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REGULAR PAPER

Lambert W Function Controller Design for Teleoperation Systems

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Abstract

Stability and transparency play key roles in a bilateral teleoperation system with communication latency. This study developed a new method of controller design, based on the Lambert W function for the bilateral teleoperation through the Internet. In spite of the time-delay in the communication channel, system disturbance, and modeling errors, this approach causes the slave manipulator tracks the master appropriately. Time-delay terms in the bilateral teleoperation systems result in an infinite number of characteristic equation roots making difficulty in the analysis of systems by traditional strategies. As delay differential equations have infinite eigenspectrums, it is not possible to locate all closed-loop eigenvalue in desired positions by using classical control methods. Therefore, this study suggested a new feedback controller for assignment of eigenvalues, in compliance with Lambert W function. Lambert W function causes the rightmost eigenvalues to locate exactly in desired possible positions in the stable left hand of the imaginary axis. This control method led to a reduction in the undesirable effect of time-delay on the communication channel. The simulation results showed great closed-loop performance and better tracking in case of different time-delay types.

Keywords Eigenvalue assignment · Lambert W function · Teleoperation systems · Time-delay

1 Introduction

In order to human operators can accomplish an action in far or perilous environments, different types of teleoperation systems were presented with a variety of application cases, ranging from underwater to space, nuclear plants, etc. [1]. Dinh et al. [2] represented a novel control method for the bilateral teleoperation system based on the sensorless force feedback joystick. An efficient force reflecting joystick controller for the nonlinear teleoperation system in construction machinery is introduced by Truong et al. [3]. Kim et al. [4] facilitated robot motion by a novel controller for bilateral teleoperation system in spite of human cognitive and operation restrictions. Baek et al. [5] improved stability and transparency of the master–slave teleoperation system employing predictive control based on the coupling matrix.

Jung et al. [6] employed a teleoperation system instead of human labor to execute a beam assembly task. Kim et al. [7] represented a position-based impedance control scheme for force tracking of a teleoperation system on the wall-climbing mobile platform. As the teleoperation system is fundamentally unstable, Liu et al. [8] suggested new nonlinear adaptive controllers for which no thorough knowledge should be gained in terms of the kinematics of the master–slave as well as of dynamics of the master–slave-operator-environment. Li and Li [9] introduced an adaptive controller for bilateral teleoperation systems in order to reduce the effect of actuator faults and time-delay. Ganjefar et al. [10] proposed a new adaptive PID controller for the nonlinear bilateral teleoperation system in order to improve the stability, transparency, and performance.

This study is a serious attempt at providing a description of a new structure controller on the basis of Lambert W Function making a strong impact which is robust, in different cases of time-delays, for the nonlinear bilateral teleoperation system. Employing the Lambert W function provides the controller designer with a critical subset of the eigenvalues for the desired locations in the left hand of the imaginary

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axis, which leads to the system stability. This new strategy involves eigenvalue assignment for the purpose of avoiding undesirable effects of time-delay and making improvements in position, transparency, and force tracking. This study is developed in the following sections: in Sect. 2, bilateral teleoperation systems are explained and Lambert W Function is introduced in Sect. 3. Section 4, presents the new control architecture based on the Lambert W function and the stability of new methods is represented is Sect. 5. In Sect. 6, simulation results of the Lambert W function-based controller for different time-delay are demonstrated and the validity of these schemes is established. Finally, Sect. 7 draws conclusions.

2 Teleoperation Systems

Generally, the bilateral teleoperation systems consist of a local site, where a hand-controller named master manipulator is driven by a human operator, a remote site, where a slave manipulator follows the master motion to perform an action in interaction with the environment, and a communication channel that connects both sites [11].

A general framework of teleoperation system is indicated in Fig. 1. The structure includes five parts: operator, master, control and communication, slave, and environment. The human operator commands via master manipulator by applying a force F_m to drive it with $X_m = [x_m\ \dot{x}_m]$ that is forwarded to the slave side via the communication block. The slave manipulator is moved by a local control T_s on the slave side. If the slave interacts a far environment and/or some external force, the remote force F_s is sent back from the slave side by force F_e that shifts slave manipulator with $X_s = [x_s\ \dot{x}_s]$ that is transmitted back to the master manipulator through the communication block. Control signal T_m or reflected force F_r is received at the master side that operator senses it. The human operator handles the local master manipulator to remotely move the slave one to perform a given act. The system must be entirely ''transparent''; thus, the operator can make a sense as if he was able to control the far environment directly.

