from 1 to 10 mm. However, when the milling depth changed from 6 to 10 mm, the stress decreased, which is consistent with the results of the analytical method. According to Eq. [\(10](#page-3-1)), when the milling depth is within 4 mm, the residual stress released is positive, and the distance to the neutral surface is negative, which makes the bending moment *M* negative and additional force F positive. Using formula (28) (28) , the residual stress on both sides of the neutral surface can be obtained, and the residual stress increases below the neutral surface and decreases above the neutral surface. When it is milled from 6 to 10 mm, the residual stress becomes negative, which makes the bending moment change sign and reversely increase. In this situation, the stress on both sides of the neutral surface increases in the opposite direction. The maximum stress variation reached 200 MPa at a milling depth of 10 mm, as shown in Fig. [7](#page-8-0)f. Equation ([18](#page-4-1)) and Fig. [6](#page-7-1) show that the bending moment varies with the increase in milling depth and maximizes at a depth of 10 mm. In Fig. [7f](#page-8-0), the standard deviation of the two methods was 67 MPa, whereas the minimum error was 27 MPa in Fig. [7e](#page-8-0).

The residual stress distributions *x*-direction in the plate with the lengthwise-stiffened panel are shown in Fig. [8,](#page-9-0) which are similar to the distributions in the panel with the crosswise stifener. Figure [8d](#page-9-0), e reveal that the maximum values of the stress and stress variation appeared at milling depths of 8 and 10 mm, respectively. The maximum standard deviation between the two curves was 55 MPa, as shown

Fig. 8 Residual stress distribution of the lengthwise-stifened panels: **a**–**f** cutting depth of 2, 4, 6, 8, 10 and 12 mm

Fig. 9 Springback value of the panel after springback in diferent cutting depths: **a** crosswise-stifened panel; **b** lengthwise-stifened panel

in Fig. [8a](#page-9-0), which is mainly caused by the bending residual stress error.

Changes of stress component in x direction in the stifeners are also presented in Fig. [8](#page-9-0). According to Eq. ([29\)](#page-5-4), the stress variation in the stifeners is mainly determined by the radius change. Thus, the stress variation trend is consistent with that in the plate. A maximum stress of 440 MPa occurred at a milling depth of 10 mm, where the plate had maximal springback, and the variation of the stress reached a maximum value of 110 MPa. In Fig. [8a](#page-9-0), b, the error distributed more uniformly along the thickness direction, which is mainly caused by initial bending residual stress error. However, when the milling depth increased, the error below the neutral surface decreased and the error above the neutral surface increased. This is because in the milling process, the neutral surface moving to the positive direction of y-axis. The distance between the concave surface and the neutral layer increases, and according to Eq. ([27\)](#page-5-5), the stress

variation caused by springback is greater. In this situation, some error can be offset. In the contrary, the error above the neutral can hardly decrease.

Compared with two types of stifened panels, the average standard deviation for crosswise-stifened panel was 46.3 MPa and for lengthwise-stifened panel was 38.4 MPa. For the lengthwise-stifened panel, the increment of errors was smaller because the larger moment of inertia reduced the panel deformation and restrained the errors. In summary, the lengthwise-stifened panel was less afected by errors than the crosswise-stifened panel.

4.3 Springback of the Stifened Panel

Taking the center of the plate as the reference point, we selected the maximum displacement after springback in the y-direction to describe the springback value. Figure [9](#page-9-1) shows the maximum springback of two types of stifened panels when the layers were removed. When the plate was milled between 2 and 4 mm, the displacements were negative. This result is attributed to the stress value of the layer and distance to the neutral surface, which have diferent signs, so the bending moment is negative, as shown in Fig. [6](#page-7-1). According to Eq. ([19\)](#page-4-2), the springback is in the convex direction. When the milling depth was 6–10 mm, the total bending moment became positive, and the springback was in the concave direction. At a milling depth of 10 mm, the springback maximized at the original neutral surface. When the plate was milled with 12 mm, the stress became positive again, and the springback value decreased. During the entire milling process, the neutral surface sustained the movement in the convex direction.