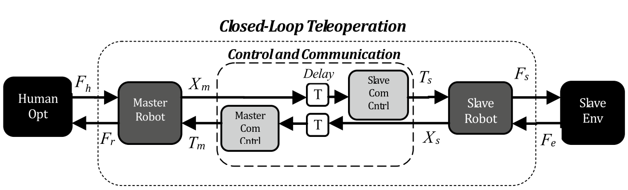

Fig. 1 Framework of closed-loop teleoperation systems

2.1 Dynamics of Teleoperation System

The motion equation for a pair of n-DOF nonlinear robotic systems in the absence of friction or other disturbances is presented as [12]:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = T \tag{1}$$

$\dot{M}(q) - C(q,\dot{q})M(q),\ C(q,\dot{q}) \in R^{n\times n}$ are symmetric, positive-definite inertia matrices and the Coriolis/centripetal vector, respectively, is skew symmetric, $G(q)$ is the gravity vector and T is the torque vector. In this study, 1-DOF for the master and the slave in the teleoperation system is presumed. A nonlinear manipulator is introduced dynamically as:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) + \frac{1}{2}mgl\sin\theta(t) = u(t) \tag{2}$$

where $J = \frac{1}{3}ml^2$ is the element inertia, m is the element mass, g is the gravity acceleration, l is the element length, $\theta(t)$ is the angle of the rotate, $u(t)$ is the control signal applied and b is the viscous friction coefficient, Proof is given in [13]. The simplified linear model is:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) = u(t) \tag{3}$$

The state space description of the master and the slave, considering as state variables of the position $x_1(t) = \theta(t)$ and the velocity $x_2(t) = \dot{\theta}(t)$ is:

$$\begin{bmatrix}\dot{x}_{m1}(t)\\ \dot{x}_{m2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{b_m}{J_m}\end{bmatrix}\begin{bmatrix}x_{m1}(t)\\ x_{m2}(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J_m}\end{bmatrix}u_m(t) \tag{4}$$

$$y_m(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_{m1}(t)\\ x_{m2}(t)\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}\dot{x}_{s1}(t)\\ \dot{x}_{s2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{b_s}{J_s}\end{bmatrix}\begin{bmatrix}x_{s1}(t)\\ x_{s2}(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J_s}\end{bmatrix}u_s(t) \tag{6}$$

$$y_s(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_{s1}(t)\\ x_{s2}(t)\end{bmatrix} \tag{7}$$

2.2 Environment

Consideration of the remote environment plays a critical role in the teleoperation systems. When the slave robot executes a given action, it can interact with the environment. This reaction must be reflected on the control system design to consider the reaction forces. The Kelvin model simplification of the environment is presented in this study [14]. (k_e) and (b_e) stand on the "stiffness" and "viscous friction", respectfully, in this simplified environment model. Therefore, the reaction force is calculated as:

$$f_s(t) = k_e\,\theta_s(t) + b_e\,\dot{\theta}_s(t) \tag{8}$$

This model shows that the reaction force, f_s, will act against the slave control signal. In order to derive the desired feedback force from the slave to the master, the matrix R_m must be:

$$R_m = [\, r_{m1} \;\; r_{m2} \,] = [\, k_f\, k_e \;\; k_f\, b_e \,] \tag{9}$$

where k_f is the force feedback gain.

3 Lambert W Function Method

It is possible to show the time-delay systems with delay differential equations (DDE). An infinite spectrum of frequencies can be produced by delay problems and they make the analysis of systems by classical methods difficult especially as regard analyzing the stability and designing the stabilizer controllers. For overcoming this difficulty, approximations such as Pade can be used indirectly. The Lyapunov strategy, algebraic Riccati equations (AREs), and linear matrix inequalities (LMIs) are other methods of controller designing [15, 16]. These techniques demand complicated equations and can cause challenging results and possibly excessive control. The estimation of this infinite frequency spectrum needs corresponding eigenvalues of characteristic equations, which is impossible. Generally, we can gain an understanding of it by applying standard methods developed for systems of linear ordinary differential equations (ODE). As the result, instead of closed-form solutions, DDE is often satisfied by applying the numerical approaches, asymptotic responses, and graphical methods principally including analysis of stability as well as controller designing.

In this section, the researcher extends this approach to obtain a complete solution for DDE system based on Lambert W function [17]. Since the response contains an analytical structure in terms of DDE parameter, it is possible to define how parameters are considered in the response and how each term affects each eigenvalue as well as on the solution. Furthermore, each "branch" of the Lambert W function relates to a particular eigenvalue. In this way, the response form is similar to the general solution structure of ODEs, and the notion of state transition matrix in ODEs can be developed with regard to DDEs by employing the idea of the matrix Lambert W function. Therefore, some analysis and control methods of ODE system, in accordance with state transition matrix notion, can potentially extend to DDE System [18]. The $W(H)$ represents the Lambert W function that satisfies:

$$W_k(H_k)e^{W_k(H_k)} = H_k \tag{10}$$

The Lambert W function [17] is complex-valued with a complex argument and has an infinite number of branches W_k, where $k = -\infty, \ldots, -1, 0, 1, \ldots, \infty$ [19]. Figure 2 shows the range of each W function branch. For instance, the principal

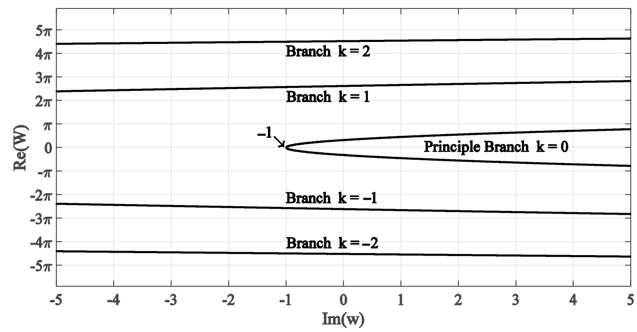

Fig. 2 Framework ranges of each Lambert W function branch of the principal branch W_0, is equal to or larger than -1

branch W_0 has the real part with a minimum value, -1. The Lambert W function technique is employed to find the roots of matrix transcendental characteristic equation like linear matrix differential equation containing delayed argument. This characteristic equation has infinite root matrices. Asymptotic stability of matrix differential equation solutions with delayed argument has the most influence in root matrices corresponding to the Lambert W function values in its neighboring and principle branch. In the time-delay systems (TDS) with real coefficients, the maximal real part of the characteristic equation roots corresponds to one real root or one pair of complex conjugate roots of the characteristic quasi-polynomial. Such a root or a pair of conjugate roots will be so-called the rightmost root. An equilibrium point of TDS is asymptotically stable if and only if the maximal real part of the characteristic equation roots is negative i.e. all the eigenvalues have negative real parts. If all eigenvalues of these root matrices have negative real parts, the solutions are asymptotically stable. In [20], a linear time-invariant (LTI) of DDEs, with a constant time-delay, T, is given as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-T) + Bu(t), && t > 0 \\ x(t) &= g(t), && t \in [-T, 0) \\ x(t) &= x_0, && t = 0 \end{aligned} \tag{11}$$

where A and A_d are $n \times n$ matrices, B is an $n \times r$ matrix, $x(t)$ is an $n \times 1$ state vector, $u(t)$ is an $r \times 1$ vector indicating the external excitation, $g(t)$ and x_0 are a specified reshape function and an initial point, respectively. The solution for Eq. (11), in term of the matrix Lambert W function is [21]:

$$x(t) = \sum_{k=-\infty}^{\infty} e^{S_k t} C_k^I + \int_0^t \sum_{k=-\infty}^{\infty} e^{S_k(t-\xi)} C_k^N Bu(\xi)d\xi \tag{12}$$

where S_K is the solution matrix described as:

$$S_k = \frac{1}{T} W_k(A_d T Q_k) + A \tag{13}$$

The coefficient C_k^I in Eq. (12), is a function of A, A_d, T and the reshape function, $g(t)$, and the initial point, x_0, while C_k^N is a function of A, A_d and T, and does not depend on $g(t)$ or x_0. The approaches for calculating the C_k^I and C_k^N were represented in [19]. The following condition is employed for deriving a solution for the unknown matrix Q_k [20]:

$$W_k(A_dTQ_k)e^{W_k(A_dTQ_k)+AT} = A_dT \tag{14}$$

The solution form in Eq. (12), shows that the eigenvalues of the matrix S_k, and the matrix e^{S_k} determine the system stability of Eq. (11) [22]. A time-delay system specified by Eq. (11) is asymptotically stable if and only if [20]:

All Eigenvalues of S_k, $k=-\infty,\ldots,-1,0,1,\ldots,\infty$, have negative real parts
Or, equivalently, in the sense of Lyapunov,
All Eigenvalues of e_k^S,$k=-\infty,\ldots,-1,0,1,\ldots,\infty$, lie within the unit circle

As the system in Eq. (11) suffers from an infinite number of eigenvalues, it is not possible to calculate matrices S_k or e^{S_k} for an infinite number of branches, $k=-\infty,\ldots,-1,0,1,\ldots,\infty$. On the other hand, all branches of eigenvalues ($k=-\infty,\ldots,-1,0,1,\ldots,\infty$) in the Lambert W function approach are distinguishable. Thus, the obtained eigenvalues related to the principal branch ($k=0$) are in the nearest distance to the imaginary axis and define the system stability [23].

Conjecture

$\text{Max}\left[\text{Re}\left\{\text{eigenvalues of } S_0\right\}\right] \geq \text{Re}\left\{\text{all other eigenvalues of } S_k\right\}$.

It has been proven, for the scalar case of DDE, [24] that the roots derived from the principal branch $k=0$ of the Lambert W function always determine stability. In designing a feedback controller for a delayed system such as teleoperation systems shown by Eq. (11), since there is an infinite number of eigenvalues for matrices S_k, $k=-\infty,\ldots,-1,0,1,\ldots,\infty$, and the number of control parameters is restricted, it is not feasible to locate them all at once [25]. Locating a selected restricted number of eigenvalues with traditional feedback controller for ODEs [26] may lead other uncontrolled eigenvalues to shift to the right half plane (RHP) [27]. According to this conjecture, the Lambert W function provides appropriate control laws without such loss of stability.