The figures also show that the analytical value is consistent with the simulated value. For the crosswise-stifened panel in Fig. [9](#page-9-1)a, the maximum positive displacement was 2.74 mm for the FEM and 2.6 mm for the analytical method when the milling depth was 10 mm. In Fig. [9](#page-9-1)b, the maximum springback value reached a value of 1.5 mm for the FEM and 1.71 mm for the analytical approach. In both pictures, the maximum springback error appeared when the milling depth was 6 mm (0.3 mm for Fig. [9](#page-9-1)a and 0.23 mm for Fig. [9](#page-9-1)b). This is because the panel occurs a reverse springback, which enlarges the error. Comparing the two types of panels, the standard deviation of the errors for each panel was 0.175 mm (crosswise) and 0.159 mm (lengthwise) respectively. The springback value and corresponding errors of the lengthwise-stifened panel are smaller than those of the crosswise-stifened panel because of the larger moment of inertia.

4.4 Efect of Diferent Milling Layer Thicknesses

The effect of different milling layer thicknesses was analyzed. The plate was fxed and milled to 12 mm with three milling layer thicknesses of 1, 2, and 3 mm. The springback value results are shown in Fig. [10.](#page-10-0) The maximum springback value for the crosswise-stifened panel was approximately 2.64 mm (FEM) when the layer was 2 mm thick, and the relative error was about 10%. The displacements varied when the layer thickness changed to 1 and 3 mm. The difference results from diferent additional bending moment and force generated per layer, which will afect the stress redistribution in the remaining plate. Therefore, the fnal total bending moments of three conditions are diferent as seen from Fig. [10](#page-10-0)b. In the analytical approach, according to Eqs. (16) (16) – (18) (18) , the total bending moment was approximately 103 N m with the layer thickness of 2 mm for the crosswisestifened panel. However, the moments were 58 and 50 N m

Fig. 10 Springback value with diferent milling layer thicknesses: **a** crosswise-stifened panel; **b** lengthwise-stifened panel

Fig. 11 Springback value with diferent initial plate thicknesses: **a** crosswise-stifened panel; **b** lengthwise-stifened panel

Fig. 12 Springback value with diferent bending radii: **a** crosswise-stifened panel; **b** lengthwise-stifened panel

for the layer thicknesses of 1 and 3 mm, respectively. The errors of bending moment ranges from 4 to 12%.

4.5 Efect of Diferent Thicknesses and Bending Radii

The milling processes of the plates with diferent initial plate thicknesses were simulated, where three diferent plate thicknesses of 18, 20 and 22 mm were designed. The results in Fig. [11](#page-11-0) shows that diferent plate thicknesses remarkably afect the springback. The maximum springback value was 5.35 mm for the crosswise-stifened panel with a thickness of 18 mm. A thinner plate has a smaller moment of inertia and makes it more liable to springback. However, the 22-mmthick plate also had larger springback than the 20-mm-thick plate, considering that the milling depth was approximately the neutral surface, which causes a larger bending moment. For the plate with diferent thicknesses, the springback value is the result of the combined action of the bending moment and moment of inertia. Comparing two methods, the maximum error of the crosswise-stifened panel was 0.35 mm for a thickness of 18 mm; the other errors were 0.11 and 0.2 mm for thicknesses of 20 and 22 mm. For the lengthwise-stifened panel in Fig. [8](#page-9-0)b, the errors were relatively small (0.1–0.3 mm).

Figure [12](#page-11-1) presents the comparison of springback because of diferent bending radii values of 200, 400 and 600 mm. The maximum springback value was 4.2 mm at the bending radius of 200 mm. After springback, the radius variation for R200 mm decreased because the larger plastic strain was contained. However, this plate has a larger curvature, so a notably small change will induce a large springback variation. For a radius of R600 mm, the springback was also larger than that of R400 mm because the plate contains substantial elastic strain, which makes the plate more prone to springback. The maximum error of the two methods for the crosswise panel was 0.5 mm for the radius of R600 mm and 0.2–0.4 mm for the lengthwise panels.