4 Lambert Controller Design Method

In Sect. 4, a Lambert W function controller is designed for locating the rightmost eigenvalues to the desired locations in the left hand of the imaginary axis when slave makes relation with the environment. In controllable ODE systems with full state feedback, the main advantage is that all the closed-loop eigenvalues can be assigned by choosing the gains. However, DDE systems suffer from an infinite number of eigenvalues, and it is not practical to locate all of them to the desired locations by using traditional approaches. Here, for controllable DDE systems, the researcher employs the Lambert W function method to determine the first matrix, S_0, correlated to the principal branch, $k=0$, as it is significant for the solution structure of Eq. (11), by designing a feedback controller and selecting the feedback gain.

Figure 3 shows a position–position structure of 1-DOF bilateral teleoperation system with a controller based on the Lambert W function controller. All the possible mutual actions which can emerge in this system are considered as follows:

- F_h, F_e: operator and environment force;
- u_m, u_s: master and slave control signal;
- X_m, X_s: vector of position and velocity for the master and slave;
- K_m, Kd_s: Lambert Controller vector in master;
- K_s, Kd_m: Lambert Controller vector in slave;
- R_m: slave–master interaction. (Force Reflection)
- R_s: master–slave interaction;

A constant time-delay, T, in the communication channel is represented by the blocks of the delay. If the master–slave manipulators are represented by nth-order linear differential equations, the general form of the matrices can be represented as follows:

$$\begin{aligned} K_m &= \begin{bmatrix} k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix} & K_s &= \begin{bmatrix} k_{s1} & k_{s2} & \cdots & k_{sn} \end{bmatrix} \\ Kd_m &= \begin{bmatrix} kd_{m1} & kd_{m2} & \cdots & kd_{mn} \end{bmatrix} & Kd_s &= \begin{bmatrix} kd_{s1} & kd_{s2} & \cdots & kd_{sn} \end{bmatrix} \\ R_m &= \begin{bmatrix} r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} & R_s &= \begin{bmatrix} r_{s1} & r_{s2} & \cdots & r_{sn} \end{bmatrix} \end{aligned}$$

where 'm' and 's' are indications of master and slave subsystem, respectively, which can take the following form:

$$\dot{X}_m(t) = A_mX_m(t) + B_mu_m(t) \tag{15}$$

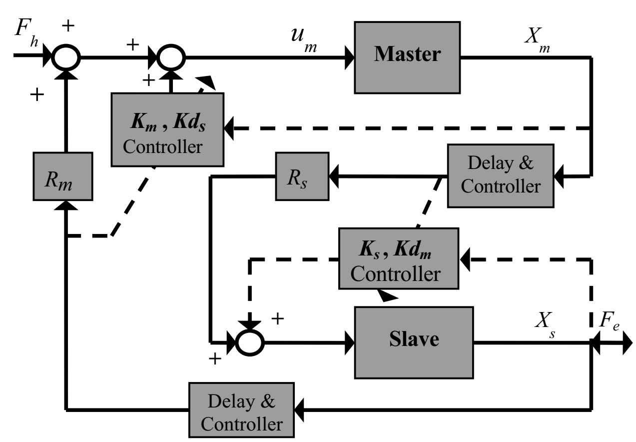

Fig. 3 Position–position structure of 1-DOF linear bilateral teleoperation system

$$Y_m(t) = C_m X_m(t) \tag{16}$$

$$\dot{X}_s(t) = A_s X_s(t) + B_s u_s(t) \tag{17}$$

$$Y_s(t) = C_s X_s(t) \tag{18}$$

In this structure of assigning unstable pole to the desired locations, three control modes take place:

$$\text{Mode 1}: \; u(t) = KX(t) \tag{19}$$

$$\text{Mode 2}: \; u(t) = K_d X(t-T) \tag{20}$$

$$\text{Mode 3}: \; u(t) = KX(t) + K_d X(t-T) \tag{21}$$

It is worth noting that we can choose either similar or different types of the controller for master and slave subsystem. According to the selected control mode in the master and slave systems, the control signal, $u(t)$, is constructed and utilized in dynamic of time-delay system shown in Eq. (11). By combining Eqs. (11) and (21), we can determine the closed-loop bilateral teleoperation system:

$$\dot{X}(t) = (A + BK)X(t) + (A_d + BK_d)X(t-T) \tag{22}$$

where

$$\dot{X}(t) = \left[\dot{X}_m(t) \; \dot{X}_s(t) \right]^T \tag{23}$$

$$X(t) = \left[X_m(t) \; X_s(t) \right]^T \tag{24}$$

$$X(t-T) = \left[X_m(t-T) \; X_s(t-T) \right]^T \tag{25}$$

The main reason of designing the controller with the new coefficients, $AA = A + BK$ and $AA_d = A_d + BK_d$, in Eq. (22) is that we can derive a solution for the matrix S_k by exploiting Eq. (12), and by assigning the rightmost eigenvalues to the desired locations.

$$\lambda_i(S_0) = \lambda_{i,desired} \quad For \quad i = 1, \ldots, n \tag{26}$$

Control signal of the master, $u_m(t)$, and the slave, $u_s(t)$, as shown in Fig. 3, can be given as follows:

$$u_m(t) = F_h - R_m X_s(t-T) + K_m X_m(t) + Kd_s X_s(t-T) \tag{27}$$

$$u_s(t) = R_s X_m(t-T) + K_s X_s(t) + Kd_m X_m(t-T) \tag{28}$$

We can replace control signal of the master and the slave in Eqs. (27) and (28) with Eqs. (15) and (17).

$$\dot{X}_m(t) = (A_m + B_m K_m)X_m(t) + B_m(Kd_s - R_m)X_s(t-T) + B_m F_h \tag{29}$$

$$\dot{X}_s(t) = (A_s + B_s K_s)X_s(t) + B_s(Kd_m + R_s)X_m(t-T) \tag{30}$$

Bilateral teleoperation system with respect to control signals dispatched from master to slave through the communication channel can be represented as follows:

$$\begin{bmatrix} \dot{X}_m(t) \\ \dot{X}_s(t) \end{bmatrix} = \begin{bmatrix} A_m + B_m K_m & 0 \\ 0 & A_s + B_s K_s \end{bmatrix} \begin{bmatrix} X_m(t) \\ X_s(t) \end{bmatrix} + \begin{bmatrix} 0 & B_m(Kd_s - R_m) \\ B_s(Kd_m + R_s) & 0 \end{bmatrix} \begin{bmatrix} X_m(t-T) \\ X_s(t-T) \end{bmatrix} + \begin{bmatrix} B_m & 0 \\ 0 & B_s \end{bmatrix} \begin{bmatrix} F_h \\ 0 \end{bmatrix} \tag{31}$$

where A_m and A_s are 2×2 matrices, B_m and B_s are 2×1 vectors, K_m and Kd_s are 1×2 Lambert controller gain matrices in master system, K_s and Kd_m are 1×2 Lambert controller gain matrices used in the slave system, X_m and X_s are 2×1 state vectors in master and slave, respectively. Success in feedback controller design depends on the control gain matrices, K and K_d, for master and slave systems in order to ensure stability in closed-loop system Eq. (22) is stable and to put in desirable performance.

The control parameters in the teleoperation system (see Fig. 3) are: $K_m = [k_{m1} \; k_{m2}]$, $K_s = [k_{s1} \; k_{s2}]$, $Kd_m = [kd_{m1} \; kd_{m2}]$, and $Kd_s = [kd_{s1} \; kd_{s2}]$. In other words, there are eight control parameters for assigning the rightmost eigenvalues to the desired locations in the left hand of imaginary axis. Four steps should be taken for the gains, K and K_d:

Step 1 Choose the desirable eigenvalues, $\lambda_{i,desired}$ for $i = 1,\ldots,n$, and set an equation so that the chosen eigenvalues appear as eigenvalues of the matrix S_0. Note that S_0 is the solution matrix resulting from the principal branch ($k = 0$) and $\lambda_i(S_0)$ which are the corresponding eigenvalues.

Step 2 Apply two updated coefficient matrices, $AA = A + B$ and $AA_d = A_d + BK_d$, in Eqs. (22)–(14), and come up with a numerical solution to calculate the matrix Q_0 for the principal branch ($k = 0$). Note that K and K_d are unknown matrices with unknown parameters, and the matrix Q_0 is a function including K and K_d.

Step 3 Substitute the matrix Q_0 in Eq. (14) with Eq. (13) to calculate S_0 and its eigenvalues as the function of the unknown matrix K and K_d.

Step 4 Equation (26) containing the matrix S_0 is calculated for the unknown K and K_d by using numerical approaches such as "fsolve" function in Matlab.

There is a restriction on the rightmost eigenvalues i.e. some values are not adequate for the rightmost eigenvalues depending on the framework or elements of the system. In this case, the mentioned method does not provide any response for K and K_d. To solve the problem, we should rerun a different set of values or lower values for desired rightmost eigenvalues and then we should arrive at a numerical solution for K and K_d matrices for a diversity of initial conditions by adopting an iterative trial and error process.

5 Scattering Theory and Stability

This section contains a theorem that describes an end-to-end model for the teleoperation system based on the scattering matrix analysis. In accordance with scattering matrix, the teleoperation systems are presented as $b = S(s)\ a$, where $a = [a_1\ a_2]^T$ and $b = [b_1\ b_2]^T$ are input and output waves of the teleoperation system, respectively.