Fig. 13 Comparison of springback between FEM and ANA for crosswise-stifened panels in diferent radius radius: **a** R=200 mm, **b** $R = 400$ mm and $c R = 600$ mm

Fig. 14 Comparison of springback between FEM and ANA for lengthwise-stifened panels in diferent radius radius: **a** R=200 mm, **b** R=400 mm and **c** R=600 m

	Lengthwise		Crosswise	
	Maximum displacement (mm)	Percent- age errors	Maximum displacement (mm)	Percent- age errors
200 mm				
Analytical	$\overline{1}$	37.5	$\overline{4}$	16.7
FEM	1.25	15.6	4.2	14.3
EXP	1.6	θ	4.8	θ
400 mm				
Analytical	1.28	15.8	2.5	13.8
FEM	1.16	23.6	2.6	10.3
EXP	1.52	0	2.9	θ
600 mm				
Analytical	1.8	18.2	3.8	9.5
FEM	1.5	31.2	3.3	21.4
EXP	2.2	0	4.2	0

Table 4 Maximum displacement and the springback errors compared with experiment

4.6 Springback Comparison with the Experiment Results

Figure [13](#page-12-0) presents the errors between experimental results and analytical results for crosswise-stifened panels in different bending radius. From both sides to the center in longitude direction, the errors turn from negative to positive. It means that the radius acquired from analytical method is a little smaller than experiment results. And the errors distribute asymmetrically since the inhomogeneous deformation of the plate. As seen from Fig. [13a](#page-12-0), disregarded the errors in stifeners, the maximum error appears on the right side of the panel, whose value was 0.8 mm. When the bending radius was 400 mm, the maximum error decreased to 0.6 mm and the distribution of the errors was more uniform. When the bending radius was 600 mm, the errors ranged from 0.3 to 0.5 mm. The percentage of the errors ranged from 11 to 17%, which were similar with the FEM results.

Figure [14](#page-12-1) shows the springback comparison between analytical results and experimental results for lengthwisestifened panels. Seen from Fig. [14a](#page-12-1), the maximum error was about 0.6 mm, and in most areas the errors were below 0.3 mm. When the bending radius was 400 mm, the maximum error was further reduced to 0.5 mm. When the radius reached 600 mm, the errors varied from −0.45 to 0.45 mm. Consistent with the FEM results, the errors of lengthwisestifened panels were smaller than that of the crosswise ones. Moreover, compared with the crosswise-stifened panels, the distribution of the errors is much symmetrical, which means the springback of the lengthwise-stifened panels were more uniform.

Table [4](#page-13-0) indicates the maximum displacements of three approaches in the y-direction for two kinds of panels. To make it more clear, the results of other two methods were compared with the experimental results to acquire the percentage errors. The maximum diference between analytical approach and experiment was 0.8 mm when the bending radius 200 mm for the crosswise-stifened panels. And for the lengthwise-stifenend panels, the displacement errors were reletively smaller, whereas the percentage errors were larger. This mainly because the moment of inertia of the panel is larger, leading to a smaller springback value after milling as shown in the table, thus the percentage of error is higher. Moreover, as seen from table, the experimental values were always larger than analytical values and FEM values. The reason is when the plate is bending, the arm of force at both sides of the plate is smaller, which results in the larger elastic strain and less plastic strain. After the bent plate is milled, this area is more liable to spingback.

5 Conclusions

An analytical method to predict the springback of a bending plate after milling into a stifened panel was proposed. The following conclusions are suggested based on the above fndings:

- 1. The analytical method, which considers the redistribution of residual stress in the remaining plate after milling, has been verifed with the FEM and experiment. The analytical results are consistent with the FEM and experimental results, which proves that the method provides an efective approach to predict the springback of the bent plate during the milling process.
- 2. The analytical and FEM results have identical variation trends of residual stress. The maximum stress variation occurred when the milling depth reached the initial neutral surface of the plate. The method can predict the residual stress distribution in the stifened panels in certain conditions according to demand.
- 3. When milling is performed to the initial neutral surface, the bending moment reaches an extreme value with maximum stress variation, whereas the maximum springback value occurs. With the decrease in plate thickness, the springback errors of the two methods increase because the calculation errors accumulate. However, because of the larger moment of inertia, the lengthwise-stifened

panel have smaller errors than the crosswise-stifened panel.

The springback prediction method presented in this paper can also be applied to other similar forming conditions. It has certain theoretical guidance meaning for milling process of the bent plate. Through the method, the residual stress and springback involved in milling process can be predicted scientifcally.

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