Theorem *Necessary and sufficient conditions for robust stability of the teleoperation system are* [28]:

(a) *$S(s)$ includes no poles in the closed RHP.*
(b) *If Δ is the structured perturbation of s:*

$$Sup_{\omega}\left[\mu_{\Delta}(S(j\omega))\right] \leq 1$$

where $\mu_{\Delta}(s)$ is the structured singular value of matrix S. An effective property of $\mu_{\Delta}(s)$ is $\mu_{\Delta}(s) \leq \bar{\sigma}(s)$, where $\bar{\sigma}(\Delta)$ is the maximum singular value of Δ and $S(s)$ is the scattering matrix.

As the structure in Fig. 3 shows, the connection between input and output waves can be declared as follows:

$$\dot{X}_m = A_m X_m + B_m u_m \tag{32}$$

$$u_m = K_m X_m + Kd_s X_s(t-T) + F_h - F_e(t-T) \tag{33}$$

If we substitute the above control signal with the state-space presentation of the master, we will have:

$$X_m = (sI - A_m - B_m K_m)^{-1} B_m [F_h + e^{-Ts} Kd_s X_s - e^{-Ts} F_e] \tag{34}$$

In slave subsystem:

$$\dot{X}_s = A_s X_s + B_s u_s \tag{35}$$

$$u_s = R_s X_m(t-T) + K_s X_s + Kd_m X_m(t-T) \tag{36}$$

If we substitute the above control signal with the state space representation of master, we obtain:

$$X_s = \alpha_{(s)} X_m \tag{37}$$

$$\alpha_{(s)} = (sI - A_s - B_s K_s)^{-1} B_s (Kd_m + R_s) e^{-Ts} \tag{38}$$

By inserting Eq. (37) into Eq. (34) and after providing a concise summary, we will have:

$$X_m = a(s) F_h + b(s) F_e \tag{39}$$

that where

$$a(s) = \beta_{(s)}^{-1} B_m \tag{40}$$

$$b(s) = -\beta_{(s)}^{-1} e^{-Ts} B_m \tag{41}$$

$$\beta_{(s)} = sI - A_m - B_m K_m - e^{-2Ts} B_m Kd_s (sI - A_s - B_s K_s)^{-1} B_s (Kd_m + R_s) \tag{42}$$

By inserting Eq. (39) into Eq. (37), we obtain:

$$X_s = c(s) F_h + d(s) F_e \tag{43}$$

where

$$c(s) = \alpha(s) a(s) \tag{44}$$

$$d(s) = \alpha(s) b(s) \tag{45}$$

As the result, for establishing a structure for teleoperation system by using Lambert W function controller, we need to estimate the scattering matrix as follows:

$$S(s) = \begin{bmatrix} a(s) & b(s) \\ c(s) & d(s) \end{bmatrix} \tag{46}$$

We calculate and scheme SVD (singular value decomposition) of $S(j\omega)$ for diverse time-delays. Figure 4 is an indication of scattering matrix norm for different time-delays. It is clear that for different time-delays $Sup_{\omega}\left[\mu_{\Delta}(S(j\omega))\right] \leq 1$ is acceptable; therefore teleoperation system controlled by Lambert W function has a stable structure and robust performance on different values of time-delay. Figure 4 shows that maximum norm occurs in interval [1, 10] frequency while minimum norm belongs to time-delay 0.1 s and maximum is possessed by time-delay 0.5 s.

6 Simulation Results

It is obvious that step response is the most popular and general dynamic test for the control system, via which we evaluate the tracking control performance with a human operator. For a better response, we take a local feedback into consideration and place the closed-loop poles in master and slave systems at locations $[-2\ -4]$, and then we calculate Lambert controller gain matrix for locating unstable poles in the

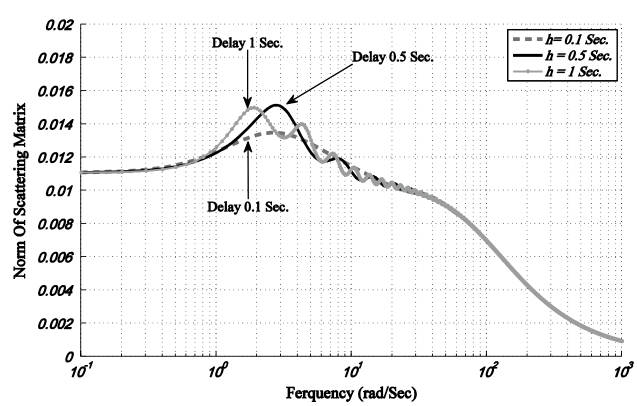

Fig. 4 S(s) SVD for different time-delays

system of Eq. (31), to the desired locations. For calculating these gains, we use the "fsolve" function in Matlab. Simulation parameters are mentioned in the following:

$$Master:\begin{cases} J_m = 1.5\,\text{kgm}^2 \\ b_m = 11\frac{\text{Nm}}{\text{rad/s}} \end{cases},\quad Slave:\begin{cases} J_s = 2\,\text{kgm}^2 \\ b_s = 15\frac{\text{Nm}}{\text{rad/s}} \end{cases}$$

$$Environment:\begin{cases} k_e = 100\,\text{Nm/rad} \\ b_e = 1\frac{\text{Nm}}{\text{rad/s}} \\ R_s = \begin{bmatrix} r_{s1} & r_{s2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \end{cases},\quad Force\ Reflection\ Gain:\ k_f = 0.1$$

$$Desired\ Poles:\ \lambda_{desired} = -1.5 \pm 0.01i$$

The obtained control parameters are as follows:Lambert controller Gain matrix in master:

$$K_m = \begin{bmatrix} 187.8993 & 144.00 \end{bmatrix} \quad Kd_s = \begin{bmatrix} -0.544 & -9.6987 \end{bmatrix}$$

Lambert controller Gain matrix in slave:

$$K_s = \begin{bmatrix} 0.6807 & 1.8218 \end{bmatrix} \quad Kd_m = \begin{bmatrix} -15.6145 & -9.9068 \end{bmatrix}$$

Figure 5 falls into three parts, in which we can see simulation results of constant time-delay, 500 ms, part (a) contains time-varying input with different amplitudes and frequencies. When this input is applied to the system and output response displays a strong performance and does good tracking, controller design is acceptable. Part (b) consists of position of the master, slave of Lambert W function, and slave of Azorin's controller [29] for this time-delay. This figure clearly exposes that the slave of Lambert W function controller predicts the performance of the master better and tracks master quicker rather than the slave of previous controller (Azorin). This part strongly demonstrates the superiority of the proposed controller in comparison with the previous controller. Master and slave control signal of proposed (Lambert) and previous (Azorin) controller are illustrated in part (c). Although proposed controller reaches master faster, it needs a lower control signal in compliance with previous controller and it is another advantage of this impressive controller. However, all of them have bounded and acceptable response with respect to the input signal.

There are three parts in Fig. 6, indicating simulation results for time-varying delay. The Internet is utilized in teleoperation systems as the communication channel. As its time-delay types are variable, we can test controller of time-varying delay shown in part (a). Part (b) demonstrates master and Lambert–Azorin slave positions being obtained by applying Fig. 5a as operator input and part (a) of this figure as the time-varying delay. This figure firmly validates the adequate performance of the Lambert-based controller. As a matter of fact, although the proposed controller is brightly robust over the noise of time-varying delay, the output of the previous controller strongly is affected by fluctuations relating to time-varying delay and presents a poor and non-robust performance. Part (c) contains master and Lambert–Azorin slave control signal for time-varying delay. This part, also, approves the excellence of the proposed controller with respect to the control signal. In fact, not only slave of previous controller suffers from a fluctuated output which negatively affects actuator but also needs a higher value of control signal which is harmful to actuator. Parts (b) and (c) in this figure completely demonstrate the unquestionable superiority of the proposed controller.

Figure 7 draws a comparison among slave positions for different time-delays while designed controller makes same Lambert gain. Figure 7 shows that the designed controller produces a good response to various constant time-delay in the communication channel and slave tracks master with a minimum error, although the proposed controller is not exactly designed for these time-delays.

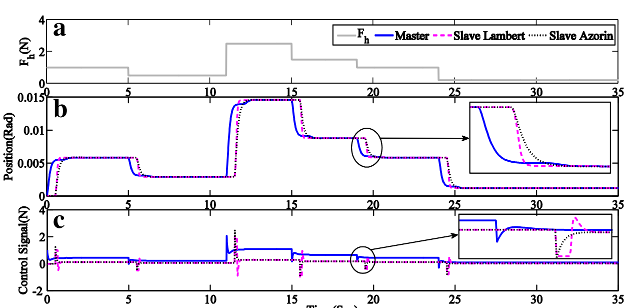

Fig. 5 Simulation results for 500 ms time-delay

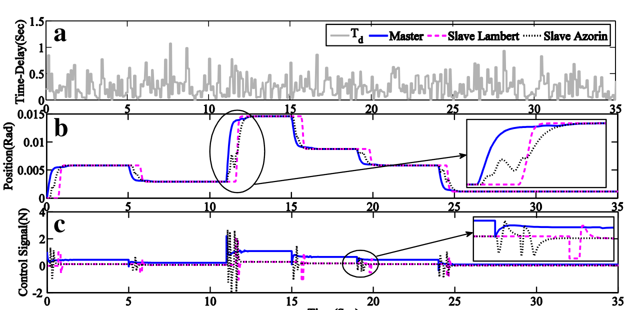

Fig. 6 Simulation results for time-varying delay

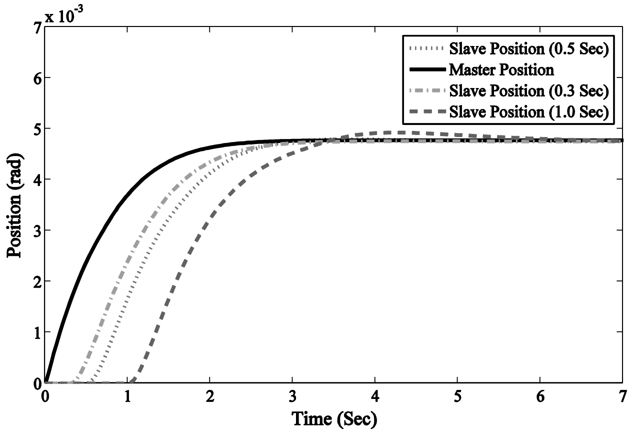

Fig. 7 Position comparison for different time-delays

Table 1 Transient response characteristics for different time-delays

Time-delay (s)	Overshoot %	Settling time (s)
0.3	0.0	2.294
0.5	0.37	2.435
1.0	3.1	5.797

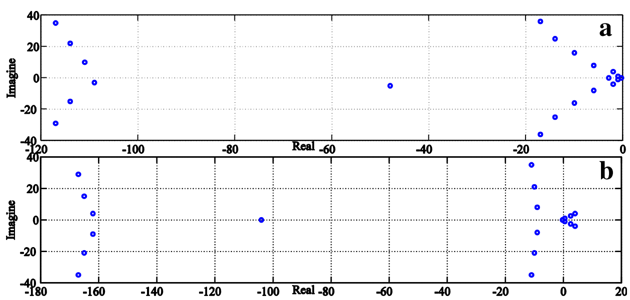

Fig. 8 Poles of the closed-loop teleoperation system **a** by using Lambert W controller **b** in the absence of Lambert W controller

Figure 7 is an indication of the fact that system even for the high amplitude of time-delay produces a favorable response, and shows that the designed controller acts in an appropriate manner.

In Table 1, there are transient response characteristics such as overshoot, and settling time for three different values of the time-delay. In fact, this table contains specifications of Fig. 7 and confirms its findings.

Figure 8 shows poles of the closed-loop teleoperation system using Lambert W function controller in part (a), and in absence of controller in seven branches of Lambert function, $(K = -3, -2, -1, 0, 1, 2, 3)$, in part (b). Compression of two parts in Fig. 8 indicates that the designed controller creates a stable system and assigns unstable poles to the desired location on the left hand of the imaginary axis.

Table 2 ESS for different types of time-delay

Delay (s)	Lambert	Azorin
500 ms	0.0253 (Rad)	0.0296 (Rad)
Time-varying delay	0.0274 (Rad)	0.0521 (Rad)

For comparison of simulation results in different schemes, the following Eq. (47) is exploited:

$$ESS = \int e^2 dt, \quad where \quad e(t) = x_{m1}(t) - x_{s1}(t) \tag{47}$$

$x_{m1}(t)$, and $x_{s1}(t)$ are master and slave positions. Table 2 numerically establishes simulation results for different types of time-delay for the proposed and the previous controller. This table like the former figures verifies the supremacy of the proposed controller in comparison with another method.

7 Conclusion

In this study, the researcher deployed a novel controller design approach based on the Lambert W function method for the teleoperation system. Time-delays in communication channel result in an infinite number of eigenvalues in teleoperation systems. They are calculated by delay differential equations and cause instability in these systems. Although due to an infinite number of eigenspectrums, the systems face difficult, by using this method a critical subset of these eigenspectrums is assigned to the desirable possible locations in the complex plane. Then, the researcher described a method for preserving stability in teleoperation systems in accordance with scattering theory. The findings demonstrated that the proposed controller of Lambert W function can improve the performance of the teleoperation systems and reduce the unfavorable influence of time-delays in them. This technique guarantees the stability, high performance, and transparency of the system and takes the time-delay as well as the slave-environment interaction into account. The model generates a state-space representation of the teleoperation system, including all the interactions that emerge in the operator–master–slave-environment set. Comparative simulation results confirmed the validity of the presented control scheme and its satisfactory performance as regards motion/force tracking. The simulation results demonstrated the effectiveness of this neoteric approach.

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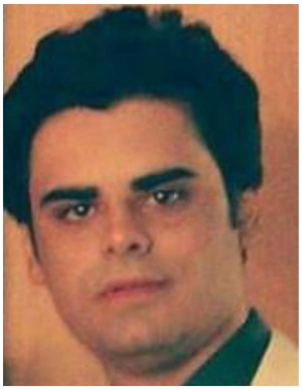

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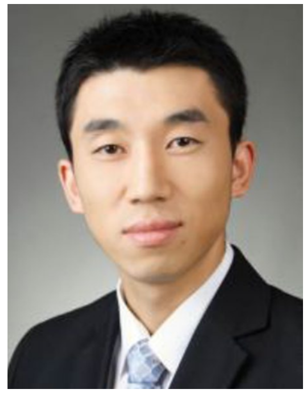

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